MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Mathematical Problems in Condensed Matter Physics

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Diese Tagung fand unter der Leitung von Herrn Jürg Fröhlich (ETH-Zürich, Schweiz) und Herrn Barry Simon (CALTECH, Pasadena, U.S.A.) statt. Vorbereitet wurde sie von J. Fröhlich, B. Simon und B. Souillard, der aber der Tagung aus gesundheitlichen Gründen fernbleiben musste.

Die Thematik unserer Tagung gehört zum Gebiet der mathematischen Physik. Die wissenschaftlichen Schwerpunkte lagen in der Quantenmechanik von Elektronen in äusseren periodischen, quasi-periodischen und zufälligen Potentialen, dem quantisierten Hall Effekt und der Theorie von Kristalloberflächen und Quasikristallen. Das Programm enthielt aber auch Vorträge über die Quantenmechanik der nicht-relativistischen Materie, integrable Systeme in der Quantenmechanik (samt Anwendungen auf die Zahlentheorie), die Herleitung der Brown'schen Bewegung aus der statistischen Mechanik, nicht-lineare Evolutionsgleichungen und dendritische Fronten, u.a..

Die mathematisch strenge Untersuchung der erwähnten physikalischen Systeme beinhaltet viel harte Analysis, Wahrscheinlichkeitstheorie, Gruppentheorie, aber auch etwas algebraische Topologie und algebraische Geometrie. Auch der Computer kommt mehr und mehr zum Zuge. Diese Vielfalt der mathematischen Methoden und die Aktualität der physikalischen Problemstellungen machten unsere Tagung ausserordentlich reizvoll und anregend. Forscher aus verschiedenen Gebieten der theoretischen und mathematischen

Physik und Mathematik konnten sich begegnen und erfahren, dass sie sich mit gutem Willen gegenseitig verstehen und vom Gespräch miteinander profitieren können. Solche Begegnungen verbreitern unsere Perspektive und regen neue Forschungsprojekte an.

Beinahe alle Teilnehmer hielten einen ein- bis eineinhalb stündigen Vortrag über ihr Arbeitsgebiet und die von ihnen in letzter Zeit gefundenen Resultate. Das durchschnittliche Niveau der Vorträge war ausgezeichnet. Die Gespräche während Mahlzeiten, in Pausen und am Abend waren anregend, und die sozialen Kontakte schienen erfreulich zu sein. Besonders geschätzt haben wir auch die Tatsache, dass unsere Tagung Forscher aus verschiedensten Ländern in Ost und West an einem Orte versammelt hat. Es gab Teilnehmer aus der BRD, der DDR, der Schweiz, Ungarn, der Sovjet Union, Italien, Frankreich, U.S.A., und ihre Nationalitäten waren noch vielfältiger. Es hat uns gefreut, Kollegen wieder zu begegnen, die wir leider nur selten sehen können. Unsere Tagung war in jeder Beziehung erfolgreicher, als der Unterzeichnete vermutet hätte. Wir sind dem mathematischen Forschungsinstitut Oberwolfach und seinen Leitern dankbar, dass sie uns die Durchführung dieser Tagung in einem sympathischen äusseren Rahmen ermöglicht haben!

Vortragsauszüge

J.E. Taylor

Singularities in Surfaces of Crystals

Grain boundaries and surfaces of crystalline materials have a surface free energy which in general depends on the normal directions of the interface or surface. Determining the shapes of surfaces which locally minimize a given surface free energy function is an interesting mathematical problem; it reduces in the case of isotropic (i.e. constant) surface energy to the minimal surface problem. A variety of results have been proved.

J.G. Conlon

Some Problems in Coulomb Systems

In this talk I discuss the stability of matter problem and related matters. We are concerned then with a large number of electrons which interact with each other and also with fixed nuclei. In 1967, Dyson and Lenard proved that the ground state energy of such a system is proportional to the total number of particles. I describe the proof of this result given by Lieb and Thirring in 1975 and also some more recent developments.

C. Itzykson

Simple Integrable Systems

We present a study of level degeneracies for the Dirichlet problem in a polyhedral box when the system is integrable. There exists a one-to-one correspondence between these cases and semi-simple Lie algebras or affine crystalline Weyl groups. Remarkable properties of the destribution of degeneracies are exhibited in low dimension, using number theoretic methods, indicating a spectacular branching of levels in the two-dimensional case.

The work is based on two recent papers:

C. Itzykson, J.M. Luck : J. Phys.A 19, **211** (1986)

C. Itzykson : Simple Integrable Systems and Lie Algebras, Saclay preprint PhT-85-172 to appear in "Journal of Modern Physics A".

J. Fröhlich

Review of Localization Theory

In this lecture I review recent rigorous results concerning the absence of transport and the existence of trapped waves (or localized vibrations) for systems of quantum mechanical particles moving under the influence of external random or certain quasi-periodic potentials and for some class of disordered, anharmonic crystal lattices. The main assumption needed in the proofs of these results is that the disorder be large. The analytical methods used in our analysis are perturbation expansions on an infinite sequence of increasing distance scales and are vaguely related to K.A.M. theory.

Results and proofs presented in this lecture are contained in the following papers: J. Fröhlich and T. Spencer, Commun. Math. Phys. **88**, 151 (1983); J. Fröhlich, F. Martinelli, E. Scoppola and T. Spencer, Commun. Math. Phys. **101**, 21 (1985); J. Fröhlich, T. Spencer and C.E. Wayne, J. Stat. Phys. **42**, 247 (1986); and in forthcoming work of T. Spencer and collaborators.

Some of our results are related to those discussed in Prof. Sinai's lecture.

Ya.G. Sinai

Anderson Localization for the Schroedinger Equation with the Quasiperiodic Potential.

We consider a general difference Schroedinger equation

$$(M(x)\psi)(n) = -\varepsilon(\psi(n+1) + \psi(n-1)) + V(T^n x)\psi(n)$$

where T is a measure-preserving transformation of a measure space (M, μ). Suppose that, for every eigenfunction ψ, one can construct an essential support $Z(\psi)$ showing a part of the lattice, where the e.f. is mainly concentrated. Then one can define a measurable object $\Phi(x)$ consisting of e.f. for which the essential support contains 0 and lies to the right from the origin. In many cases, it is natural to expect that the whole spectrum consists of $\bigcup_k S^k\Lambda(T^k x)$, where S is the shift. The case of the rotation of the circle is discussed in detail.

K.B. Efetov

Supersymmetry in Disordered Systems

Supersymmetry methods based on the use of both commuting and anticommuting variables are developed. Several problems of localization theory are solved.

A. Klein

A Rigorous Replica Trick Approach to Anderson Localization

We use a supersymmetric replica trick to prove estimates on the Green's functions of random potentials.

F. Wegner

Anderson Transition—a Critical Phenomenon

The description of the Anderson transition in terms of a model of interacting matrices (nonlinear σ-model) will be reviewed. On the basis of recent three-loop (with D. Höf) and four-loop (preliminary) calculations in an $\epsilon = d - 2$ expansion for this model it will be argued that not all of the critical exponents (for conductivity, crossover, participation ratio) can be "simple" expressions in ϵ.

B. Simon

Regularity of the IDS in the Anderson Model

After reviewing previous results I discuss joint work with M. Taylor, proving $k(E)$ is C^∞ in the Anderson model, with a potential density $d\lambda$ obeying $d\lambda(x) = F(x)dx$, where F has compact support and lies in the Sobolev space L^1_α.

W. Kirsch

Lifshitz Tails for Random Schrödinger Operators

We discuss the high- and low-energy behaviour of the density of states $N(E)$ for a random Schrödinger operator H_ω. For E near a band edge of H_ω Lifshitz predicted a behavior of $N(E)$ like

$$c_1 e^{c_2 (E-E_0)^{-\frac{d}{2}}}$$

(now known as "Lifshitz tails"). We discuss recent mathematically rigorous works that confirm Lifshitz' conjecture.

D. Szász

Spectral gap in the Markov approximation of Brownian motion

The following result is part of a unified dymanical theory of Brownian motion connecting its various mathematical models (Wiener process, Ornstein-Uhlenbeck process). Denote by $Q_M(t)$ the displacement of a particle of mass M interacting with an ideal gas of point particles of mass 1 via elastic collisions. Then, for $\gamma > \frac{3}{5}$, $A^{-\frac{1}{2}} Q_{A^\gamma}(At)$ converges to a Wiener process whose variance coincides with the Sinai-Soloveychik-Szász-Tòth lower bound $\underline{\sigma}^2 = \lim_{M\to\infty} \liminf_{A\to\infty} \; Var\, t^{-\frac{1}{2}} Q_M(t)$.

P. Kramer

Quasi-Crystallography Associated With the Icosahedral Group

Following the Penrose [*1*] paradigm of non-periodic tilings of E^2 and its global analysis by de Bruijn [*2*], new crystallographic ideas are developed on the basis of [*3*] for the description of non-periodic tilings of E^3. The mathematical technique uses induced and subduced representations of point and space groups in E^3 and E^N , $N > 3$, associated with the icosahedral group. Quasilattices in E^3 are analyzed with respect to elementary and composite cells and point symmetry. Global properties include layer and nematic structures and the structure factor for diffraction. A zone description is introduced to discuss the generation of the quasilattice from an initial point.

[1] R. Penrose *Bull.Inst.Math.Appl.*, **10** (1974) 266

[2] N.G. de Bruijn *Indagationes Math.*, **43** (1981) 39 and 53

[3] P. Kramer and R. Neri *Acta Cryst. A*, **40** (1984) 580

[] P. Kramer *Z. Naturf.*, **40a** (1985) 775

A. Katz

Theory of Quasi-Periodic Tilings and Quasicrystals (joint work with M. Duneau)

A general method is proposed to build quasiperiodic tilings and more general quasi-periodic patterns. Such tilings are obtained by the projection in $\mathbb{R}^3$ of suitable 3-dim. surfaces built up with facets of a higher dimensional lattice. As an example icosahedral quasiperiodic structures are produced, which are hoped to give a useful template for real quasicrystals. Our method allows the study of local properties (classification of vertex neighbourhoods) as well as global properties, and an analytic expression of the Fourier transform is derived, exhibiting the quasiperiodicity.

C. Radin

The Spatial Symmetry of Classical Ground States

The talk will be a survey of attempts to understand the tendency of many body systems to exhibit spatial periodicity or quasiperiodicity at low temperature.

J.-P. Eckmann

The Existence of Dendritic Fronts

In this joint work with P. Collet, we study the fourth order equation on $\mathbb{R}^2$

$$\partial_t u(x,t) = (\varepsilon - (1+\partial_x^2)^2)u(x,t) - \varepsilon u^3(x,t).$$

We show that these equations have propagating front solutions which leave a periodic pattern with wavelength ω in the laboratory frame. They move with speeds $c > 4$.

J. Bellissard

The KAM Theorem in Quantum Mechanics.

Several problems in quantum mechanics exhibit small divisors: The problem of metal-insulator transition in incommensurate systems, stability of quantum systems periodically perturbed in time, etc.. The KAM algorithm can be adapted to treat these cases. The method and the applications will be explained.

F. Martinelli

Anderson Localization in a Binary Alloy.

We will give on outline of the proof of Anderson localization for a one dimensional tight-binding random Hamiltonian when the probability distribution of the random potential is a Bernoulli measure. We will also discuss some consequences for the integrated density of states. Multidimensional cases will be also discussed.

H. Englisch

Exact Kohn-Sham Equations

The density functional formalism (DF) became a standard method for the calculation of ground state energies. The V-representability of the densities, the differentiability of the density functionals and the only vague knowledge of the exchange and correlation functional are difficult problems for the mathematical foundation of practical DF-calculations. The Kohn-Sham equations have been modified such that, for E-V-representable densities (a dense set of one-particle densities), for which the differentiability of Lieb's exact functional was shown, their solutions coincide with the ground-state densities searched for.

Reference:

H. Englisch, R. Englisch "Exact Density Functionals for Ground-State Energies", phys. stat. sol. (b) 123, 711, 124, 343 (1984) and paper in preparation.

Y.-S. *Wu*

Fractional Quantum Statistics in Two Dimensions

Fractional statistics, which depends on a continuous angular parameter Θ and is intermediate between the usual Bose-Einstein and Fermi-Dirac statistics, may exist in two-dimensional physical systems. Recent progress and present status of the theoretical understanding of these exotic statistics are summarized. Several open questions which need more efforts from the side of mathematical physics for their solution are emphasized.

E. Trubowitz

Algebraic Geometry of Periodic Schrödinger Operators

It is shown that, for the operator $-\Delta + v(x)$ on $L^2(\mathbb{R}^2)$, with $v(x)$ a periodic potential, the density of states generically determines the potential v, up to the obvious ambiguities. Related results for periodic Schrödinger operators on $L^2(\mathbb{R}^3)$ are also sketched. An indication of the flavour of the proofs which involve topological and algebraic-geometric methods is given.

J. Hajdu

Introduction to the Quantum Hall Effect

The experimental results which led to the discovery of the quantum Hall effect and a phenomenological theoretical description of that effect are reviewed. Some of the basic theoretical ansätze used to explain the integral quantum Hall effect are briefly described.

H. Kunz

Integral quantum Hall Effect

We show that the Hall conductivity, resulting from an electric field applied statically, is not quantized. For an electric field applied adiabatically, we prove that when the Fermi energy is in a gap or in a region of localised states, the Hall conductivity is quantised, and we compute the corresponding integer in some models. We also prove that, in such cases, the localisation length diverges at some energies.

B. Neudecker

A Simple Model for the Fractional Quantized Hall Effect

Following the ideas of Haldane we investigate a model for N electrons in a strong magnetic field on a sphere, interacting via shortranged repulsive interaction. We find that for filling factors $\nu \leq \frac{1}{3}$ the Laughlin wavefunction is a true eigenstate with zero energy. Numerical calculations have been carried out for $N \leq 6$. At $\nu = \frac{1}{3}$ a jump in the chemical potential occurs. The picture of Laughlin and Haldane for the excitation spectrum is in good agreement with our numerical data. This statement holds also in the case of repulsive Coulomb forces which we considered for $N \leq 6$.

J.E. Avron

Generic Properties of Finite Hall Hamiltonians and Diophantine Equations

In the talk I reviewed the works with R. Seiler and R. Seiler and B. Shapiro on generic properties of Hall Hamiltonians of finite systems; work with Dana and Zak on diophantine equations for the Hall conductance of non-interacting electrons in a periodic potential and a work with L. Yaffe on a diophantine equation for the Hall conductance of interacting electrons on a two-dimensional torus.

R. Seiler

Adiabatic Theorems and the QHE

The adiabatic limit of a quantum system is analyzed for time dependent Schrödinger operators satisfying a gap condition. An adiabatic time evolution which approximates the physical evolution is defined. As an application Kubo's formula is shown to hold for the Hall conductivity. It is related to the Chern class of a line bundle.

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