

MATHEMATISCHES FORSCHUNGSIINSTITUT OBERWOLFACH

Tagungsbericht 23/1986

Lokale Algebra und lokale analytische Geometrie  
25.5. bis 31.5. 1986

Die Tagung fand unter der Leitung von J. Lipman (LaFayette, Indiana), R. Berger (Saarbrücken) und G. Scheja (Tübingen) statt.

Gegenstand der Tagung war die Verbindung zwischen algebraischer und analytischer Geometrie in Fragen, die zu gemeinsamen Begriffsbildungen der (lokalen) kommutativen Algebra geführt haben. In den Vorträgen wurden u.a. Aspekte der Auflösung und der Deformation von Singularitäten behandelt; außerdem standen lokale topologische Eigenschaften der Singularitäten im Vordergrund des Interesses.

Von den Teilnehmern kamen 29 aus Deutschland und anderen europäischen Ländern, 10 aus Nordamerika und 4 aus Japan.

Vortragsauszüge

L. AVRAMOV:

Applications of homotopy to commutative algebra

We introduce an algebraic notion of homotopy groups.

Definition: If  $R$  is a commutative ring, and  $k$  the residue field of  $R_p$  for some prime  $p$ , the graded vector space

$(I^{(2)})_{p^1} \subset \text{Tor}^R(k, k)^V$  is called the homotopy of  $R$  at  $p$ . Here  $I = \text{Ker}(\text{Tor}^R(k, k) + k)$ ,  $I^{(2)} = I^2 + \text{the span of the divided powers}$

$\gamma^j(x)$  ( $x$  of even degree  $> 0$ ,  $j \geq 2$ ), and  $(\cdot)^V$  denotes vector-space duals.

Theorem: Let  $R \rightarrow S \rightarrow T$  be local homomorphisms of local rings, such that  $\phi, \psi$  and  $\psi\phi$  are essentially of finite type. Then  $\phi$  and  $\psi$  are l.c.i. maps if and only if  $\psi\phi$  is l.c.i. and flat  $\dim_S T < \infty$ .

Theorem (with H.B.Foxby): Let  $\phi : R \rightarrow S$  be a homomorphism of finite flat dimension. Then  $I_S = I_R \cdot I_F$  holds for the Bass series  $(I_S(t) = \sum \dim_{\mathbb{R}} \text{Ext}_R^i(k, k)t^i)$ .

R.O. BUCHWEITZ:

#### Characterizing Algebraic Cycles on Projective Hypersurfaces

Let  $X$  be any smooth variety over  $\mathbb{C}$ ,  $K(X)$  its Grothendieck group,  $H^*(X, \mathbb{C})$  its cohomology ring. Then there is a Chern character  $ch_X : K(X) \rightarrow H^*(X, \mathbb{C})$ . The problem is to describe its image.

Among other results we give the following:

Assume  $X^{2m} \rightarrow \mathbb{P}^{2m+1}$  is an even dimensional, smooth hypersurface.

Let  $R = \Gamma_*(\mathcal{O}_X)$ ,  $S = \Gamma_*(\mathcal{O}_{\mathbb{P}})$  be the homogeneous coordinate rings.

$R = \frac{S}{(f)}$ ,  $\deg f = d$ . A graded MCM  $M$  over  $R$  is given by a matrix factorisation  $(\phi, \psi) : F \rightarrow G$ ,  $\psi : G(-d) \rightarrow F$ ;  $G, F$  free graded  $S$ -Modules such that  $\phi\psi = f \cdot \text{id}_{G(-d)}$ ;  $\psi\phi = f \cdot \text{id}_F$  (D.Eisenbud).

Theorem:  $\exists$  a universal constant  $c(d, m)$  such that  $\gamma(M)$  is

given by  $\gamma(M) = c(d, m) \cdot \frac{\text{Trace}((d\phi \wedge d\psi)^{m+1})}{\text{volume element of } S} e^{(\frac{R}{J(f)})_{(m+1)(d-2)}}$

$$= H^{m,m}(X)^0$$

where  $\gamma$  is the composite map  $D^b(R) / D^b(S) \rightarrow D^b(X) / D^b(\mathbb{P}) \rightarrow K(X) / K(\mathbb{P}) \rightarrow \bigoplus H^{p,p}(X)^0$ .

M. CONTESSA:

#### Rings with double infinite chain condition (DICC)

A DICC is a commutative ring with unity in which every double in-

finite chain of ideals  $\dots \subseteq \mathfrak{a}_{-1} \subseteq \mathfrak{a}_0 \subseteq \mathfrak{a}_1 \subseteq \dots$  stabilizes either to the right or to the left or to both sides. A reduced DICC is Noetherian. For the nilradical  $\underline{n}$  of a non-noetherian DICC we have i)  $\text{Ass}(\underline{n})$  consists of finitely many maximal ideals; ii)  $\underline{n}$  is nilpotent; iii)  $\underline{n}$  is DCC. We call a min/max ideal a prime ideal which is both minimal and maximal.

Theorem 1: Let  $S$  be a non-Noetherian ring with no min/max ideals. Then  $S$  is DICC iff 1)  $S_{\text{red}}$  is Noetherian; 2)  $\underline{n}$  is nilpotent; 3)  $\underline{n}$  is DCC; 4)  $\forall x \in S - \underline{n} : \frac{\underline{n}}{Sx \cap \underline{n}}$  or equivalently  $\frac{\underline{n}}{x \cdot \underline{n}}$  has finite length.

Theorem 2: A ring  $R$  is DICC iff either  $R$  is Noetherian or  $R \cong A \times S$  where  $S$  is a DICC with no min/max ideals and  $A$  is an Artinian ring.

Cor.: A non-Noetherian DICC with no min/max ideals has a unique minimal prime.

#### D. EISENBUD:

##### Linear Sections of determinantal Varieties

Let  $V, W$  be vector spaces of dimensions  $v, w$  over a field  $F$ .

Proposition-Definition: For a subspace  $M \subset \text{Hom}(V, W)$  with associated pairing  $\mu: V \otimes W^* \rightarrow M^*$  and Matrix of linear forms  $L$ , the following are equivalent:

- 1) The annihilator  $M^\perp \subseteq \text{Hom}(W, V) = \text{Hom}(V, W)^*$  meets the rank  $\leq k$  locus only in 0.
- 2) No sum of  $\leq k$  pure vectors in  $V \otimes W^*$  goes to zero under  $\mu$ .
- 3) Even after row and column operations, any  $k$  of the elements of  $L$  are linearly independent.

When these conditions are satisfied, we say that  $M$  (or  $\mu$  or  $L$ ) is  $k$ -generic.

Theorem 1: Let  $L$  be an  $(n-k)$ -generic matrix of linear forms. The  $k \times k$ -minors of  $L$  generate a prime ideal of generic height, and this remains so modulo any  $\leq k-2$  linear forms.

Theorem 2: (-, Koh, Stillman): If  $X$  is a reduced, irreducible curve of genus  $g$ , and  $\mathcal{L}_1, \mathcal{L}_2$  are line bundles of degree  $\geq 2g+1$ ,

(and distinct if both have degree  $2g+1$ ), then  $I(X)$  is generated by the  $2 \times 2$  minors of  $L(\mathcal{L}_1, \mathcal{L}_2)$ .

R. M. FOSSUM:

There exist indecomposable rank two Gorenstein modules (d'apres D.Weston)

Let  $(A, \mathfrak{m})$  be a local noetherian ring with maximal ideal  $\mathfrak{m}$ . A finitely generated module  $G$  is a Gorenstein module if i)  $\text{inj dim}_A G < \infty$ , ii)  $\text{Hom}_A(G, G)$  is free and iii)  $\text{Ext}_A^i(G, G) = 0$  for  $i > 0$ . The following facts are known: 1) If  $A$  has a Gorenstein module, then a)  $A$  is CM and b)  $A$  has Gorenstein formal fibres. 2) If  $G$  is a Gorenstein module, then  $\text{rk}_A \text{Hom}_A(G, G) = t^2$ . The integer  $t$  is called the rank of  $G$ . 3) If  $G$  is a Gorenstein module of rank 1, then  $G$  has a dualizing module. 4) If  $A$  has a Gorenstein module, then it has a unique indecomposable Gorenstein module  $G$  such that any Gorenstein module is a direct sum of copies of  $G$ . 5) If  $A$  has a Gorenstein module of odd rank, then  $A$  has a dualizing module. 6) If  $A$  is a Gorenstein module over itself, then  $A$  is Gorenstein.

Theorem (D.Weston Univ. Ill. Thesis 1986). There exists an analytically normal two dimensional factorial local domain  $A$  that has an indecomposable Gorenstein module of rank 2.

Such an  $A$  has Gorenstein formal fibres, but no dualizing complex.

A. V. GERAMITA:

Linear Systems of Curves in  $\mathbb{P}^2$  with Prescribed Multiplicity

$k = \bar{k}$ ,  $P_1, \dots, P_s$  points in  $\mathbb{P}^2(k)$ ,  $R = k[x_0, x_1, x_2]$ ,  $\mathcal{I}_i \subseteq R$ ,

$\mathcal{I}_i \leftrightarrow P_i$ ,  $a_1 \geq \dots \geq a_s \geq 1$  integers and  $I = \mathcal{I}_1^{a_1} \cap \dots \cap \mathcal{I}_s^{a_s} = \oplus I_d$ .

Theorem (A.Gimigliano, Ph.D.thesis): Let  $P_1, \dots, P_s$  lie on a non-singular curve of degree  $d$  and suppose  $\mathcal{L}^1(dE_0 - E_1 - \dots - E_s) = 0$

on  $X$  (the blow-up of  $\mathbb{P}^2$  at  $P_1, \dots, P_s$ ). Let  $D_t = tE_0 - \sum_{i=1}^s \alpha_i E_i$  where  $t \geq \sum_{i=1}^s \alpha_i$ . Then i) if  $s \neq \binom{d+2}{2} - 1 \Rightarrow h^1(D_t) = 0$ ; ii) if  $s = \binom{d+2}{2} - 1$  and  $D_t \neq m \cdot d E_0 - \sum_{i=1}^s m E_i$  then  $h^1(D_t) = 0$ ; iii) if  $s = \binom{d+2}{2} - 1$ ,  $D_t = m d E_0 - \sum_{i=1}^s m E_i$  then  $h^1(D_t) = 0$  if the

points  $P_1, \dots, P_s$  are sufficiently general.

Cor.: If  $P_1, \dots, P_s$  are sufficiently general and lie on a nonsingular curve of degree  $d$  then if  $t \geq \sum_{i=1}^d \alpha_i \Rightarrow H(A, t) = e(I) = \sum \binom{\alpha_i + 1}{2}$  where  $H(A, t) = \dim_k R_t - \dim_k (R/I)_t$ .

S. GOTO:

#### On maximal Buchsbaum modules

Let  $R$  be a local ring and  $M$  a finitely generated  $R$ -module.

Then  $M$  is said to be Buchsbaum, if the difference

$I_R(M) = l_R(M/\eta_M) - e_{\eta}(M)$  is independent on the choice of parameter ideals  $\eta$  for  $M$ . (Hence  $M$  is CM iff  $M$  is Buchsbaum and

$I_R(M) = 0$ .) A Buchsbaum  $R$ -module  $M$  is called maximal if  $\dim_R M = \dim R$ . We show that a regular ring  $R$  possesses only finitely many isomorphism classes of indecomposable maximal Buchsbaum modules. In a certain special case (e.g., when  $R = \mathbb{C}[X_1 - X_n]^G$  with  $G$  finite group) the converse is also true.

J.W. HOFFMAN:

#### Topology of the infinitesimal site and the Hodge Conjecture

We interpret the general problem of Hodge as to give those conditions on a  $C^\infty$  complex vector bundle  $E$  over a projective nonsingular variety  $X$  over  $\mathbb{C}$  so that  $E^m \oplus \mathbb{C}^n$  has a holomorphic structure for some integers  $m, n$ . More generally, if  $X$  is any complex manifold, we introduce a Grothendieck topology called the holomorphic

topology of  $X$ , whose topos is denoted by  $(X)_{\text{hol}}$ , and a canonical morphism of topos  $f: X \rightarrow (X)_{\text{hol}}$ , whose construction mirrors that of the crystalline site in Algebraic Geometry. It has the property that a holomorphic structure on  $E$  is equivalent to giving a crystal  $E_1$  in  $(X)_{\text{hol}}$  such that  $E = f^*E_1$ . When interpreted in the language of differential operators, the equivalence between holomorphic structures on  $E$  and crystalline structures on  $E$  is seen to be a restatement of a form of Nirenberg's complex Frobenius theorem.

R. HARTSHORNE:

Rational surfaces in  $\mathbb{P}^4$

This talk is a report on recent work of Ch. Okonek and J. Alexander on possible rational surfaces in  $\mathbb{P}^4$ . Aside from the classically known rational surfaces of degree  $\leq 6$  in  $\mathbb{P}^4$ , Okonek found new ones of degree 7 and 8, and suggested another one of degree 8. Alexander proved the existence of the surface of degree 8 and found another of degree 9. He shows also that any non-special rational surface (i.e. with  $H^1(\mathcal{O}_X(1)) = 0$ ) is among those now known. It is still open whether there exist special rational surfaces of degree  $\geq 9$ .

S. IKEDA:

The Gorensteinness of Rees algebras and associated graded rings

Let  $(A, m, k)$  be a Noetherian local ring and  $I$  an ideal of  $A$ . We put  $R(I) = \bigoplus_{n \geq 0} I^n$  and call this ring the Rees algebra of  $I$ . We give a criterion for  $R(I)$  to be a Gorenstein ring.

Theorem. Let  $G(I) = \bigoplus_{n \geq 0} I^n/I^{n+1}$ . Suppose that  $\text{grade}(I) \geq 2$ .

Then the following conditions are equivalent:

- (1)  $R(I)$  is Gorenstein
- (2)  $K_A = A$  and  $K_{G(I)} \cong G(I)(-2)$ ,

where  $K_A$  and  $K_{G(I)}$  are the canonical modules of  $A$  and  $G(I)$  respectively.

We construct a local ring  $(A, m, k)$  such that  $R(m)$  is Gorenstein but  $A$  is not Cohen-Macaulay.

K. YAMAGISHI:

Unconditioned strong d-sequences and some applications to generalized Cohen-Macaulay rings

A sequence  $a_1, a_2, \dots, a_s$  of elements in a commutative ring  $A$  is called an unconditioned strong d-sequence on an  $A$ -module  $E$  if every power  $a_1^{n_1}, a_2^{n_2}, \dots, a_s^{n_s}$  ( $n_i > 0$ ) and every permutation of them form a d-sequence on  $E$ . The reason why we discuss this sequence is that we want to find a good sequence property which unifies the behaviours of s.o.p.'s for Buchsbaum rings and moreover generalize Cohen-Macaulay rings. Main conclusion is that we can describe the local cohomology modules  $H_a^i(E)$ 's ( $i < s$ ) in terms of quotient modules concerning  $a_i$ 's and also the local cohomology modules of  $R(E)$  and  $G(E)$ , the Rees module and the associated graded module of  $E$  w.r.t.  $a_i$ 's, in terms of  $H_a^i(E)$ 's. This sequence has a closed relation with a pS-sequence, in fact they form a pS-sequence on  $E$  iff they form a u. s. d-sequence on  $E$  under suitable assumptions.

U. KARRAS:

Equimultiplicity of  $\mu$ -constant deformations

We gave a report on the present knowledge concerning the question: Does topological equimultiplicity of isolated hypersurface singularities imply equimultiplicity? Actually we are interested in the slightly weaker problem whether the multiplicity does not change along the  $\mu$ -constant stratum where  $\mu$  denotes the Milnor number of the singularity. Our new approach uses the concept of deformations of embedded resolutions. Then our main result characterizes equi-

multiplicity of  $\mu$ -constant deformations of 2-dimensional isolated hypersurface singularities in terms of certain cohomological vanishing conditions. We can apply this criterion to the class of hypersurface singularities for which the Newton polyhedron gives rise to an embedded resolution via toroidal embeddings.

J. LIPMAN:

Topological Invariants of Quasi-ordinary Singularities

A d-dimensional irreducible hypersurface singularity  $x \in X \subset \mathbb{C}^{d+1}$  is called quasi-ordinary (q.o.) if it admits, locally, a finite projection into  $\mathbb{C}^d$  with discriminant locus  $\Delta$  having only normal crossings. Such a singularity can be parametrized by a fractional power series, the case  $d=1$  being the classical Puiseux parametrization. For curves one knows how the "characteristic pairs" of a parametrization control the local topology (via knot theory). In higher dimensions, one associates to fractional power series parametrizations analogues of the characteristic pairs, and naturally asks what the relation of these to the local topology of  $(X, x)$  is. A big step towards the answer is the following. For each component  $\Delta_i$  of  $\Delta$ , ( $1 \leq i \leq c$ ) let  $z_i := \pi^{-1}(\Delta_i)$ . Then  $z_i$  is irreducible, and we can let  $m_i := \deg(\pi|_{z_i})$  be the branching order of  $\pi$  at a generic point  $z_i$  of  $z_i$ . Lemma: With suitable ordering of the  $i$  we have

$$m_c | m_{c-1} | \dots | m_1 = \deg(\pi_x) \quad ("| = \text{divides}).$$

Theorem: Locally homeomorphic q.o. singularities have the same  $m_i$  ( $i = 2, \dots, c$ ). An important role in the proof is played by the local homology group  $H_{2d-2}(X, X-x)$ , which is finite, of order  $m_2 \dots m_c$ . (This is proved via the group of rational equivalence classes of codimension-one cycles in  $\mathcal{O}_{X,x}$ , which maps naturally to  $H_{2d-2}(X, X-x)$ : cycle  $\longleftrightarrow$  analytic cycle on  $X \rightarrow$  fundamental class. Thm: This map is an isomorphism.)

G. LYUBEZNIK:

On the number of equations defining algebraic sets in  $A_k^n$

The following theorem is proved:

Theorem: Let  $V \subseteq A_k^n$  an algebraic set of irreducible components of positive dimensions. Assume that either

- i)  $V$  is locally a complete intersection, or
- ii)  $\text{char } k = p > 0$  and  $V$  arbitrary.

Then  $V$  can be defined by  $n-1$  equations.

This theorem generalizes earlier results of Ferrand - Szpiro  
- Boratynski - Mohan Kumar - Cowsik - Nori.

G. R. PELLIKAAN:

Nonisolated hypersurface singularities

Let  $f : (\mathbb{C}^{n+1}, 0) \rightarrow (\mathbb{C}, 0)$  be a germ of an analytic function,  $\Sigma$  its singular locus. Def.:  $A_\infty := f(x, y_1, \dots, y_n) = y_1^2 + \dots + y_n^2$ ,  $\Sigma = V(y_1, \dots, y_n)$ ;  
 $D_\infty := f(x, y_1, \dots, y_n) = xy_1^2 + y_2^2 + \dots + y_n^2$ ,  $\Sigma = V(y_1, \dots, y_n)$   
 $A_1 := f(x_0, y_1, \dots, y_n) = y_0^2 + \dots + y_n^2$ ,  $\Sigma = \{0\}$ ;

Let  $I_f$  be the Jacobi ideal of  $f$ ;  $I = \text{rad}(I_f)$ . Assume from now on  $\Sigma$  is 1-dimensional.

Prop.: If  $\Sigma$  is a complete intersection and  $\dim_{\mathbb{C}} \frac{I}{I_f} < \infty$ , then there exists a deformation  $\{f_t, \Sigma_t\}$  of  $(f, \Sigma)$  s.t. for all small  $t \neq 0$ : i)  $\Sigma_t$  is a smooth curve; ii)  $f_t$  has only  $A_1$  singularities outside  $\Sigma_t$  and only  $A_\infty$  or  $D_\infty$  singularities on  $\Sigma_t$ . We prove the conjecture of Siersma that  $\dim_{\mathbb{C}} \frac{I}{I_f} = \#A_1 + \#D_\infty$  by using

iii) of the following

Prop.: Let  $R$  be a commutative noeth. ring. Let  $I \supseteq J$  be ideals in  $R$ . i) Assume that  $R$  is a local CM Ring,  $\text{ht } I = n$ ,  $J$  is generated by  $(n+1)$  elements,  $J = I \cap \sigma$   $\text{ht } \sigma \geq n+1$ . Then  $\frac{I}{J}$  is CM. ii) If  $I$  is perfect of grade  $n+1$  and  $\text{grade}(\frac{I}{J}) \geq n+1$  and  $J$  is gen. by  $(n+1)$  elements, then  $\frac{I}{J}$  is perfect. iii) If  $I$  is gen. by a  $R$ -sequence of length  $n$  and  $J$  is gen. by  $m$  elements,  $m \geq n$ , and  $\text{grade} \frac{I}{J} \geq m$  then  $\frac{I}{J}$  has a free resolution of length  $m$ .

L. SZPIRO:

Surfaces arithmétiques elliptiques

Nous considérons dans ce qui suit des courbes elliptiques semi-stables  $f: X \rightarrow T$  où  $T$  est soit une courbe projective et lisse de genre  $g$  sur un corps  $k$ , soit le spectre d'un anneau d'entiers d'un corps de nombre algébriques. Nous noterons  $S$  l'ensemble-fini de points de  $T$  dont la fibre n'est pas lisse.

Th. 1 (situation géométrique  $T = \text{courbe}/k$ ) Soit  $\Delta_X$  le discriminant de  $X$  sur  $T$  alors

$$\deg \Delta_X \leq 6(2g - 2 + \deg S)p^e$$

où  $p = \text{char}(k)$  et  $p^e$  est le degré d'inseparabilité de morphisme  $f$ .

Th. 2 (situation arithmétique  $T = \text{Spec } \mathcal{O}_k$ ) Munissons le dual de l'algèbre de Lie  $\omega_X$  de la métrique d'Arakatov qui satisfasse à la formule d'ajonction sur les surfaces arithmétiques. Alors on a  $12 \deg \omega_X = Pg \text{ Norme}(\Delta_X)$ .

Conjecture (Situation arithmétique) Soit  $N$  le conducteur de la courbe elliptique semi-stable  $f: X \rightarrow \text{Spec } \mathcal{O}_k$  et soit  $\epsilon$  un réel positif, alors il existe une constante  $C(k, \epsilon)$  telle que

$$\text{Norme}(\Delta_X) \leq C(k, \epsilon)N^{6+\epsilon}$$

Cette Conjecture implique clairement la suivante conjecture': Dans le même situation il existe une constante

$$c(k) \text{ telle que } \text{Norme}(\Delta_X) \leq N^{c(k)}$$

L'intérêt de ces conjectures est magnifié par la construction de G.Frey: Soient  $a, b, c$  des entiers tels que  $a + b = c$  alors la courbe elliptique  $y^2 = x(x - a)(x - c)$  est semi-stable si  $v_2(a) \geq 4$  et  $c \equiv -1(4)$ .

Comme corollaire d'une de ces conjectures et de la construction de G.Frey on voit que pour toute équation à coefficients entiers  $(*) ax^n + by^m = cz^p$  telle que  $a + b \neq \pm c$  il existe une constante  $C(a, b, c)$  telle que si  $\inf(n, m, p) \geq C(a, b, c)$  l'équation  $(*)$  n'a pas de solution entière non triviale.

W. VASCONCELOS:

Koszul homology and the structure of low codimension ideals

We indicate how, in low codimension (3,4) , the structure of CM ideals is mirrored in its Koszul homology. Let  $R$  be a regular local ring and  $I = (x_1, \dots, x_n)$  a CM ideal. Denote by  $H_i(I)$  the Koszul homology modules of  $I$  using the given generating set.  $I$  is said to be linked to  $J$  if there exists a family of links  $I \sim L_0 \sim \dots \sim L_r \sim J$ .

Theorem 1: If  $I$  has codimension 3 then " $H_1(I)$  is Cohen Macaulay" is an invariant of the full linkage class of  $I$ . It does not extend to the next Koszul module, nor to higher codimension. In case  $I$  has a pure resolution  $0 \rightarrow R^{b_3}(-d-a-b) \rightarrow R^{b_2}(-d-a) \rightarrow R^{b_1}(-d) \rightarrow I \rightarrow 0$ .

Theorem 2 (Villarreal): If  $I$  is, besides, generically a complete intersection,  $a \geq b$  and  $b_1 \geq 6$ , then  $H_1(I)$  is not CM.

For Gorenstein ideals of codimension 4:

Theorem 3:  $H_1(I)$  is CM  $\Leftrightarrow I/I^2$  is CM.

J. WAHL:

$T^1$  for quasi homogeneous surface singularities

If  $A$  is a finitely generated (over  $\mathbb{C}$ ) graded normal domain of dimension 2, then the module  $T_A^1 = \text{Ext}_A^1(\Omega_A, A)$  is graded; one tries to compute the graded pieces in terms of the geometry of  $A$ . In case  $A$  is the cone over a projectively normal embedding of a curve  $C \subseteq \mathbb{P}^N$ , we prove:

Theorem If  $C$  has general moduli,  $g(C) \geq 50$ , then for

$$\deg L \geq 4g - 2 \quad (L = \mathcal{O}_C(1)) , \quad T_{-1}^1 = 0 .$$

Theorem If  $C$  has general moduli,  $g(C) \geq 50$ , then for the canonical cone one has  $T_{-1}^1 = 0$ .

One any  $\mathbb{C}$ -scheme, with line bundle  $L$ , one defines

$$\phi_L : \Lambda^2 H^0(L) \rightarrow H^0(\Omega^1 \otimes L^2) \quad \text{by the "formula"} \quad \phi_L^*(f \wedge g) = f dg - g df .$$

$\phi_K$  is surjective for the general curve of genus  $\geq 50$ . The previous result then follows from the

Theorem For a canonical cone,  $(T_{-1}^1)^* \cong \text{Coker } \phi_K$ .

K. WATANABE:

Gorenstein ASL (algebras with straightening laws) domains of dimension 3 and 4

Problem: Given a poset  $H$ , is there (1) a Gorenstein ASL on  $H$ ?  
(2) an ASL which is an integral domain on  $H$ ?

In case (1) we call  $H$  to be weakly Gorenstein and in case (2) we call  $H$  integral. We give the classification of the following posets

- i) Integral (weighted) posets of rank 1
- ii) weakly Gorenstein posets of rank 1
- iii) integral, weakly Gorenstein posets of rank 2.

In each case, we assume our ASL to be graded over a field and in case iii) we assume our ASL to be homogeneous (generated by deg. 1 elements). In particular, we can show that the coordinate rings of Del Pezzo surfaces of degree  $\geq 4$  by anti-canonical embedding are ASL. Finally, we classify  $H$  of rank 3, with unique minimal element  $T$ , which is integral and  $H' = H - \{T\}$  defines a triangulation of a 2-sphere (that is the simplicial complex  $\Delta(H')$  associated to  $H'$  has  $S^2$  as underlying topological space). There are 18 such posets.

J. WUNRAM:

Reflexive modules on quotient surface singularities

Let  $(X, x)$  be a germ of an analytic quotient surface singularity.

Let  $\pi: \tilde{X} \rightarrow X$  be the minimal desingularization of  $X$  with exceptional system  $\{E_i\}_{1 \leq i \leq r}$  and the fundamental cycle  $Z = \sum_{i=1}^r r_i E_i$ . For

each reflexive module  $M$  on  $X$  the sheaf  $\tilde{M} := \pi^*M/\text{torsion}$  is locally free on  $\tilde{X}$  and the first Chern class is represented by a divisor which is transversal to the exceptional set  $E$  of  $\pi$ .

The subject of this talk is a generalization of the theorem of Artin + Verdier and the multiplication formula of Esnault + Knörrer on the McKay correspondence for rational double points to the case of an

arbitrary quotient surface singularity:

Thm. i) For each  $E_i$  there is exactly one indecomposable reflexive module  $M_i$  on  $(X, x)$  with  $c_1(\tilde{M}_i) \cdot E_j = \delta_{ij}$ ,  $1 \leq i, j \leq r$ , and  $R^1\pi_*(\tilde{M}_i^\vee) = 0$ . The rank of  $M_i$  is  $r_i$ .

ii) If  $0 \rightarrow \tau(M) \rightarrow N_M \rightarrow M \rightarrow 0$  is an almost split exact sequence

$$\text{then } c_1(N_M) = \begin{cases} c_1(\widetilde{\tau(M)}) + c_1(\widetilde{M}) & \text{if } M \neq M_1, \dots, M_r \\ c_1(\widetilde{\tau(M)}) + c_1(\widetilde{M}) + E_i & \text{if } M = M_i \end{cases}$$

The fundamental sequence  $0 \rightarrow \omega_X \rightarrow N_{\mathcal{O}_X} \rightarrow \mathcal{O}_X \rightarrow \mathbb{C} \rightarrow 0$  induces  $c_1(N_{\mathcal{O}_X}) = c_1(\omega_{\tilde{X}}) - z$ .

Berichterstatter: T. Lehmkühl

Tagungsteilnehmer

Prof. Dr. L. L. Avramov  
Institute of Mathematics  
Bulgarian Academy of Sciences  
P.O.Box 373  
1090 Sofia  
Bulgarien

Prof. Dr. R. Berger  
Fachbereich Mathematik  
Universität des Saarlandes  
Bau 27  
6600 Saarbrücken

Prof. Dr. J. Bingener  
Fakultät für Mathematik  
Universität Regensburg  
Universitätsstr. 31  
8400 Regensburg

Prof. Dr. E. Böger  
Mathematisches Institut  
Universität Bochum  
Universitätsstr. 150  
4630 Bochum - Querenburg

Prof. Dr. W. Bruns  
Universität Osnabrück  
Abteilung Vechta  
Fachbereich 3  
Driverstr. 22  
2848 Vechta

Prof. Dr. R.-O. Buchweitz  
Institut für Mathematik  
T. U. Hannover  
Welfengarten 1  
3000 Hannover 1

Prof. Dr. Maria Contessa  
Dipartimento di Matematica  
Università di Roma  
"La Sapienza"  
P.le Aldo Moro, 5  
I-00185 Roma

Prof. Dr. E. D. Davis  
Mathematics Department  
S.U.N.Y.A.  
Albany, N.Y. 12222  
USA

Prof. Dr. D. Eisenbud  
Department of Mathematics  
Brandeis University  
Waltham, MA 02254  
USA

Prof. Dr. D. Ferrand  
U.E.R. Mathématiques  
Université de Rennes I  
Campus de Beaulieu  
F-35042 Rennes - Cedex  
Frankreich

Prof. Dr. R. M. Fossum  
Matematisk Institut  
Københavns Universitets  
Universitetsparken 5  
DK-2100 København Ø  
Dänemark

Prof. Dr. H.-B. Foxby  
Matematisk Institut  
Københavns Universitets  
Universitetsparken 5  
DK-2100 København Ø  
Dänemark

Prof. Dr. A. V. Geramita  
Department of Mathematics  
Queen's University  
Kingston, Ontario K7L 3N6  
Kanada

Prof. Dr. J. W. Hoffman  
Department of Mathematics  
LSU  
Baton Rouge, LA 70803  
USA

Prof. Dr. S. Goto  
Department of Mathematics  
Nihon University  
Sakura-Josui  
Setagaya-ku  
Tokyo 156 / Japan

Reinhold Hübl  
Fakultät für Mathematik  
Universität Regensburg  
Universitätsstr. 31  
8400 Regensburg

Prof. Dr. G.-M. Greuel  
Fachbereich Mathematik  
Universität Kaiserslautern  
Erwin-Schrödinger-Str.  
6750 Kaiserslautern

Prof. Dr. S. Ikeda  
Mathematisches Institut  
Universität Köln  
Weyertal 86-90  
5000 Köln 41

Prof. Dr. R. Hartshorne  
Mathematics Department  
UC Berkeley  
Berkeley, CA 94720  
USA

Prof. Dr. U. Karras  
Fachbereich Mathematik  
Universität Dortmund  
Postfach 500500  
4600 Dortmund 50

Prof. Dr. M. Herrmann  
Mathematisches Institut  
Universität Köln  
Weyertal 86-90  
5000 Köln 41

Dr. M. Kersken  
Ruhr-Universität Bochum  
Fakultät für Mathematik  
Universitätsstr. 150  
Gebäude NA  
4630 Bochum 1

Prof. Dr. J. Herzog  
Universität Essen - GHS  
FB 6 - Mathematik  
Universitätsstr. 3  
4300 Essen 1

Prof. Dr. K. Kiyek  
Fachbereich 17 - Mathematik  
Universität Paderborn  
Warburger Str. 100  
4790 Paderborn

Prof. Dr. E. Kunz  
Fakultät für Mathematik  
Universität Regensburg  
Universitätsstr. 31  
8400 Regensburg

Dr. R. Pellikaan  
Wiskundig Seminarium der  
Vrije Universiteit  
De Boelelaan 1081  
Postbus 7161  
NL-1007 MC Amsterdam

Thomas Lehmkühl  
Mathematisches Institut  
Universität Tübingen  
Auf der Morgenstelle 10  
7400 Tübingen

Prof. Dr. O. Riemenschneider  
Mathematisches Seminar  
Universität Hamburg  
Bundesstr. 55  
2000 Hamburg 13

Frau Professor  
Dr. Monique Lejeune-Jalabert  
Institut Fourier  
Dépt. de Mathématiques  
Université de Grenoble  
B.P. 74  
F-38402 Saint Martin-d'Hères

Prof. Dr. G. Scheja  
Mathematisches Institut  
Universität Tübingen  
Auf der Morgenstelle 10  
7400 Tübingen 1

Prof. Dr. J. Lipman  
Department of Mathematics  
Purdue University  
West Lafayette, IN 47907  
U S A

Prof. Dr. F.-O. Schreyer  
Fachbereich Mathematik  
Universität Kaiserslautern  
Erwin-Schrödinger-Str.  
6750 Kaiserslautern

Prof. Dr. G. Lyubeznik  
Department of Mathematics  
Purdue University  
West Lafayette, IN 47907  
U S A

Gerhard Seibert  
Fakultät für Mathematik  
Universität Regensburg  
Universitätsstr. 31  
8400 Regensburg

Prof. Dr. H.-J. Nastold  
Mathematisches Institut  
Universität Münster  
Einsteinstr. 62  
4400 Münster

Prof. Dr. U. Storch  
Ruhr-Universität Bochum  
Institut für Mathematik  
Postfach 102148  
4630 Bochum 1

Prof. Dr. L. Szpiro  
E. N. S.  
45, rue d'Ulm  
F-75005 Paris

Prof. Dr. K.-i. Watanabe  
Dept. of Mathematical Sciences  
Tokai University  
Hiratsuka  
Kanagawa 259-12  
Japan

Prof. Dr. W. V. Vasconcelos  
Department of Mathematics  
Rutgers University  
New Brunswick, N.J. 08903  
U S A

Jürgen Wunram  
Mathematisches Seminar  
Universität Hamburg  
Bundesstr. 55  
2000 Hamburg 13

Prof. Dr. U. Vetter  
Fachbereich Mathematik  
Universität Osnabrück  
Abteilung Vechta  
Driverstr. 22  
2848 Vechta

Prof. Dr. K. Yamagishi  
Mathematisches Institut  
Universität Köln  
Weyertal 86-90  
5000 Köln 41

Prof. Dr. J. M. Wahl  
Department of Mathematics  
University of North Carolina  
Chapel Hill, N. C. 27514  
U S A

