

MATHEMATISCHES FORSCHUNGSIINSTITUT OBERWOLFACH

Tagungsbericht 12/1986

Darstellungstheorie endlichdimensionaler Algebren

23.3. bis 29.3.1986

Die Tagung fand unter der Leitung von Herrn Prof. Dr. Peter Gabriel (Zürich) und Herrn Prof. Dr. Claus M. Ringel (Bielefeld) statt. Neben dem zentralen Gegenstand dieser Tagung, der Darstellungstheorie endlichdimensionaler Algebren über einem Körper, wurden auch in vielfältiger Weise Zusammenhänge der Darstellungstheorie mit benachbarten Gebieten der Mathematik diskutiert, sowie die bisher bei den endlichdimensionalen Algebren entwickelten Methoden auf allgemeinere Situationen in der Darstellungstheorie ausgedehnt.

Die Mehrzahl der Vorträge beschäftigte sich mit Strukturfragen von Algebren über einem Körper und der Beschreibung der Kategorie ihrer unzerlegbaren Moduln. Darunter wurden speziell Algebrenepimorphismen, zweireihige Algebren und Ueberlagerungstheorie, derivierte Kategorien und Kippalgebren, sowie Darstellungen von Halbordnungen diskutiert.

Invariantentheoretische Fragestellungen und deren Anwendung auf Deformationen von Algebren und Darstellungen von Köichern bildeten einen weiteren Schwerpunkt dieser Tagung.

Des weiteren wurden in einigen Vorträgen Zusammenhänge zwischen der Darstellungstheorie von zahmen Algebren und kohärente Garben bzw. Vektorbündeln auf projektiven Räumen dargestellt.

Die Methoden der Darstellungstheorie fanden ausserdem Anwendung in der Topologie von CW-Komplexen, der Klassifikation von zahmen Blöcken von Gruppenalgebren und in der Singularitätentheorie.

Das Bestreben, die Methoden der Darstellungstheorie endlichdimensionaler Algebren auf allgemeinere Situationen auszudehnen, stand schliesslich im Mittelpunkt einer Reihe von weiteren Vorträgen. Dabei ging es um Ringe von isolierten Singularitäten höherer Dimension, sowie Ordnungen über vollständigen Dedekindringen. Diskutiert wurden in diesem Zusammenhang die Kategorien von Cohen-Macaulay-Moduln, Existenz von beinahe zerfallenden Sequenzen, sowie die Struktur von Auslander-Reiten Köchern.

Die freie Zeit zwischen den Vorträgen wurde von den Teilnehmern zu regem Gedankenaustausch und informellen Gesprächen genutzt. Sicherlich regte diese Tagung alle Teilnehmer zu weiteren Forschungstätigkeiten im Zusammenhang mit der Darstellungstheorie endlichdimensionaler Algebren an.

### Vortragsauszüge

#### I. ASSEM:

##### Iterated tilted algebras of type $\tilde{A}_n$

This is a report on a joint work with A. Skowroński.

We classify the iterated tilted algebras of type  $\tilde{A}_n$ , that is to say, those algebras which come from a hereditary algebra of type  $\tilde{A}_n$  by a finite number of tilts. These algebras are classified by their bound quivers using the representation theory of biserial algebras, tilting theory and covering techniques. In the representation-finite case, we also give a characterisation of these algebras in terms of their Auslander-Reiten quivers.

M. AUSLANDER:

(based on work with Idun Reiten)

Almost split sequences in dimension 2, II

A unified approach to proving the following theorems is given.

- a) Let  $X$  be a connected projective Cohen-Macaulay curve over an infinite field. An indecomposable Cohen-Macaulay sheaf  $\mathcal{X}$  is locally free if and only if there exists an almost split sequence  $0 \rightarrow \mathcal{F} \rightarrow \mathcal{G} \rightarrow \mathcal{X} \rightarrow 0$  in the category of Cohen-Macaulay sheaves.
- b) Let  $k$  be an infinite field and let  $S = k[x_0, \dots, x_n]/I$  with  $I$  a homogeneous ideal such that Krull dimension of  $S$  is 2 and  $S$  is Cohen-Macaulay. Then a graded indecomposable module  $H$  in the category of degree zero maps has the property that  $H_{\underline{p}}$  is  $S_{\underline{p}}$ -free for all prime ideals  $\underline{p}$  such that  $\text{ht } \underline{p} \leq 1$  if and only if there is an almost split sequence  $0 \rightarrow F \rightarrow G \rightarrow H \rightarrow 0$  in the category of graded Cohen-Macaulay modules with degree zero maps.
- c) Let  $S$  be a complete 2-dimensional local ring which is Cohen-Macaulay. Then an indecomposable nonprojective module  $H$  has the property that  $H_{\underline{p}}$  is  $S_{\underline{p}}$ -free for all nonmaximal prime ideals  $\underline{p}$  if and only if there is an almost split sequence  $0 \rightarrow F \rightarrow G \rightarrow H \rightarrow 0$  in the category of Cohen-Macaulay modules.

D. BAER:

Wild algebras and projective varieties

We present a comparison theorem relating the representation theory of certain finite dimensional algebras to the theory of coherent sheaves on weighted projective spaces. This theorem describes a generalized tilting procedure from sheaves to modules and leads - for usual projective spaces - to a more elementary understanding of Beilinson's result that shows the coincidence of the

respective derived categories.

We explain our results in detail for the projective plane and the corresponding quiver  $\begin{array}{c} X_0 & X_0 \\ \downarrow & \downarrow \\ X_1 & X_1 \\ \downarrow & \downarrow \\ X_2 & X_2 \end{array}$ . with relations  $X_i X_j = X_j X_i$ . Furthermore, in this case, there are reductions to the hereditary situation: For instance, every coherent sheaf on the projective plane is - up to shift - uniquely determined by a representation of the quiver  $\begin{array}{c} X_0 \\ \downarrow \\ X_1 \\ \downarrow \\ X_2 \end{array}$ .

### C. CIBILS:

#### Hochschild cohomology of algebras

Let  $k$  be a field and  $Q$  be a finite narrow quiver, where narrow means that between two vertices there is at most one oriented path. Let  $I$  be an admissible ideal of the quiver algebra  $kQ$  and let  $\Lambda$  be the quotient algebra  $kQ/I$ .

Let  $H^*(\Lambda, M)$  denote the Hochschild cohomology groups of  $\Lambda$  with coefficients in a  $\Lambda$ -bimodule  $M$ .

We prove that  $H^*(\Lambda, \Lambda) = 0$  if  $*$   $\geq 2$ . If  $Q$  is connected,  $\dim_k H^1(\Lambda, \Lambda) = 1 - \chi(Q)$ .

The fact that  $H^2(\Lambda, \Lambda) = 0$  implies that the orbit of an algebra  $\Lambda$  is open if its Gabriel quiver is narrow.

### E. DIETERICH:

#### Reduction of isolated singularities

Let  $R$  be a commutative noetherian complete regular local ring with maximal ideal  $m$ ,  $\Lambda$  an  $R$ -algebra which is finitely generated free as  $R$ -module, and denote by  $\text{mod}_R^\Lambda$  the category of finitely generated  $R$ -projective  $\Lambda$ -modules.

I pose the problem of describing the structure of the stable Auslander-Reiten quiver  $\Gamma_s(\Lambda)$  of  $\text{mod}_R \Lambda$ , in case  $\Lambda$  is an isolated singularity (i.e.  $\text{gldim} \Lambda \neq \dim R$ , and  $\text{gldim}_{\Lambda_p} \Lambda_p = \dim_{R_p} R_p$  for all  $p \in \text{Spec}(R) \setminus \{\mathfrak{m}\}$ ).

In this respect I can prove the following results.

Proposition 1. Let  $\Lambda$  be an isolated singularity of finite type, and  $\mathcal{C} \subset \Gamma_s(\Lambda)$  a connected component containing a periodic point. Then  $\mathcal{C} = \mathbb{Z}\Delta/G$ , where  $\Delta$  is Dynkin and  $G \subset \text{Aut } \mathbb{Z}\Delta$ .

Proposition 2. Let  $\Lambda$  be the complete local ring of an affine algebraic variety at an isolated singular point (over an algebraically closed field), such that  $\Lambda$  is Cohen-Macaulay. Let  $\mathcal{C} \subset \Gamma_s(\Lambda)$  be a connected component containing a periodic point. Then either  $\mathcal{C} = \mathbb{Z}\Delta/G$ ,  $\Delta$  and  $G$  as above, or  $\mathcal{C} = \mathbb{Z}\Lambda_\infty/\langle \tau^r \rangle$ ,  $r \in \mathbb{N}$ .

Proposition 3. Let  $\Lambda = \mathbb{C}[X_0, \dots, X_d]/(f(X_0, \dots, X_d))$  be an isolated hypersurface singularity of infinite type. Then  $\Gamma_s(\Lambda) = \coprod_I \mathbb{Z}\Lambda_\infty/\langle \tau^r \rangle$ , where  $r \in \{1, 2\}$  and  $I$  is some index set. In addition, if  $d$  is even, then  $r \equiv 1$ .

The proofs are based on the construction of an  $\mathfrak{m}$ -primary ideal  $a$  in  $R$  which annihilates the functor  $\text{Ext}_\Lambda^1(\cdot, \cdot) : \text{mod}_R \Lambda \times \text{mod} \Lambda \rightarrow \text{mod } R$ . This gives rise to a reduction functor  $\Lambda/\mathfrak{a} \otimes_\Lambda : \text{mod}_R \Lambda \rightarrow \text{mod}(\Lambda/\mathfrak{a})$ , where  $\Lambda/\mathfrak{a}$  is an artinian factor algebra of  $\Lambda$ , such that methods from representation theory of artin algebras can be applied. As side-results one obtains that in the situation of Proposition 2 the first Brauer-Thrall conjecture holds, and that in the situation of Proposition 3 even the second Brauer-Thrall conjecture holds.

P. DRAEXLER:

#### Fibred sums over $\Lambda$ in representation-finite algebras

Let  $A$  be a representation-finite algebra over a field  $k$  and  $\hat{U}$  a module

in  $A\text{-mod}$  with  $\text{Ext}_A(\hat{U}, \text{Sub}\hat{U}) = 0 = \text{Ext}_A(\hat{U}, \text{Fac}\hat{U})$ . Then there exists  $\hat{V}$  in  $A\text{-mod}$ , such that  $\mathfrak{v} := \text{Add}\hat{V}$  is the subcategory of  $A\text{-mod}$  consisting of all finite direct sums of indecomposable modules  $V$  with  $\text{Hom}_A(\hat{U}, V) \neq 0$  and  $\text{Ext}_A(V, \text{Fac}\hat{U}) = 0$ . The cokernels of homomorphisms with domain in  $\mathfrak{v}$  and source in  $A := \text{Add}\hat{U}$  are called fibred sums over  $A$ . We have the following results:

1. Each indecomposable module  $M$  in  $A\text{-mod}$  with  $\text{Hom}_A(\hat{U}, M) \neq 0$  is a fibred sum over  $A$ .
2. There is an injective module  $Q$  in  $\text{End}_A(\hat{V})\text{-mod}$ , such that  $\text{Sub}Q$  is equivalent to the factor category  $A\text{-mod}/\mathcal{F}$ , where  $\mathcal{F}$  denotes the subcategory of all  $N$  in  $A\text{-mod}$  with  $\text{Hom}_A(\hat{U}, N) = 0$ .
3.  $A\text{-mod}/\mathcal{F}$  is equivalent to the category of the representations of some representation-finite partially ordered set iff  $A\text{-mod}/\mathcal{F}$  has directed Auslander-Reiten quiver and  $k$  is splitting field for  $A\text{-mod}/\mathcal{F}$ .

K. ERDMANN:

#### Reconstructing tame blocks from the stable Auslander-Reiten quiver

It is known that a p-block  $B$  of a group algebra is of tame representation type if and only if  $p = 2$  and a defect group of the block is either dihedral or semidihedral or generalized quaternion.

The topic of the lecture is the classification of these blocks (mainly the semidihedral case) with the help of Auslander-Reiten theory, by the following strategy:

- 1) Find the graph structure of the stable Auslander-Reiten quiver  $\Gamma_S(B)$  of the block  $B$
- 2) More generally: classify algebras  $\Lambda$  with  $\Gamma_S(\Lambda) \approx \Gamma_S(B)$  (satisfying

appropriate regularity conditions).

3) Find the blocks in the list.

W. GEIGLE:

Coherent sheaves and representation theory

Let  $\Lambda = \Lambda(p, \lambda)$  be a canonical algebra (c.f. C.M. Ringel, Tame Algebras and Integral Quadratic Forms, LNM 1099). We want to interrelate finite dimensional modules over  $\Lambda$  with coherent sheaves over a weighted projective curve  $C = C(p, 2)$ .

There exists a tilting sheaf  $\mathcal{L} \in \text{coh}(C)$  (i.e.  $\text{Ext}^1(\mathcal{L}, \mathcal{L}) = 0$ ) and the indecomposable direct summands of  $\mathcal{L}$  form a basis of the Grothendieck group  $K_0(\text{coh}(C))$  with  $\text{End}(\mathcal{L}) \cong \Lambda$ .

We obtain subcategories  $\text{coh}^+(C)$ ,  $\text{coh}^-(C)$  of  $\text{coh}(C)$  and subcategories  $\text{mod}^{\geq}(\Lambda^{\text{op}})$ ,  $\text{mod}^{<}(\Lambda^{\text{op}})$  of  $\text{mod}(\Lambda^{\text{op}})$  in such a way that

$\text{Hom}(\mathcal{L}, -) : \text{coh}^+(C) \rightarrow \text{mod}^{\geq}(\Lambda^{\text{op}})$  and

$\text{Ext}^1(\mathcal{L}, -) : \text{coh}^-(C) \rightarrow \text{mod}^{<}(\Lambda^{\text{op}})$

are equivalences of categories which are exact with respect to exact sequences formed in  $\text{coh}(C)$  and  $\text{mod}(\Lambda^{\text{op}})$ , respectively. We note that every indecomposable module over  $\Lambda$  is contained in  $\text{mod}^{\geq}(\Lambda^{\text{op}})$  or  $\text{mod}^{<}(\Lambda^{\text{op}})$ , thus the representation theory of  $\Lambda$  is controlled by the sheaf theory of  $C$ .

E.L. GREEN:

On the cohomology ring of a finite dimensional algebra

Let  $\Lambda$  be a finite dimensional factor of the path algebra  $k\Gamma$ , where  $\Gamma$

is the quiver of  $\Lambda$ . Let  $\underline{r}$  denote the Jacobson radical of  $\Lambda$ . Let  $E(\Lambda)$  denote the Ext-algebra  $\coprod_{i=0}^{\infty} \text{Ext}_{\Lambda}^i(\Lambda/\underline{r}, \Lambda/\underline{r})$  with multiplication given by the Yoneda product. Note that  $E(\Lambda)$  is a graded algebra,  $E(\Lambda) = E_0 + E_1 + \dots$  where  $E_i = \text{Ext}_{\Lambda}^i(\Lambda/\underline{r}, \Lambda/\underline{r})$ . By  $E^2(\Lambda)$  we mean the Ext-algebra of  $E(\Lambda)$ ,  $\coprod_{i=0}^{\infty} \text{Ext}_{E(\Lambda)}^i(E_0, E_0)$ . The structure of  $E(\Lambda)$  and  $E^2(\Lambda)$  are discussed in relation to the structure of  $\Lambda$ , in the case when  $\Lambda$  is a zero-relation algebra. When  $\Lambda$  is a zero-relation algebra, we show for example, that  $\Lambda \cong E^2(\Lambda)$  if and only if the ideal of relations for  $\Lambda$  in  $k\Gamma$  can be generated by paths of length 2. Other results are presented.

#### D. HAPPEL:

##### Iterated tilted algebras of affine type

Let  $A$  be a finite-dimensional associative algebra with 1 over an algebraically closed field  $k$ . The algebra  $A$  is called piecewise hereditary if there exists a finite quiver  $\tilde{\Delta}$  without oriented cycle such that the derived categories of bounded complexes  $D^b(A)$  and  $D^b(k\tilde{\Delta})$  are triangle-equivalent.

The main result describes these algebras in case the underlying graph is an affine diagram of the form  $\tilde{A}_n, \tilde{D}_n, \tilde{E}_6, \tilde{E}_7, \tilde{E}_8$ . In fact, we will show that a piecewise hereditary algebra of affine type  $\tilde{\Delta}$  is an iterated tilted algebra of affine type  $\tilde{\Delta}$ .

It was known before that the same characterization holds in the case of Dynkin diagrams.

H.W. HENN:

p-local CW-complexes of low stable dimension and a generalization of results of Gelfand-Ponomarev, Ringel a.o....

Let  $p$  be an odd prime. The homotopy classification of  $p$ -local CW-complexes which satisfy:  $\tilde{H}_j(X)$  is of finite type over  $\mathbb{Z}_{(p)}$ , the integers localized at  $p$ , and  $\tilde{H}_j(X, \mathbb{Z}_{(p)}) = 0$  for  $j < n$ ,  $j > n+4p-5$  ( $n \gg p$ ) leads to the following problem:

Let  $M = \{0, 1, \dots, m\}$   $N = \{\bar{0}, \bar{1}, \dots, \bar{n}\}$  (i.e.  $M$  and  $N$  are supposed to be disjoint) and  $\tau$  be a fixed point free involution of a subset  $K \subset M \cup N$ .

Classify finite dimensional vectorspaces  $A$  which are equipped with two filtrations

$$0 = F^0 \subset F^1 \subset \dots \subset F^m = A$$

$$0 = F^{\bar{0}} \subset F^{\bar{1}} \subset \dots \subset F^{\bar{n}} = A$$

and isomorphisms  $\Phi_k : F_k/F_{k-1} \rightarrow F_{\tau k}/F_{\tau k-1}$  such that  $\Phi_k \circ \Phi_{\tau k} = \text{id}$ .

This covers the classification of finite dimensional  $k\langle x, y \rangle/(xy, yx)$ - and  $k\langle x, y \rangle/(x^2, y^2)$ -modules (Gelfand-Ponomarev and Ringel).

H. LENZING:

Stable and semi-stable modules over canonical algebras

For representations over a canonical algebra of type  $(p_0, \dots, p_n)$  in Ringel's sense we introduce the notions of stability and semi-stability by transfer from the attached category of coherent sheaves on the corresponding parameter curve. Always the category  $\mathcal{C}$  of all semi-stable objects has a decomposition

$\mathcal{C} = \bigcup_{\mu \in Q} \mathcal{C}_\mu$ , where  $\text{Hom}(X_\mu, X_\nu) = 0$  for  $X_\mu \in \mathcal{C}_\mu$ ,  $X_\nu \in \mathcal{C}_\nu$  and  $\mu > \nu$  in  $Q$ .

For  $(p_0, \dots, p_n)$  Dynkin or tubular, all indecomposables are stable or

semi-stable resp.. Here, the  $\mathcal{C}_\mu$  - which are abelian length categories in

general - decompose into uniserial subcategories. Finally we present a Riemann-Roch theorem for parameter curves as a classification tool.

L.A. NAZAROVA:

Tame problems

Let  $K$  be a category of finite-dimensional vector-spaces having a finite number of objects. We call  $K$  special if it has a multiplicative basis for which all non-identical basis morphisms have rank 1. A (linear) subcategory  $L \subset K$  is semifull if, for all  $X, Y \in L$ ,  $L(X, Y)$  contains all morphisms of rank 1 in  $K(X, Y)$ . We prove that  $K$  is wild iff it contains a wild special semi-full subcategory. In order to characterize the wild special categories, we attach a "special" poset  $S(L)$  to each special  $L$ . Then  $L$  is wild iff  $S(L)$  contains a "critical" poset. There are ten "critical" posets. In similar way we characterize the tame special categories of infinite growth.

J.A. DE LA PEÑA:

Epimorphisms of Algebras

Let  $k$  be a commutative ring and let  $A$  be a  $k$ -algebra. A  $k$ -algebra homomorphism  $\varphi : A \rightarrow B$  is called an epimorphism iff for any two morphisms  $B \xrightarrow[\psi]{\chi} C$  with  $\chi\varphi = \psi\varphi$  we have  $\chi = \psi$ . Two epimorphisms  $A \xrightarrow{\varphi} B$ ,  $A \xrightarrow{\varphi'} B'$  are equivalent if there is an isomorphism  $j : B \rightarrow B'$  with  $\varphi' = j\varphi$ . The class  $\bar{\varphi}$  of  $\varphi$  is called an epiclass.

Theorem: There is a bijective correspondence between the epiclasses  $\bar{\varphi}$ ,  $(\varphi : A \rightarrow B)$  and the full subcategories of  $\text{Mod } A$  which are closed under the construction of isomorphic objects, products, direct sums, kernels,

and cokernels.

Among other things we prove:

Theorem: Let  $k$  be an algebraically closed field and  $A$  a representation-finite finite-dimensional  $k$ -algebra. Let  $\varphi: A \rightarrow B$  be an epimorphism. Then, the number of simples (up to isomorphism) of  $B$  is smaller or equal to the number of simples of  $A$ .

Z. POGORZALY and A. SKOWROŃSKI:

Biserial algebras

During our talk we shall outline the proofs of the following two theorems:

Theorem 1. Every finite dimensional biserial algebra  $A$  over an algebraically closed field is tame and  $\beta(A) \leq 2$ .

Theorem 2. Let  $A$  and  $B$  be finite dimensional algebras over an algebraically closed field. Assume that  $A$  is biserial and  $B$  is stably equivalent to  $A$ . Then  $B$  is biserial.

C. PROCESI:

An inverse to the Cayley Hamilton theorem

One proves in Char. 0 that the validity of the Cayley Hamilton theorem in degree  $n$  characterizes the algebras with trace that can be embedded in  $n \times n$  matrices over a commutative ring.

I. REITEN:

(based on work with M. Auslander)

Almost split sequences in dimension 2 , I

Let  $k$  be an algebraically closed field and  $T = k[[x_1, \dots, x_n]]$ . Let  $\Lambda$  be a  $T$ -order, and denote by  $L(\Lambda)$  the category of  $\Lambda$ -lattices, which is known to have almost split sequences. We discuss the special features of the case when  $\dim R = 2$ , which essentially have to do with the fact that for indecomposable projective  $\Lambda$ -lattices there are sequences (fundamental sequences) similar to almost split sequences. In particular we discuss the shape of the AR-quiver in the case of finite type, in the commutative case and for maximal orders of finite type classified by Artin.

K.W. ROGGENKAMP:

Subhereditary orders

Let  $R$  be a complete Dedekind domain,  $K = \text{frac}(R)$  and  $\Lambda$  an  $R$ -order in a separable  $K$ -algebra; by  $\mathcal{A}(\Lambda)$  we denote the Auslander-Reiten quiver  $d\Lambda$ .  $\Lambda$  is said to be subhereditary, provided there exists a hereditary order  $\Gamma$  with  $\text{rad } \Gamma \subset \Lambda \subset \Gamma$ . In this case  $\mathcal{D} = \begin{pmatrix} \Gamma/\text{rad } \Gamma & \Gamma/\text{rad } \Gamma \\ 0 & \Lambda/\text{rad } \Gamma \end{pmatrix}$  is a socle projective algebra, and every socle projective algebra without simple ring direct factors arises this way. Denote by  $\mathcal{S}(\mathcal{D})$  the category of socle projective  $\mathcal{D}$ -modules with  $A$ -R-quiver  $\mathcal{A}(\mathcal{S}(\mathcal{D}))$ . Then for subhereditary  $\Lambda$ ,  $\mathcal{A}(\Lambda)$  is uniquely determined by  $\mathcal{A}(\mathcal{S}(\mathcal{D}))$  and the permutation associated to  $\Gamma$ . Moreover, for an arbitrary order  $\Lambda$  of finite lattice type the following are equivalent : (i) every oriented cycle in  $\mathcal{A}(\Lambda)$  passes through a vertex with only projective successors. (ii)  $\Lambda$  is subhereditary and  $\mathcal{A}(\mathcal{S}(\mathcal{D}))$  has preprojective components. Generalizations to higher

dimensions are discussed.

W. RUMP:

Finite dimensional Algebras and Invariant Theory

An approach to the structure of finite dimensional algebras by means of classical invariant theory.

For a given finite dimensional associative algebra  $A$  with  $1$  over some p.i.d.  $R$  ( $A$  free as an  $R$ -module), we decompose  $A$  linearly as  $A = R \oplus B$  and analyse the induced algebra structure on  $B$ , that is, for  $x, y \in B$ ,  $x \cdot_A y = \phi(x, y) + x \cdot_B y$  with a bilinear form  $\phi : B \times B \rightarrow R$ , and  $x \cdot_B y \in B$ . Then the associativity of  $A$  turns  $B$  into a "preassociative"  $R$ -algebra which is determined up to homotopy, where a homotopy  $B \cong B'$  is given by a linear form  $x \in B^*$  which changes the product in  $B$  to  $x \cdot_{B'} y = x \cdot_B y + x(y)x + x(y)x$ . Moreover, if  $\dim B \geq 2$ , then  $\phi$  is uniquely determined by the preassociativity of  $B$ .

If  $V$  is the underlying  $R$ -module of  $B$  and  $T_0$  the set of algebra structures  $\gamma \in T = V \otimes V \otimes V^*$  on  $V$  with  $\gamma \cong 0$ , then  $T/T_0$  is a free  $R$ -module on which  $GL(V)$  acts, and the set  $\mathcal{A}$  of preassociative  $\gamma \in T$  is a closed  $GL(V)$ -invariant variety in  $T$  with  $T_0 \subset \mathcal{A}$ . Thus if  $\dim V = n$ , we obtain a 1-1 correspondence between the iso-classes of  $(n+1)$ -dimensional associative  $R$ -algebras with  $1$  and the  $GL(V)$ -orbits of the closed variety  $\mathcal{A}' \subset T/T_0$  induced by  $\mathcal{A}$ .  $\mathcal{A}'$  is defined by simultaneous vanishing of two concomitants  $A_2, A_3$  of resp. degree 2 and 3. A geometrical interpretation is given for  $n = 3$ .

M. SCHAPS:

Deformations of Algebras and their idempotents

Let  $A$  be a finite dimensional, unitary algebra over a field  $k$ ,  $k = \bar{k}$ . Let  $e$  be a complete, primitive, orthogonal idempotent set, and  $x$  a filtered basis, multiplicative with respect to multiplication by elements of  $e$ , and containing  $e$ . Let  $Q(A)$  be the diagram with vertices  $e_i$  and arrows which are non-idempotent  $x_j$ .  $\bar{Q}(A)$  will be the same diagram with arrows weighted by their position in the radical flag. If  $A$  specializes to  $B$ , we show that  $Q(A)$  degenerates to  $Q(B)$  by coallescing idempotents, adding loops to replace vanishing vertices.  $B$  cannot be uniserial if the coallescing idempotents are connected by an arrow. For  $A$  of finite representation type, sufficient conditions are given for  $B$  to be uniserial with filtered multiplicative basis. As an application we consider deformations of six-dimensional algebras.

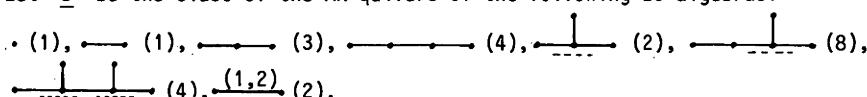
G. TODOROV:

(based on work with Igusa, Platzeck and Zacharia)

Auslander algebras of finite representation type

We classify all finite Auslander-Reiten quivers of finite representation type.

Let  $\underline{B}$  be the class of the AR-quivers of the following 25 algebras:



First we show that all quivers in the class  $\underline{B}$  are of finite representation type.

Second we show that if an AR-quiver is of finite representation type, then

its universal covering consists of the quivers from B and any two of them are either disjoint, one point pasted or a triangle (triangular mesh) pasting, and this decomposition is unique.

Third we give a complete list of possible triangle pastings which preserve finite representation type. For point pastings we give an easy numerical condition for the pasted quiver to be of finite representation type.

Finally we show that the above gives necessary and sufficient conditions for a finite AR-quiver to be of finite representation type.

L. UNGER:

An induction principle for wild concealed algebras

This is a report on some recent joint work with D.Happel.

Let  $\tilde{\Delta}$  be a representation-infinite quiver and  $T_{k\tilde{\Delta}}$  a preprojective tilting module over the path algebra  $k\tilde{\Delta}$ . Then  $A = \text{End}(T)$  is called a concealed algebra of type  $\tilde{\Delta}$ . The main result asserts that this class of algebras is well-suited for induction.

Theorem: Let  $\tilde{\Delta}$  be a wild quiver with underlying graph a tree and  $A$  be a concealed algebra of type  $\tilde{\Delta}$ . Then there exists a primitive idempotent  $e \in A$  such that  $A' = A/\langle e \rangle$  is a concealed algebra of type  $\tilde{\Delta}'$  such that  $\tilde{\Delta}'$  is a full subquiver of  $\tilde{\Delta}$ .

If  $\tilde{\Delta}$  is an affine diagram the classification of the concealed algebras is well-known. Using similar methods one also gets the classification of the concealed algebras of the minimal wild hereditary algebras.

A. WIEDEMANN:

The Auslander-Reiten quivers of the lattice finite Gorenstein orders

Let  $R$  be a complete Dedekind domain, and let  $\Lambda$  be a Gorenstein order of finite lattice type over  $R$ . If we assume that  $\Lambda$  is neither hereditary nor that the stable Auslander-Reiten quiver  $\mathcal{A}_s(\Lambda)$  of  $\Lambda$  is discrete - in this case  $\Lambda$  is a Bäckström-order -, it is known that  $\mathcal{A}_s(\Lambda) \cong \mathbb{Z}\Delta/G$  for a Dynkin diagram  $\Delta$ . Moreover,  $\mathcal{A}(\Lambda)$  is determined by the three ingredients  $\Delta$ , the automorphism group of  $\mathbb{Z}\Delta$  and the set  $C$  of successors of the projective vertices in the universal cover  $\mathbb{Z}\Delta$  of  $\mathcal{A}_s(\Lambda)$ .  $C$  is called the configuration of  $\Lambda$ . In the case where  $\Delta = A_n, D_n, E_n$  ( $n \geq 2$ ) and  $D_n$  ( $n \geq 4$ ), two conditions are given which characterize the possible configurations for a lattice-finite Gorenstein order  $\Lambda$  of class  $\Delta$ . Finally, we indicate the basic ideas leading to the result saying that for a given configuration  $C \subset (\mathbb{Z}\Delta)_0$  any admissible automorphism group  $G'$  of  $\mathbb{Z}\Delta$  stabilizing  $C$  occurs as the fundamental group of the AR-quiver of a Gorenstein order  $\Lambda'$ , i.e.  $\mathbb{Z}\Delta/G' \cong \mathcal{A}_s(\Lambda')$ .

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