MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 16/1986

Elasticity Theory

13.4. bis 19.4.1986

The meeting was organized by J.M. Ball (Heriot-Watt University, Edinburgh) with the aid of much valuable advice from W. Jäger (Heidelberg). It was the first meeting at Oberwolfach on this topic. The participants included, as well as experts in elasticity, researchers in the fields of partial differential equations, the calculus of variations, and numerical analysis. Approximately half the lectures concerned the full equations of nonlinear elastostatics/elastodynamics in more than one space dimension. The remaining lectures covered various rod and shell theories and relevant problems of analysis. The main trends in current research highlighted at the meeting were as follows:

1. The study of nonlinear elastostatics when the stored-energy function of the material is not elliptic. This is the case for elastic crystals due to their symmetry properties. For some models of crystals, when the stored-energy function is invariant under appropriate shears, this leads to a prediction of fluid-like behaviour for minimizing sequences that raises questions about the appropriateness of the model and/or of current definitions of metastability. Another feature induced by nonellipticity is the fine phase mixing commonly observed in solid-solid phase transformations (e.g. fine twinning). A related property is the infinitely fine folding possible in an ideal membrane when bending energy is ignored. An important rôle is played in such problems by relaxation methods from the calculus of variations.

2. The investigation of fundamental properties of solutions in nonlinear elasticity, for example the global invertibility of solutions when contact with an obstacle and/or self-contact are allowed, surface instabilities and the rôle of the complementing condition, cavitation and the field theory of the calculus of variations, and the relationship between the underlying stored-energy to that of a cellular elastic material (e.g. foam rubber).

3. The investigation of various rod and shell problems, for example the buckling of spherical shells, torsion of a rod, systems of many balloons. One set of important questions concerns justifying the passage from 3-D to 1 or 2-D problems, e.g. by means of the centre manifold theorem. Other work concerns the existence and stability of dynamic solutions to various plate and membrane problems.

4. The analysis of elasticity theory with additional physical ingredients (e.g. thermoelasticity, viscoelasticity, elasto-plasticity, electro-magneto-thermoelasticity).

5. Mathematical techniques of relevance for problems in elasticity, e.g. weak convergence and compensated compactness, the analysis of shock waves, partial regularity theory, the Lavrentiev phenomenon.

6. The numerical computation of solutions in nonlinear elasticity by means of appropriate finite-element schemes.

J. Ball

Vortragsauszüge

S. ANTMAN:

Nonlinear problems of elasticity theory

Part I. Steady-State Problems of Electro-magneto-thermo-elasticity. It is shown that the problems of the title can be given a simple and general formulation in material coordinates leading to a quasilinear elliptic system of functional-partial differential equations. Existence and regularity results, based upon direct methods of the calculus of variations, pseudomonotone operators and variational inequalities, are presented.

Part II. Global Buckling of Nonlinearly Elastic Spherical Shells. A geometrically exact theory for the axisymmetric deformation of spherical shells under hydrostatic pressure is constructed. A very delicate analysis shows that bifurcating solution branches are globally characterized by the number of simultaneous zeros of the shear and bending strains. This nodal structure is novel.

J. M. BALL:

Fine phase mixtures as minimizers of energy

I report on joint work with R. D. James (Minnesota) that attempts to explain certain microstructural features involving fine mixtures of the phases in solid-solid phase transformations. In martensitic transformations one such feature is a plane interface which separates one homogeneous phase, austerite, from a very fine mixture of twins of the other phase, martensite. The idea is roughly the following. Suppose that for energetic reasons a body prefers to be deformed in, say, 3 states with constant deformation gradients A, B and C. Assume that conditions of geometric compatibility are satisfied across an interface separating regions deformed with gradients A and B; i.e. that there are vectors a, n such that $B-A = a \otimes n$, but that compatibility cannot be maintained across an interface separating A and C or B and C, i.e. $A-C, B-C \neq$ a rank-one tensor. Then it may be possible to arrange a very fine mixtures of layers A/B/A/B/A ... on one side of an appropriately oriented interface such that the deformation is almost compatible with the gradient C on the other side. By increasing the fineness of the mixture one obtains better and better compatibility and a minimizing sequence for the energy. Some details are given for Indium-Thallium, which undergoes a face-centred cubic → face-centred tetragonal transition at a temperature θ_o. The interfacial planes predicted agree with those observed very close to θ_o.

H. BECKERT:

The bending of plates and their stability region

In the first part we determine the static stability and instability region of a plate in the Banach space $C^1(\sigma_{ij})$ of the stress components. In the second part the initial value problem of the general dynamic equations in plate theory is solved. We apply a difference scheme in the time direction with a convergence proof according to the author's theory in case of the general dynamic equations in nonlinear elasticity theory (Zamm (1982), Bd. 62, S. 357-369).

P. G. CIARLET:

Unilateral boundary conditions and global injectiveness in three-dimensional nonlinear elasticity

In 1977, J. Ball has obtained a series of very important existence results in three-dimensional hyperelasticity. The purpose of this talk, which is based on recent joint work with J. Nečas, is to extend J. Ball's results in two directions, first by considering unilateral boundary conditions corresponding to contact without friction, secondly by including a global injectiveness condition in the set of admissible deformations.

I. FONSECA:

Remarks on liquid crystals

In recent years Ericksen proposed a continuum theory based on thermoelasticity to analyze questions related to equilibrium stability, twinning and phase transitions of solid crystals. The invariance of the energy density with respect to change of the lattice basis together with frame indifference give rise to highly unstable variational problems that cannot be treated with the usual techniques of the calculus of variations.

Stability of configurations held in a dead loading device are studied. It is proved that only residual stresses can provide global minima of the total energy and that possible metastable configurations are subjected to severe restrictions.

In joint work with L. Tartar, necessary and sufficient conditions for the existence of Lipschitz minimizers when the boundary is fixed and in the presence of body forces with the energy density replaced by its lower quasi-convex envelope are obtained.

M. FUCHS:

Vector valued obstacle problems

We analyse the partial regularity properties of vector valued functions $u:\mathbb{R}^n \supset \Omega \to \mathbb{R}^N$, $n,N \geq 2$, which are weak minimizers of energy type integrals $\int_\Omega a_{\alpha\beta}(x)B^{ij}(x,u)\partial_\alpha u^i \partial_\beta u^j\,dx$ under the side condition $u(\Omega) \subset \overline{M}$ for a (bounded) smooth domain $M \subset \mathbb{R}^N$, and we may also impose an additional integral constraint of the form $\int_\Omega g(.,u)\,dx \equiv$ const on the comparison maps. The following results were obtained in collaboration with F. Duzaar.

A. (interior regularity) u a local minimizer $\Rightarrow \mathbb{H}\text{-dim}(\Omega \cap \text{Sing } u) \leq n-3$; $n=3$: The interior singularities are isolated.

B. (boundary regularity) if u minimizes for smooth boundary values $\partial\Omega \to \overline{M} \Rightarrow$ u is regular on a full neighborhood of $\partial\Omega$.

C. (removable singularities) if $B^{ij} = \delta^{ij}$ and if M is star shaped $\Rightarrow \text{Sing } u = \emptyset$.

The last result immediately applies to graph obstacles $u^N \geq f(u^1,\dots,u^{N-1})$, moreover we could extend A-C to obstacle problems in Riemannian manifolds.

J. GWINNER:

A remark on nonlinear-elastic unilateral problems

In the monograph "Inequalities in Mechanics and Physics" by Duvaut and Lions the Signorini problem is treated for a linear-elastic body loaded by external and internal forces with no prescribed displacements. There it is shown that solutions to this unilateral problem can be obtained as weak limits of solutions of related boundary value problems where friction takes place on the normal displacement and the friction coefficient goes to infinity. To extend this approximation result to nonlinear elasticity we use J. Ball's approach to nonconvex hyperelasticity and a decomposition technique for semicoercive problems. The unilateral constraint is described by a convex cone such that the more general set constraint considered in the recent existence results of Ciarlet and Nečas can be replaced by a handier functional constraint. Thus the approximation result becomes a convergence theorem for a penalty method which is known to be even exact in convex situations.

B. KAWOHL:

Remarks on a problem raised by Saint Venant

In his famous paper "Memoire sur la torsion des prismes ..." Saint Venant observes among other things the following: If D is a twodimensional domain which is symmetric with respect to the x- and y-axis, and if $\Delta u = -1$ in D and $u = 0$ on ∂D holds, then $|\nabla u|$ attains its maximum in those points of ∂D which have minimal distance to the origin. These points are of physical interest because they indicate the location of the onset of plasticity. As early as 1859 Saint Venant recognized that his observation was not universally valid. Nevertheless there is a "proof" of Boussinesq from 1870 for it. His proof was found to be faulty by Filon in 1900.

Under additional hypotheses on the geometry of D, however, Saint Venant's observation can be proved even for nonlinear materials. The proof uses the moving plane method a la Gidas, Ni and Nirenberg.

J. KACUR:

On an approximate solution of higher order evolution equations in viscoelasticity

Evolution equations $\sum_{k=0}^{m} A_k(t) \frac{d^k}{dt^k} u(t) = g(t,u,\dots,\frac{d^{m-1}u}{dt^{m-1}})$ with $A_k(t) \in L(V,V^*)$ $(k=1,\dots,m-1)$ $A_m(t) \in L(H,H)$ are transformed to quasiparabolic or to quasihyperbolic equations with Volterra type operators on right hand side. These equations can be effectively solved by the method of Rothe (method of lines). Convergence of the used method is proved. Some properties of the solution are established.

C. KLINGENBERG:

Hyperbolic conservation laws in two space dimensions

I will discuss progress I have made towards a better analytic understanding of solutions to hyperbolic conservation laws in two space dimensions in order to aid the numerical computation of these equations. Among other things I will discuss solutions to the nonlinear scalar equation

$$u_t + f(u)_x + g(u)_y = 0$$

which are invariant under the transformation

$$(x,y,t) \to (cx,cy,ct), \quad c > 0,$$

the socalled two-dimensional Riemann problem.

R. V. KOHN:

Relaxation of variational problems

An analogy is drawn between the multigrid method in numerical analysis and the notion of relaxation. Then two examples are presented, in which the relaxed problem ("quasi-convexification") can be computed exactly. The first example, due to A. C. Pipkin, is the case of a two-dimensional rubber membrane in $\mathbb{R}^3$. The second is a variational method recently proposed for impedance computed tomography; its relaxation was joint work with M. Vogelius.

A. MIELKE:

Elastic deformations of an infinite strip

In [Systems of Nonlin. PDE, Nato Series C 111] Ericksen pointed out, that for treating the Saint-Venant's Principle it is of great interest to construct all possible deformations of a given prism having bounded strains along the whole body. Here we deal with the analogous two-dimensional problem, the infinite strip.

The arising quasilinear partial differential equations can be reduced to a system of six ordinary differential equations using the center-manifold theory. These ordinary differential equations contain the full information about all deformations having sufficiently small strains. Using the four invariants of resultant forces, momentum and energy per cross-section all possible deformations can be characterized qualitatively.

V. MIZEL:

Lavrentiev's phenomenon in deterministic and stochastic model problems

Consider the problem (P) of minimizing $\int_a^b f(t,x(t),\dot{x}(t))\,dt$, $x \in \{y \in W^{1,1}(a,b): y(a)=\alpha,\ y(b)=\beta\} =: \mathcal{A}$, where $f = f(t,x,p) \in C^\infty$ satisfies: (a) $f_{pp} > 0$, (b) $f(t,x,p) \geq \varphi(p)$, $\lim_{|p|\to\infty} \frac{\varphi(p)}{|p|} = \infty$. The Lavrentiev phenomenon occurs if for some p, $1 < p \leq \infty$, one has

$$m_1 := \inf_{x\in\mathcal{A}} I(x) < m_p := \inf_{x\in\mathcal{A}\cap W^{1,p}} I(x) \qquad (lav_p).$$

Ball & Mizel recently showed that there are problems (P) for which (lav_p) holds - in fact with m_p actually attained by some "pseudo-minimizer" $\bar{x} \in \mathcal{A} \cap W^{1,p}$. There may be 3-dimensional elasticity problems where the presence of Lavrentiev's phenomenon (hence of a singular absolute minimizer) signals the initiation of fracture. In any event there are serious computational implications since most numerical schemes would at best lead to $\bar{x}$ and m_p - with no indication to the unwary of the existence of a lower energy absolute minimizer x^*.

A model problem $f(t,x,p) = (x^2+t^2)p^6$, $a=-1$, $b=0$, $\alpha \in \mathbb{R}$, $\beta=0$, is analyzed by Heinricher & Mizel and shown to possess (lav_2) [here (a), (b) fail along the parabola $x^2+t=0$]. Then a stochastic control problem (P^ε) which corresponds to injecting a small amount of Gaussian whitenoise into (P) is analyzed and the limit as $\varepsilon\downarrow 0$ of the cost m_ε of (P^ε) is obtained. It is shown that (P^ε) also has a Lavrentiev type gap, with $\lim_{\varepsilon\downarrow 0} m_\varepsilon = m_1$, $\lim_{\varepsilon\downarrow 0} \bar{m}_\varepsilon = m_2$.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

I. MÜLLER:

Simulation of a Phase Transition in a System of Many Rubber Balloons

Rubber balloons exhibit a non-monotone pressure-radius characteristic that suggests a non-trivial stability problem, if several balloons are interconnected. For instance, for two balloons: Can both lie on the downward part of the characteristic, or can one lie there, if only the other one lies on the upward part? This question is answered and we proceed to look at increasingly more balloons. It turns out that the pressure as a function of the volume traces out a hysteresis loop that is akin to those of pseudoelastic bodies.

S. MÜLLER:

Cellular elastic materials and homogenization

Let us consider a hyperelastic material with a periodic structure of period ε. The total energy for a deformation $u:\Omega \subset \mathbb{R}^3 \to \mathbb{R}^3$ of such a material is given by

$$I^{\varepsilon}(u) = \int_{\Omega} W(\tfrac{x}{\varepsilon}, Du(x))\, dx.$$

For small ε the material should macroscopically behave almost like a homogeneous material. More precisely I show that for $\varepsilon \to 0$, I^{ε} converges to a functional

$$I(u) = \int_{\Omega} \overline{W}(Du(x))\, dx,$$

provided W satisfies polynomial growth conditions with respect to the deformation gradient. The convergence I use is convergence of minimizers or more technically Γ-convergence in the sense of De Giorgi.

The homogenized stored energy density $\overline{W}$ is given by

$$\overline{W}(F) = \inf_{k\in\mathbb{N}} \inf_{w\in W^{1,p}_{periodic}(Y)} \frac{1}{k^N} \int_{kY} W(y, F+Du(y))\, dy,$$

where $Y = [0;1]^3$.

This generalizes a result of Marcellini (1978) who assumed that W is convex in the deformation gradient in which case the infimum over k can be replaced by $k = 1$.

J. NEČAS:

Displacement-traction-contact problems with plasticity effects

One considers $I(\underset{\sim}{\psi}) \to \min$, where $I(\underset{\sim}{\psi}) = \int_\Omega W(\nabla\underset{\sim}{\psi}, \mathrm{Adj}\nabla\underset{\sim}{\psi}, h)\,dx + \int_\Omega W^\infty(\underset{\sim}{0},\underset{\sim}{0},k)\,d|\mu^s|$, $\mu = \mu^a + \mu^s$, $d\mu^a = h\,dx$, $d\mu^s = k\,d|\mu^s|$ in the set of admissible functions; for pure displacement problem $\int_\Omega \underset{\sim}{f}\underset{\sim}{\psi}\,dx$ is added. The growth conditions to W admit the Det $\nabla\underset{\sim}{\psi} = \mu$ is a Borel measure, for example $\frac{9}{4} < k < 3$: $W(F,H,\delta) \geq c(\|F\|^k + \delta)$. W^∞ is the recession function. It exists a minimum and it is possible to handle contact problems if $k \geq 3$. In that case, a sufficient condition of Fichera's type is considered, provided there is no displacement prescribed.

R. RACKE:

Local and global solutions in nonlinear thermoelasticity

Existence and uniqueness of solutions to the equations in nonlinear thermoelasticity have been obtained in the case of a bounded reference configuration for a homogeneous isotropic medium within a special one-dimensional model by M. Slemrod in 1981. He got local existence, and global existence for small data. The boundedness of the domain and the type of boundary conditions ([traction free, constant temperature] or [rigidly clamped, insulated] were essential for his approach. Recently (1985) C. M. Dafermos and L. Hsiao investigated the development of singularities for a special one-dimensional model, taking the whole real line as reference configuration and proving that for large data a smooth solution blows up in finite time.

We shall prove that for arbitrary (physically reasonable) boundary conditions smooth solutions always exist locally for both unbounded and bounded domains in one dimension; in the latter case global solutions exist for small data.

J. SIVALOGANATHAN:

Applications of the field theory of the calculus of variations in elastostatics

The field theory of the Calculus of Variations is concerned with finding sufficient conditions for a solution of the Euler-Lagrange equations of an integral functional to be minimizer of it. We outline the basic idea of the theory, in the classical context this involves the use of null Lagrangians and we are able to give an explicit representation for the general null Lagrangian.

We describe how the phenomenon of cavitation may be treated within the framework of the classical field theory. In general we prove a stability result for strongly elliptic stored energy functions which shows that any equilibrium solution is globally minimising in certain classes of deformations. As an application of this we prove a uniqueness theorem for finite elastostatics due originally to Knops and Stuart and a partial result on the question of whether rank one convexity implies quasiconvexity.

S. J. SPECTOR:

Surface Instabilities in Elasticity

Consider the problem of minimizing the integral

$$I(f) = \int W(x,\nabla f(x))\, dx, \quad f:\Omega \subset \mathbb{R}^n \to \mathbb{R}^n.$$

The classical necessary condition for a smooth function f to be a weak relative minimizer is that $C = \partial^2 W/\partial(\nabla f)^2$ satisfy the Legendre-Hadamard condition at every interior point.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

We determine a condition at the boundary, Agmon's condition, that is a further necessary condition for f to be a weak relative minimizer and relate this new condition to Ball & Marsden's recent work on quasiconvexity at the boundary.

We also prove that given the strict positivity of the second variation of I at f, $\delta^2 I$, necessary and sufficient conditions for the uniform positivity of $\delta^2 I$ with respect to the H^1-norm are: That C be strongly elliptic and that the natural linearized boundary condition, $C_n = 0$, satisfy the complementing condition.

Finally, we show the relevance of these results to problems of bifurcation and surface wrinkling in elasticity.

E. STEIN:

Consistent Linearizations in Nonlinear Elasticity Theory and Finite Element Analysis

Starting with kinematics and the weak formulation of equilibrium in material and spatial coordinates, a hyperelastic material is used whose free Helmholtz energy is to be polyconvex (Ball). This leads to Ogden-material (Ciarlet) and includes Mooney-Rivlin- and Neo-Hookean-materials.

Consistent linearizations in the 1. variation of the energy functional are made by developping the direction derivatives at a given configuration. So, one gets the consistent tangential material tensors for a Newton method in both descriptions.

The following finite-element-analysis is performed in the current configuration because of its efficiency for large strains. Spatial 8-node and plane 4-node isoparametric elements (with linear shape functions) were used for the following examples. Due to the isoparametric concept, the metric coefficients in the material tensor are δ_{ij}. The update of the actual geometry has to be performed in each iteration step in order to get nearly quadratic convergence near the solution point. Some examples for tension, compression and simple shear with large deformations of order 1, using Neo-Hookean material, were calculated with only one increment needing 4-6 iteration steps for the residual norm of equilibrium $O(\|R_n\|) = 10^{-5}$.

Finally some remarks about contact algorithms for contact of several elastic bodies are made

L. TARTAR:

Weak convergence, oscillations, and related questions

In studying a minimization problem $\inf f(x)$ one is often led to change the topology on X in order to make it compact with keeping f lower semi-continuous; this has often led to the use of weak topologies with hypothesis of convexity.

Some problems have no solutions and their minimizing sequences show often oscillations: It is then important to understand such phenomena either to show that they appear or that they do not, in which case existence of a minimum will hold.

One of the tools used for that study is the compensated compactness theory, developped in collaboration with F. Murat.

The talk presented some aspects of a program of research following these lines.

W. VON WAHL:

Nonlinear vibrations of the clamped plate and quasilinear hyperbolic systems

First we treat the equations $u_{tt}+\Delta^2 u-[u,v] = f$, $\Delta^2 v+[u,u] = 0$, $u|\partial\Omega = \frac{\partial u}{\partial n}|\partial\Omega = v|\partial\Omega = \frac{\partial v}{\partial n}|\partial\Omega = 0$, $u(0) = \varphi$, $u_t(0) = \psi$ for the clamped plate. $\Omega \subset \mathbb{R}^2$ is the plate, u its displacement, v is the Airy stress function. We show that there is a local (in time) solution $(u,v) \in (C^{3+\varepsilon}(\bar{\Omega}))^2$. If this solution ceases to exist at a finite time $T(\varphi,\psi)$ then $\|K(t)\|_{L^{1+\delta}(\Omega)} \to \infty$ if $t \uparrow T(\varphi,\psi)$

where K(t,x) is the Gaussian curvature of the bended plate in (t,x). As for quasilinear hyperbolic systems $u_{tt} - a_{\alpha\beta}(t,x,u,u_t,\nabla u)D^{\alpha+\beta}u = f(t,x,u,u_t,\nabla u)$ on $[0,T]\times\bar{\Omega}\subset\mathbb{R}^4$ with $u|\partial\Omega = 0$, $u(0) = \varphi$, $u_t(0) = \psi$, we assume that strong ellipticity holds, i.e. $a_{\alpha\beta}(t,x,u,p,q)\xi^\alpha\xi^\beta \eta\eta^* \geq c(M)|\xi|^2|\eta|^2$ if $(|u|^2+|p|^2+|q|^2)^{1/2} < M$, and that $a_{\alpha\beta} = a^*_{\beta\alpha}$. If there are initial values φ,ψ fulfilling the compatibility conditions of order 1 and 2 and the inequality $(|u|^2+|p|^2+|q|^2)^{1/2} \leq M-\delta$ for some $\delta > 0$, then the problem above is well-posed. We can also construct a maximal interval of existence and characterise it.

P. WEIDEMAIER:

Exponential stability of the O-solution for a weakly damped nonlinear-membrane equation

Existence of a global solution (in $\{u|u\in C^4(\mathbb{R}^+,L^2(\Omega))$, $\frac{d^l}{dt^l}u(t)\in H^{4-l}(\Omega)\cap \overset{\circ}{H}{}^1(\Omega)$, $\frac{d^l}{dt^l}u(\cdot)\in C^0(\mathbb{R}^+,H^{4-l}(\Omega))$, $0\leq l\leq 3\}$) of the initial-boundary value problem

$$u_{tt} - \partial_i\left(\frac{\partial_i u}{\sqrt{1+|\nabla_x u|^2}}\right) + 2ku_t = 0,\quad \Omega\subset\mathbb{R}^n,\ n\in\{2,3\},\ k>0$$

$$u|_{\partial\Omega} = 0,$$

$$u(0) = u_0,\ u_t(0) = u_1$$

is proved, provided k is small, $u_0\in H^4\cap\overset{\circ}{H}{}^1$, $u_1\in H^3\cap\overset{\circ}{H}{}^1$, u_0 and u_1 satisfy the usual compatibility conditions; this solution is unique and decays exponentially:

$$\sum_{k=0}^{4}\left\|\frac{d^k}{dt^k}u(t)\right\|_{H^{4-k}} \leq Me^{-kt}\qquad (t\geq 0).$$

The decay result is (for small k) better than results obtained by Matsumura (Publ. RIMS, Kyoto Univ. 13(1977), 349-379).

Berichterstatter: P. Weidemaier

Tagungsteilnehmer

Prof. Stuart Antman
Department of Mathematics
University of Maryland
College Park, MD 20742
U S A

Prof. Gabriele Anzellotti
Dipartimento di Matematica
Universitá di Trento
38050 P o v o (TN)
I t a l y

Prof. J. M. Ball
Department of Mathematics
Heriot-Watt University
Edinburgh EH14 4AS
S c o t l a n d

Prof. Dr. Herbert Beckert
Raschwitzerstr. 12
7113 Markkleeberg
D D R

Prof. Dr. Dietrich Braess
Institut für Mathematik
Ruhr-Universität
4630 B o c h u m

Prof. Philippe Ciarlet
Analyse Numérique, T. 55
Université Pierre et Marie Curie
4 Place Jussieu
75005 P a r i s
F r a n c e

Prof. Dr. Martin Costabel
Mathematisches Institut
der Universität
Schloßgartenstr. 7
6100 D a r m s t a d t

Prof. Bernard Dacorogna
Dept. de Mathématiques
EPFL
1015 L a u s a n n e
S w i t z e r l a n d

Dr. Ulrich Dierkes
Fachbereich Mathematik
der Universität des Saarlandes
Bau 36
6600 S a a r b r ü c k e n

Prof. Irene Fonseca
C.M.A.F.
Av. Prof. Gama Pinto, 2
1699 L i s b o a Codex
P o r t u g a l

Prof. Dr. Jens Frehse
Institut für Angew. Mathematik
der Universität
Beringstr. 4-6
5300 B o n n

Dr. Martin Fuchs
Mathematisches Institut
der Universität
Universitätsstr. 1
4000 D ü s s e l d o r f

Dr. Joachim Gwinner
Fachbereich Mathematik
der Techn. Hochschule
Schloßgartenstr. 7
6100 D a r m s t a d t

Prof. Dr. Stefan Hildebrandt
Mathematisches Institut
der Universität
Wegelerstr. 10
5300 B o n n

Dr. Stephan Luckhaus
Institut für Angew. Mathematik
SFB 123
Im Neuenheimer Feld 294
6900 H e i d e l b e r g

Prof. Dr. W. Jäger
Institut für Angew. Mathematik
der Universität
Im Neuenheimer Feld 294
6900 H e i d e l b e r g

Prof. Dr. E. Meister
Fachbereich Mathematik
der Techn. Hochschule
Schloßgartenstr. 7
6100 D a r m s t a d t

Prof. Jozef Kačur
Ůstav aplikovanej matematicy
Mlynska dolina
842 15 B r a t i s l a v a
C S S R

Dr. Alexander Mielke
Institut für Mathematik
der Universität
Pfaffenwaldring 57
7000 S t u t t g a r t 80

Dr. Bernhard Kawohl
SFB 123
Universität Heidelberg
Im Neuenheimer Feld 194
6900 H e i d e l b e r g

Prof. Victor J. Mizel
Department of Mathematics
Carnegie-Mellon University
Pittsburgh, PA. 15217
U S A

Dr. Christian Klingenberg
Institut für Angew. Mathematik
der Universität
Im Neuenheimer Feld 294
6900 H e i d e l b e r g

Prof. Dr. Ingo Müller
Fachbereich 9 der
Technischen Universität
Straße des 17. Juni 135
1000 B e r l i n 12

Prof. R. J. Knops
Department of Mathematics
Heriot-Watt-University
Edinburgh EH14 4AS
S c o t l a n d

S. Müller
Department of Mathematics
Heriot-Watt-University
Riccarton, Edinburgh EH14 4AS
S c o t l a n d

Prof. Robert Kohn
Courant Institute
251 Mercer Street
10012 New York, NY
U S A

Prof. Francois Murat
Laboratoire d'Analyse Numerique
Tom 55-65, 5. etage
Université Paris VI
75252 P a r i s Cedex 05
F r a n c e

Prof. Jindrich Nečas

Mathematical-Physical Faculty
of Charles University
Malostrznshi M. 25

P r a h a 1

C S S R

Dr. Reinhard Racke

Institut für Angew. Mathematik
der Universität
Wegelerstr. 10

5300 B o n n 1

Prof. Dr. M. Reeken

Fachbereich Mathematik
Bergische Universität - GHS
Gaußstr. 20

5600 W u p p e r t a l

Dr. J. Sivaloganathan

School of Mathematics
University of Bath

Claverton down, Bath

Great Britain

Prof. Scott J. Spector

Department of Mathematics
Southern Illinois University

Carbondale, IL 62901

U S A

Prof. Dr. Erwin Stein

Institut für Baumechanik
und Numerische Mechanik
Universität Hannover
Callinstr. 32

3000 H a n n o v e r

Prof. C. A. Stuart

Dept. de Mathématiques
EPFL

1015 L a u s a n n e

S w i t z e r l a n d

Prof. Luc Tartar

Centre d'Etudes de Limeil-
Valenton, DMA/ MCN
B.P. 27

94190 Villeneuve Saint-Georges

F r a n c e

Prof. Dr. Friedrich Tomi

Mathematisches Institut
der Universität
Im Neuenheimer Feld 288

6900 H e i d e l b e r g

Prof. Dr. Waldemar Velte
Institut für Angew. Mathematik
der Universität
Am Hubland

8700 W ü r z b u r g

Prof. Dr. Wolf von Wahl

Mathematisches Institut
der Universität
Postfach 10 12 51

8580 B a y r e u t h

Dr. Peter Weidemaier

Mathematisches Institut
der Universität
Postfach 101251

8580 B a y r e u t h

Prof. Dr. Michael Wiegner

Mathematisches Institut
der Universität
Postfach 101251

8580 B a y r e u t h

Prof. Dr. E. Zehnder

Abteilung für Mathematik
der Universität
Universitätsstr. 150

4630 Bochum-Querenburg