MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 18/1986

Ringe und Moduln

27.4. bis 3.5.1986

An dieser von den Herren G.Michler (Essen) und L.W.Small (San Diego) geleiteten Tagung über Ringe und Modulen nahmen 46 Fachvertreter aus 10 Ländern teil; Herr A.Rosenberg (Ithaca, N.Y.), ein weiterer Organisator der Tagung, konnte leider nicht nach Oberwolfach kommen.

In 33 Vorträgen wurde über die wesentlichen Fortschritte auf den Gebieten der Noetherschen Ringe, der Ringe mit Polynomidentitäten und der Fixringe unter der Operation von endlichen Gruppen bzw. endlich dimensionalen Hopf-Algebren auf Ringen ein Überblick gegeben. Außerdem wurde über Gruppenringe, Galoistheorie, einhüllende Algebren von endlich dimensionalen Lie Algebren, Darstellungstheorie endlich dimensionaler Algebren und ihre Beziehung zur allgemeinen Ringtheorie berichtet. Diesmal legten die Tagungsleiter großen Wert darauf, daß die Tagungsteilnehmer einen möglichst breiten Überblick über die neueren Entwicklungen in der Ringtheorie und Anwendungen ring- und modultheoretischer Methoden in anderen Gebieten der Algebra und Computer-Algebra erhalten konnten.

Die Diskussionen zwischen den Teilnehmern profitierten besonders von der Anwesenheit der russischen Kollegen Bokut, Kharchenco und Zalesskii; hierdurch konnten sich die Teilnehmer aus den westlichen Ländern nach langer Zeit erstmals einen breiten Überblick über die an den russischen Instituten derzeit aktuellen Fragestellungen und Methoden verschaffen.

Wie immer trug die freundliche Atmosphäre des Hauses wesentlich zum Gelingen der Tagung bei.

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Vortragsauszüge

D.J. Anick: USING COMBINATORIAL PROPERTIES OF RINGS TO BUILD PROJECTIVE RESOLUTIONS

Let Γ be a directed graph on a finite number of vertices and let $A = k\Gamma/I$ be a homomorphic image of the path algebra $k\Gamma$. One approach to the algebra A is combinatorial: choose a set of paths on Γ which surjects to a basis for A, and study its properties. Another approach is algebraic: consider and classify modules over A, paying particular attention to their homological properties.

In this joint work with Ed Green (Virginia Polytechnic Institute), we have blended the two approaches. Combinatorial invariants derived from the map $f:k\Gamma \to A$ are assembled into projective resolutions for the vertex-associated simple A-modules. These resolutions are fairly small and they are practical for specific calculations. Applications include the construction of interesting examples of rings having finite global dimension.

C. Bessenrodt: OVERRINGS OF RIGHT CHAIN RINGS AND A CONSTRUCTION METHOD FOR CHAIN RINGS

The lattice of overrings of a classical valuation ring is easily described and this description generalizes to chain domains. With only the chain condition for right ideals, the situation is much more complicated as can be seen from an example which is discussed in detail. For certain overrings T of a right chain domain R of rank 1 we obtain results on the relationship between the ideal lattices of R and T. If R and T are also assumed to be right invariant, we can describe T in terms of right invariant right chain domains contained in R.

To show how to construct chain domains, a method due to Dubrovin is presented. Using this construction Dubrovin produced the first (and so far only) example of a prime chain ring with nilpotent elements. As there is a mistake in the proof of a key lemma, a more direct approach is suggested in which a kind of weak Ore condition is used.

(Joint work with G. Törner)

L.A. Bokut: EMBEDDINGS OF RINGS, PI-RINGS, FREE PRODUCTS AND JORDAN HOMOMORPHISMS

I New examples of noninvertible rings embeddable in groups (L.A.Bokut, A.I. Valitskas)

II On the embedding of rings into Jacobson radical rings (A.I.Valitskas)

III Radical PI-algebras (A.I. Valitskas)

IV On the embedding of rings into matrix algebras over commutative rings (A.Z. Anan'in)

V On the structure of a variety over a field of zero characteristic (A.R. Kemer)

VI Free products of Λ-rings (V.N. Gerasimov)

VII Jordan homomorphisms (L.A. Lagutina)

A. Braun: LOCALIZATION IN PI RINGS

Given R a prime noetherian PI ring and P a prime ideal in R. Let T(R) be the "trace ring of R" and $P_1,\dots,P_r$ the finite number of all prime ideals in T(R) contracting to P. We say that P satisfies condition (*) if the following implication holds: If $Q \in \operatorname{spec} T(R)$ with $Q \cap Z(T(R)) = P_i \cap Z(T(R))$ for some $1 \leq i \leq r \Rightarrow Q = P_j$ for some $j \in \{i,\dots,r\}$.
The following theorem is proved

Theorem (Braun-Warfield): Let R be a prime noetherian PI ring, $P \in \operatorname{spec} R$.
Then the following are equivalent

1) P satisfies condition (*)
2) P is left (right) localizable.

This answers an old classical question of Goldie(1967) for finding a finite criterion determining the Ore localization at a prime ideal, for a prime noetherian PI ring.

K. Brown: REPRESENTATION THEORY OF SOLVABLE ENVELOPING ALGEBRAS

I describe work of Warfield and myself giving information about the "layers" of an indecomposable injective module over a Noetherian ring R satisfying the strong second layer condition. This provides a particularly detailed description in the case where R is polynomial, and so offers the prospect to begin develop a representation theory for enveloping algebras of solvable Lie algebras of finite dimension over $\mathbb{C}$.

In joint work with F. du Cloux which I shall outline, I have taken the first steps in this direction, with applications to determining cohomology groups for solvable Lie groups in mind.

P.M. Cohn: THE RATIONAL HULL OF A SEMIFIR

Let R be a semifir, containing a (skew) field K . The tensor R-ring on a set X centralizing K , $R_K\langle X\rangle$ is defined as the ring generated by R and X with defining relations $\alpha x = x\alpha(\alpha\in K, x\in X)$. If one forms $R_K\langle X\rangle = F$ with an infinite set X and localizes at the set Σ of all "full" matrices that are "totally coprime" to matrices over R , one obtains a simple semifir F_Σ in which R is inertly embedded, and by choosing R appropriately one obtains right principal Bezout domains (the first such examples were obtained in P.M. Cohn and A.H. Schofield, Bull. London Math. Soc. 17(1985), 25-28). E.g. starting from Jategaonkar's ring one obtains right principal Bezout domains of arbitrary Krull dimension.

S.C. Coutinho: GENERATING MODULES EFFICIENTLY

We show that a simple proof of the Forster-Swan Theorem (on the minimal number of generators of a module) for right noetherian rings can be obtained if one uses the so-called basic dimension of a module. This dimension is obtained from the Krull dimension by "ignoring" modules which have non-zero annihilators.

E. Formanek: THE INVARIANTS OF n×n MATRICES

Let K be a field of characteristic zero and let $C(n,r)$ be the ring of invariant functions $f: M_n(K)^r \to K$. The first fundamental theorem of matrix invariants says that $C(n,r)$ is generated by traces. The second fundamental theorem gives all multilinear relations among traces.

Let $d(n)$ be the minimal degree of a generating set for $C(n,r)$. Procesi has shown that $d(n)$ is also the minimal degree of nilpotence in the Nagata-Higman Theorem. That is, $d(n)$ is the least integer such that $(K\langle X\rangle/J)^{d(n)} = 0$, where $K\langle X\rangle$ is a free associative algebra (without unit) and J is the T-ideal generated by X^n.

Kostant used the second fundamental theorem to give a proof of the Amitsur-Levitzki Theorem, and I recently used it to prove a conjecture of Regev that a certain matrix polynomial is nonzero.

W. Geigle: THE PREPROJECTIVE ALGEBRA OF A TAME HEREDITARY ALGEBRA

Let k be an arbitrary field and Λ be a finite dimensional tame hereditary k-algebra. Then the preprojective algebra $\Pi(\Lambda)$ is defined by $\Pi(\Lambda) = \bigoplus_{n\geq 0} \tau^{-n}\Lambda$, where τ denotes the Auslander-Reiten transformation. We obtain the following result:

Theorem: $\Pi(\Lambda)$ is a left-noetherian prime PI-algebra of Krull dimension 2.

The existence of a polynomial identity yields lots of consequences on the structure of $\Pi(\Lambda)$ and Λ. In particular, if Q denotes the unique indecomposable torsionfree divisible Λ-module and D the division ring $End_\Lambda(Q)$, then D is finite dimensional over its center C and $trdeg_k C = 1$.

A. Goldie: MIDDLE ANNIHILATOR PRIMES

A middle annihilator ideal $M(\in MA)$ is defined in a ring R to be $M = \langle x \in R \mid AxB = 0 \rangle = \mathrm{mid}(A,B)$ if $AB \neq 0$ where A,B are given ideals of R. There is an outstanding problem, namely in a noetherian ring R is the number of middle annihilator prime ideals finite? This is known for factor rings of $U(\mathfrak{g})$, $\mathfrak{g}$ f.d. Lie algebra and FG the group algebra of a polycyclic-by-finite group. It also holds for subrings of artin rings and hence holds for fully bounded noetherian rings. It is considered likely that a counterexample to $|MAP| < \infty$ exists, but it will be complicated.

K.R. Goodearl: PATCH-CONTINUITY OF NORMALIZED GOLDIE RANKS

Stafford's patch-continuity theorem, for normalized ranks of finitely generated modules at prime ideals of a noetherian ring, will be discussed. A new, simpler, proof will be sketched, together with applications concerning the global dimension of fully bounded noetherian rings, via the patch-continuity of certain Ext and Tor groups.

R. S. Irving: THE SOCLE FILTRATION FOR VERMA MODULES

I will give a vague description of Verma modules, Kazhdan-Lusztig polynomials and K.L. conjecture, and projectives in the category $\mathcal{O}$. Also I will discuss the Jantzen filtration and conjecture. All of this will be as non-technical as possible. I will then discuss Loewy length of the Verma modules and selfdual projectives in $\mathcal{O}$, and relation to K.L. conjecture. Then I will show how the information just reviewed on Loewy length allows one to introduce another filtration on Verma modules, and allows one to prove that (i) it has all the properties that the Jantzen conjecture implies for Jantzen filtration, (ii) it is the socle filtration, (iii) in particular, multiplicities of simples in layers of socle filtration are coefficients of K.L. polynomials. Thus, Loewy length information alone yields Loewy series information.

C.U. Jensen: APPLICATION OF MODEL THEORY IN FIELD THEORY

For a finite group G and a field K let $\nu(G,K)$ be the number of non-K-isomorphic separable normal extensions of K with G as a Galois group. Some results are announced concerning the distribution of the values of $\nu(G,K)$. For instance, there exists a field F of arbitrarily prescribed characteristic such that every finite simple group but not every finite group can be realized as a Galois group over F.

A. Joseph: RINGS IN THE $\mathcal{O}$ CATEGORY

Let $\mathfrak{g}$ be a complex semisimple Lie algebra and $\mathcal{O}$ the category of "highest weight" modules introduced by Bernstein-Gelfand-Gelfand. A ring A is said to be an $\mathcal{O}$ ring if $\mathfrak{g}$ acts by derivations in A and the resulting module lies in the $\mathcal{O}$ category. For example,

(i) The $\mathfrak{g}$ ring End E where E is a finite dimensional $\mathfrak{g}$ module.

(ii) The ring of local sections on the big cell for the structure sheaf of the flag variety associated to a parabolic subgroup.

(iii) An appropriate combination of (i), (ii).

Any $\mathcal{O}$ ring satisfies a polynomial identity.

We analyse $\mathcal{O}$ subrings A of $\mathrm{Fract}(U(\mathfrak{g})/P) : P \in \mathrm{spec}\, U(\mathfrak{g})$. If P is completely prime, then A is of type (ii) and this implies that P can be induced from the corresponding parabolic subalgebra. We ask if significantly different $\mathcal{O}$ rings, that is not of type (iii), can occur in general. Our analysis shows that this is false, where the last step was provided by Braun, Goodearl and Small during the meeting.

V.K. Kharchenco: NEW STRUCTURE THEORY AND DERIVATIONS OF SEMIPRIME RINGS

Theorem 1: Let R be a semiprime ring and L be a Lie ∂-algebra of the outer for Martindale ring of quotients Q derivations. Suppose that L is finitely generated as a module over C. Then there exists a one-to-one correspondence between Lie ∂-subalgebras in L, which are finitely generated over C, and intermediate rational-complete subrings.

The proof is based on the meta-theorem: If the formulation of some theorem can be written as the truth of the Horn formula of a signature Ω, then from the truth of this theorem on almost all stalks of the correct sheaf there follows its truth on the system of global sections.

L. Le Bruyn: COUNTEREXAMPLES TO THE KAČ CONJECTURES

A representation V of an arbitrary quiver $Q = (Q_o, Q_1, t,h)$ is called a Schur representation (following Roiter) if its endomorphism ring is reduced to $\mathbb{C}$. The corresponding dimension vector $\underline{\dim}\, V \in \mathbb{N}^n$ is called a Schur root. Kač conjectured that a dimension vector $\alpha \in \mathbb{N}^n$ is a Schur root iff there is no decomposition $\alpha = \beta+\gamma$ with $\beta,\gamma \in \mathbb{N}^n \setminus \{(0,\dots 0)\}$ s.t. $R(\beta,\gamma)$, $R(\gamma,\beta) > 0$ (R is the Ringel bilinear form). The easiest counterexamples are the roots $\alpha = (k,\ell,k)$ where $\frac{n}{n^2-1} k < \ell < \frac{2}{n} k - \frac{2}{n}$ for the quiver

$\underset{1}{\circ} \overset{n}{\rightrightarrows} \underset{2}{\circ} \overset{n}{\rightrightarrows} \underset{3}{\circ}$ on 3 vertices with n arrows from 1 to 2 and from 2 to 3. The main reason why the conjecture fails is that Schur roots are preserved under reflection functors whereas the notion of an indecomposable root is not.

L. Makar-Limanov: ON SYMMETRIC NONCOMMUTATIVE POLYNOMIALS

Let K be a commutative polynomial ring with n generators and S be the symmetrics group of permutations on n letters. There is a natural representation of S as automorphisms of K and the elements of K^S are called symmetric elements of K. As is well known one can easily give examples of polynomials from $K^S[t]$ which vanish on the generators of K.

I am planning to present some new results on the existence of such polynomials when K is replaced by a free algebra with n generators.

K. Masaike: LOCALIZATION AND ARTINIAN RINGS WITH DUALITY

Let V be a left module over a ring R. Assume $T = \mathrm{End}({}_RV)$. Let τ_1(resp. τ_2) be the hereditary torsion theory cogenerated by $I({}_RV)$ (resp. $I(V_T)$). Then the following conditions are equivalent, where $I({}_RV)$ is the injective hull.

(i) ${}_RV$ is τ_1-finitely generated Δ-injective R-module.

(ii) T is right Artinian, V_T is finitely generated cogenerator and $E(V_T)$ is a ring of quotients w.r.t. τ_1.

(iii) The bimodule ${}_RV_T$ defines a duality between the category of τ_1-finitely generated τ_1-quotient modules and the category of finitely generated right T-modules.

S. Montgomery: CROSSED PRODUCTS OF HOPF ALGEBRAS

Let H be a finite-dimensional Hopf algebra acting on the k-algebra A. We study the relationship between A and the subring A^H of H-invariants by using the semi-direct product $A\#H$. In particular, if $A\#H$ is simple, then A is a fin. gen. projective A^H-module; and also A^H is simple $\Leftrightarrow$ $\hat{f}(A) = A^H$. Here $\hat{f} : A \to A^H$ is a trace-like function coming from an integral $f \in H$; define $\hat{f}(a) = f \cdot a$, any $a \in A$. More generally, if $A\#H$ is semiprime and every $0 \neq I \triangleleft A\#H$ satisfies $I \cap A \neq 0$, then A is Goldie $\Leftrightarrow$ A^H is Goldie. These results (joint with J. Bergen) give a common generalization of known results for finite groups of X-outer automorphisms, finite-dimensional restricted Lie algebras of X-outer derivations, and certain algebras graded by finite groups.

B. J. Müller: LOCALIZATION AT INFINITE CLANS

We discuss the proof of our theorem, that any affine noetherian PI-algebra R over a field k can be Ore-localized at every clique. Technically one has to verify the intersection condition for (right) ideals I . Noetherian induction puts one into a situation where R is semiprime, and there are two elements $a,b \in I$ such that $a \in \mathcal{C}(P)$ or $b \in \mathcal{C}(P)$ for each prime P of the clique. One wants to find $\lambda \in k$ with $a\lambda+b \in \cap\, \mathcal{C}(P)$. Due to the countability of cliques, this is fairly trivially possible if k is uncountable. For countable k, the strategy is to construct ring homomorphisms $\varepsilon_P : R \to M_{n(P)}(K)$ with $\ker \varepsilon_P = P$, for every P, with an algebraically closed field K. Moreover one needs a subfield K^*, closed under extensions of degree $\leq N$ but with $k \not\subset K^*$, such that $\varepsilon_P(s) \in M_{n(P)}(K^*)$ for all s in a generating set S of R. Once such ε_P exist, $a\lambda+b \in \mathcal{C}(P)$ yields $\det \varepsilon_P(a) + \lambda\varepsilon_P(b) = 0$, and since this is a polynomial of degree n ($\leq N$) over K^*, one concludes $\lambda \in K^*$. Thus any $\lambda \in k \setminus K^*$ works.

To construct ε_P, one descends from $R = k\{S\}$ to $R^* = k^*\{S\}$, where $k^* = k \cap K^*$. One needs to know that all links arise from factors of finitely many ideals N_i - here the $N_i = \cap \{P : \text{PI degree } (P) \leq i\}$ work. Taking the integer N large enough, K^* incorporates the data describing the N_i as well as the matrix entries of $\varepsilon_{P_o}(s)$ for all $s \in S$ and one prime P_o. Arguing along the links, a fairly straightforward dimension counting argument, followed by a lifting procedure from R^* to R, allows one to construct all the ε_P.

D. S. Passman: PRIME IDEALS IN ENVELOPING RINGS AND CROSSED PRODUCTS

We describe the prime ideals in enveloping rings and crossed products. Let $U(L)$ be the enveloping algebra of the Lie algebra L over K. If L acts on the K-algebra R as derivations then we can form the Lie algebra smash product $R\#U(L)$. Similarly if the group G acts as automorphisms on the ring R, then we can form the crossed product $R*G$. In each case we relate the primes P of the larger ring to the primes of R under suitable Noetherian hypotheses. In particular we assume that R is right Noetherian, that $\dim_K L < \infty$ with $\operatorname{char} K = 0$ and that G is polycyclic-by-finite. Thus both $R\#U(L)$ and $R*G$ are Noetherian. In the key special case when R is prime, we extend R to $S = Q_s(R)$, the symmetric Martindale ring of quotients of R. We then determine the centralizer of S in $S\#U(L)$ and in $S*G$ and show that the primes P with $P \cap R = 0$ come from primes of this well understood centralizer.

C. M. Ringel: THE SPECTRUM OF A TUBULAR ALGEBRA

Let A be a tubular k-algebra, and $A \to M_n(D)$ a (categorical) epimorphism, where $M_n(D)$ is the full $n \times n$-matrix ring over a division ring D . Since all finite-dimensional indecomposable A-modules are known, we may assume that D is infinite-dimensional over k . In this case, $D = k(t)$, the field of rational functions in one variable, and there are countably many equivalence classes of such epimorphisms, indexed in a natural way by $[0,1] \cap \mathbb{Q}$. This result follows from a general theorem which characterizes a particular infinite-dimensional module over any canonical algebra.

K. W. Roggenkamp : UNITS IN GROUP RINGS

Let P be a finite p-group and V the normalized units in the p-adic group ring $\hat{\mathbb{Z}}_p P$. We say that the Sylow theorems hold in V , provided, for every finite p-subgroup U in V there exists $v \in V$ with ${}^vU < P$.

Theorem I: If $p = 2$, then the Sylow theorems hold in V .

For a pro p-group X we denote by $H^*(X,\mathbb{F}_p)$ the even dimensional cohomology ring for the continuous cohomology, and $\mathcal{V}(X)$ denotes its variety.

Theorem II: The following are equivalent:

(i) The Sylow theorems hold in V .

(ii) For every $H < P$, the variety $\mathcal{V}(N_V(H)/H)$ is connected.

Some consequences and aspects of the proof will be discussed.
(Joint work with L.L. Scott)

W. Schelter: GRADED ALGEBRAS OF FINITE GLOBAL DIMENSION

If A is an algebra $k\{X_1,\dots,X_r\}/\{f_1,\dots f_t\}$ with $\deg X_i = 1$, and f_i homogeneous, then we say A is regular if

1) gl dim $A = d < \infty$ (graded modules)

2) gk dim $A < \infty$

3) $\mathrm{Ext}_A^q(k,A) = \begin{cases} k & \text{if } q = d \\ 0 & \text{if } q \neq d \end{cases}$

Regular algebras of dim 2 and 3 are classified. A discussion of the classification in deg 3 was given: The relationship between skew polynomial rings and j-invariants was given. Also a discussion of Type A algebras and the relation with elliptic curves was given. Finiteness of the algebras over their centers in the Type A case is equivalent to finiteness of the order of σ, an automorphism of the elliptic curve.

A. Schofield: RELATIVE INVARIANTS OF QUIVERS

Let $\vec{Q}$ be a quiver, and $\underline{d}$ a dimension vector for the quiver. The orbit space for the action of an algebraic group, $GL(\underline{d})$, on a certain vector space, $V(\underline{d})$, classifies the isomorphism classes of representations of dimension vector $\underline{d}$. It is important to describe the relative invariants for this action. I shall describe a class of relative invariants that in many cases are all the relative invariants, and describe the answer it gives to questions of Kač.

R. Schulz: PERIODIC MODULES OVER QF ALGEBRAS

Eisenbud proved that if $R = KG$ (K field, G finite group), then any bounded R-module is periodic. Tachikawa proved that if $R = KG$ (G finite p-group), then any R-module without large self-extensions is projective. We give an example of a bounded, nonperiodic, nonprojective module without self-extensions over a QF-algebra R. Also, we show:

Theorem 1: For a f.g. bounded module M over a QF algebra R, the following are equivalent:
(a) M is periodic,
(b) $Ext^*_R(M,X)$ is a noetherian $Ext^*_R(M,M)$-module for all f.g. X_R.

Theorem 2: If modules over a QF algebra R satisfy (b), then any module without large self-extensions is projective.

By Even's Theorem, (b) holds for modules over KG. Hence, as a Corollary, we get Eisenbud's result with an alternative proof and Tachikawa's result without restriction on G. Looking at the QF algebra $R_\rho = K\cdot 1 + K\cdot x + K\cdot y + K\cdot xy$ (where $x^2 = y^2 = yx - \rho xy = 0$, $0 \neq \rho \in K$ fixed), we show that R_ρ satisfies (b) iff ρ is a root of unity.

S. P. Smith: RINGS OF DIFFERENTIAL OPERATORS

This will be a survey talk intended to give the flavour of the subject, and in particular to illustrate how techniques of non-commutative ring theory are used, and conversely how the subject of differential operators motivates questions and techniques in non-commutative ring theory. Some topics to be mentioned include:

(a) differential operators on non-singular varieties / $\mathbb{C}$

(b) differential operators on singular varieties / $\mathbb{C}$, particularly curves

(c) differential operators on non-singular varieties in positive characteristic

(d) differential operators and primitive ideals in enveloping algebras

M. Sweedler: FROM DIFFERENTIAL GEOMETRY TO DIFFERENTIAL ALGEBRA

Say A is a commutative K algebra of positive characteristic p. There is a pair (T,t) where T is a commutative A algebra and $t : T \to T$ is a K-linear derivation with $t^p = 0$ and for any other such pair (T', t') there is a unique A algebra map $\phi : T \to T'$ with $t'\phi = \phi t$. This T is graded. $T_o = A$. $T_1 \cong \Omega$ the Kähler module of A over K where $t|T_o : A \to T_1$ is the universal derivation. $t : T \to T$ is homogeneous degree 1.

If A has a p-basis over K - for example $A = k[x_1,\dots,x_n]$ and $K = k[x_1^p,\dots,x_n^p]$ - the complex:

$$A \overset{t}{\to} T_1 \xrightarrow{t^{p-1}} T_p \overset{t}{\to} T_{p+1} \xrightarrow{t^{p-1}} T_{2p} \overset{t}{\to} \dots$$

has zero homology in positive degree and the degree zero homology is K.

(T,t) can be used to classify intermediate fields if A is a purely inseparable exponent one extension field of K. In this case there is a one to one correspondence:

$$\left\{\begin{array}{l}\text{intermediate fields } B\\ \text{with } K \subset B \subset A\end{array}\right\} \longleftrightarrow \left\{\begin{array}{l}A \text{ subspaces } V \text{ of } T_1, \text{ with}\\ t^{p-1} V \subset \sum_{i=1}^{p-1} T_i t^{p-1-i}(V)\end{array}\right\}$$

$$B \longrightarrow At(B)$$

$$\{a \in A \mid t(a) \in V\} \longleftarrow V$$

This result is an analog to the differential ideal formulation of the Frobenius theorem on integral submanifolds. V corresponds to the differential ideal; B corresponds to the functions which are constant on integral submanifolds.

The vanishing cohomology is an algebra analog, in positive characteristic, to the Poincaré Lemma.
(Joint work with M. Takeuchi)

J.-P. Tignol: VALUATIONS ON SKEW FIELDS

Let D be a central finite-dimensional division algebra over a field F, and assume F has a Henselian valuation v. Then v extends uniquely to a valuation on D, with abelian value group. In the case where D is totally ramified and tame over F (i.e. $\overline{D} = \overline{F}$ and char $\overline{F}$ does not divide $[D:F]$), multiplicative commutators are used to define a canonical symplectic structure on the quotient group of value group $v(D^*)/v(F^*)$, and it is shown how the algebraic structure of D is related to the symplectic structure on $v(D^*)/v(F^*)$. As applications, it is shown that, if F is algebraically closed, then every finite-dimensional splitting field of a totally ramified tame division algebra D over F contains a maximal subfield of D, and an example of two division algebras of odd degree having no common subfield but whose tensor product is not a division algebra is given.
(Joint work with S. Amitsur and A. Wadsworth)

F. Van Oystaeyen: PROJECTIVE REPRESENTATIONS OF FINITE GROUPS AND THE BRAUER SPLITTING THEOREM OVER COMMUTATIVE RINGS

The theorem of the Schur multipliers depends on the finiteness of $H^2(G,K^*)$ i.e. on the fact that one considers projective representations over algebraically closed fields. Over connected commutative rings one can obtain a lifting theorem for projective representations as well as a version of the Brauer splitting theorem allowing projective separable extensions of the groundring (note that arbitrary Azumaya algebras need not be splittable by such extensions) whereas the central extension of the group used in the lifting property is still finite (of order dividing the exponent of the given group). Allowing connected commutative groundrings makes it possible to consider infinite groups. We provide an interesting example: Clifford representations of $(\mathbb{Z} / 2\mathbb{Z})^n$, describing projective representations determined by Clifford algebras.

R. B. Warfield: BLOCK THEORY FOR NOETHERIAN RINGS

If R is an Artinian ring and S and T are simple right modules, then one can write $S \rightsquigarrow T$ if $\mathrm{Ext}(S,T) \neq 0$. If M and N are the corresponding maximal ideals, then one writes $M \rightsquigarrow N$ under the same circumstances, and one notes that this is equivalent to $M \cap N/MN \neq 0$. The equivalence classes generated by this relation are the "blocks" of simple modules. The blocks, of course, correspond to a decomposition of R into its indecomposable factors.

If R is a commutative Noetherian ring, then the nonisomorphic simple modules never have nontrivial extensions, so one should consider the blocks as singletons. There is again a way of finding a new ring corresponding to each block - namely the localization at the corresponding maximal ideal.

What I am calling the "block theory" for Noetherian rings starts from both of these points. The spectrum of a Noetherian ring can be made in a natural way into a directed graph - the "graph of links" of the ring. (If M and N are maximal ideals such that R/M and R/N are Artinian, then we will write $M \rightsquigarrow N$ if $M \cap N/MN \neq 0$, as in the Artinian case above. The general description is more complicated.) The components of this graph (which should probably be called "blocks" of primes) are called <u>cliques</u>. An important continuity theorem of J.T. Stafford's implies that the cliques are at most countable. We want to address the significance of this graph for the representation theory of the ring, and the extent to which one can find localizations corresponding to the cliques.

I. Representation Theory and the Graph of Links. The representation theoretic significance of the graph of links is more subtle than might at first appear. We illustrate the point (as an application) by constructing a critical module whose annihilator is not a prime ideal. The graph of links is particularly useful for those rings satisfying the "second layer condition". We will illustrate how this condition can be verified for a large class of rings. Finally, we will discuss the relation between the graph of links and the indecomposable injective modules over such rings.

II. Localization and the Graph of Links. If P is a prime ideal in a Noetherian ring R, then the least we should require of a "localization at R" would be an Ore set $\mathcal{C}$ such that if $R' = R\mathcal{C}^{-1}$ then (i) R'/PR' is the Goldie quotient ring of R/P, and (ii) R' has as few simple modules as possible. It turns out that to localize at a prime in this sense, one must plan on localizing at all primes in the clique containing P. If this clique is finite then the resulting theory is fairly complete. If not, there are additional difficulties, mostly concerning appropriate "general position" arguments. We will discuss the current state of progress on these questions, and also the "local" properties which the localized rings seem to have. As applications we will discuss (i) recent results on the relation between the Krull and global dimensions of PI rings, and (ii) a recent symmetry result on prime PI rings which illustrates the interplay between the study of Ore sets and representation theory. (In particular, for such rings, a prime ideal is left localizable if and only if it is right localizable.)

A.E. Zalesskii: IDENTITIES OF CERTAIN GRADED RINGS

An interesting class of graded algebras is algebras with commutative 0-component. Important examples of such algebras are matrix algebras M_n for each n. For those the following analogue of the known theorem of Amitsur and Levitski holds [1]:

Theorem. Let P be a field of charcteristic $p > 0$ and let $s_r = \Sigma x_{\sigma(1)} \cdots x_{\sigma(n)}$ be the symmetrical element of the free algebra $P[x_1, \dots, x_n]$ (here the sum is taken with all permutations σ of $1, \dots, n$). Then $M_n(P)$ satisfies the identity $s_{np} = 0$ and does not satisfy any identities $s_r = 0$ for $r < np$.

Conjecture. Let A be an algebra over P graded by a group of order n. If 0-component A_o is commutative, then A satisfies the identity $s_{np} = 0$.

This conjecture is true for $n = 2$ (see [2]).

Ref.: [1] Zalesskii, A.E., Symmetrical analogue of Amitsur-Levitski's theorem, Vesci AN BSSR, ser. fis.-mat., 1985, N2, p. 108-110

[2] Zalesskii, A.E., Prime varieties of algebras over fields of positive charactersistic, Dokl. AN BSSR, 1985, vol. 29, N11, p. 965-968.

B. Zimmermann-Huisgen: ON THE GENERALIZED NAKAYAMA CONJECTURE AND THE CARTAN DETERMINANT PROBLEM

For Artin algebras allowing certain filtered module categories the generalized Nakayama Conjecture is confirmed with the aid of a "filtered Grothendieck module". The result applies in particular to all positively graded Artin algebras and to those Artin algebras whose radical cube is zero. For the corresponding class of left artinian rings it is shown that finite global dimension forces the determinant of the Cartan matrix to be 1 .

Berichterstatterin: Christine Bessenrodt

Tagungsteilnehmer

Professor D. Anick
Department of Mathematics
M. I. T.
Cambridge, MA 02139
U. S. A.

Prof. Dr. K. A. Brown
Department of Mathematics
University of Glasgow
University Gardens
Glasgow G12 8QW
Grossbritannien

Dr. Christine Bessenrodt-Timmerscheidt
FB 11 - Mathematik
Universität Duisburg
Lotharstr. 65
4100 Duisburg 1

Prof. Dr. M. Chamarie
Dépt. de Mathématiques
Université Claude Bernard Lyon I
F-69622 Villeurbanne-Cédex
Frankreich

Prof. Dr. L. A. Bokut
Institute of Mathematics
Novosibirsk 90
USSR

Prof. Dr. P. M. Cohn
Department of Mathematics
University College London
Gower Street
London WC1E 6BT
Grossbritannien

Prof. Dr. K. Bongartz
Bergische Universität, GH
FB 7 - Mathematik
Gaußstr. 20
5600 Wuppertal 1

Prof. Dr. S. C. Coutinho
School of Mathematics
Leeds University
Leeds LS2 9JT
Grossbritannien

Prof. Dr. W. Borho
Bergische Universität, GH
FB 7 - Mathematik
Gaußstr. 20
5600 Wuppertal 1

Prof. Dr. E. Formanek
Mathematics Department
Penn. State University
University Park, PA 16802
U. S. A.

Prof. Dr. A. Braun
Department of Mathematics
University of Haifa
Mount Carmel
Haifa 31999
Israel

Dr. W. Geigle
Universität-GH-Paderborn
FB 17 - Mathematik/Informatik
Warburger Str. 100
4790 Paderborn

Prof. Dr. A. W. Goldie
Dept. of Pure Mathematics
Leeds University
L e e d s LS2 9JT
Grossbritannien

Prof. Dr. G. R. Krause
Dept. of Math. & Astronomy
The University of Manitoba
Winnipeg, Manitoba R3T 2N2
Canada

Prof. Dr. K. R. Goodearl
Department of Mathematics
University of Utah
Salt Lake City, Utah 84112
U. S. A.

Prof. Dr. L. Le Bruyn
Department of Mathematics
U. I. A.
Universiteitsplein 1
B-2610 W i l r i j k
Belgien

Prof. Dr. R. S. Irving
Department of Mathematics
University of Washington
S e a t t l e , WA 98195
U. S. A.

Prof. Dr. H. Lenzing
Universität-GH-Paderborn
FB 17 - Mathematik
Warburger Str. 100
4790 P a d e r b o r n

Prof. Dr. C. U. Jensen
Matematisk Institut
Universitet Kopenhagen
Universitetsparken 5
DK-2100 K o p e n h a g e n
Dänemark

Dr. T. Levasseur
Université Paris VI
Mathématiques
4, Place Jussieu
F-75230 P a r i s - Cédex 05
Frankreich

Prof. Dr. A. Joseph
Dept. of Theoretical Mathematics
Weizmann Institute of Science
R e h o v o t 76100
Israel

Prof. Dr. L. G. Makar-Limanov
Department of Mathematics
Wayne State University
646 Mackenzie Hall
D e t r o i t , MI 48202
U. S. A.

Prof. Dr. V. K. Kharchenco
Institute of Mathematics
N o v o s i b i r s k 90
USSR

Frau Professor
Dr. Marie Paule Malliavin
10 Rue Saint Louis en l'Ile
F-75004 P a r i s
Frankreich

Prof. Dr. K. Masaike
Department of Mathematics
Tokyo Gakugei University
Koganei

T o k y o / Japan

Prof. Dr. D. S. Passman
Mathematics Department
University of Madison
Van Vleck Hall
480 Lincoln Drive

M a d i s o n , WI 53706 U.S.A.

Prof. Dr. J. C. McConnell
School of Mathematics
University of Leeds

L e e d s LS2 9JT

Grossbritannien

Prof. Dr. C. M. Ringel
Fakultät für Mathematik
Universität Bielefeld
Universitätsstr. 1

4800 B i e l e f e l d 1

Prof. Dr. G. Michler
FB Mathematik
Gesamthochschule Essen
Universitätsstr. 1

4300 E s s e n 1

Prof. Dr. K. W. Roggenkamp
Universität Stuttgart
Mathematisches Institut B/3
Pfaffenwaldring 57

7000 S t u t t g a r t 80

Prof. Dr. Susan Montgomery
Dept. of Mathematics DRB 306
University of Southern Calif.

Los Angeles, CA 90089-1113

U. S. A.

Prof. Dr. W. Schelter
Department of Mathematics
University of Texas at Austin

A u s t i n , Texas 78712

U. S. A.

Prof. Dr. B. J. Mueller
Department of Mathematics
Mc Master University

Hamilton, Ontario L8S 4K1

Canada

Dr. A. H. Schofield
Department of Mathematics
University College
Gower Street

GB-L o n d o n WC1E 6BT

Grossbritannien

Prof. Dr. F. van Oystaeyen
Department of Mathematics
University of Antwerpen, UIA
Universiteitsplein 1

B-2610 W i l r i j k
Belgien

Prof. Dr. R. Schulz
Mathematisches Institut
der Universität München
Theresienstr. 39

8000 M ü n c h e n 2

Prof. Dr. L. W. Small
Department of Mathematics
University of California
at San Diego

L a J o l l a , CA 92093
U. S. A.

Prof. Dr. R. B. Warfield
Department of Mathematics
University of Washington

S e a t t l e , WA 98195
U. S. A.

Dr. S. P. Smith
Mathematics Institute
University of Warwick

C o v e n t r y CV4 7AL

Grossbritannien

Dr. A. Wiedemann
Universität Stuttgart
Mathematisches Institut B/3
Pfaffenwaldring 57

7000 S t u t t g a r t 80

Prof. Dr. R. L. Snider
Department of Mathematics
V. P. I.

Blacksburg, VA 24061

U. S. A.

Prof. Dr. A. E. Zalesskii

Institute of Mathematics
of AN BSSR

220604 M i n s k

USSR

Prof. Dr. M. Sweedler
I B M
Thomas J. Watson Research Center
P.O. Box 218

Yorktown Heights, N.Y. 10598
U. S. A.

Frau Professor
Dr. Birge Zimmermann-Huisgen
Fakultät für Mathematik u. Inf.
Universität Passau
Innstr. 27

8390 P a s s a u

Prof. Dr. J.-P. Tignol
Dépt. de Mathématiques
U. C. L.
Chemin du Cyclotron, 2

B-1348 Louvain-la-Neuve

Belgien