MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 19/1986

General Inequalities

4.5. bis 10.5.1986

The fifth International Conference on General Inequalities was held from May 4 to May 10 at the Mathematisches Forschungsinstitut Oberwolfach.
The organizing committee consisted of W.N.Everitt (Birmingham, England), L.Losonczi (Debrecen, Hungary) and W. Walter (Karlsruhe, BRD). Dr.A.Kovacec served efficiently and enthusiastically as secretary of the conference. The meeting was attended by 50 participants from 16 countries.

In his opening address, W.Walter had to report on the death of five colleagues who had been active in the area of inequalities and who had served the mathematical community: P.R.Beesack, G.Polya, D.K.Ross, R.Bellman, G.Szegö. He made special mention of G.Polya, who had been the last surviving author of the book Inequalities (Cambridge University Press, 1934), who died at the age of 97 years and whose many and manifold contributions to mathematics will be recorded elsewhere, in due course.

Inequalities continue to play an important and significant role in nearly all areas of mathematics. The interests of the participants to this conference reflected the many different fields in which both classical and modern inequalities continue to influence developments in mathematics. In addition to the established fields, the lectures clearly indicated the importance

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of inequalities in functional analysis, eigenvalue theory, convexity, number theory, approximation theory, probability theory, mathematical programming and economics.

On the occasion of this conference special attention was payed to the recent solution of the Bieberbach conjecture. In two carefully prepared lectures, given on the invitation of the committee, Dr.N.Steinmetz (Karlsruhe) reviewed the history, and proof of the correctness of the conjecture. His excellent presentation showed the importance of a number of inequalities required for the proof, and how an inequality for a solution of a linear system with constant coefficients could significantly simplify part of the proof as a whole.

The problems and remarks sessions yielded many new ideas and intriguing conjectures.

All the participants came under the influence of the remarkable atmosphere now such an established feature of the Institute.

The conference was closed by W.N. Everitt, who, in paying tribute to all those who had contributed to the progress of the meeting, asked that the best thanks of all the participants be presented to the staff of the Institute for their unique contribution in the form of excellent hospitality, and quiet and effective service.

Abstracts

J.ACZEL: Entropies, generalized entropies, inequalities and the maximum entropy principle.

Inequalities for the Shannon (and Hartley-) entropies and their generalizations have been used in applications and they served as building blocks for their characterizations. After a short survey of such results the idea is put forward that the maximum entropy principle (also an inequality) may be used not only to justify probability distributions but also to justify entropies.

R.P. AGARWAL: Linear and nonlinear discrete inequalities in n independent variables.

We introduce a discrete analogue of Riemann's function and use it to study discrete Gronwall-type inequalities in n independent variables. Next we provide an estimate on Riemann's function and use it to obtain Wendroff type estimates.

C. ALSINA: On the stability of a functional equation arising in probabilistic normed spaces.

Motivated by a problem on probabilistic normed spaces we study the inequality

$$(*) \qquad d_L(\tau(F(\tfrac{j}{a}),F(\tfrac{j}{b})),\ F(\tfrac{J}{a+b})) \leq \epsilon$$

where $\epsilon > 0$ is fixed, d_L is the modified Lévy metric, F is an arbitrary function in Δ^+, a, b are arbitrary positive real numbers, and τ is a nondecreasing continuous binary operation on Δ^+ to be found.

We show the following

Thm: A continuous nondecreasing binary operation on Δ^+ satisfies (*), if and only if

$$\hat{d}_L(\tau,\ \tau_u) = \sup\ \{d_L(\tau(F,G),\ \tau_u(F,G))\ :\ F,G \in \Delta^+\} < \epsilon,$$

where $\tau_u(F,G)(x) = \sup\{\mathrm{Min}(F(u),G(v))\ :\ u+v = x\}.$

A. BEN-ISRAEL: $\mathfrak{F}$-convexity.

Let $\mathfrak{F}$ be a family of functions: $\mathbb{R}^n \to \mathbb{R}$. $f:\mathbb{R}^n \to \mathbb{R}$ is $\mathfrak{F}$-convex in $S \subseteq \mathrm{dom}\ f$ if for all $x \in S$ there exists $F \in \mathfrak{F}$ such that (i) $f(x) = F(x)$, (ii) $f(z) \geq F(z)$ for all $x \neq z$ $(x,z \in S)$.

Examples: (i) Convex functions. Here $\mathfrak{F}$ is the family of affine functions: $\mathfrak{F} = \{F : F(x) = \langle x^*, x\rangle - \eta,\ x^* \in \mathbb{R}^n,\ \eta \in \mathbb{R}\}$

(ii) Sub-$\mathfrak{F}$-functions in the sense of Beckenbach, e.g. Bull.Amer. Math.Soc. 43 (1937), 363-371.

For concreteness let $\mathfrak{F} = \{F : F(x) = F(x^*, \eta_i x)\}$ be a family of differentiable functions, depending continuously on n+1 parameters $(x^*, \eta) \in X^* x Y \subseteq \mathbb{R}^n x \mathbb{R}$. The correspondence $X^* x Y \leftrightarrow \mathfrak{F}$ is assumed 1:1. Results include: 1st order characterizations (gradient inequality), 2nd order characterizations (Hessian) of $\mathfrak{F}$-convexity. Applications to mathematical economics, numerical analysis, optimization.

Reference: A.Ben-Tal, A.Ben-Israel; J.Austral.Math.Soc.XXI A (1976), 341-361.

C.BENNEWITZ: The HELP inequality in a regular case.

Starting in 1972 Everitt, and later others, studied a generalization of the well known Hardy Littlewood Polya inequality

$$\left(\int_0^\infty |u'|^2\right)^2 \leq 4 \int_0^\infty |u|^2 \int_0^\infty |u''|^2$$

In general the problem is to decide whether there is a finite K such that

$$\left(\int_a^b (p|u'|^2 + q|u|^2)\right)^2 \leq K^2 \int_a^b |u|^2 \int_a^b |-(pu')' + qu|^2$$

for any u. Here p and q are given so that the differential expression

$(-pu')' + qu$ is regular at a but singular at b. It was thought that no inequality was possible if both, a and b were regular. I shall describe a counterexample to this and then give conditions reasonably close to being necessary and sufficient for such an inequality to hold.

B. CHOCZEWSKI: A linear iterative functional inequality of third order.

The inequality in question has the form

(1) $\Phi(f^3(x)) + b_2\Phi(f^2(x)) + b_1\Phi(f(x)) + b_0\Phi(x) < 0.$

A description of continuous solutions of inequality (1) will be presented as well as some conditions under which solutions of (1) yield solutions of a Schröder functional equation.

Results are due to Mrs.Maria Stopa from Kraków and to the speaker.

In equation (1) f^k denotes the k-th iterate of a given function f, and b_0, b_1, b_2 are given constants.

A.CLAUSING: Experimenting with operator inequalities.

G. Polya was not only one of the founders of inequality theory but was also very active in making the inductive process of mathematics an explicit topic.

This talk tries to report, in Polyas spirit, on some computer experiments, done with an APL workspace, which are concerned with Polya operators, a class of linear differential operators defined by an inverse positivity condition.

Three results are given, all of which had first been found experimentally by studying examples.

Thm 1: The nonzero coefficients of the basic functions of a standard Polya operator alternate.

Thm 2: The Greens kernel of a standard Polya operator is quasiconcave.

Thm 3: The ν-th eigenvalue of a Polya operator is an isotonic function of the absolute value of the second coefficient of the boundary conditions.

Conjecture: This function is also convex.

W. EICHHORN: Tax progression and measurement of income inequality.

Let $T:\mathbb{R}_+\to\mathbb{R}$, $y\mapsto T(y)$, y a pre-tax income be a feasible (i.e. $T(y) < y$ for all $y\in\mathbb{R}_+$) and incentive preserving ($y<y^*$ implies $y-T(y)\leq y^*-T(y^*)$) tax function and let $I_n:\mathbb{R}^n_+\to\mathbb{R}$, $x\mapsto I(x)$ be a strictly Schur-convex "μ-measure" of inequality of incomes $\underline{x}$, i.e. a strictly Schur-convex function that satisfies

$$I_\mu(\underline{x}) = I_\mu(\underline{x}+\tau(\mu\underline{x}+(1-\mu)\underline{1}))$$

for all $\underline{x}\in\mathbb{R}^n_+$ and $\tau\in\mathbb{R}$ satisfying $\underline{x}+\tau(\mu\underline{x}+(1-\mu)\underline{1})\in\mathbb{R}^n_+$, where $\mu\in[0,1]$ is fixed. The functional equation shows for which income distributions the income inequality is preserved.

The following two statements are equivalent:

(i) $I_\mu(y_1-T(y_1),\dots, y_n-T(y_n) < I_\mu(y_1,\dots,y_n)$ for all $y\in\mathbb{R}^n_+$ such that $(y_1,\dots,y_n) \neq (a,a,\dots,a)$.

(ii) $T(y)/(\mu y+(1-\mu)\underline{1})$ is strictly increasing in $y\in\mathbb{R}_+$.

This result obtained by my Ph.D. student Andreas Pfingsten generalizes a result (μ=1, I_μ the Lorenz measure of inequality) that I presented at the 1983 meeting on Allgemeine Ungleichungen.

Corollary: An inequality preserving tax function T is necessarily an affine-linear function.

M. ESSEN: Rearrangements and optimization problems for certain linear second-order differential equations.

For coefficients q in certain classes of integrable functions on the interval $[0,T]$, we determine sup y(T) and inf y(T) when q varies in the class and y is a solution of one of the equations

$$y'' \pm qy = 0, \quad y(0) = 1, \quad y'(0) = \alpha, \quad t \in [0,T].$$

In the proofs, we use a kind of calculus of variations and a partial order between functions introduced by Hardy, Littlewood and Polya. The infimum problem arose in the study of growth problems for subharmonic functions. There are applications to problems where we try to maximize the first eigenvalue of a certain Sturm-Liouville problem when q varies in a certain class (there are also generalizations to higher dimensions).

W.N.EVERITT: An example of the Hardy-Littlewood type of integral inequalities.

At the General Inequalities meeting held at Oberwolfach in May 1985 it was reported that the inequality

$$\left(\int_0^\infty \{f'^2 + (x^2-1)f^2\}dx\right)^2 \leq 4 \int_0^\infty f^2\, dx \int_0^\infty \{f'' - (x^2-1)f\}^2 dx$$

is best possible with equality if and only if $f(x) = k \exp[-x^2/2]$ (when both sides are equal but zero).

It was conjected that the inequality

$$\left(\int_0^\infty \{f'^2 + (x^2-3)f^2\}dx\right)^2 \leq K \int_0^\infty f^2\, dx \int_0^\infty \{f'' - (x^2-3)f\}^2 dx \qquad (*)$$

is valid with K = 4 and with equality if and only if $f(x) = kx \exp[-x/2]$ (again with both sides equal but zero).

This conjecture is false. The inequality (*) is valid but with

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$4 < K \simeq 4.1094$. There are two cases of equality; the first when $f(x) = kx \exp[-x^2/2]$ (with both sides equal but zero), and a second case when both sides of (*) are not zero but when the equalising function is more complicated.

The lecture will report on joint work with W.D.Evans, W.K.Hayman, and S.Ruscheweyh.

F.FEHER: p-estimates for ultraproducts of Banach lattices.

A Banach lattice L is said to satisfy a lower p-estimate ($1\leq p<\infty$) iff there exists a constant $K>0$ such that for each finite sequence $f_1, f_2, \dots, f_n \in L$ the inequality

$$\left(\sum_{h=1}^{n} \|f_k\|^p\right)^{1/p} \leq K \left\| \sum_{h=1}^{n} |f_k| \right\|$$

holds. Moreover, L is said to satisfy an upper p-estimate, iff there exists a constant $M > 0$ such that for each finite sequence $f_1, f_2, \dots, f_n \in L$ one has

$$\left\| \sup_{1\leq k\leq n} |f_n| \right\| \leq M \left(\sum_{h=1}^{n} \|f_n\|^p\right)^{1/p} .$$

The purpose of the lecture is to show, in what sense these p-estimates carry over to ultraproducts of Banach lattices. An application is given to the problem of superreflexivity of Banach lattices.

I.FENYÖ: Inequalities concerning convolutions of kernels of integral equations.

Let $\Lambda \subset \mathbb{R}$ be a measurable set with $0<|\Delta|<\infty$, $P,Q:\Lambda\times\Lambda\to\mathbb{R}$ integrable functions. Let p be a number $0<p\leq 2$ and q its adjoint ($1/p + 1/q = 1$.). For an arbitrary function $Z:\Lambda\times\Lambda\to\mathbb{R}$ as above define following norm (if it exists)

$$\|Z\|_p = \{\int_\Lambda(\int_\Lambda|Z(s,t)|^p\,dt)ds\}^{1/p} .$$

The main result is following statement: If $\|P\|_q < \infty$, and $\|Q\|_q < \infty$, then $\int_\Lambda P(s,r)Q(r,t)\,dr$ exists, its p norm is bounded, and

$$\|\int_\Lambda P(s,r)Q(r,t)\,dr\|_p \leq |\Lambda|^{\frac{q-2}{q}} \|P\|_p\|Q\|_q$$

$$\|\int_\Lambda P(s,r)Q(r,t)\,dr\|_p \leq |\Lambda|^{\frac{q-2}{q}} \|P\|_q\|Q\|_q .$$

R. GER: Subadditive multifunctions and Hyers-Ulam stability.

Let $(S,+)$ be an Abelian semigroup and let $(Y,\|\,,\|)$ denote a (real or complex) Banach space. Consider a multifunction F from S into the family of all nonempty closed convex subsets of Y, fulfilling the subadditivity condition

$$F(x+y) \subset F(x) + F(y), \quad x,y \in S.$$

If

$$\sup\{\operatorname{diam} F(x) : x \in S\} < \infty$$

then F admits an additive selection, i.e. a homomorphism a of $(S,+)$ into the additive group $(Y,+)$ such that $a(x) \in F(x)$ for all $x \in S$.

Abstract version of this result is also possible. The problem is strictly related to the question about the behaviour of solutions of the functional inequality

$$\|f(x+y) - f(x) - f(y)\| < \varepsilon, \quad x,y \in S,$$

considered for mappings $f: S \to Y$ (Hyers-Ulam stability problem).

M. GOLDBERG: Multiplicativity and mixed-multiplicativity for operator-norms and matrix-norms

Let V be a normed vector space over $\mathbb{C}$, let L(V) denote the algebra of bounded linear operators on V, and let N be an arbitrary norm on L(V). In this talk we discuss multiplicativity factors for N, i.e., constants $\mu > 0$ for which

$$N(AB) \leq \mu N(A)\, N(B) \;\forall A,B \in L(V).$$

We shall examine several finite and infinite dimensional examples, as well as certain generalizations of the above concepts.

W. HAUßMANN: Uniqueness inequality and best Harmonic L^1-approximation

For a given measure space, let $f \in L^1(X)$ and V a subspace of $L^1(X)$. Given two best L^1-approximants v_1, $v_2 \in V$ to f, then we prove the following inequality:

$$(*) \qquad (f-v_1)(f-v_2) \geq 0 \quad \text{a.e. in } X.$$

This inequality is very useful in order to prove uniqueness of a best L^1-approximant in the case when the occuring functions are continuous on an appropriate X.

We consider the approximation of subharmonic functions f by a space V of harmonic functions and give a sufficient condition for a best L^1-approximant. Under mild assumptions, (*) yields the uniqueness of a best L^1-approximant.

C.O. IMORU: On a generalization of Steffensens inequality

In a recent paper, Pečarić obtained the following interesting generalization of Steffensen's inequality:

Let $f: [0,1] \to R$ be a non-negative and non-decreasing function and let $g: [0,1] \to R$ be an integrable function such that $0 \le g(x) \le 1$, for every $x \in [0,1]$. If $p \ge 1$, then

$$\left[\int_0^1 f(t)g(t)dt\right]^p \le \int_0^\lambda f(t)^p dt$$

where

$$\lambda = \left[\int_0^1 g(t)dt\right]^p .$$

The purpose of this talk is to prove a considerably more general result, which is an extension of Steffensen's inequality.

H.H.KAIRIES: Inequalities for q-factorial functions

The q-factorial function $\gamma_q : \mathbb{R}_+ \to \mathbb{R}$, given by

$$\gamma_q(x) = (1-x)\log(1-q) + \log \prod_{n=0}^{\infty} (1-q^{n+1})(1-q^{n+x})^{-1}, \quad q \in (0,1)$$

can be characterized as a Krull principal solution of its difference equation

(D) $\quad f(x+1) - f(x) = \log(q^x-1)/(q-1).$

Moreover, inequalities are obtained which give detailed information on the behaviour of γ_q near 1.

THEOREM. a) Assume $f : \mathbb{R}_+ \to \mathbb{R}$ to be convex, to satisfy (D) for some $q \in (0,1)$ and $f(1) = 0$. Then $f = \gamma_q$.

b) Let $F(x) := f(x) + f(1/x)$. Then $1 \le x < y$ implies $F(x) \le F(y)$.

c) Let $F_t(x) := f(x) + f(1-t(x-1))$, $x \in [1,1+t^{-1})$ and $\tau(x,q) := \log(2-q^{x-1})/\log q^{1-x}$.

Then $F_t(x) > 0$ if $x \in (1,1+1/t)$ and $t \ge \tau(x,q)$.

H.KÖNIG: A strict inequality for projection constants

The projection constant of a finite dimensional Banach space X is $\lambda(X) := \sup\{\inf\{\|P\| \mid P:Z \xrightarrow{\text{onto}} X \subset Z \text{ projection}\} \mid Z \text{ Banach}, X \subset Z\}$. A well-known result of Kadets is that $\lambda(X) \leq \sqrt{\dim X}$ holds. This yields that for all $N \in \mathbb{N}_{\geq 2}$ there is $\varepsilon_N > 0$ such that for all X with $\dim X = N$ one has $\lambda(X) \leq \sqrt{N} - \varepsilon_N$. There are spaces X with $\lambda(X) > \geq \sqrt{N} - \frac{1}{\sqrt{N}}$, thus necessarily $\varepsilon_N < \frac{1}{\sqrt{N}}$.

A. KOVAČEC: On an extension of the Bruhat Order of the symmetric group

The Bruhat Order on S_n can be defined by saying that $\pi \leq \sigma$ iff there exists a sequence $\pi = \pi_0, \pi_1, \pi_2, \pi_3, \ldots, \pi_k = \sigma$ of permutations $\pi_i \in S_n$, such that π_{i+1} is obtained from π_i by an order-generating transposition. For example $54132 \leq 13524$, as we have the sequence $54132 \to 53142 \to 13542 \to 13524$. One can ask for nice criteria in order that $\pi \leq \sigma$. In connection with a conjecture of G. Lusztig in Lie Representation Theory, R.Proctor found an answer:
$\pi \leq \sigma$ in S_n if and only if for all $r,s \in \{1,2,\ldots,n\}$ there holds the inequality $|\{i : i \geq r, \pi(i) \geq s\}| \leq |\{i : i \geq r, \sigma(i) \geq s\}|$.
The same answer was found independently by this author in dealing with a refinement of an inequality of Hardy Littlewood and Polya. In this talk we generalize the viewpoint that a permutation is a bijection between the (very simple) partially ordered set $1<2<3<\ldots<n$ and itself. By appropriate definition of the term "Order generating transposition" one may obtain similar results as above for bijections between suitable partially ordered sets.

N. KUHN: Almost t-convex functions.

Let $\emptyset \neq I \subset \mathbb{R}$ denote an interval and fix $t \in [0,1]$. A function $f: I \to \mathbb{R}$ is called almost t-convex iff

$$f(tu+(1-t)v) \leq tf(u) + (1-t)f(v) \quad \text{holds for almost all } (u,v) \in I^2$$

(in Lebesguemeasure on $\mathbb{R}^2$). Furthermore we define $K_a(f) :=$
$:= \{t \in [0,1]: f \text{ is almost t-convex}\}$.

Theorem: If $K_a(f) \neq \{0,1\}$, then

$$K_a(f) = [K_a(f)] \cap [0,1],$$

where $[K_a(f)]$ denotes the field generated by $K_a(f)$.
The proof is based on a related result for t-convex functions and on a construction of Kuczma.

M.K. KWONG: Norm inequalities between a function and its derivatives

We report on work done jointly with A.Zettl. There is a discrete analogue of the classical Landau inequality $\|\Delta x\|_p^2 \leq C\|x\|_p\|\Delta^2 x\|$ where x is either a semiinfinite or biinfinite sequence in L^∞ and Δ is the difference operator. The central problem is to determine the best constant C in terms of p.

Various extensions of the inequality, both in the discrete and continuous cases, are also discussed. This includes the extensions to m-dissipative operators, higher orders, Everitts generalization and weighted norms.

Although analogous results hold in the discrete case, the proofs are often significantly different from the corresponding ones in the continuous cases.

L. LOSONCZI: Nonnegative trigonometric polynomials and related quadratic inequalities

Inequalities of the form

$$\lambda \sum_{j=o}^{n} |x_j|^2 \leq \Sigma\ |x_j \pm x_{j+k}|^2 \leq \Lambda \sum_{j=o}^{n} |x_j|^2$$

are considered, where $x_o,\ldots,x_n \in \mathbb{R}$ (or $\mathbb{C}$), λ, Λ are constants and the sum in the middle means one of the following ones:

(i) $\sum_{j=o}^{n-k}$, (ii) $\sum_{j=o}^{n}$ with $x_{n+1}=\ldots= x_{n+k} = 0$,

(iii) $\sum_{j=-k}^{n-k}$ with $x_{-k}= \ldots = x_{-1} = 0$,

(iv) $\sum_{j=-k}^{n}$ with $x_{-k}=\ldots=x_{-1}=x_{n+1}=\ldots=x_{n+k}= 0$.

In all cases (with both signs +and-) the exact constants λ, Λ are given. They are minimal and maximal eigenvalues of suitable Hermitian matrices. The case k=1 has been known.

E.R. LOVE: An inequality for geometric means.

Cochran and Lee [Math.Proc.Camb.Phil.Soc.96(1984)1-7]obtained the inequality

$$\int_0^\infty x^r \exp\left(\frac{p}{x^p}\int_0^x t^{p-1} \log f(t)dt\right)dx < e^{r+1/p}\int_0^\infty x^r f(x)dx$$

for r and p real, p>0 and $f(x) \geq 0$. This generalizes an old inequality of Knopp (r=0, p=1). They also obtained a discrete analogue.

The exponential in the integral on the left is the geometric mean of f on (0,x) with weight function t^{p-1}. It is proposed to present a corresponding inequality with a general weight function and to consider possible discrete analogues.

G. LUMER: Parabolic maximum principles, diffusion equations, and population dynamics

We give general parabolic maximum principles for L-subharmonic functions u ($Lu \geq 0$) on space time $\underline{\Omega} = \Omega \times I$ (I an interval), L being a locally dissipative, parabolic, local operator. We consider general open sets $\underline{V}$ in $\underline{\Omega}$ and an appropriate closed boundary $B_p(\underline{V})$ for $\underline{V}$. The linear maximum principle then says that $\sup_{\underline{V}} u \leq \sup_{B_p(\underline{V})} u^+$, (where $u^+ = \sup\{u,0\}$). Similarly, for the semilinear case we have comparison theorems (with Lipschitz or locally Lipschitz nonlinearities). There are many applications to parabolic 2-nd order PDE, but also to situations where L is a more complicated object than a PD operator (for instance in transmission problems, or the construction of highly non-differentiable extensions of PD operators as intermediate tools).

The mentioned maximum principles play an essential role in obtaining unique global solutions of problems of the type

$$Lu + Nu = 0 \quad \text{in } \underline{V}_s := \{(x,t) \in \underline{V} : t > s\}$$
$$u(x,s) = f(x) \quad (f \text{ initial value at } t=s)$$
$$u|\Gamma_s = 0 \quad (\Gamma_s \text{ an appropriate lateral boundary}),$$

assuming this problem is solvable for $\underline{V} = \underline{\Omega}$ and an L-barrier can be constructed for $\underline{V}$. Such results apply in particular to second order parabolic PD operators with merely continuous coefficients (real, c(x,t) independent term ≤ 0), in general open noncylindrical domains $\underline{V}$. In particular one gets unique global solutions for generalized timedependent Kolmogorov-Petrovskii-Piskunov equations important in population dynamics.

A.W. MARSHALL: Extensions of Markovs inequality for random variables taking values in a linear topological space

The usual assumption is that $A \leq B$ means A and B are real numbers but there are many other possibilities: A familiar example is that A and B are Hermitian matrices and

(1) $A \leq B$ means $B-A$ is positive semi-definite. Analogues of some classical inequalities are known, where the quantities compared are not real numbers.

Example 1. If X and Y are $k \times n$ complex matrices and XX^* is nonsingular, then $YX^*(XX^*)^{-1}XY^* \leq YY^*$ in the sense of (1); this reduces to Cauchy's inequality when $k=1$.

Example 2. If X is a random Hermitian matrix such that $EX = u$ is positive definite and $P(X \geq 0)=1$, then for every positive definite matrix ϵ,

(2) $P(X \geq \epsilon) \leq$ minimal root of $\epsilon^{-1/2} u \epsilon^{-1/2}$

The new result to be presented is a similar version of Chows extension of Markov's inequality but for random variables taking values in a linear topological space. The proof will not be given but to illustrate methods, a proof of (2) will be given in that more general setting.

H.W. Mc LAUGHLIN: Inequalitites arising from discrete curves

After defining the notion of a discrete curve (a geometrically defined set of discrete points) one has to characterize the curve with discrete analogues of the classical notions from differential geometry. Since there are no differentilable functions available with which to compute, for example, error estimates, one has to rely solely on classical discrete inequalities. This leads to an

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investigation of the behaviour of inequalities under the influence of recursion. An example is: how does the ratio of the geometric mean to the arithmetic mean vary as the weights are recursively changed?

R.N. MOHAPATRA: <u>On an analogue of Hardy's Inequality for sequence space $l(p_n)$</u>.

Let w be the vector space of all real sequences and (p_n) be a sequence of positive real numbers. For x in w, let us write

$$l(p_n) = \{x \in w: \Sigma_k |x_k|^{p_x} < \infty\}$$

$$\{\sigma_n(x)\} = \{\frac{1}{n} \sum_{k=1}^{n} x_k\}$$

$$ces(p_n) = \{v \in w: \sigma(|x|) \in l(p_n)\}.$$

If $(p_n) = p$ for all n, then $l(p_n)$ and $ces(p_n)$ reduce to well-known sequence spaces l_p and ces_p.

Hardy's inequality for sequences essentially shows that $l_p \subset ces_p$ $(1 < p \leq \infty)$. It is known that this inclusion is strict.

If we assume (p_n) to be bounded then $l(p_n)$ and $ces(p_n)$ can be shown to be paranormed sequence spaces. It is natural to wonder if it is possible to have an analogue of Hardy's Inequality for the sequence space $l(p_n)$. If that were the case then it should be possible for us to show that for bounded (p_n), $l(p_n) \subseteq$ $\subseteq ces(p_n)$.

In the present paper we have shown that such an inclusion among these paranormed sequence spaces fails to hold. We have also discussed some open problem in this connection.

R.J. NESSEL: Approximation theory in the space of Riemann integrable functions

The following notion of a sequential convergence is suggested for the space $R[a,b]$ of Riemann integrable functions ($\int_a^b f(x)dx :=$ upper Riemann integral).

Definition: A sequence $\{f_n\} \subset R[a,b]$ is called Riemann convergent to $f \in R[a,b]$ if

$$\text{(i)} \quad \sup_{a\leq x\leq b} |f_n(x)| = O(1), \qquad \text{(ii)} \quad \int_a^b [\sup_{k\geq n} |f_k(x) - f(x)|]dx = o(1).$$

It turns out that with this notion of convergence $R[a,b]$ is not only complete, but continuous functions are also dense in $R[a,b]$. This enables one to discuss approximation in $R[a,b]$. For example, convergence criteria of Banach-Steinhaus-type are developed, extending basic work of Pólya (1933) on the convergence of quadrature formulas. The lecture is a survey of joint work with W.Dickmeis, H. Mevissen and E.van Wickern.

C.T. NO: A functional inequality

It is shown that a function $f:I\to\mathbb{R}$ on an interval I satisfies the inequality $f(x) + f(y) \leq f(\lambda x+(1-\lambda)y) + f((1-\lambda)x+\lambda y)$ for all $x,y \in I$ and all $\lambda \in [0,1]$ if and only if f is the sum of a convex function C (i.e.: $\lambda C(x) + (1-\lambda)C(y) \leq C(\lambda x+(1-\lambda)y)$) and an additive function A.

Equivalently speaking, $\Phi(x,y) = f(x)+f(y)$ is Schur convex on I^2 if and only if f is the sum of a convex function on I and an additive function.

Z. PALES: How to make fair decisions?

The purpose of the lecture is to give the complete description of decision functions, i.e. functions $f:S \to [-\infty, +\infty]$ satisfying

(i) $\min\{f(s),f(t)\} \leq f(s+t) \leq \max\{f(s),f(t)\}$

(ii) $\lim_{n\to\infty} f(ns+t) = f(s)$

for all $s,t \in S$, where S is an arbitrary commutative semigroup.

J. RÄTZ: Some remarks about Cauchy-Schwarz inequality

The best known method of proof is the one representing the Cauchy-Schwarz-deficit as the discriminant of a real quadratic form. This method heavily rests on divisions and on commutativity of the domain of scalars. The following statement avoids divisons and commutativity of multiplication and therefore also includes the quaternionic case.

Thm: If K is a ring with 1, $-:K\to K$ an involutorial automorphism, and $\tilde{K} := \{\lambda \in K;\ \bar{\lambda}=\lambda\}$ contained in the center of K and made into a totally ordered ring so that $0 \leq \lambda\bar{\lambda}$ $(\forall \lambda \in K)$, if X is a left K-module and $f: X\times X \to K$ is a positive semidefinite hermitian form, then $f(x,y)\overline{f(x,y)} \leq f(x,x)f(y,y)$ $(\forall x,y \in X)$. Equality occurs if and only if $\exists(\lambda,\mu) \in K\times K\setminus\{(0,0)\}$ with $f(\lambda x+\mu y, \lambda x+\mu y) = 0$.

B. SAFFARI: Refinements of norm inequalities for functions of mean value zero.

Let f be a real-valued bounded measurable function on $[a,b]$ such that $f \not\equiv 0$ and $\int_a^b f(x)dx = 0$. Then

$$\frac{1}{b-a}\int_a^b |f(x)|dx \leq \frac{1}{(b-a)^2}\int_a^b\int_a^b |f(x)-f(y)|dxdy \leq \frac{2hH}{h+H},$$

where $H = \text{ess sup } f$, $h = \text{ess inf } f$. Similar results hold for other L^p-norms and for $[a,b]$ (with measure $(b-a)^{-1}dx$) replaced by any probability space. The proof involves the decreasing rearrangements of functions f^+ and f^-.

P. SCHÖPF: Zwei Ungleichungen für konvexe bzw. sternförmige Funktionen

Sei $B \subset \mathbb{R}^d$ die euklidische Einheitskugel, $f: B \to \mathbb{R}$, $f \geq 0$, geeignet integrierbar und $0 < m \leq n$. Die dann gültige klass. Ungleichung

$$\left(\frac{1}{\mathrm{vol}(B)} \int_B f^m\right)^{1/m} \leq \left(\frac{1}{\mathrm{vol}(B)} \int_B f^n\right)^{1/n}$$

wird zunächst für die Klasse der Funktionen mit $f(0) = 0$ und sternförmigen Epigraphen bzgl. $(0,0) \in \mathbb{R}^{d+1}$ verschärft, und anschließend wird eine dimensionsfreie, maßtheoretische Verallgemeinerung der verschärften Ungleichung angegeben.

Der zweite Abschnitt des Vortrages betrifft konvexe Mino- bzw. Majoranten von monoton fallenden Funktionen und Ungleichungen zwischen deren sogenannten k-Momenten.

B. SMITH: Convolution orthogonality

Use $\int^a$ for integration over the set $\{\theta : |\theta - \frac{k}{a}| \leq \frac{1}{2a}, (k,a,) = 1\}$. Use $\lambda_a = \sum_{a=0}^{a-1} \frac{1}{a} \delta(\frac{k}{a})$, where δ is the Dirac point mass. Put $D_x = \sum_{\frac{x}{2} < \theta \leq x} e^{2\pi i h\theta}$. Put $\lambda_a \circledast \lambda_b = \lambda_{ab}$. $k \approx A$ means $k \in [\frac{A}{2}, A] \cap F$ where F is arbitrary. Put $G = \sum_{c \approx C} \lambda c$.

Theorem. For all $\epsilon > 0$ there exists $\delta > 0$, such that if G has bounded multiplicative transform, $A \leq B$, $A^2 B \leq X^{1-\epsilon}$. Then

$$\sum_{a \approx A} \int^a \left(\lambda_a \circledast \sum_{c \approx C} \lambda_c\right) \left(\sum_{b \approx B} \sum_{d \approx D} (\lambda_b \circledast \lambda_d) * D_x\right) e(\theta) d\theta \ll \frac{x \log^2 x}{A\delta}$$

R. SPERB: Optimal inequalities in a semilinear boundary value problem on a two dimensional Riemannian manifold

Consider the semilinear boundary value problem

$$(*) \quad \Delta u+\lambda f(u) = 0 \quad \text{in } \Omega \subset M, \; u = 0 \text{ on } \partial\Omega$$

where Ω is a domain on a two-dimensional Riemannian manifold. Suppose that $f(0)>0$ and f is convex and increasing and λ is a positive parameter. Then it is known that $(*)$ has a positive solution for $\lambda \in (0,\lambda^*)$ and no solution for $\lambda>\lambda^*$.

Let Ψ be the solution of

$$(**) \quad \Delta\Psi+1 = 0 \text{ in } \Omega, \quad \Psi=0 \quad \text{on } \partial\Omega.$$

Using $\chi(s(\Psi(x)))$ as a supersolution where $\chi(s)$ and $s(\Psi)$ are to be chosen in an optimal way isoperimetric estimates for λ^* and other quantities of interest can be derived.

N. STEINMETZ: The Biebertach Conjecture

In two lectures a survey has been given on the developments which lead to a proof of the Bieberbach Conjecture, including: I. Historical remarks; II. Löwner Theory; III.De Branges proof.

P.M. VASIĆ: Interpolation of some inequalities

Assume, that $I \subset N$ (I nonvoid and finite) and that an inequality $F(I) \geq 0$ is given. We have "interpolations" of the inequality $F(I) \geq 0$, provided $F(I) \geq F(J) \geq 0$ whenever $I \supset J$. An important case is that $F(I_n) \geq F(I_{n-1}) \geq \ldots \geq F(I_2) \geq F(I_1) = 0$, where $I_n = \{1,2,\ldots,n\}$. We show more inequalities of the cited type. For example, for the inequality of Levinson,

$$F(I) = \sum_{i\in I} p_i \{ f\left(\frac{\sum_{i\in I} p_i x_i}{\sum_{i\in I} p_i}\right) - f\left(\frac{\sum_{i\in I} p_i x_i'}{\sum_{i\in I} p_i}\right)\} - \{\sum_{i\in I} p_i f(x_i')\} \geq 0$$

if $x_i + x_i' = 2a$, $0 < x_i < a$, $p_i > 0$ and f is convex of order 2 in $[0,2a)$ one has $F(I \cup J) \geq F(I) + F(J)$ if $I \cap J = \emptyset$; hence

$$F(I_n) \geq F(I_{n-1}) \geq \ldots \geq F(I_2) \geq F(I_1) = 0 .$$

P.VOLKMANN: <u>Ein Existenzsatz für gewöhnliche Differentialgleichungen in geordneten Banachräumen</u>

Es wird ein Existenzsatz für gewöhnliche Differentialgleichungen in Banachräumen bewiesen, wobei die rechte Seite der Differentialgleichung eine bzgl. eines Kegels wachsende Funktion ist. (Arbeit mit Roland Lemmert und Raymond M.Redheffer).

R.J.WALLACE: <u>Sequential search for zeroes of $2(2^n-1)$-th derivatives</u>

How might simple real zeroes of real valued continuous k-th derivatives $f^{(k)}$ be efficiently approximated, given that there is to be recourse solely to values of f? A standard approach to these questions entails successively selecting points to be the abscissae for sequences of k-th divided differences whose signs are then used to locate the zeroes; see Wallace [1] and the references therein to the work of S.Johnson, J.Kiefer, R.S.Booth and others. Of central importance is the particular rule (or strategy) by which these points are chosen. In this talk the speaker shows how analysis of a class of restricted subadditive inequalities has enabled him to determine the most efficient strategy for each of the special cases $k=2,6,14,30,\ldots, 2(2^n-1),\ldots$. Illustrations are given, and the suggestion made that similar analysis should lead to analoguous results for other even k.

<u>Reference</u>: R.J.WALLACE: Sequential search for zeroes of derivatives, in General Inequalities 4: Proc.of the Fourth Internat. Conference, Oberwolfach, 1983, W.Walter, ed.(Birkhäuser, Basel, 1984), 151-167.

C.L.WANG: Inequalities and mathematical programming III

Three equivalent mathematical programming problems concerning monotone infinite sequences with suitable constraints are solved by the establishment of pertinent inequalities. The continuous version of the inequalities as well as some variants of discrete and continuous inequalities are also studied.

K.ZELLER: Positivity in absolute summability

Positivity considerations are useful not only for ordinary summability, but also for absolute summability. In the latter case it is quite natural to employ two positivity concepts and two types of summing operators. Thereby one meets matrices having properties like diapositivity. Applications concern Cesàro methods (factors for absolutely summable series). Several modifications and extensions are indicated.

A. ZETTL: Norm inequalities for derivatives and differences

In this joint work with M.K.Kwong we consider the inequalities (1) $\|y'\|_p^2 \leq K\|y\|_p\|y''\|_p$ and (2) $\|\Delta x\|_p^2 \leq C\|x\|_p\|\Delta^2 x\|_p$. The norms here are the classical $L^p(J)$, $l^p(M)$ norms, respectively, with $J=\mathbb{R}=(-\infty,\infty)$ or $J=\mathbb{R}^+=(0,\infty)$ and $M=\mathbb{Z}=\{\ldots-2,-1,0,1,2,\ldots\}$ or $M=\mathbb{N}=\{0,1,2,\ldots\}$, $1\leq p\leq\infty$. The constants $K=K(p,J)$, $c=c(p,M)$ denote the smallest constants in (1) and (2) respectively. It is known that $c(p,\mathbb{Z}) = K(p,\mathbb{R})$ and $c(p,\mathbb{N}) = K(p,\mathbb{R}^+)$ when $p = 1,2$ or ∞. For other p the values of these constants are not known and it is not known if these equalities hold for other p. Here we present "good" upper and lower bounds for $K(p,\mathbb{R})$: $L(p) \leq K(p,\mathbb{R}) \leq M(p)$

with explicit formulas for L and M in terms of p for $p \geq 2$. Also an elementary proof is given for a result of Ljubič to show that if u_i, $i=0,1,2$ are positive numbers satisfying $u_1^2 < K(p,J)u_o u_2$ then there exists a y in $L^p(J)$ such that $\|y^{(i)}\|_p = u_i$, $i = 0,1,2$.

<u>Berichterstatter</u>: A. Kovačec.

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