MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 11/1986

Diophantische Approximationen

16.3. bis 22.3.1986

Die diesjährige Tagung stand unter der Leitung von P. Bundschuh (Köln) und R. Tijdeman (Leiden). 38 Teilnehmer aus 13 Ländern berichteten in 35 Vorträgen über die neueren Entwicklungen im Bereich der diophantischen Approximation. Schwerpunkte bildeten dabei die Themen: Gleichverteilung, Irrationalität, Transzendenz und algebraische Unabhängigkeit sowie diophantische Gleichungen in algebraischen Zahlkörpern und in Funktionenkörpern. In der traditionellen "Problem Session", die P. Erdös leitete, wurden einige offene Probleme zur Diskussion gestellt. Dieses Bericht enthält Vortragsauszüge und Probleme. Die Leiter danken Herrn B.M.M. de Weger für die Herstellung dieses Berichts.

Vortragsauszüge

R.C. BAKER:

Recent progress on the Davenport-Birch-Ridout theorem

Let $Q(x_1,\ldots,x_n)$ be a quadratic form equivalent over the real numbers to $X_1^2 + \ldots + X_r^2 - X_{r+1}^2 - \ldots - X_n^2$. It is required to solve $|Q(\underline{x})| < 1$ in integers $\neq 0$. This is known to be possible if $n \geqq 21$ after work of Davenport, Birch and Ridout in 1956-9. Recently, H.P. Schlickewei and the speaker managed to prove some further cases of the theorem, for instance $n = 18, r = 9$. The method is an elaboration of that of Davenport and Ridout and depends on the application of recent work of Schlickewei to find small zeros of certain auxiliary integral quadratic forms.

J. BECK:

Two discrepancy type theorems

A more than 50 years old conjecture of Erdös states that for any function $f(n) \in \{-1,+1\}$, $n \in \mathbb{N}$,

$$\sup_{d\geq 1} \sup_{k\geq 1} \left| \sum_{i=1}^{k} f(id) \right| = \infty .$$

Since this problem is hopeless at present, we investigated the following easier question: instead of the arithmetic progression $d, 2d, \ldots, kd$ consider the "almost arithmetic progression" $[\delta], [2\delta], \ldots, [k\delta]$ (integral part) where $\delta \geqq 1$ real. We have proved the following theorem:

$$\sup_{k\geq 1} \left| \sum_{i=1}^{k} f([i\delta]) \right| = \infty$$

for all real $\delta \geqq 1$ except of a set of Lebesgue measure zero. The proof uses, among others, a quantitative version of the following well-known result of H. Weyl: given any infinite sequence $1 \leq n_1 < n_2 < \ldots$ of integers, the sequence $\{n_i\alpha\}$ of fractional parts is uniformly distributed in $[0,1)$ for almost all $\alpha \in [0,1)$.

In 1959 Leo Moser raised the following question: does there exist a universal function $g(x)$ satisfying $g(x) \to \infty$ as $x \to \infty$, such that any convex region of area x can be placed on the plane so to cover $\geq x + g(x)$ lattice points? Recently I gave an affirmative answer to this question. In fact, I proved that $g(x) > x^{1/9}$ if $x > c_1$, where c_1 is a "noneffective" constant. The reason is that we apply a deep "noneffective" theorem of W. Schmidt.

F. BEUKERS:

A Diophantine equation over function fields

We give an effective version of the Manin-Grauert theorem on Mordell's conjecture over function fields which reads as follows. Let $P(X,Y,Z) = 0$ be a homogeneous equation of a non-singular plane projective curve C over K , where K is a function field of transcendence degree 1 over an algebraically closed field k of characteristic 0. Suppose genus $C \geq 2$ and C is not projectively equivalent to a curve over k . Then C contains only finitely many K-rational points $(x:y:z)$ and for each such point we have $h(x,y,z) \leq n^{13}(h(P)+3g_K)$, where g_K is the genus of K and h denotes the projective height of finite sets in K .

B. BRINDZA:

Catalan equation over finitely generated domains

Let G be a finitely generated extension field of the rational number field $\mathbb{Q}$. Then G can be written in the form $G = \mathbb{Q}(z_1,\dots,z_q,u)$, ($q \geq 0$) where $\{z_1,\dots,z_q\}$ is a transcendence basis of G over $\mathbb{Q}$ and u is integral over the polynomial ring $\mathbb{Z}[z_1,\dots,z_q]$. Let $s(\alpha)$ denote the

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"size" of a non-zero element $\alpha \in G$. Further, let $R = \mathbb{Z}[u_1,\ldots,u_n]$ be a finitely generated subring of G .

Theorem. There exists an effectively computable number C which depends only on G and R such that all solutions of the equation $x^p - y^q = 1$, in $x, y \in R$, $p, q \in \mathbb{N}$ with x, y not roots of unity and $p > 1, q > 1$, $pq > 4$ satisfy $\max\{s(x),s(y),p,q\} < C$.

In the proof I used some arguments from Tijdemans's and van der Poorten's proof concerning the Catalan equation. Moreover, I used Győry's specialization method and Mason's inequality.

W.D. BROWNAWELL:

A local Nullstellen inequality and applications

If $P, Q \in \mathbb{Z}[x]$ are relatively prime polynomials with height $\leq H$, degree $\leq D$, then for any $\omega \in \mathbb{C}$, $\max\{|P(\omega)|,|Q(\omega)|\} \geq (2DH)^{-2D}$.

We generalize this result to polynomials in several variables. The general result has many consequences for transcendence theory, in particular for algebraic independence.

W.D. BROWNAWELL:

Vanishing sums in function fields

Recently D.W. Masser and I have generalised a fundamental result of R. Mason's on the height of a vanishing sum of S-units. For simplicity we consider polynomials $P_1, \ldots, P_n \in K[x]$, K an algebraically closed field of characteristic zero. We are interested in the case that $P_1 + \ldots + P_n = 0$ but $(P_1,\ldots,P_n) = 1$ and, more importantly, any proper subset of $\{P_1,\ldots,P_n\}$ is K-linearly independent.

A. If the zeros of all P_i lie in $Z \subseteq K$, then $\deg P_i \leq \frac{1}{2}(n-1)(n-2)(|Z|-1)$.

B. If Z_i denotes the set of zeros of P_i without multiplicity, then $\deg P_i \leq (n-2)(1+|Z_1|+\ldots+|Z_n|)$.

W.W.L. CHEN:

The Roth phenomenon in the theory of irregularities of distribution

It was shown by Roth that if one considers irregularities of point distribution with respect to aligned rectangles in the unit square, one will get a greater discrepancy if the set of points is replaced by a (finite) sequence of points (in a certain sense). We now investigate the analogous problem on irregularities of point distribution with respect to discs in the unit torus. (Work with J. Beck).

S. ECKMANN:

Über die lineare Unabhängigkeit bei Polylogarithmen und verwandten Funktionen

Hauptziel des Vortrages war eine kurze Beweisskizze des folgenden Resultats:

Satz. Seien $b_1,\ldots,b_\rho \in \mathbb{Q} \setminus \mathbb{N}$, $b_\lambda - b_\mu \notin \mathbb{Z}$ für $\lambda \neq \mu$; $n_1,\ldots,n_\rho \in \mathbb{N}$, $s := n_1+\ldots+n_\rho$ und $1 =: \beta_0,\beta_1,\ldots,\beta_r \in \mathbb{Q} \setminus \{0\}$ paarweise verschieden.

Für die durch $F(\lambda,i_\lambda;x) := \sum_{\nu=1}^{\infty} (\nu-b_\lambda)^{-i_\lambda} x^\nu$ in $|x| < 1$ definierten Funktionen gilt: $1, F(\lambda,i_\lambda;\beta_j b/a)$ $(\lambda=1,\ldots,\rho;\ i_\lambda=1,\ldots,n_\rho;\ j=0,\ldots,r)$ sind linear unabhängig über $\mathbb{Q}$, wenn die ganzen Zahlen $b > 0$ und a der Ungleichung $|a| > cb^{(r+1)s+1}$ genügen. Dabei kann die Konstante $c = c(r,s,b_1,\ldots,b_\rho,n_1,\ldots,n_\rho,\beta_1,\ldots,\beta_r) > 0$ völlig explizit angegeben werden.

Der Bewies erfolgt durch explizite Konstruktion von Padé-Approximationen und verallgemeinert die Methode von Nikisin (On irrationality of the values of the functions $F(x,s)$, Math. USSR Sb. 37 (1980), 381-388), welcherobige Behauptung für die Polylogarithmen $\sum_{\nu=1}^{\infty} \nu^{-k} x^{\nu}$, $k = 1, \ldots, s$ an rationalen Stellen $x = b/a < 0$, $(a,b) \in \mathbb{Z} \times \mathbb{N}$ mit $|a| > c(s) b^{s+1}$ zeigte.

R. ENDELL:

Interpolation auf algebraischen Gruppen

Sei K ein algebraisch abgeschlossener Unterkörper von $\mathbb{C}$. G/K sei eine quasiprojektive, zusammenhängende, kommutative, algebraische Gruppe der Dimension $n \geq 1$. Diese Gruppe kann in einen projektiven Raum $\mathbb{P}^N$ mit projektiven Variablen $X_0, \ldots, X_N$ eingebettet werden. Ferner sei A eine analytische Untergruppe von G der Dimension $d \geq 1$ und Γ sei eine endlich erzeugte Untergruppe von G vom Rang $\ell \geq 0$. $\{\gamma_1, \ldots, \gamma_m\}$ sei ein Erzeugendensystem von Γ . O.B.d.A. können wir annehmen, dass Γ die durch $X_0 = 0$ definierte Hyperebene in $\mathbb{P}^N$ nicht trifft. Schliesslich definieren wir Derivationen $s_1, \ldots, s_d$ und Zahlen ρ_r (Masser-Wüstholz) und σ_r (Wüstholz) für $0 \leq r \leq n$. Setzt man $\Gamma(S) = \{s_1\gamma_1 + \ldots + s_m\gamma_m,\ 0 \leq s_i \leq S,\ s_i \in \mathbb{Z}\}$ und $\mathbb{N}^d(T) = \{\tau = (\tau_1, \ldots, \tau_d) \in \mathbb{N}^d ,\ \tau_1 + \ldots + \tau_d < T\}$ für positive reelle Zahlen S und T und bezeichnet man mit $x(\gamma)$ die affinen Koordinaten von $\gamma \in \Gamma$, dann gilt das folgende

Theorem. Es gibt eine positive reelle Konstante $c > 0$ mit folgender Eigenschaft: zu allen reellen Zahlen $S \geq 0$ und $T \geq 1$ und jeder Abbildung a aus $K^{\mathbb{N}^d(T) \times \Gamma(S)}$ existiert ein Polynom P aus $K[X_1/X_0, \ldots, X_N/X_0]$ vom Gesamtgrad höchstens $c \max_{0 \leq r < n} S^{(\ell-\rho_r)/(n-r)} T^{(d-\sigma_r)/(n-r)}$, so dass

$s^{\tau} P(x(\gamma)) = a(\tau, \gamma)$ für alle $(\tau, \gamma) \in \mathbb{N}^d(T) \times \Gamma(S)$.

Für $T > 1$ ist dies eine Verallgemeinerung eines Interpolationsergebnisses von Masser.

J.-H. EVERTSE:

Equal values of binary forms at integral points

This is an outline of some joint work with Győry, Shorey and Tijdeman.

Let $F_1, \dots, F_r, G_1, \dots, G_s$ be (possibly constant) pairwise coprime binary forms with coefficients in $\mathbb{Z}$. Let $\mathcal{F}, \mathcal{G}$ be collections of binary forms defined by $\mathcal{F} = \{F_1^{a_1} \dots F_r^{a_r} : a_i \in \mathbb{N}\}$, $\mathcal{G} = \{G_1^{b_1} \dots G_s^{b_s} : b_i \in \mathbb{N}\}$. For any binary form $F \in \mathbb{Z}[X,Y]$, $\omega(F)$ denotes the maximal number of pairwise nonproportional linear forms which divide F in $\mathbb{C}[X,Y]$. The following result holds:

Theorem. Let $F \in \mathcal{F}$, $G \in \mathcal{G}$, $x, y \in \mathbb{Z}$ with $(x,y) = 1$. Then

(i) if $F(x,y) = G(x,y)$, $\omega(FG) \geq 3$, then $\max(|x|,|y|) \leq C$,

(ii) if $F(x,y) \mid G(x,y)$, $\omega(F) \geq 3$, then also $\max(|x|,|y|) \leq C$,

where C is an effectively computable number, depending on $F_1, \dots, F_r, G_1, \dots, G_s$ only.

This result can be derived from an effective result on the Thue-Mahler equation (Coates 1970, Shorey et.al. 1977) by using certain properties of resultants. An analogous statement can be proved about the number of integers x, y satisfying (i) or (ii) for some $F \in \mathcal{F}$, $G \in \mathcal{G}$. We shall briefly discuss some extensions and consequences of the theorem.

K. GYÖRY:

Finiteness criteria for decomposable form equations

(Joint work with J.-H. Evertse). Let K be a field of characteristic 0, $F(\underline{X}) = F(X_1,\dots,X_m) \in K[X_1,\dots,X_m]$ $(m \geq 2)$ a decomposable form which factorizes into linear factors over some finite extension G of K, $\mathcal{L}_0$ a maximal set of pairwise linearly independent linear factors of F, $\mathcal{L} \supseteq \mathcal{L}_0$ a finite set of pairwise linearly independent linear forms with coefficients in G . A subspace $V \neq (\underline{0})$ of K^m is called L_0-non-degenerate if L_0 contains at least 3 forms which are linearly dependent on V , but pairwise linearly independent on V . V is called L-admissible if no form in L vanishes identically on V .

Theorem 1. The following two statements are equivalent:

(a) Every L-admissible subspace of K^m of dimension ≥ 2 is L_0-non-degenerate.

(b) For every $b \in K \setminus \{0\}$ and every subring $R \subset K$ finitely generated over $\mathbb{Z}$, the equation

(*) $F(\underline{x}) = b$ in $\underline{x} \in R^m$ with $\ell(\underline{x}) \neq 0$ for all $\ell \in L \setminus L_0$

has only finitely many solutions.

Of particular interest is the special case $L = L_0$.

Theorem 2. For every $b \in K \setminus \{0\}$ and every subring $R \subset K$ finitely generated over $\mathbb{Z}$, the solutions of (*) are contained in finitely many L-admissible, L_0-non-degenerate subspaces of K^m .

In the course of the proofs the following implications are shown.

{Finiteness theorem on unit equations (due to Evertse, and van der Poorten and Schlickewei)} $\Rightarrow$ {Theorem 2} $\Rightarrow$ {(a)$\Rightarrow$(b) in Theorem 1} $\Rightarrow$ {Finiteness theorem (above) on unit equations} .

When K, G and the coefficients of the forms in L have appropriate effective representations, an algorithm is given to decide whether in Theorem 1 condition (a) holds.

Theorems 1 and 2 have several applications to Thue equations, norm form equations, discriminant form equations and index form equations.

H.G. KOPETZKY:

Ein Dichtemass für Folgen

Die von H. Niederreiter als Dichtemass für Folgen eingeführte "Dispersion" ist für Folgen (x_n) , $n = 1, 2, \ldots$ aus dem Einheitsintervall I definiert als $d_N = \sup_{x \in I} \min_{n \leq N} |x - x_n|$. Bei der Behandlung der speziellen Folgen $(\{n\alpha\})$, α irrational, wird die Untersuchung zahlentheoretisch interessant. Es ergeben sich Resultate analog zu solchen über Diophantische Approximationen. So existiert ein "Dispersionsspektrum" für die Werte von $D(\alpha) = \limsup Nd_N$ in Analogie zum bekannten Markoffspektrum. Als kleinster Häufungspunkt des Spektrums ergibt sich zum Beispiel hier der Wert $(1+2\sqrt{2})/3$. Die Untersuchungen werden mit Hilfe von regulären Kettenbrüchen durchgeführt.

M. LAURENT:

A conjecture on polynomial-exponential equations

I give a conjecture which is a generalization and a formalization of some classical conjectures like Pisot's one: if a linear recurrent sequence $(u_n)_{n \geq 0}$ has values in $\mathbb{N}^2$ then there exists some linear recurrent sequence (v_n) such that $u_n = v_n^2$, $n \geq 0$.

R.C. MASON:

Norm form equations

In 1983 a result was established on a general unit equation in several variables. This had important applications for norm form equations over function fields. Later developments include applications to decomposable form equations and to rational function solutions. There has also been progress on the analogous problem in positive characteristic and to that of non-degenerate modules.

M. MIGNOTTE:

Equations involving linear recursive sequences

We solve completely the two following equations: $u_n = 2^k$ and $u_m = \pm u_n$ for a certain ternary linear recursive sequence.

H. NIEDERREITER:

Low-discrepancy point sets

Let $\Delta_N = ND_N^*$, where D_N^* is the usual star discrepancy of N given points in $[0,1)^s$, $s \geq 2$. Various constructions of point sets with small Δ_N are presented. The first construction is based on the theory of good lattice points and yields improvements of results of Hlawka and Korobov, e.g. $\Delta_N \leq C_s(\log N)^s + O_s(1)$ with $\lim_{s\to\infty} C_s = 0$. Two more constructions use points $\underline{x}_n = (x_{n1},\ldots,x_{ns})$ with digit expansions $x_{ni} = \sum_{j=1}^{m} x_{ni}^{(j)} b^{-j}$ to a base $b \geq 2$. Let b be prime, $N = b^m - 1$, $m \geq 2$, $\gcd(N,m) = 1$, and let (y_n) be an m th order maximal period sequence mod b . Then we define $x_{ni}^{(j)} = y_{m(n+i-2)+j}$ and show that a suitable choice of (y_n) yields $\Delta_N \leq C_s(\log N)^{s+1}\log\log N$ with $\lim_{s\to\infty} C_s = 0$. In the last construction, take b prime, $N = b^m$, $\mathbb{F}_N = \mathbb{F}_b(\sigma)$ the finite field of order N , and choose an $s \times m$ matrix $B = (\beta_{ij})$ over $\mathbb{F}_N$. For $1 \leq n \leq N$ let $n-1 = \sum_{r=0}^{m-1} a_r(n)b^r_{m-1}$ be the digit expansion in base b . Then we define $x_{ni}^{(j)} = \mathrm{Tr}(\sum_{r=0}^{m-1} \sigma^r \beta_{ij} a_r(n))$, where $\mathrm{Tr} : \mathbb{F}_N \to \mathbb{F}_b$ is the trace. Averaging over all B , we can achieve $\Delta_N \leq C_s(\log N)^s + O_s((\log N)^{s-1})$ with $\lim_{s\to\infty} C_s = 0$. For individual B , let $\rho(B)$ be the smallest cardinality of a non-empty system $\{\beta_{11},\ldots,\beta_{1d_1},\ldots,\beta_{s1},\ldots,\beta_{sd_s}\}$ that is linearly dependent over $\mathbb{F}_b$. If $b > 2$ then

$$\Delta_N \leq b^{m+1-\rho(B)} \sum_{i=0}^{s-1} \binom{\rho(B)+i-1}{i} [b/2]^i .$$

If $b = 2$ then

$$\Delta_N \leq 2^{m+1-\rho(B)} \sum_{i=0}^{\min(\rho(B),s)-1} \binom{\rho(B)-1}{i} .$$

K. NISHIOKA:

Algebraic independence of Liouville numbers

Let $f(z) = \sum_{k=1}^{\infty} z^{k!}$. Then it is conjectured that, if $\alpha_1, \ldots, \alpha_n$ are algebraic numbers with $0 < |\alpha_1|, \ldots, |\alpha_n| < 1$, and no α_i/α_j $(1 \leq i < j \leq n)$ a root of unity, then $f(\alpha_1), \ldots, f(\alpha_n)$ are algebraically independent. The conjecture for $n = 3$ is proved and a p-adic analogy of the conjecture is completely solved. If we take $\sum_{k=1}^{\infty} (2/3)^{(k+1)!} z^{k!}$ instead of $\sum_{k=1}^{\infty} z^{k!}$, the conjecture for the complex number field is valid. These are proved by estimating a lower bound of $\beta_1\alpha_1^{k!} + \ldots + \beta_n\alpha_n^{k!}$ for infinitely many positive integers k .

P. PHILIPPON:

Mesures d'indépendance algébriques

On expliquera d'abord les notions de mesures d'indépendance algébrique que l'on utilise en faisant le lieu avec les notions autérieurement introduites. Puis on illustrera sur un example précis une méthode permettant d'etablir de telles mesures d'indépendance algébrique. On essaiera également de répertorier les différents résultats qui ont été réceinment obtenus, par divers auteurs.

A.J. VAN DER POORTEN:

Zeros of p-adic exponential polynomials

(Joint work with R.S. Rumely). Let $b(t) = \sum_{i=1}^{m} B_i(t) \exp t\eta_i$, $B_i \in \mathbb{C}_p[t]$, $\eta_i \in \mathbb{C}_p$ denote a p-adic exponential polynomial of order $n = \sum_{i=1}^{m} (1+\deg B_i)$, with $\operatorname{ord}_p \eta_i > 1/(p-1)$ $(i=1,2,\dots,m)$. We note (this is evident, but seems not to be mentioned in the literature) that $b(t)$ has only finitely many zeros in its domain of definition, and provide an explicit bound for that number in the most general context. The special case of greatest interest and relevance is $p \geq n > 2$ and $b(t)$ defined over $\mathbb{Q}_p$ with $\eta_i = \log \beta_i$, where $\beta_i \equiv 1 \bmod p$: then $\operatorname{ord}_p \eta_i > 1$ and $b(t)$ is defined for the disc $\operatorname{ord}_p t > -1 + 1/(p-1)$. In that situation it was well known that there are at most $n-1$ zeros in the unit disc of $\mathbb{C}_p$; we show there are at most $p(n-2)$ zeros in the entire domain of definition. We have examples to show that some such bound is correct: a construction with $\simeq p\sqrt{n}$ zeros.

G. RHIN:

New irrationality measures of some numbers

It is well known that Padé-approximants provide good rational approximations of some numbers as $\log(1-\zeta)$ ($\zeta \in \mathbb{Q}$ sufficiently small), $\pi/\sqrt{3}$ etc. In 1982 Choodnovsky announced better results for the irrationality measures of $\log 2$ and $\pi/\sqrt{3}$. We give a method which improves Choodnovsky's results ($\operatorname{mes}_{irr}(\log 2) = 4.077$ instead of 4.134 and $\operatorname{mes}_{irr}(\pi/\sqrt{3}) = 4.971$ instead of 5.817). For $\log(1-\zeta)$ we replace the study of $\int_0^1 t^n(1-t)^n/(1-\zeta t)^{n+1} dt$ by the study of $\int_0^1 H_n(t)/(1-\zeta t)^{n+1} dt$ where H_n is a polynomial which is in the n th power of a suitable ideal of $\mathbb{Z}[t]$. For π , using the two-dimensional analogue of this method, we prove that $\operatorname{mes}_{irr}(\pi) = 15.37..$, improving Mignotte-Choodnovsky-Reyssat's results: $\operatorname{mes}_{irr}(\pi) \simeq 20$.

A. SCHINZEL:

The number of terms of the square of a polynomial

It is proved that if $f \in \mathbb{C}[x]$ has $T > 1$ non-zero coefficients then f^2 has at least $3 + \log_2(1+\frac{1}{5}\log_2(T-1))$ non-zero coefficients, where $\log_2$ is the logarithm to the base 2.

H.P. SCHLICKEWEI:

Quadratic Geometry of Numbers

Let $F(\underline{X}) = \sum_{i=1}^{n} \sum_{j=1}^{n} f_{ij}X_iX_j$ be a quadratic form with rational integer coefficients $f_{ij} = f_{ji}$. Put $F = (\sum_{i=1}^{n} \sum_{j=1}^{n} f_{ij}^2)^{1/2}$. Suppose F vanishes on a d-dimensional rational subspace where $0 < d < n$. In a recent paper I proved that then F has an integral zero $\underline{x}$ satisfying $|\underline{x}| \ll F^{(n-d)/2d}$, where $|\underline{x}| = (x_1^2+\ldots+x_n^2)^{1/2}$. The case $d = 1$ had been done by Cassels in 1955. Here a report will be given of the following more general result. It was proved in a joint paper with W.M. Schmidt.

Theorem. Let F be as above. Suppose F has rank $> n - d$. Then there exist d-dimensional sublattices of $\mathbb{Z}^n$ say $\Gamma_0, \ldots, \Gamma_{n-d}$ with the following properties:

(i) For each $i = 0, \ldots, n-d$, F vanishes on Γ_i.

(ii) For each $j = 1, \ldots, n-d$, $\Gamma_0 \cap \Gamma_j$ has dimension $d - 1$.

(iii) The union of $\Gamma_0, \Gamma_1, \ldots, \Gamma_{n-d}$ spans E^n, the n-dimensional space.

(iv) For each $j = 1, \ldots, n-d$, $\det \Gamma_0 \cdot \det \Gamma_j \ll F^{n-d}$.

J.H. SILVERMAN:

Integral points on abelian surfaces are widely spaced

Let K be a number field, A/K an abelian variety, and $h : A(K) \to \mathbb{R}$ a logarithmic height function corresponding to a fixed embedding $A \subset \mathbb{P}^n$.

As is well known, $\#\{P \in A(K) : h(p) \leq X\} \sim C_A \sqrt{X}^{\text{rank } A(K)}$ as $X \to \infty$.

Now let $U \subset A$ be an affine open subset of A, and let R be the ring of integers of K. The Lang has conjectured that the set of R-integral points of U (for any embedding $U \subset \mathbb{A}^n$) is finite. We show that for abelian surfaces, this set $U(R)$ is at least quite sparsely distributed inside $A(K)$.

Theorem. With notation as above, assume that A is an abelian surface (i.e. $\dim A = 2$) . Then $\#\{P \in U(R) : h(P) \leq X\} \leq C_A' \log(X)$ as $X \to \infty$.

The proof involves a computation with local height functions, a lemma of Mumford, and an application of Falting's proof of the Mordell conjecture to eliminate certain exceptional cases.

C.L. STEWART:

Effective approximations to cubic irrationals

We shall discuss some recent work of D. Easton and some work of A. Baker and the speaker concerning effective measures of irrationality for numbers of the form $\sqrt[3]{a}$ where a is a positive integer which is not a perfect cube.

R.F. TICHY:

Uniform distribution of recurring sequences

At the last conference on "Diophantine Approximation" W. Narkiewicz has given a lecture on uniform distribution in residue classes. He presented several interesting results and formulated a list of problems. In this lecture a solution of one of these problems is discussed; all uniformly distributed linear recurrences of third order with integer coefficients are determined (due to G. Turnwald). Furthermore several generalizations to recurrences in

Dedekind domains are presented (due to R.F. Tichy and G. Turnwald). Finally the distribution behaviour of the quotients (a_{n+1}/a_n) of a real-valued linear recurring sequence (a_n) is investigated.

R. TIJDEMAN:

On prime factors of sums of integers

Let $\omega(n) = \prod_{p|n} 1$. Let W be a subset of $\{(i,j) \mid 1 \leq i \leq k, 1 \leq j \leq \ell\}$. Let $|W|$ denote the cardinality of W. Győry, Stewart and I proved that if $\omega(\prod\prod_{(i,j)\in W} (a_i+b_j)) = s$, then $s \gg \log |W|(|W|-k)/k\ell^2$. In particular, if $k = \ell$, then $|W| = k^{3/2+\varepsilon}$ implies $s \gg_\varepsilon \log k$. Here ε is any positive number. We conjecture that 3/2 can be replaced by 1. The conjecture of Erdös that $|W|/k \to \infty$ implies $s \to \infty$ is false.
Masser conjectured that all relatively prime positive integers a, b satisfy $a + b \ll_\varepsilon G^{1+\varepsilon}$ where $G = \prod_{p|ab(a+b)} p$. Stewart and I proved: $\log(a+b) \ll G^{15}$, but there are infinitely many coprime pairs a, b such that $a+b \gg_\delta G^{1+\eta}$ where $\eta = (4-\delta)(\log G)^{-1/2}(\log\log G)^{-1}$. An interesting numerical example due to de Weger is that $3^2 5^6 7^3 + 11^2 = 2^{21} 23$. In this case $a+b \simeq G^{1.626}$.

J.D. VAALER:

Automatic vanishing of polynomials with low height

Let K be a number field, $g_1(X), \ldots, g_s(X)$ distinct, irreducible polynomials in $K[X]$ and $m_1, \ldots, m_s$ positive integers. Let S be the vector space of polynomials F in $K[X]$ such that $\deg(F) < N$ and $\prod_{j=1}^{s} \{g_j(X)\}^{m_j} \mid F$. Then S has dimension $N - \sum_{j=1}^{s} m_j \deg(g_j)$, which we assume to be positive. Let F be a nonconstant polynomial in $K[X]$. We describe a method for showing that: if $f \in S$ is nontrivial and has low height then f must have F as a factor with relatively high multiplicity. Some special cases will be discussed in which $K = \mathbb{Q}$ and $F = F_n$ is the n th cyclotomic polynomial.

C. VIOLA:

Some results on effective measures of irrationality for algebraic numbers

Let $P \in \mathbb{C}[x_1,x_2]$, P not identically zero, with $\deg_{x_j} P \leq d_j$, and let $\xi_1, \ldots, \xi_m \in \mathbb{C}^2$, $\xi_h = (\xi_{h1},\xi_{h2})$ be distinct points such that

$$\frac{\partial^{i_1+i_2} P}{\partial x_1^{i_1} \partial x_2^{i_2}}(\xi_h) = 0 \text{ for all } (i_1,i_2) \text{ satisfying } \theta_1 \frac{i_1}{d_1} + \theta_2 \frac{i_2}{d_2} < t_h \ (h=1,\ldots,m) ,$$

where $\theta_1, \theta_2 > 0$ and $t_1, \ldots, t_m > 0$ are given numbers. Let

$\phi(t) = \int_0^1 \int_0^1_{\theta_1 z_1+\theta_2 z_2 \leq t} dz_1 dz_2$. The the so-called Dyson's lemma states that if the points $\xi_1, \ldots, \xi_m$ are admissible, i.e. such that $\xi_{h1} \neq \xi_{h'1}$ and $\xi_{h2} \neq \xi_{h'2}$ for $h \neq h'$, then $\sum_{h=1}^{m} \phi(t_h) \leq 1 + \max(m/2-1,0)\min(d_1/d_2,d_2/d_1)$.

Bombieri (Acta Math. 148 (1982), 255-296) combined this inequality with a suitable form of Siegel's lemma to obtain effective measures of irrationality for all the generators of some number fields. I extend Bombieri's effective results to all numbers of the same fields (not necessarily generators), by replacing the admissibility condition for $\xi_1, \ldots, \xi_m$ in Dyson's lemma with a suitable upper bound for $t_1, \ldots, t_m$ (see Annali Scuola Norm. Sup. Pisa (4) 12 (1985), 105-135). The proof requires algebraic geometry.

M. WALDSCHMIDT:

A refinement of the six exponentials theorem

Let x_1, x_2 be two linearly independent complex numbers, y_1, y_2, y_3 be three $\mathbb{Q}$-linearly independent complex numbers, and $a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}$ be six algebraic numbers. If the six numbers $\exp(x_i y_j - a_{ij})$, $(i=1,2,\ j=1,2,3)$ are all algebraic, then $x_i y_j = a_{ij}$ for all $i=1,2$ and $j=1,2,3$.

R. WALLISER:

On the computation of all imaginary quadratic fields of class number one

Let d be an odd discriminant of an imaginary quadratic field with class number one. Stark showed: If $d < -10^4$, then $h(12d) \leq 2\sqrt{|d|}$, $h(24d) \leq 2\sqrt{|d|}$ and $|h(24d)\ln(5+2\sqrt{6}) - 2h(12d)\ln(2+\sqrt{3})| < 50 \exp(-\frac{\pi}{24}\sqrt{|d|})$.
We estimate this linear form for "large" $|d|$ with the aid of the quantitative version of Schneider's α^β theorem by Mignotte and Waldschmidt. In the "medium large" region $2.10^4 \leq |d| \leq 10^{40}$ my student Cherubini showed by computing the begin of the continued fraction of $\frac{\ln(5+2\sqrt{6})}{\ln(2+\sqrt{3})}$, that the above relations of Stark cannot hold.

L. WANG:

p-adic continued fractions

In this talk I shall introduce an algorithm of p-adic simple continued fractions, and give some criteria of algebraic independence for a system of p-adic numbers expanded in p-adic simple continued fractions, and establish the transcendence for the value ξ^η of power exponent function in p-adic numbers ξ and η under some condition by using a theorem on linear forms in logarithms in the p-adic case due to van der Poorten. The last result is a p-adic analogue of a theorem due to Bundschuh.

B.M.M. DE WEGER:

Algorithms for solving exponential diophantine equations

The method of Gelfond-Baker yields explicitly known, but usually very large upper bounds for the solutions of many types of diophantine equations. Computational methods from diophantine approximation theory can be used to reduce these bounds considerably in many cases. A powerful algorithm for solving (complex or p-adic) multi-dimensional approximation problems is the recent "L^3-Basis-Reduction-Algorithm" of L. Lovász.

We shall present algorithms for solving the following problems. Let S be the set of positive integers that are composed of primes from some fixed set. We consider:

(i) $x^2 + k \in S$, (k constant, $x \in \mathbb{Z}$) , the "generalized Ramanujan-Nagell equation" (joint work with A. Pethö);

(ii) $0 < x - y < y^{\delta}$, ($\delta \in (0,1)$ fixed, $x, y \in S$);

(iii) $x + y = z$, ($x, y, z \in S$) .

In (ii) and (iii) the L^3-algorithm is used.

J. WOLFART:

Values of hypergeometric functions

Theorem. Let $a, b, c \in \mathbb{Q}$; $a, b, c, a-c, b-c \notin \mathbb{Z}$,

$F(z) := F(a,b,c;z) = 1 + \frac{ab}{c}z + \frac{a(a+1)b(b+1)}{2!c(c+1)}z^2 + \dots$ a classical hypergeometric function with monodromy group Δ . Then

I) Δ finite $\iff$ $F(z)$ algebraic function $\iff$ $F(z) \in \overline{\mathbb{Q}}$ for all $z \in \overline{\mathbb{Q}}$.

II) Δ arithmetic (:= norm-1-group of a quaternion algebra) $\iff$ there is an exceptional set of arguments $E = E(a,b,c)$, E and $\overline{\mathbb{Q}} - E$ dense in $\overline{\mathbb{Q}}$, such that $F(z) \in \overline{\mathbb{Q}}$ for all $z \in E$, $F(z) \notin \overline{\mathbb{Q}}$ for all $z \in \overline{\mathbb{Q}} - E$ (84 cases).

III) Δ nor finite nor arithmetic $\Longleftrightarrow$ $F(z) \notin \overline{\mathbb{Q}}$ for almost all $z \in \overline{\mathbb{Q}}$

(in 'most' cases, $z = 0$ is the only exception).

Ideas of proof: 1) linear independence of abelian integrals (Wüstholz);

2) construction and decomposition of Jacobians of Fermat curves (Koblitz, Rohrlich) and other algebraic curves;

3) Triangle functions and automorphic functions (Shimura).

K. YU:

A generalization of Mahler's classification to several variables

A classification of all points in $\mathbb{C}^n$ into $3n+1$ disjoint nonempty classes, termed A^n, T^n_t, S^n_t, U^n_t , $t = 1, 2, \ldots, n$, is introduced. We prove that this classification possesses the following invariant property: if $\underline{\xi} = (\xi_1,\ldots,\xi_n)$ and $\underline{\eta} = (\eta_1,\ldots,\eta_n)$ in $\mathbb{C}^n$ satisfy $\overline{\mathbb{Q}(\xi_1,\ldots,\xi_n)} = \overline{\mathbb{Q}(\eta_1,\ldots,\eta_n)}$, then $\underline{\xi}$ and $\underline{\eta}$ belong to the same class. (For a field F , $\overline{F}$ denotes its algebraic closure.) When $n = 1$, this classification coincides with that introduced by Mahler in 1932. It is also proved that T^n_n can be further classified into continuum many disjoint classes, among which there exist infinitely many ones being nonempty, and that this classification also has the invariance property.

Y. ZHU:

Arithmetic properties of gap series with algebraic coefficients

Let $s, t \in \mathbb{N}, \geq 1$. Suppose that $F_\nu(z) = \sum_{k=0}^{\infty} f_{\nu,k} z^k$ $(\nu=1,2,\ldots,s)$ are s power series satisfying the following conditions: for any ν $(1 \leq \nu \leq s)$

(i) $f_{\nu,k} \in \overline{\mathbb{Q}}$ ($k = 1, 2, \ldots$);

(ii) there exist two sequences $\lambda_{\nu,n}$ and $\mu_{\nu,n} \in \mathbb{N}$, $\uparrow \infty$, satisfying

$0 = \lambda_{\nu,1} \leqq \mu_{\nu,1} < \lambda_{\nu,2} \leqq \mu_{\nu,2} < \dots < \lambda_{\nu,n} \leqq \mu_{\nu,n} < \dots$ such that $f_{\nu,k} = 0$ ($\mu_{\nu,n} < k < \lambda_{\nu,n+1}$) , but $f_{\nu,\lambda_{\nu,n}} \neq 0$ and $f_{\nu,\lambda_{\nu,n+1}} \neq 0$ $(n \geqq 1)$;

(iii) The radius of convergence of series $R_\nu > 0$.

In the present paper the algebraic independence of values of $F_\nu(z)$ at algebraic points is considered.

F. BEUKERS:

We call $(n_1, n_2, \ldots, n_m)$ admissible if there exist non-constant polynomials $P_1, P_2, \ldots, P_m \in \mathbb{C}[t]$ such that $\gcd(P_i, P_j) = 1$ for all $i \neq j$ and $P_1^{n_1} + P_2^{n_2} + \ldots + P_m^{n_m} = 0$. It follows from Mason's inequality that for $m = 3$ all admissible triples are given by $(2,2,n)$ ($n \geq 1$ arbitrary), $(2,3,3)$, $(2,3,4)$ and $(2,3,5)$. Characterize the admissible quadruples (e.g. known are $(5,5,5,5n)$ for arbitrary $n \geq 1$; not known are $(6,6,6,6)$ and $(4,4,8,8)$).

P. ERDÖS:

1. Some problems on irrationality of series. I proved that if $a_1 < a_2 < \ldots$ is a sequence of integers for which $a_{n+1} - a_n \to \infty$ then $\sum a_n / 2^{a_n}$ is irrational. Probably the assumption $a_n/n \to \infty$ suffices. On the other hand I never succeeded in finding a sequence of integers for which $\overline{\lim}(a_{n+1} - a_n) = \infty$ and for which $\sum a_n / 2^{a_n}$ is rational.

Let $q_1 < q_2 < \ldots$ be the sequence of squarefree numbers. I could not prove that $\sum q_n / 2^{q_n}$ is irrational.

I proved that $\sum 1/(2^n - 1) = \sum d(n)/2^n$ is irrational, but I could not prove that $\sum 1/(2^n - 3)$, $\sum \phi(n)/2^n$, $\sum \omega(n)/2^n$, $(\omega(n) = \sum_{p|n} 1)$ are all irrational. Also I could not prove that $\sum 1/(n! - 1)$ is irrational.

Stolarsky has the following nice conjecture: Let $a_1 < a_2 < \ldots$ satisfy $\sum 1/a_n < \infty$. Then there is an integer t for which $\sum_{t < a_n} 1/(a_n - t)$ is irrational.

2. Some problems on uniform distribution. Let E be a set of positive measure $m(E)$ in $(0,1)$, and α an irrational number. Is it true that the logarithmic density of the integers m for which $m\alpha - [m\alpha] \in E$ is $m(E)$ for almost all α , i.e.

$$\frac{1}{\log x} \sum_{\substack{m<x \\ m\alpha-[m\alpha]\in E}} 1/m \to m(E) , \quad x \to \infty ?$$

Is it true that the logarithmic density of the integers n for which the number of integers $m < n$ for which

$$\sum_{m=1}^{n} (m\alpha - [m\alpha] - \tfrac{1}{2}) > 0$$

is $\frac{1}{2}$ for almost all α ?

Is it true that $p\alpha - [p\alpha]$ is never well distributed where p runs through the set of primes? In fact it seems certain that for even α and k there are k consecutive primes $p_{r+1}, \ldots, p_{r+k}$ for which

$$0 < p_{r+i}\alpha - [p_{r+i}\alpha] < \tfrac{1}{2}$$

for every i with $1 \leq i \leq k$. Unfortunately I could not even prove the existence of a single such α .

H. NIEDERREITER:

For any positive integer $m \geq 2$ put $K_m = \min_{\substack{(a,m)=1 \\ 1\leq a\leq m}} \max_{1\leq i\leq r} a_i$ where $[a_0, a_1, \ldots, a_r]$ denotes the continued fraction expansion of a/m . Zaremba conjectured $K_m \geq 5$ for all $m \geq 2$. I could only prove: $K_m \geq 3$ if m is a pure power of 2 or 3 , $K_m \geq 5$ if m is a pure power of 5 , $K_m = O(\log m)$ as $m \to \infty$.

A. SCHINZEL:

1. Let $N(f)$ denote the number of terms of a polynomial f . Is it true that for every $F \in \mathbb{C}[x] \setminus \mathbb{C}$ and every sequence f_n , where $f_n \in \mathbb{C}[x]$ $(n = 1, 2, \ldots)$ $\lim_{n\to\infty} N(f_n) = \infty$ implies $\lim_{n\to\infty} N(F(f_n)) = \infty$?

2. For $k > 1$, $\underline{n} = (n_1, n_2, \ldots, n_k) \in \mathbb{Z}^k$ let $h(\underline{n}) = \max_{1 \le i \le k} |n_i|$. Is it true that for every $\underline{n} \in \mathbb{Z}^k$ there exists $\underline{p}, \underline{q} \in \mathbb{Z}^k$ such that $\underline{n} = u\underline{p} + v\underline{q}$, $u, v \in \mathbb{Q}$ and $h(\underline{p})h(\underline{q}) \le h(\underline{n})^{(k-2)/(k-1)}$?

J.H. SILVERMAN:

The number of solutions of the S-unit equation. Let $S = \{p_1, \ldots, p_s\}$ be a finite set of primes, $\mathbb{Z}_S^* = \{ \frac{a}{b} \in \mathbb{Q}^* : a$ and b are products of primes in $S \}$, and $N(S) = \text{card}\{ u \in \mathbb{Z}_S^* : 1 - u \in \mathbb{Z}_S^* \}$. Evertse has proven that $N(S) \le 3 \times 7^{3+2s}$; while C.L. Stewart has shown that there are sets of primes S such that for all $\varepsilon > 0$, $N(S) \gg_\varepsilon \exp(s^{\frac{1}{2}-\varepsilon})$. Let $\Sigma_s = \{2, 3, \ldots, p_s\}$ be the set consisting of the first s primes. Based on a heuristic argument, Stewart suggests the following.

<u>Conjecture (C.L. Stewart):</u> (a) For all $\varepsilon > 0$ and all sets of primes with $\text{card}(S) = s$, $N(S) \ll_\varepsilon \exp(s^{2/3+\varepsilon})$.

(b) For all $\varepsilon > 0$, $N(\Sigma_s) \gg_\varepsilon \exp(s^{2/3-\varepsilon})$.

On the other hand, numerical evidence of P. Vojta and R. Gross for $3 \le s \le 8$ suggests that $(\log N(\Sigma_s))/s \to 1$ as $s \to \infty$.

<u>Questions:</u> (1) Does there exist an $\varepsilon > 0$ such that $N(S) \ll_\varepsilon \exp(s^{1-\varepsilon})$ for all sets of primes S with $\text{card}(S) = s$?

(2) Does there exist an $\varepsilon > 0$ such that $N(\Sigma_s) \gg_\varepsilon \exp(s^\varepsilon)$ for all $s \ge 1$?

(3) Is it true that for all $s \ge 1$, $\max_{\text{card}(S)=s} \{N(S)\} \ll N(\Sigma_s)$?

C.L. STEWART:

Let $(u_n)_{n=0}^\infty$ be an integer linear recurrence sequence. Then we have

$$u_n = f_1(n)\alpha_1^{\,n} + \ldots + f_t(n)\alpha_t^{\,n} , \quad n = 0, 1, 2, \ldots ,$$

where $f_1, \ldots, f_t$ are polynomials in n and $\alpha_1, \ldots, \alpha_t$ are algebraic integers and we shall suppose henceforth that $\alpha_1 \ldots \alpha_t \neq 0$ and $f_1 \ldots f_t \neq 0$. We say that the sequence is non-degenerate if $t > 1$ and α_i/α_j is not a root of unity for $1 \leq i < j \leq t$. Is it true that if $(u_n)_{n=0}^{\infty}$ is a non-degenerate linear recurrence sequence with $f_1, \ldots, f_t$ constant polynomials and u_n is the pure power of an integer (i.e. square, cube, etc.) for infinitely many integers n that then for some integer $q \geq 2$ there exists a linear recurrence sequence $(v_n)_{n=0}^{\infty}$ such that $u_n = v_n^q$ for $n = 0, 1, 2, \ldots$?

R. TIJDEMAN:

Prove or disprove: $\prod_{p|x(x-1)(x-2)} p > x$ for all $x \in \mathbb{Z}_{>2}$.

B. VOLKMANN:

Consider for each integer n the unique expansion in Fibonacci numbers $n = F_k + c_{k+1}F_{k+1} + \ldots + c_{k+\ell}F_{k+\ell}$ with $c_j \in \{0,1\}$ and $c_j c_{j+1} = 0$ for all j. Put $\Lambda(n) = k$. Is it true that $\Lambda(2^m)$ is unbounded?

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