MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 22/1986

Optimale Steuerung mit partiellen Differentialgleichungen: Theorie und Verfahren.

18.5. bis 24.5.1986

Die Tagung fand zum zweiten Mal unter der Leitung von Herrn K.-H. Hoffmann (Augsburg) und W. Krabs (Darmstadt) statt. Sie wurde simultan mit einer Tagung über "Inverse Probleme" unter der Leitung der Herren J.R. Cannon (Pullman, USA) und U. Hornung (Neubiberg) veranstaltet, was sich sowohl organisatorisch als auch inhaltlich als sehr sinnvoll erwies. Organisatorisch wurde die Zweigleisigkeit dadurch bewältigt, daß vormittags Übersichtsvorträge aus den Gebieten beider Tagungen für alle Teilnehmer gehalten wurden und nachmittags Parallelsitzungen stattfanden, bei denen sich die Teilnehmer für eines der Gebiete der beiden Tagungen entscheiden konnten. Inhaltlichen Gemeinsamkeiten von Steuerungsproblemen und inversen Problemen waren zwei Abenddiskussionen gewidmet, bei denen trotz dezidiert geäußerter Meinungsverschiedenheiten festgestellt wurde, daß sich die beiden Problemkreise nicht nur ergänzen, sondern auch strukturelle Gemeinsamkeiten aufweisen. Von einigen Fachleuten für inverse Probleme wurde allerdings auch hervorgehoben, daß aus der Sicht der Modellbildung Unterschiede erkennbar sind, insofern, als die Formulierung eines inversen Problems sehr viel stärker in die Modellbildung eingebunden werden muß, als das in der Steuerungstheorie der Fall ist. Das trifft allerdings wohl nur zu, wenn man die Steuerungstheorie primär als eine Optimierungstheorie betrachtet und ihren systemtheoretischen Anteil vernachlässigt.

Die Themen aus der Steuerungstheorie waren diesmal breit gestreut. Einige Vorträge befaßten sich direkt mit technisch-physikalischen Anwendungen, wie z.B. der optimalen Kühlung bei der Herstellung von Walzstahl, der Optimierung des Signal-Rausch-Verhältnisses bei einer zylindrischen Antenne oder Problemen der optimalen Formgebung in der Hydrodynamik und Mechanik. Einige Vorträge waren numerischen Methoden zur Lösung optimaler Steuerungsprobleme gewidmet,

Gefördert durch DFG Deutsche Forschungsgemeinschaft

wie z.B. der Anwendung der Methode der finiten Elemente auf parabolische Steuerungsprobleme mit Zustandsbeschränkungen oder Quasi-Newton-Methoden bei unrestringierten Steuerungsproblemen. Auch freie Randwertprobleme waren diesmal wieder vertreten. Daneben gab es einige stärker theoretisch orientierte Beiträge über Konvergenz suboptimaler Steuerungen, Randsteuerungen bei der Wärmeleitungsgleichung mit nicht-linearen Randbedingungen, optimale Bereichssteuerung bei einseitigen Randwertproblemen, die Rechtfertigung von notwendigen Optimalitätsbedingungen oder Sensitivitätsanalyse bei konvexen optimalen Steuerungsproblemen mit verteilten Parametern.

An der Vielfalt der angebotenen Themen wurde wieder einmal deutlich, daß eine einheitliche Steuerungstheorie für partielle Differentialgleichungen wohl kaum möglich ist, da die einzelnen Probleme in sich außerordentlich komplex und untereinander sehr verschieden sind. Ebenso deutlich zeigte sich aber eine immer stärker werdende Tendenz,Probleme mit einem direkten Anwendungsbezug zu behandeln, was sich nicht zuletzt auch für die Theoriebildung als sehr fruchtbar erweist.

Vortragsauszüge

H.W. Alt:

ON A MINIMUM PROBLEM WITH FREE BOUNDARY ARISING IN FLUID MECHANICS

Optimizing the shape of the blade of a turbine after a cylindrical cross-section will lead to the following periodic 2-dim problem: Find a stream-function u with

$$(*) \quad u = k \text{ on } E^k, \quad u(x,y) - y \text{ periodic in } y$$

$$(**) \quad \Delta u = 0 \text{ outside } E, \quad \nabla u(x,y) \to (s_{\pm}, 1) \quad \text{for } x \to \pm\infty$$

where $E = \bigcup_{k \in Z} E^k$ is the cross-section of the blade. We want to prescribe the velocity distribution λ on ∂E. Taking the average over a family of flows, we are looking for a minimizer of

$$I(u,E) := \int_{\mathbb{R}^2} (\int_{\Omega} (|\nabla u_s - e_s|^2 - \lambda^2 \chi_E)) \, d\mu(s)_E$$

with side condition (*) and $E \supset E_x$, where E_* is given, and e_s a divergence free vector field satisfying $e_s(x,y) \to (s_{\pm}, 1)$ for $x \to \pm\infty$. Existence and regularity results and numerical computations are presented.

T.S. Angell:

ON A PROBLEM OF OPTIMAL DESIGN IN HYDROMECHANICS

In the description of bodies, either partially or totally submerged in an inviscid, irrotational fluid and subjected to a periodic vertical displacement, certain functionals dependent on the velocity potential of the wave pattern are of physical interest. These functionals, for example the "added mass", are dependent on the geometry of the body.

We report on some results concerning the choice of the shape of the body which is to be optimal in the sense of minimizing the added mass for the case that the body is totally submerged in a fluid of finite depth. The problem is cast in terms of a boundary integral equation in which the "control parameter" is the boundary of the body.

M. Brokate:

OPTIMAL CONTROL OF AN AGE-DEPENDENT POPULATION

We consider optimal control of a population which depends on age and time. The dynamics of the system are defined by the Gurtin-Mac Camy equations. We state the Pontryagin principle and draw some conclusions concerning the switching structure. Finally we formulate a semidiscrete version of the optimal control problem and discuss convergence of their solutions to a solution of the continuous problem.

A.G. Butkovskiy:

SURVEY OF SOME PROBLEMS IN THE THEORY OF DISTRIBUTED PARAMETER SYSTEMS

Some problems and results are described in three different parts of the theory of distributed parameter systems:

1) Structural theory

2) Mobile control

3) Control in quantum-mechanical processes.

A.G. Butkovskiy:

GEOMETRICAL REPRESENTATION FOR DYNAMIC SYSTEMS WITH CONTROL

The notion of phase-space (state-space) portrait of CDC described by differential inclusion (DI) is introduced. This notion is an extension of a well-known concept of phase-space portrait for ordinary differential equations. Phase-space portraits for two-dimensional CDS are considered in more detail. The connection between CDS and continuous media is also considered. Formulae for the Laplace operator in these media are given.

F. Colonius:

A PROBLEM IN OPTIMAL PERIODIC CONTROL

We consider nonlinear functional differential equations and concentrate on an analysis of local properness. An optimal steady state is called locally proper if it is not an optimal solution of the corresponding periodic problem (i.e. if the system behaviour can be improved by introducing oscillations). This problem can be analysed using second order necessary optimality conditions. Furthermore, local properness is related to dynamic properties by discussing its relation to Controlled Hopf Bifuncations. Finally an example is analysed.

H.W. Engl:

AN INVERSE PROBLEM CONNECTED WITH CONTINUOUS CASTING OF STEEL

The mathematical modelling of the continuous casting process leads to a nonlinear boundary value problem for a nonlinear heat equation. If one wants to control the front of solidification by regulating the pressure of the cooling water, this leads to an inverse problem for this boundary value problem.

In the first part of the talk we report about the implementation of an algorithm for approximating a solution of this inverse problem in an industrial environment (VOEST-Alpine AG, Linz).

In the second part of the talk we present some theoretical results: uniqueness for the inverse problem; existence, uniqueness and continuous dependence for the direct problem; restoration of stability for the inverse problem under a-priori assumptions. Unfortunately, this last result is only a qualitative result, no modulus of continuity is known so far.

Joint work with P. Manselli (second part) and T. Langthaler.

H.O. Fattorini:

CONVERGENCE OF SUBOPTIMAL CONTROLS

We consider optimal problems for general nonlinear, nonconvex input-output systems. Using methods based on Ekeland's variational principle, we show that in many cases sequences of suboptimal controls converge in L^p topologies ($p \geq 1$) to optimal controls. We consider both the point target case and the set target case. The results are applied to systems defined by nonlinear partial differential equations (both in the distributed and boundary control case), ordinary differential equations, functional differential equations, etc. We discuss concrete methods of computing convergent sequences of suboptimal controls.

N. Kenmochi:

ASYMPTOTIC STABILITY OF SOLUTIONS OF TWO PHASE STEFAN PROBLEMS WITH FLUX CONTROL ON THE FIXED BOUNDARIES

The problem is to find a function $u = u(t,x)$ on $J \times [0,1]$, $J = R_+$ or R, and a curve $x = \ell(t)$, $0 < \ell(t) < 1$, on J satisfying

(E) $\rho(u)_t - u_{xx} = 0$ in $\{0 < x < \ell(t),\ t \in J\}$ and $\{\ell(t) < x < 1,\ t \in J\}$,

$$(BC)_0 \begin{cases} u(t,0) \geq g_0(t),\ u_x(t,0+) \leq 0 \quad \text{for } t \in J, \\ u_x(t,0+) = 0 \quad \text{if } u(t,0) > g_0(t). \end{cases}$$

$(BC)_1$ $u(t,1) = g_1(t)$ for $t \in J$,

$$(FBC) \begin{cases} u(t,\ell(t)) = 0 \quad \text{for } t \in J, \\ \ell'(t) = -u_x(t,\ell(t)-) + u_x(t,\ell(t)+) \quad \text{for } t \in J, \end{cases}$$

where g_0, g_1 are given functions on J.

In this talk the asymptotic behavior of solutions are investigated in the following three cases (a), (b) and (c):

(a) g_0, g_1 asymptotically converges as $t \to \infty$;

(b) g_0, g_1 are periodic in t with same period;

(c) g_0, g_1 are almost periodic in t on R.

A. Kirsch:

SOME REMARKS CONCERNING A NONQUADRATIC ANTENNA PROBLEM

The author considers the optimization of the signal-to-noise ratio of an arbitrary cylindrical antenna array. Existence of an optimal solution and convergence of finite dimensional approximations is shown. The necessary conditions are used to compute optimal solutions for some numerical examples.

U. Mackenroth:

FINITE ELEMENT APPROXIMATIONS OF STATE CONSTRAINT PARABOLIC OPTIMAL CONTROL PROBLEMS

A system which is governed by the following PDE is considered: $\frac{\partial y}{\partial t} + Ay = 0$, $\alpha y_{|\Sigma} + \beta \frac{\partial y}{\partial n_A} = u$, $y(0) = 0$. The problem is to find a control u such that a certain quadratic but not necessary coercive objective functional is minimized under a control constraint, $u \in U_{ad}$, and a state constraint, $y(t) \in C(t)$. A discretization which uses the finite element method is introduced and error estimates for $|\min(P_h) - \min(P)|$, $\|u_h - u_0\|$ are derived (u_h, resp. u_0, is the optimal solution of the discrete problem (P_h), resp. the continuous problem (P)). The estimates for $\|u_h - u_0\|$ work only in the coercive case. The bang-bang case requires a detailed analysis in which the structure of u_0 is investigated. This leads to a further convergence result for the u_h.

K. Malanowski:

SENSITIVITY ANALYSIS OF CONVEX OPTIMAL CONTROL PROBLEMS FOR DISTRIBUTED PARAMETER SYSTEMS

We consider a convex optimal control problem for a system described by a linear mapping from $L^2(\Omega)$ (control space) into another Hilbert space (output space). Control functions are subject to pointwise convex constraints. All data of the system depend on a vector parameter.

It is shown that under some regularity conditions the solutions of the problem as well as the associated Lagrange multipliers are Lipschitz continuous and directionally differentiable functions of the parameter.

The right-differentials are characterized as the solutions and the associated Lagrange multipliers of some auxiliary quadratic optimal control problems subject to linear constraints.

Sufficient conditions of continuations Gâteaux differentiability are derived.

P. Neittaanmäki:

ON THE CONTROL OF THE SECONDARY COOLING IN THE CONTINUOUS CASTING

In the continuous casting the water spray cooling is used to accelerate steel solidification and to strenghten the solified shell. The strand is to be cooled down according to the pattern which depends on steel quality, casting speed and product size. The problem of optimal cooling strategy is formulated as an optimal control problem subject to nonlinear parabolic state equations including the phase changes (solid, mushy, liquid) and with certain constraints in state. Discretization together with a solution algorithm are given. Numerical examples are presented.

P. Neittaanmäki, J. Haslinger, D. Tiba:

OPTIMAL SHAPE CONTROL OF THE DOMAIN IN UNILATERAL BOUNDARY VALUE PROBLEMS

We give a general existence theorem for an optimal control problem where the control is domain in $\mathbb{R}^n$ and where the system is governed by partial differential equations with classical or with unilateral boundary conditions. Discretization by finite element method with a numerical algorithm is presented. Application to design problems in elasticity are given.

In the second part we consider an abstract optimal design problem with constraints in state and control. A variational inequality approach is given for this design problem.

I. Pawlow:

OPTIMAL CONTROL TO STEFAN PROBLEMS

The talk is concerned with numerical methods of solving optimal control problems for two-phase Stefan processes, possibly of mixed elliptic-parabolic type. In the first part of the talk the approximation method is presented. The method uses a variational inequality formulation of the Stefan problem. The inequality and the associated control problem are discretized by applying piecewise linear elements in space and finite differences in time. A gradient type algorithm is proposed to solve control problems numerically.

In the second part of the talk a computer-generated movie on the simulation of boundary control of Stefan problems is presented.

E. Sachs:

QUASI NEWTON METHODS AND UNCONSTRAINED OPTIMAL CONTROL PROBLEMS

Quasi Newton methods play an important role in the numerical solution of problems in unconstrained optimization. Optimal control problems in their discretized form can be viewed as optimization problems and therefore be solved by quasi Newton methods. Since the discretized problems do not solve the original infinite-dimensional control problem but rather approximate it up to a certain accuracy, various approximations of the control problem need to be considered. It is known that an increase in the dimension of optimization problems can have a negative effect on the convergence rate of the quasi Newton method which is used to solve the problem. The purpose of this paper is to investigate this behavior and to explain how this drawback can be avoided for a class of optimal control problems. We show how to use the infinite dimensional original problem to predict the speed of convergence of the BFGS-method for the finite-dimensional approximations.

G. Schmidt:

BOUNDARY CONTROL OF THE HEAT EQUATION WITH NON-LINEAR BOUNDARY CONDITIONS

Let D be a domain in R^n having boundary ∂D and let υ denote an outward pointing normal. Thinking of D as a body we let u(x,t) denote the temperature at x in D and time t. We suppose that the evolution of u is governed by the heat equation with a non-linear boundary condition $\frac{\partial u}{\partial \upsilon} = g(u,f)$ where g is monotone decreasing in u and increasing in f with g(u,f) = 0 when u = f. Treating f (the ambient temperature) as a control we consider several control problems and the technical problems associated with them. We apply notions of total positivity to obtain a bang-bang principle for a special case.

T. Seidman:

JUSTIFICATION OF NECESSARY CONDITIONS FOR OPTIMALITY

Consider minimization of an integral functional of the form: $I := \int_Q f(\cdot,x,u)$ subject to an operator relation x = G(u) and perhaps a finite number of scalar constraints $\xi_j(u) = 0$ (or ≥ 0). The basic assumption is that f is "smooth where finite" but no other requirements are imposed - other than the a priori hypothesis that a minimum $[\bar{x},\bar{u}]$ is known to exist. We present an approach to the rigorous justification of the formally obtained (necessary) optimality conditions. On the setting in which G is given, e.g., by a partial differential equation $\dot{x} - \Delta x = g(x,u)$, $x(0) = x_0$, $x|_{\partial\Omega} = 0$, with the constraints $\|x(T) - x_1\| \leq \alpha$, the problem is one of distributed parameter control theory.

F. Tröltzsch:

FINITE ELEMENT DISCRETIZATION OF PARABOLIC BOUNDARY CONTROL PROBLEMS - CONVERGENCE OF OPTIMAL CONTROLS

In this talk a class of parabolic boundary control problems with time-dependent control and convex objective is discussed. Using a semigroup-approach for the treatment of the boundary condition the structural behaviour of the optimal control (bang-bang properties for non-coercive objectives, switching points in the coercive case) is investigated. Combining these statements with recent results on Ritz-Galerkin-approximation of parabolic equations new convergence theorems for the optimal controls of finite element approximations of the control problem are presented. In particular, the convergence of a certain type of switching points can be proved. The possibilities for the numerical application of switching point techniques are briefly outlined.

J. Zabczyk:

EXIT THEOREMS FOR STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS

Let an infinite dimensional system

(1) $\dot{z} = A(z), \quad z(0) = x \in E$

evolve on a Banach space E and let a stochastic equation

(2) $dX = A(X)dt + \varepsilon dW_t, \quad X(0) = x,$

where (W_t) is a Wiener process and $\varepsilon > 0$, be a perturbed version of (1). Let us assume that system (1) attracts to 0 an open set D, $(0 \in D)$. Due to the additive nature of the disturbances, trajectories of (2), sooner or later, will reach the boundary ∂D of D. Exit theorems give some informations about the limit behaviour of the exit time and the exit place as $\varepsilon \downarrow 0$. Theorems for finite dimensional systems are due to M. Freidlin and A. Wentzell and in the talk we describe some infinite dimensional extensions of their results. Solution to the exit problem is closely linked with a minimum-energy problem for the controlled system $\dot{y} = A(y) / Q^{1/2}u$, Q being the covariance operator of W_1.

Berichterstatter: W. Krabs

Tagungsteilnehmer

Prof. Dr. H. W. Alt
Institut für Angew. Mathematik
Universität Bonn
Wegelerstr. 6

5300 Bonn 1

Prof. Dr. Th. S. Angell
Dept. of Mathematical Sciences
University of Delaware

Newark, Delaware 19716

USA

Dr. M. Brokate
Institut für Mathematik
Universität Augsburg
Memminger Str. 6

8900 Augsburg

Prof. Dr. R. Bulirsch
Mathematisches Institut der
Technischen Universität München
Arcisstr. 21
Postfach 202420

8000 München 2

Prof. Dr. A. G. Butkovskiy
Institut Problem Upravlenija
A. N. SSSR
ul. Profsoyuznaja 65

117 342 Moskau

Sowjetunion

Prof. Dr. L. Collatz

Eulenkrugstr. 84

2000 Hamburg 67

Prof. Dr. F. Colonius
Fachbereich 3 - Mathematik
Universität Bremen
Postfach 330 440

2800 Bremen 33

Prof. Dr. H. Engl
Institut für Mathematik
Kepler-Universität

A-4040 Linz

Österreich

Prof. Dr. H. O. Fattorini
University of California
Department of Mathematics

Los Angeles, California 90024

USA

Dr. Zhicheng Guan
Institut für Mathematik
Universität Augsburg
Memminger Str. 6

8900 Augsburg

Prof. Dr. K.-H. Hoffmann
Institut für Mathematik
Universität Augsburg
Memminger Str. 6

8900 Augsburg

Qiyuan Jiang
Institut für Mathematik
Universität Augsburg
Memminger Str. 6

8900 Augsburg

Prof. Dr. N. Kenmochi
Department of Mathematics
Faculty of Education
Chiba University

Chiba-Shi 260

Japan

Prof. Dr. Andreas Kirsch
Georg-August-Universität
Institut für Numerische und
Angewandte Mathematik
Lotzestr. 16-18

3400 G ö t t i n g e n

Dr. P. Knabner
Institut für Mathematik
Universität Augsburg
Memminger Str. 6

8900 A u g s b u r g

Prof. Dr. W. Krabs
Fachbereich Mathematik
Technische Hochschule Darmstadt
Schloßgartenstr. 7

6100 D a r m s t a d t

Prof. Dr. F. Lempio
Lehrstuhl für Angew. Mathematik
Universität Bayreuth
Postfach 3008

8580 B a y r e u t h

Priv.-Doz. Dr. U. Mackenroth
Mathematisches Institut
Universität Bayreuth
Postfach 3008

8580 B a y r e u t h

Prof. Dr. K. Malanowski
Systems Research Institute of
the Polish Academy of Science
Ul. Newelska 6

01-447 W a r s c h a u

Polen

Prof. Dr. P. Neittaanmäki
Department of Mathematics
University of Jyväskylä
Seminaarinkatu 15

SF-40100 J y v ä s k y l ä 10

Finnland

Dr. Irena Pawlow
Systems Research Institute of
the Polish Academy of Science
Ul. Newelska 6

01-447 W a r s c h a u

Polen

Prof. Dr. E. Sachs
Universität Trier
Fachbereich IV - Mathematik
Postfach 3825

5500 T r i e r

Prof. Dr. G. Schmidt
Department of Mathematics
and Statistics
McGill University

M o n t r e a l , Quebec

Canada

Prof. Dr. Th. I. Seidman
Dept. of Math. & Computer Sc.
University of Maryland Balti-
more County

C a t o n s v i l l e , MD 21228

U S A

Prof. Dr. J. Stoer
Institut für Angew. Mathematik
und Statistik
Universität Würzburg
Am Hubland

8700 W ü r z b u r g

Dr. F. Tröltzsch
Technische Hochschule
Sektion Mathematik
PSF 964

DDR - 9010 Karl-Marx-Stadt

Prof. Dr. J. Zabczyk
Institute of Mathematics
Polish Academy of Sciences
Sniadeckich 8

00-950 W a r s c h a u

Polen