MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Variationsrechnung

20.4 - 26.4.1986

Die Tagung fand unter der Leitung der Herren M. Giaquinta (Florenz), J. Jost (Bochum) und F. Tomi (Heidelberg) statt.

Sie hatte 42 Teilnehmer, von denen 23 Vorträge hielten. Bei der Zusammenstellung des Vortragsprogrammes wurde wiederum darauf geachtet, daß ausreichend Gelegenheit für wissenschaftliche Gespräche gegeben war.

Schwerpunkte stellten Variationsprobleme aus Differentialgeometrie und mathematischer Physik dar, insbesondere Ergebnisse aus der Theorie der Minimalflächen und der Flächen vorgeschriebener mittlerer Krümmung, der harmonischen Abbildungen sowie Variationsprobleme für allgemeinere elliptische Integranden. Probleme aus Anwendungsbereichen wie z. B. Elastizitätstheorie, Thermodynamik sowie Anwendungen von Variationsmethoden zur Behandlung nichtelliptischer Differentialgleichungen wurden ebenfalls vorgestellt. Eine größere Rolle als bei früheren Tagungen spielten diesmal auch Methoden der geometrischen Maßtheorie.

Von besonderem Interesse war eine Problemsitzung, bei der unter anderem offene Fragen aus dem Gebiet der Minimalflächen sowie der harmonischen Abbildungen und ihrer Bedeutung bei der Behandlung wichtiger Probleme aus der Differentialgeometrie und der algebraischen Geometrie vorgestellt und diskutiert wurden.

Der Leitung des Instituts sowie dem Personal des Hauses gilt der besondere Dank aller Teilnehmer für die vorbildliche Betreuung.

Vortragsauszüge

W.K. Allard: An integrality theorem and a regularity theorem for hypersurfaces whose first variation with respect to a parametric elliptic integrand is controlled.

Consider a hypersurface which is stationary with respect to the area integrand in $C = \{(x,y) \in \mathbb{R}^n \times \mathbb{R}: |x| < 2\}$ and which lies close to $D = \{(x,y): |x| < 2$ and $y = 0\}$. The following two theorems are proved im my 1972 Annals paper.

Theorem The area of S in $\{(x,y): |x| < 1\}$ is nearly an integer times the area of the unit disc in $\mathbb{R}^n$.

Theorem Suppose the area of S in $\{(x,y): |x| < 1\}$ is nearly the area of the unit disc in $\mathbb{R}^n$. Then $S \cap \{(x,y): |x| < 1\}$ equals the graph of a function whose derivative can be estimated by a constant times $\left(\int |y|^2 \, d \, \|S\| \, (x,y) \right)^{1/2}$.

In the present work we extend these theorems to arbitrary elliptic integrands.

F. Almgren: Optimal isoperimetric inequalities

We wish to make precise and indicate proofs for the following:

(1) **Optimal isoperimetric inequality**. Corresponding to each m dimensional closed surface T in $\mathbb{R}^{n+1}$ there is an m+1 dimensional surface Q having T as boundary such that

$$|Q| \leq \gamma(m+1) |T|^{(m+1)/m}$$

with equality if and only if T is a standard round m sphere (of some radius) and Q is the corresponding flat m+1 disk. The equality defines $\gamma(m+1)$.

(2) **Optimal isoperimetric inequality for mappings of manifolds with boundary.**

(3) **Area-mean curvature characterization of standard spheres.** Suppose V is an m dimensional surface in $\mathbb{R}^{n+1}$ without boundary. If the mean curvature vectors of V do not exceed in length those of a standard round m sphere S^m of unit radius, then the m area of V (actually of the extreme points of V) is not less than the m area of S^m . Furthermore, equality holds if and only if V is such an S^m .

(4) **Mean curvature regularity theorem for combinatorial cycles.**

R. Böhme: <u>Konforme Struktur berandeter Minimalflächen</u>

Wir beweisen elementar folgenden Satz: Ist R eine kompakte Riemannsche Fläche vom Geschlecht g , und F der berandete Rest von R mit Randkurve α_o , wenn eine Kreisscheibe aus R entfernt wird, so gilt: Es gibt $(6g - 3)$ reelle Parameter, die unter allen möglichen Veränderungen von ∂F in R entscheiden, ob die neue Randkurve α_1 ein zu F konform äquivalentes Gebiet berandet. Dieser Satz stellt einen Zusammenhang zwischen dem Indexsatz von Atiyah-Singer, dem Hauptsatz der Teichmüller-Theorie und dem Buch "Dirichlets principle, conformal mapping and minimal surfaces" von Courant her. Damit folgt dann auch der Indexsatz von Böhme und Tromba für höheres Geschlecht allgemein.

J.M. Coron: On a nonlinear elliptic equation involving the critical Sobolev exponent (joint work with A. Bahri)

Let Ω be a bounded regular open set in $\mathbb{R}^N$. We consider the following equation

$$(*) \qquad \begin{aligned} -\Delta u &= u^{\frac{N+2}{N-2}} \\ u &> 0 \text{ in } \Omega \\ u &= 0 \text{ on } \partial\Omega \end{aligned}$$

The solutions of (*) are the non trivial critical points of the functional I defined on $H_0^1(\Omega)$ by

$$I(u) = \frac{1}{2}\int |\nabla u|^2 - \frac{N-2}{2N}\int (u^+)^{\frac{2N}{N-2}}\, dx$$

I does not satisfy the Palais smale condition at the level pS where S is some real number which depends only on N and $p \in \mathbb{N}^+$.
Following an idea introduced by A. Bahri in his paper on the Weinstein's conjecture we compute the change of topology at the level pS due to this lack of compactness.
We give sufficient conditions on the topology of Ω for the existence of a solution to (*). In particular when $N = 3$ we prove that if Ω is connected but not contractible in itself then (*) has at least one solution.

J. Eells: Strings (Joint work just begun with Simon Salomon)

Let (M,g) and (N,h) be pseudo-Riemannian manifolds, where M is a surface with signature $g = (1,1)$ and N with signature $h = (p,q)$. The physicists call a string a conformal harmonic map $\varphi: M \to N$. On an isothermal chart on M, conformality is expressed by $|\varphi_u|^2 + |\varphi_v|^2 = 0 = \langle \varphi_u, \varphi_v \rangle$, and harmonicity by

$$\varphi^\gamma_{uu} - \varphi^\gamma_{vv} + \Gamma^\gamma_{\alpha\beta}(\varphi^\alpha_u \varphi^\beta_u - \varphi^\alpha_v \varphi^\beta_v) = 0 \qquad (1 \le \gamma \le p+q) .$$

The Grassmannian $G_{1,1}(\mathbb{R}^{r,s})$ of subspaces of signature (1,1) in $\mathbb{R}^{r,s}$ has tangential decomposition

$$T\, G_{1,1}(\mathbb{R}^{r,s}) = (L_+ \otimes W) \oplus (L_- \otimes W)$$

with respect to which we can define an almost product structure. With it - and in analogy with our twistor construction in the definite case (Ann. Scuola Nor. Sup. Pisa 1986) - we can classify strings $\varphi\colon M \to N$ via their twistor transform into the Grassmann bundle:

$$\begin{array}{ccc} \tilde\varphi : M & \longrightarrow & G_{1,1}(N) \\ & \searrow^{\varphi} & \downarrow \\ & & N \end{array}$$

M. Fuchs: <u>The obstacle problem for energy minimizing maps</u>

Let X^n, Y^k be Riemannian manifolds, Y embedded in $\mathbb{R}^N$, and consider a bounded smooth domain $M \subset Y$ not touching ∂Y. We define ($\Omega :=$ Int X) $H^1(\Omega,\bar M) := \{u \in H^1(\Omega,\mathbb{R}^N) : u(x) \in \bar M\}$ and (the set of local minimizers) $\mathcal{C} := \{u \in H^1(\Omega,\bar M) : E(u) := \int_\Omega \|du\|^2 \le E(v)$, $v \in H^1(\Omega,\bar M)$, $\operatorname{spt}(u-v) \subset\subset \Omega\}$. The following regularity theorems were obtained in collaboration with F. Duzaar.

A. (1^{st} interior partial regularity): if $u \in \mathcal{C} \Rightarrow \mathcal{H}^{n-2}(\Omega \cap \text{Sing } u) = 0$.

B. (optimal partial regularity): if $u \in \mathcal{C} \Rightarrow \mathcal{H}$ - $\dim(\Omega \cap \text{Sing } u) \le n-3$;

if $n = 3$: the interior singularities are isolated.

C. (boundary regularity): if $u \in H^1(\Omega,\bar M)$ minimizes for fixed smooth boundary values $\partial\Omega \to \bar M$ $\Rightarrow$ $\text{Sing}(u) \subset\subset \Omega$.

D. (removable singularities): if $M \subset B(p)$ for a regular ball B and if M is star shaped w.r.t. $p \Rightarrow \text{Sing}\, u = \emptyset$ for $u \in \mathcal{C}$.

Here we use the notation $\text{Reg}\, u = \{x \in \bar{\Omega}: u \in C^{1,\alpha} \cap H^{2,p}$ for all $p < \infty$ near $x\}$. In the Euclidean case $(\Omega \subset \mathbb{R}^n, M \subset \mathbb{R}^N)$, A. holds for $H^1(\Omega,\bar{M})$ - minima of $Fu := \int_\Omega a_{\alpha\beta}(x,u)\, B^{ij}(x,u)\, \partial_\alpha u^i\, \partial_\alpha u^j\, dx$, and B, C. are true if $\partial_u a \equiv 0$. Moreover, in all cases we can impose an additional integral constraint of the form $\int_\Omega g(x,u)\, dx = \text{const}$ on the comparison functions.

M. Grüter: Minimal surfaces with free boundaries

We consider the problem of minimizing n-dimensional area among currents T whose boundary (or part of it) is supposed to lie in a given hypersurface of $\mathbb{R}^{n+k}$. In the case of codimension one ($k=1$) we prove that the singular set of T, i.e. the set of points where $\text{spt}\, T$ is not an embedded submanifold (with boundary), has codimension at least seven. An example shows that this result is optimal. For $k > 1$ we show the finiteness of the mass of the free boundary and estimate the upper $(n-1)$-dimensional density of the free boundary. An important ingredient is a regularity result for stationary varifolds with a free boundary by Jost and myself. Applications include regularity results for minimal hypersurfaces with prescribed homology class of the boundary, boundary regularity for solutions of a partitioning problem, and the existence of a minimal embedded disk inside a given convex body in $\mathbb{R}^3$ (joint work with Jost).

R. Gulliver: The approach of a harmonic mapping to its homogeneous limit

We study harmonic maps $f: M^m \to N^n$ between Riemannian manifolds, where N is considered to be isometrically embedded in Euclidean space $\mathbb{R}^d$, and assumed compact. Consider a singular point $0 \in M$; this may occur only if $n \geq 2$ and $m \geq 3$. If f minimizes energy on a neighborhood of 0 , then as $\lambda \to 0$, a subsequence of any blowup sequence $f_\lambda(x) := f(\lambda x)$ converges weakly to a homogeneous harmonic mapping $f_o: \mathbb{R}^m \to N$ which locally minimizes energy; f_o is called a homogeneous tangent mapping to f at the singular point 0 . Suppose that one homogeneous tangent mapping f_o is smooth. Then L. Simon has shown that f_o is unique and $f_\lambda \to f_o$ in $C^2(S^{m-1})$; but his theorem gives no estimate on the rate of convergence. However, an earlier result of Allard and Almgren may be adapted to prove $\|f_\lambda - f_o\| \leq c\lambda^\alpha$ for some $\alpha > 0$, under the hypothesis that every harmonic Jacobi field φ along f_o is actually the derivative of a one-parameter family of harmonic mappings $f^t: S^{m-1} \to N$. We show that this hypothesis always holds if $m = 3$ and $n = 2$. The proof shows that f_o is actually conformal and φ is conformal Jacobi. Finally, we construct explicit stationary examples for all remaining cases $m \geq 3$ and $n \geq 3$, for which $\|f_\lambda - f_o\| = A(B - 2\log\lambda)^{-\frac{1}{2}}$. In particular, there is no majorant of the form $c\lambda^\alpha$. (Joint work with Brian White).

O. John: The counterexample to the regularity of nonlinear parabolic sytems.

In my talk I communicated (a joint result) with Jana Stará and J. Malý. We constructed a parabolic system

$$\frac{\partial u^i}{\partial t} - D_\alpha (A^{ij}_{\alpha\beta}(z,u)\, D_\beta\, u^j) = 0 \ , \quad i,j,\alpha,\beta = 1,\ldots,3 \ ,$$

for which the weak solution of the initial - boundary value problem with Lipschitzian data on the boundary develops a singularity at an interior point of the parabolic cylinder Q_t .

J. Jost: Harmonic maps and Teichmüller theory

In this talk, we present a new approach to Teichmüller theory based on the existence and uniqueness of harmonic diffeomorphisms. We consider such harmonic maps in their dependence on domain and image metric. The effect of variations of the image structure was computed by M. Wolf, the effect of those of the domain by the author. This approach allows to recover the essential structures of Teichmüller theory, namely the topological, differentiable, complex and metric structure, and to compute the curvature of the Weil-Petersson metric in an easy way.

S. Luckhaus: Partial regularity for energy minimizing p-harmonic maps

A simple proof for Hölder regularity of function and gradient for energy minimizing p-harmonic maps between Riemannian manifolds outside a singular set of Hausdorff dimension at most $n-p$ is given.

The main step is to get Hölder continuity for the function. This also works for general functionals $\int G(x,u,\nabla u)$ if G is uniformly continuous, convex in ∇u and fulfils $c^{-1}|q|^p - 1 < G(x,u,q) < c|q|^p + 1$ and if solutions to the blow up equations in Euclidean space $\operatorname{div}(\partial_q F(\nabla v)) = 0$, $F(q) = \lim_{\alpha\to\infty} \alpha^{-p}\, G(x,u,\alpha q)$ are always Hölder continuous.

The proof relies on the strong convergence of the blow up sequence in regular points. In order to get this one needs a lemma, which states:

If the normalized Dirichlet integral of u in B_ρ is small, or the Dirichlet integral is bounded and the mean oscillation is small, then one can modify u in the interior of B_ρ such that the oscillation of $\tilde{u}$ in $B_{(1-\lambda)\rho}$ is small and the energy increases by a factor $(1+\lambda)$ only.

L. Modica: <u>Phase Transitions in Fluids and their interpretation in the Calculus of Variations</u>

Suppose that a fluid is confined to a container $\Omega \subset R^n$ and assume that the energy of the fluid is given by

$$E_\epsilon(u) = \int_\Omega [\epsilon^2 |Du|^2 + W(u)]\, dx + \int_{\partial\Omega} \epsilon\sigma(u)\, dH_{n-1}$$

where u is the density of the fluid, W is the Gibbs free energy (for unit volume) and σ is the contact energy with the container walls (for unit surface area). The parameter ϵ is a small positive parameter introduced (Van der Waals - Cahn - Hilliard theory) for taking into account some interfacial energy. We prove that the minima u_ϵ of $E_\epsilon(u)$ under the constraint $\int_\Omega u\, dx = m$ (m = total mass of the fluid) converge as $\epsilon \to 0+$ to a configuration u_o which takes only two values (two phases transition) and solves a variational problem related to the minimal surfaces problem (the liquid-drop problem). Moreover the form of this limit variational problem is such that it includes the boundary layers observed experimentally.

F. Morgan: Area-minimizing surfaces in Grassmannians

In joint work with H. Gluck and W. Ziller, we look for area-minimizing representatives of the homology of the Grassmannian $G_m\mathbb{R}^n$ of oriented unit m-planes in $\mathbb{R}^n$. Subgrassmannians, which are totally geodesic, are particularly good candidates.

Theorem: Consider $S = G_m\mathbb{R}^{m+k} \subset G_m\mathbb{R}^{m+\ell}$. If m is odd, then S is homologous to 0 over $\mathbb{Q}$. If m is even, then S is homologically area-minimizing.

The method of proof is to "calibrate" S by an invariant differential form φ, such as the Euler form. The hard part is to verify that the comass $\|\varphi\|^*$ is one.

However, we show that $G_2\mathbb{R}^4 \subset G_3\mathbb{R}^6$ is not homologically area-minimizing by presenting another surface $S \sim G_2\mathbb{R}^4$ with less area. S is not totally geodesic, and it has two conical singularities. This surface S suggests new families of canonical subvarieties of Grassmannians.

J. Pitts: Minimal surfaces and variational methods in the large

In joint work with J.H. Rubinstein, variational methods in the large and geometric measure theory are used to construct smooth minimal surfaces in manifolds. Typically one is able to bound a priori both the topological type and the index of instability of the minimal surfaces so obtained. For example, one has the following.

Theorem. Let Σ be a smooth, compact, connected, oriented, three dimensional Riemannian manifold with Heegard genus H. Then Σ supports a nonempty, smooth, compact, embedded, two dimensional, minimal submanifold M such that $\mathrm{genus}(M) \leq H$ and

$$\mathrm{index}(M) \leq 1 \leq \mathrm{index}(M) + \mathrm{nullity}(M) .$$

(Here $\mathrm{index}(M)$ and $\mathrm{nullity}(M)$ are the index and nullity of the usual elliptic differential operator associated with the second variation of area of M .) Using these methods, one constructs many new examples of minimal surfaces of geometric and topological interest.

F. Sauvigny: On the Morse Index of Minimal Surfaces in $\mathbb{R}^p$ with Polygonal Boundaries

Let $\Gamma \subset \mathbb{R}^p$ $(p \geq 3)$ denote a Jordan polygon with $N+3$ $(N \geq 1)$ vertices. Then the minimal surfaces x spanning Γ correspond to the critical points $\overset{0}{\tau}$ of an analytic function $\theta: T \to \mathbb{R}$ in N variables. This function was introduced by M. Shiffman and the regularity of θ has been investigated by E. Heinz.

Now we are interested in the correspondence of the second order: The second derivative of θ is given by its Hessian $M(\overset{0}{\tau})$, the second variation of the minimal surface x is described by the Schwarz operator S_x . We first study the eigenvalue problem of this singular differential operator by variational methods. Counting the negative eigenvalues of S_x we obtain the Morse index m_x of x .

As central result we prove: The morse index m_x of the minimal surface x coincides with the Morse index $m(\overset{0}{\tau})$ of θ (the number of negative eigenvalues of $M(\overset{0}{\tau})$) at the corresponding critical point $\overset{0}{\tau} \in T$. This is achieved by comparison of the finite and infinite dimensional eigenvalue problems of $\overset{o}{M}(\tau)$ and S_x, using certain simultaneous variations of the minimal surface.

J. Souček: Constitutive inequalities in elastostatics

Let Φ be a diffeomorphism $\Omega \subset R^3 \to R^3$, $D\Phi = F = RU$ polar decomposition of its gradient considered as an element of the general linear group GL_+, $(\det F > 0)$. The behavior of the material is specified by the stored energy density $\mathcal{E}: GL_+ \to R$. The natural constitutive assumption of convexity of $\mathcal{E}$ is unrealistic (see notations - $\mathcal{E}(RU) = \mathcal{E}(U)$, isochoric deformations for an almost incompressible material - see Ball's paper). Usually convexity is very weakened (quasi-, poly-, 1-rank- convexity). We suggest another solution to this trouble: the convexity depends crucially on coordinates chosen. We propose to disregard completely the linear structure of GL_+ (as $GL_+ \subset R^9$) and instead of it to rely on the group structure of GL_+ - i.e. to consider the composition (and not the sum) of two deformations as a basic structure. The group structure induces the right invariant metric on GL_+ in which geodesics are of the form $c(t) = (\exp At) \cdot B$, $B \in GL_+$, $A \in gl$.

Then we require $\mathcal{E}$ to be convex on each geodesic $c(t)$ - the global form reads $\mathcal{E}(AB) + \mathcal{E}(A^{-1}B) \geq 2\mathcal{E}(B)$, $\forall\, A,B \in GL_+$. The inspection of above examples shows that this is quite a reasonable assumption. The square distance $\rho^2(1,F) = \mathrm{tr}(\lg^2 U) = I_o$ on GL_+ is an invariant and is geodesically convex. The ("ideal") elasticity will be given by $\mathcal{E}(F) = f(I_o)$ with $f', f'' \geq 0$ - it depends only on the geometry of GL_+. The usual weak convergence $F_u(x) \to F(x)$ must be changed for a new one defined in an intrinsic way in which the stored energy functional is lower semicontinuous.

J. Stará: Variational inequalities - eigenvalues and solvability.

In our communication we gave some properties of the set $\sigma_K(A)$ of eigenvalues of a variational inequality

$$\lambda \in \mathbb{R} \ , \ u \in K \quad (\lambda u - Au \ , \ v - u) \geq 0 \ , \quad \forall \ v \in K \ ;$$

where K is a closed convex cone in a real Hilbert space and A a self-adjoint, positive and compact operator on H. Some conditions were given , under which the inequality is solvable for arbitrary right hand side and it was shown that a Fredholm alternative generally does not hold.

M. Struwe: Free boundary problems for surfaces of constant mean curvature

Let S be a surface contained in a ball $B_R(0)$ in $\mathbb{R}^3$ and C^4-diffeomorphic to the standard sphere S^2. A disc-surface of constant mean curvature H supported by S and meeting S orthogonally along its boundary induces a nonconstant solution $X \in C^2(B;\mathbb{R}^3) \cap C^1(\bar{B};\mathbb{R}^3)$ of the system

(1) $\Delta X = 2H \, X_u \wedge X_v$ in B ,

(2) $|X_u|^2 - |X_v|^2 = 0 = X_u \cdot X_v$ in B ,

(3) $X(\partial B) \subset S$,

(4) $\frac{\partial}{\partial n} X(w) \perp T_{X(w)} S \ , \ \forall \ w \in \partial B$.

Here, $B = \{w = (u,v) \in \mathbb{R}^2 \mid u^2 + v^2 < 1\}$, $X_u = \frac{\partial}{\partial u} X$, "$\wedge$" denotes exterior product in $\mathbb{R}^3$, n is a unit normal on ∂B , "$\perp$" means orthogonal and T_pS denotes the tangent space to S at p .

Theorem: For (Lebesque) almost every H with $|H|R < 1$ there exists a nonconstant solution to (1)-(4) .

The result extends an earlier result by the author for minimal surfaces $H = 0$ (Inv. Math., 1984).

For the proof one analyzes the evolution problem associated with (1)-(4) using methods developed for harmonic mappings of surfaces (Struwe, Comm. Math. Helv., 1985) .

P.A. Vuillermot: Spatially localized free vibrations for certain semilinear wave equations on $\mathbb{R}^2$: recent results and open problems

We prove the nonexistence of free vibrations of arbitrary period with polynomially decreasing profiles for a large class of semilinear wave equations in one space dimension. Our class of admissible models includes examples of non integrable wave equations with certain polynomial nonlinearities, as well as examples of completely integrable ones with exponential nonlinearities related to Mikhailov's equations. Our results also prove a particular case of the so-called EKNS- Conjecture. Our class of admissible nonlinearities does not contain the Sine-Gordon equation, which is known to possess a 2π-periodic solution in time with exponential fall-off in the spatial direction. Our results are complementary to recent results by Coron and Weinstein. Our arguments are entirely global, and rest upon methods from the calculus of variations.

G. Warnecke: The homogeneous Dirichlet problem for the nonlinear Boussinesq equation

The point of the talk is to show that variational methods used for proving the existence of weak solutions to semilinear elliptic equations can be applied to certain nonelliptic equations.

As an example the mountain pass lemma is applied to the Boussinesq equation

$$v_{tt} - av_{xx} - bv_{xxxx} + cv_{xx}v_x = 0 \ , \ b > 0 \ , \ a,c \in \mathbb{R} \ , \ (x,t) \in G \subset \mathbb{R}^2$$

to obtain the existence of a nontrivial weak solution on bounded domains. The solution is obtained in an anisotropic Sobolev space. It satisfies generalized homogeneous boundary conditions.

H. Wente: Twisted immersed tori of constant mean curvature in R^3

Let $\omega(u,v)$ be a solution to $\Delta\omega + \sinh\omega\cosh\omega = 0$ doubly periodic with respect to a parallelogram with sides $\bar{p}_1 = \langle a,0\rangle$ and $\bar{p}_2 = \langle c,b\rangle$. If the first fundamental form is $ds^2 = e^{2\omega}(du^2+dy^2)$, the mean curvature $H = 1/2$, and the lines of curvature correspond to two orthogonal families of parallel lines in the $u - v$ plane one of which makes an angle β with the u-axis, then the second fundamental form is determined and there is an essentially unique map $\bar{x}(u,v)\colon R^2 \to R^3$

which is a conformal representation of a surface of constant mean curvature $H = 1/2$ (generally not closed).

There exist Euclidean motions $\mathcal{E}_1, \mathcal{E}_2$ which commute and satisfy $\bar{x}(w + \bar{p}_i) = \mathcal{E}_i \circ \bar{x}(w)$, $i = 1,2$. It follows that there exists an axis ℓ so that $\mathcal{E}_i$ is a rotation about ℓ through an angle θ_i followed by a translation $\tau_i \bar{e}$ parallel to ℓ (here $\bar{e}$ is a unit vector parallel to ℓ).

There are 4 control variables $\{a,b,c,\beta\}$ and 4 variables to be determined $\{T_1,\theta_1,T_2,\theta_2\}$. Under suitable conditions we show that the map $\Phi(a,b,c,\beta) = (T_1,\theta_1,T_2,\theta_2)$ is smooth and locally invertible. One can then show that there exist (a_o,b_o,c_o,β_o) with $\Phi(a_o,b_o,c_o,\beta_o) = (0,2\pi r_1,0,2\pi r_2)$ where r_1 , r_2 are non-zero rational numbers. This gives us a closed immersed torus with a twist.

M. Wiegner: Zur Regularität von Variationsproblemen mit nichtkonvexem Hindernis

Es werde das Dirichletintegral $\int_\Omega |\nabla u|^2\, dx$ in der Klasse

$$\mathcal{M} = \{u \in u_o + \overset{o}{H}{}^1_2(\Omega)^N \,|\, u^N(x) \geq g(x,u^1(x),\dots,u^{N-1}(x)) \text{ a.e.}\}$$

minimiert, u_o und $\partial\Omega$ seien glatt. Hierbei seien $g(x,y): \Omega \times \mathbb{R}^{N-1} \to \mathbb{R}$ sowie auch $D_y g(x,y)$ von der Klasse C_1 , aber nicht notwendig konvex. Unter gewissen Voraussetzungen (z.B. $g = g(x,|y|^2)$ mit $\frac{\partial g}{\partial t}(x,t) \geq 0$ oder $N = 2$, $\frac{\partial g}{\partial y}(x,y) < 0$) wird gezeigt, daß ein Minimum $u \in H^1_2(\Omega)^N$ beschränkt ist und zur Klasse $C_{1,\alpha}(\bar{\Omega})$, $\alpha > 1$, gehört, wobei die Norm a-priori abgeschätzt werden kann.

R. Ye: <u>Existence and finiteness results in the free boundary value problem for minimal hypersurfaces</u>

Some of the finiteness results in this paper are the following:

<u>Theorem 1:</u> Let $\mathcal{M}$ be a compact, real analytic Riemannian 3-manifold whose boundary $\partial\mathcal{M}$ is nonvoid and has non-negative mean curvature with respect to the inward normal. Suppose that $\Gamma \subset \overset{\circ}{\mathcal{M}}$ is a homotopically nontrivial curve. Then there are only finitely many Γ - minimizing disks except that $\mathcal{M}$ is filled with Γ - minimizing disks and that either $\mathcal{M}$ is a solid torus or a solid Klein bottle. (A Γ-disk is a disk type surface whose boundary lies on $\partial\mathcal{M}$ and is linked with Γ and a Γ - minimizing disk is a Γ-disk with minimal area.)

<u>Theorem 2:</u> Let $\mathcal{M}$ be a compact, real analytic Riemannian n-manifold with $\partial\mathcal{M} = \emptyset$ and $n \leq 7$. Let γ be a non-zero element in $H_{n-1}(\mathcal{M})$, the (n-1)-dimensional integral homology group. Set

$$S_\gamma^g = \{ S : S \text{ is a connected regular hypersurface of topological type } g \text{ appearing in some } \gamma\text{-minimizing current}\} .$$

Then either $\# S_\gamma^g < \infty$ or $\mathcal{M}$ is a bundle over S^1 whose fibers are elements of S_γ^g.

<u>Theorem 3:</u> Let $\mathcal{M}$ be a compact, real analytic Riemannian n-manifold with $\partial\mathcal{M} \neq \emptyset$ and $n \leq 7$. Let γ be a non-zero element in $H_{n-1}(\mathcal{M},\partial\mathcal{M})$. S_γ^g is defined in the same way as in Theorem 2. Then either $\# S_\gamma^g < \infty$ or $\mathcal{M}$ is a bundle over S^1 whose fibers are elements of S_γ^g. We note that an element in S_γ^g is a minimal hypersurface meeting $\partial\mathcal{M}$ orthogonally along its boundary.

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