MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 21/1986

Inverse Probleme

18.5. bis 24.5.1986

Die Tagung fand unter der Leitung von Herrn J. Cannon (Pullman/Wash.) und Herrn U. Hornung (Neubiberg) statt. Im Mittelpunkt des Interesses stand die Frage der Rekonstruktion von unbekannten Koeffizienten in Differential- und Integralgleichungen aus gemessenen Daten. Dabei wurden die Existenz und Eindeutigkeit von Lösungen dieser inversen Probleme diskutiert, spezielle Lösungsalgorithmen wurden vorgeschlagen und Möglichkeiten zur Regularisierung der im allgemeinen schlecht gestellten Probleme untersucht.

Vortragsauszüge

R.S. ANDERSSEN:

The Linear Functional Strategy for Improperly Posed Problems

For the solution of specific inverse problems which in one way or another can be identified with indirect measurement problems, all that is required in the final analysis are simple unambiguous indicators which can be used for decision making purposes. In fact, all the practitioner requires is: confidence in the utility of the indicators; clearly defined interpretations in terms of the problem context; simple procedures for evaluating the indicators. But, because simple questions do not necessarily have equally simple answers, it is often necessary to utilize "deep" results in mathematics in order to achieve these goals.

In the talk, two different situations are examined:
(i) the direct use of indirect measurements; (ii) the use of indicators corresponding to bounded linear functionals on the solution. The consequences of these ideas are discussed for the Abel integral equation, the foliage angle distribution problem and the transmissivity problem, including the transformation of functionals defined on the solution into functionals defined on the data.

When the functionals required are point estimates of the solution, this often leads to the need to differentiate the available data numerically. The talk concludes with a discussion of stabilized multi-point finite difference formulas for the numerical differentiation of observational data.

V. BARCILON:

Inverse Problem for the Vibrating Beam

The talk is devoted to questions associated with the solution to the fourth order inverse eigenvalue problem

$$(r(x)\, u_n'')'' = w_n^2\, p(x)\, u_n$$

After reviewing the question of uniqueness, I discuss at length the question of existence which differs greatly from that for the second order case. Indeed, whereas interlacing and simple asymptotic trends are the only conditions which two sequences of numbers must satisfy in order to qualify as the spectra of a vibrating string, the spectra for a vibrating beam must satisfy much stringer conditions. The results for the fourth order operator can be generalized to a broader class of operators.

J.R. CANNON:

An Inverse Problem for an Elliptic Partial Differential Equation

We demonstrate uniqueness and local existence of the unknown coefficient $a = a(x)$ in the elliptic equation $\Delta u - a(x)u = 0$ in the quarter plane $x > 0$, $y > 0$ which is subject to the boundary conditions $u(0,y) = f(y)$, $u_x(0,y) = g(y)$ and $u(x,0) = h(x)$. The proof consists of the derivation of an integral equation for $a(x)$ utilizing transformations of Gel'fand-Levitan type. The work is joint with William Rundell.

G. CHAVENT:

A Sufficient Condition for the Uniqueness of Local Minima of a Linear Optimization Problem

We consider the problem

(1) Find $\hat{x} \in C$ such that $J(x) \leqslant J(x) \quad \forall x \in C$

where

(2) $J(x) = ||\varphi(x) - z||^2$

where C is a connected, regular subset of a vector space E, φ is a C^2-mapping from C into a prehilbert space F, and z a given point of F. Such problems occur in parameter estimation (x = parameter, z = data, φ = parameter → output mapping, C = set of admissible parameters) or in control problems.

The purpose of the paper is to find conditions on φ and C such that (1) has at most one unique solution as soon as z is close enough to $\varphi(C)$. The found condition will involve evaluations of first and second derivatives of φ along pathes connecting any couple of points of the boundary of C. One numerical example will be given.

M. CHENEY:

Three-Dimensional Inverse Scattering

We consider the problem of obtaining information about an inaccessible region of space from scattering experiments. Inverse scattering theory for the time-independent Schrödinger equation

$$[\Delta + k^2 - V(x)]\, \psi(x) = 0$$

is summarized. It is most easily understood by considering the associated hyperbolic equation

$$[\Delta - \partial_{tt} - V(x)]u(t,x) = 0 .$$

Particular attention is paid to those aspects of the theory that also hold for the wave equation

$$[\Delta - n^2(x)\partial_{tt}]u(t,x) = 0 .$$

K. DRIESSEL:

An Isospectral Gradient Flow

I consider the following problem: Given a real symmetric matrix, find its eigenvalues. I use the theory of ordinary differential equations to solve this problem. In particular, I describe a spectrum preserving ("isospectral") dynamical system on symmetric matrices that flows "downhill" towards diagonal matrices.

P. DuCHATEAU

Parabolic Inverse Problems

Approaches to inverse problems can be roughly described as those that exactly compute functions that approximately solve the problem and those that approximately compute functions that exactly solve the problem. This talk describes an approach of the latter type which proceeds according to the following program:

1. The Direct Problem
2. The Inverse Problem
3. Approximation of the Solution to the Inverse Problem

The direct problem is treated in the weak formulation, and detailed information about the solution is extracted by use of specially contrived test functions.

The emphasis in part 2 is on establishing uniqueness of the solution to the inverse problem.

The ideas are illustrated by means of a simple example.

R.E. EWING:

Parameter Estimation for Fluid Flow Problems

The process of determining unknown parameters, such as porosity and permeability, which are necessary for mathematical models used in reservoir simulation is very complex, especially for multiphase flow problems arising in enhanced oil recovery techniques. A brief survey of the difficulties involved in these processes is given emphasizing the complex interaction of various sources of errors from the mathematical modeling methods. Since the associated least squares minimization problem lacks uniqueness and is highly ill-conditioned, techniques for obtaining a better initial guess will be presented togehter with preliminary numerical results. These techniques involve a direct marching procedure in the space direction away from Cauchy data on a time boundary and require a stabilization process.

R. GORENFLO:

Approximation of Discrete Probability Distributions in Spherical Stereology

We consider the "tomato salad problem". From a solid opaque medium in which spheres with random radii r are embedded a slice of thickness s>0 is cut out so thin as to be transparent to perpendicularly throughfalling light projecting the parts of spheres cut out as circles with radii ρ onto a plane of observation. We also treat the limiting case s = 0 (classical stereology). From the distribution function of ρ one obtains that of r by solving an Abel integral equation of second kind, if s>0, of first kind, if s = 0. If r obeys a discrete probability law the maximum norm is inappropriate for estimating the approximation error. Under the assumption that there is a known finite upper bound to r we show the backward Euler method (equidistant in r^2) to converge in L^1-norm, also in the more general case of the distribution function of r being a linear combination of Heaviside functions superimposed on a Lipschitz-continuous background.

R. KRESS:

Integral Equations of the First Kind in Inverse Acoustic Scattering Problems

For the solution of the exterior Dirichlet problem for the Helmholtz equation an approximation method is described which seeks the solution in the form of an acoustic single-layer potential with a distribution extended over an internal surface. This leads to an ill-posed integral equation of the first kind which can be approximately solved by the Tikhonov regularization technique. It is illustrated how this approach can be employed to approximately solve the inverse problem: Determine the shape of a scatterer from the far-field pattern of the scattered wave for one (or more) incident (plane) waves.

KUNISCH:

Output Least Squares Stability for Elliptic Systems

We consider the estimation of the scalar valued diffusion coeffient a = a(x) in

$$- \operatorname{div}(a \operatorname{grad} u) + cu = f \text{ in } \Omega$$
boundary conditions

from an "observation" $z \in L^2(\Omega)$ in the output least squares formulation

$$(P)_{z^0} \quad \min_{Q_{ad}} |u(a) - z^0|^2 ,$$

where $Q_{ad} = \{a \in H^2(\Omega) : a(x) \geq \alpha, |a|_{H^2} \leq \gamma\}$, $\alpha > 0$,

and $\Omega \subset \mathbb{R}^2$ or $\mathbb{R}^3$. The parameter a is called output least squares stable (OLSS) at the solution a^* of $(P)_{z^0}$, if there exist neighbourhoods $V(z^0)$ and $V(a^*)$ such that for every $z \in V(z^0)$ there exists a solution $a_z \in V(a^*)$ of $(P)_z$ and all solutions of $(P)_z$ in $V(a^*)$ depend Hölder continuously on z. OLSS can only be expected and proved to hold under restrictive assumptions on $(P)_{z^0}$. Therefore we subsequently consider a regularized problem and prove OLSS for a well specified range of regularization parameters. The technical tool to obtain OLSS are lower bounds on the Lagrangian associated with $(P)_{z^{\delta}}$. The necessary estimates give valuable insight in the illposed nature of $(P)_{z^{\delta}}$.

J.K. McLAUGHLIN:

Inverse Spectral Theory Using Nodal Positions as Data

The present work is motivated by the inverse spectral problem for the beam. What is considered is the zero of spectral data which consists of positions of nodal points of mode shapes. A uniqueness theorem is presented to show that in a second order problem the position of a single node (judiciously chosen) from each mode shape determines a material parameter uniquely. Furhter existence, uniqueness results as well as constructive techniques are presented to show that nodal positions, positions of maximal deflection, and measurements of mode shapes at the midpoint can be used to reconstruct a material parameter.

M.M. LAVRENTIEV:

Inverse Problems for Equations of Mathematical Physics and Integral Geometry

By inverse problems we mean the problems of determining coefficients of a differential equation by a given set of functionals of solutions of the equation. Many inverse problems, for example, for hyperbolic equations, quasistationary equations, such as the Helmholtz equations, reduce to the problems of integral geometry.

Recently, at Novosibirsk Scientific Center a number of new results have been obtained in integral geometry. These results relate to various classes of curvilinear manifolds such as ellipsoids, surfaces of parabolic types, and also to the problems with discrete sets of manifolds.

M.Z. NASHED:

Operator Extremal Problems and Constrained Minimization for Linear Relations

The first part of this talk deals with two problems in representation and compensation of systems (or operators). For example, let $A : X \to X$ be a bounded linear operator on a Banach space X and assume that each of $N(A)$ and $R(A)$ has a topological complement, say M and S, respectively, in X, and let P, Q denote the induced projectors on M and R(A). Let

$A^+ = A_{P,Q}^+$ denote the generalized inverse of A. In general,

$$(*)\quad (T^{-1}AT)^+ \neq T^{-1}A^+T,$$

where T is an invertible operator. However, $(T^{-1}AT)^+_{P',Q'} = T^{-1} A_{P,Q}^+ T$, where $P' := T^{-1} PT$ and $Q' = T^{-1} QT$. The question of equality in (*) leads to several problems, e.g.: Given A, characterize all invertible T which commute with P and Q. Given T, characterize all A for which equality in (*) holds.

The second part deals with characterizations and existence of restricted LSS solutions (least-squares) of the inclusion $h \in L(x)$ with respect to S, where L is a linear manifold in $H_1 \oplus H_2$, $S = g + N$, N is a subspace of H, $g \in \mathrm{Dom}\, L$, $h \in H_2$. We report on joint work with S.J. Lee.

F. NATTERER:

An Inverse Problem for Radon's Integral Equation

In emission tomography one has to solve the integral equation

$$\int_{x\cdot\theta=s} f(x)\, e^{-D\mu(x,\theta)} dx = g(\theta,s),$$

$$D\mu(x,\theta) = \int_0^\infty \mu(x+t\theta)dt.$$

If μ is unknown, one has to determine μ prior to the computation of f. This can be done using the consistency conditions

$$\int_0^{2\pi} \int_{-\infty}^{+\infty} e^{\frac{1}{2}(I+iH)R\mu}\, s^m e^{ik\varphi} g(\theta,s)ds d\varphi = 0,\quad k > m \geq 0$$

where I is the identity and H is the Hilbert transform. We report on several numerical attempts to compute μ from these equations.

C. PAGANI:

Existence Results for the Inverse Problem of the Volume Potential

Let G be a homogeneous material body whose shape is unknown. We want to determine the figure of G from measurments of the Newtonian potential created by it taken either on ∂G or on a spherical surface surrounding G. We prove, in both cases, the (local) existence of an exact solution.

S. PEREZ ESTEVA:

An Inverse Problem Related to the Heat Equation

We consider the problem of determining the unknown source $F = F(x,t)$ in the heat conduction equation from overspecified data. For $F = f(t)\chi_D(x)$ we state uniqueness and continuous dependence, where $D \subset \mathbb{R}^n$ and χ_D is the characteristic function of D.

V. RUNDELL:

Some Numerical Schemes for the Determination of an Unknown Reaction Term in a Reaction Diffusion Equation

We consider the equation $C(x,t,u,\nabla u)u_t - u_{xx} = F(x,t,u,\nabla u)$ where the functions C or F are not completely known and have to be determined in addition to $u(x,t)$. Primary bondary conditions, sufficient to solve the direct problem, are given in a domain $\Omega \times [0,T]$. Overposed data $u(x_0,t)$ for $x_0 \in \Omega$ is prescribed, and it is shown that the unknown elements are able to be uniquely recovered. The technique used is a fixed point method and leads to a constructive algorithm. Several numerical results are presented.

P.C. SABATIER:

Ambiguities in Reconstruction

In the one-dimensional scattering problems governed by the Schrödinger equation or by the impedance equation, there exists a class of potentials (resp. impedances) that is bijectively related to a class of spectral data (for potentials,

$$L_1{}' = \{V: \int_{-\infty}^{+\infty} (1+|x|)|V(x)|dx < \infty\}).$$

When there is no bound state, these data reduce to the reflection coefficient as a function of energy for all positive energies. However, this class is not the largest one consistent with scattering phenomena, and several authors showed examples of different potentials that are consistent with a given reflection coefficient and no true bound state. The lecture presents a complete study of these ambiguities:

(1) They are related to a Darboux-type transformation which is defined on the set of potentials (resp. impedances), leaves invariant the Schrödinger (resp. impedance) equation, whereas the reflection coefficient is flipped $R^+(k) \rightarrow -R^+(k)$, without modifying the transmission coefficient, and depends on an arbitrary parameter, say c. Hence, if we start from the potential $V(x)$ (resp. the impedance factor $\alpha(x)$) which yields $R^+(k)$ and apply the transformation $T(c)$, we obtain $V^T(x,c)$ (resp. $\alpha^T(x,c)$) which yield $-R^+(k)$, i.e. an infinity of "equivalent" potentials (resp. impedances).

(2) Let us define a class $P(l_-, l_+)$ of potentials by their asymptotic behaviour $\frac{l_\pm(l_\pm+1)}{x^2} + u$ as $x \to +\infty$ where $u \in L_1'$, and a class (p,q) of impedance factors by their asymptotic behaviour. The transformation takes a potential (resp. an impedance factor) from one class to another one.

(3) The transformation introduces or suppresses zero energy bound states on half bound states so that the invariance of $T(k)$ is not a true isospectral property.

(4) All the known ambiguities are described.

T. SUZUKI:

Gel'fand-Levitan's Theory and Related Inverse Problems

The first object is to give a detailed study about the structure of Gel'fand-Levitan's theory. Then, this is applied to inverse spectral problems and identifiability of evolution equations. Some phenomenon peculiar to inverse problems is found by this method.

G. TALENTI:

Recovering a Function from a Finite Number of Moments

Let $\mu_0, \mu_1, \ldots, \mu_N, \epsilon, E$ be given numbers. Suppose a function u obeys

$$\sum_{k=s}^{N} \left(\int_D^1 x^k u(x)dx - \mu_k\right)^2 \leq \epsilon^2$$

and

$$\int_D^1 (u')^2 dx \leq E^2.$$

We show that such a function can be recovered within the following tolerance:

$$\text{(2E)}\ [(\epsilon/E)^2\, e^{3.5(N+1)} + \frac{1}{4(N+1)^2}]^{1/2}.$$

We also present algorithms and examples.

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