MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 24/1986

Topologische Methoden in der Gruppentheorie

1.6. bis 7.6.1986

Die Tagung fand unter der Leitung von R. Bieri (Frankfurt), K.S. Brown (Ithaca) und K.W. Gruenberg (London) statt. Mit fast 50 Teilnehmern war die Tagung stark belegt; zahlreiche Interessenten konnten nicht aufgenommen werden.

Es fanden 37 Vorträge statt. Der zentrale methodische Aspekt quer durch alle Vorträge war die Betrachtung von Gruppenoperationen auf CW-Komplexen (und Λ-Bäumen).

Inhaltlich läßt sich der überwiegende Teil der Vorträge einem der folgenden drei Themenkreise zuordnen:

1. Explizite Berechnung der (Co)homologie, Euler-Charakteristik etc. von spezifischen Gruppen: Automorphismengruppen, Abbildungsklassengruppe, arithmetische Gruppen, nilpotente Gruppen...
2. Beiträge zur Klassifikation unendlicher Gruppen nach Endlichkeitsbedingungen: virtuelle cohomologische Dimension, endlich erzeugte Auflösungen, endliche Präsentierbarkeit, Akzessibilität...
3. Beiträge zur Cohomologietheorie endlicher Gruppen.

Weitere Vorträge beschäftigten sich mit Fragestellungen aus der K-Theorie, der kombinatorischen Gruppentheorie und der niedrigdimensionalen Topologie.

Vortragsauszüge

H.ABELS:

Finiteness properties of S-arithmetic groups in the function field case

Result: $SL_n(\mathbb{F}_q[t])$ is FP_{n-2}, not FP_{n-1} for q sufficiently large depending on n.

P. ABRAMENKO:

FP_* - properties of $SL_n(\mathbb{F}_q[t])$

The talk is concerned with the following result:

Proposition:

$$q \geq \binom{n-2}{\left\lceil \frac{n-2}{2} \right\rceil} \Longrightarrow SL_n(\mathbb{F}_q[t]) \text{ is of type } FP_{n-2}, \text{ but not } FP_{n-1}.$$

A sketch of the proof is given and it is tried to explain the curious restriction for q.

The proof is based on a topological criterion of K.S. Brown. It yields some information about the FP_*-properties of a group Γ acting on a CW-complex X which is suitably filtered (in our context Γ is $SL_n(\mathbb{F}_q[t])$ and X is the Bruhat-Tits-building belonging to $SL_n(\mathbb{F}_q[t])$). Applying the criterion, the connectedness-properties of the links occuring with the filtration become crucial. They are isomorphic to subcomplexes of the spherical Tits building associated with $\mathbb{F}_q^n$. Eventually, the proposition is a consequence of

Lemma: Suppose $V \cong \mathbb{F}_q^n$, $0 < E < V$, $\dim E = k$ and $q \geq \binom{n-2}{k-1}$.

Then the subcomplex of the Tits building defined by

$T(E,V) := \{ 0 < U < V \mid U \cap E = 0 \;\vee\; U + E = V \}$ is

is (n-3)-connected.

J.M. ALONSO:

Growth functions of amalgams

Each group G under consideration will be assumed to contain a finite subset which generates the group as a semigroup (and does not contain 1). Such a subset provides the group with a (word) length. This length is used to filter the group algebra RG of the group G (R denotes a commutative ring with unit). The associated graded R-algebra will be denoted grRG.

We consider the case when $G = G_1 *_A G_2$, and give a condition on the inclusions $A \hookrightarrow G_i$ $(i = 1,2)$ under which

$$grRG = grRG_1 \coprod_{grRA} grRG_2 \qquad \text{as (connected) graded R-algebras.}$$

Combining this with a result of J.-M. Lemaire gives the formula

$$\frac{1}{G(z)} = \frac{1}{G_1(z)} + \frac{1}{G_2(z)} - \frac{1}{A(z)}$$

expressing the growth function G(z) of the group G in terms of those of the subgroups A, G_1 and G_2. This is applied to the computation of a few examples.

R. ALPERIN:

Groups, Graphs and Property τ

Applications of groups acting on trees to computer science.

D. ARLETTAZ:

On the homology of the special linear group over a number field

Let F be a number field and SL(F) its infinite special linear group. The integral homology groups of SL(F) are in general not finitely generated. We prove that, for all $i \geq 0$, $H_i(SL(F); \mathbb{Z})$ is the direct sum of a torsion group and a free abelian group of finite rank. The proof is based on the description of the homotopy groups of the infinite loop space $BSL(F)^+$ and the fact that the Postnikov invariants of infinite loop spaces are cohomology classes of finite order.

We then deduce from this result an upper bound for the order of the Chern classes of representations of discrete groups over a number field, without any finiteness condition on the group.

A.H. ASSADI:

Homotopy actions and cohomology of groups

Let X be a connected G-space, where G is a finite group, and let $M \equiv \bigoplus_{i>0} H_i(X)$ be the associated $\mathbb{Z}G$-module. Assume that for each maximal p-elementary abelian $A \subseteq G$, the spectral sequence $E_A \times_A X \longrightarrow BA$ collapses. Then the $\mathbb{Z}G$-module M is $\mathbb{Z}G$-projective iff $M \mid \mathbb{Z}A$ is $\mathbb{Z}A$-projective.

Applications: Let $G \supset \mathbb{Z}_p \times \mathbb{Z}_p$ or Q_8.

(1) Then there exists a $\mathbb{Z}G$-module M such that M is not isomorphic to the homology of any Moore G-space.

(2) There exists a ZG-module M' satisfying (1), but $M \cong H_*(Y)$ for some G-space Y.

(3) There exists a $\mathbb{Z}G$-module $M'' = M_1 \oplus M_2$, $M_i \neq 0$, such that $M'' \ncong H_*(X)$ for any G-space X

(4) The projectivity criterion above gives a criterion for replacing a given homotopy action by some topological action equivalent to it.

H. BASS:

Length functions of group actions on Λ-trees

Attached to an action (without inversions) of a group G on a Λ-tree X (Λ an ordered abelian group) is the hyperbolic length function $l = l_X : \Gamma \longrightarrow \Lambda$. If $s \in G$ has a fixed point, $l(s) = 0$; otherwise $l(s)$ is the translation length of s along the "s-axis". Like the character of a representation $G \longrightarrow SL_2$, l_X determines X (with the G-action) provided that X is "minimal" and that l_X is not abelian, i.e. does not factor through $G^{ab} = G / G'$. Thus non abelian minimal Λ-tree actions can be parametrized by their length functions, whence the possibility of speaking about "nearby Λ-tree actions".

D. BENSON:

Semidihedral groups and Bott periodicity

Let A_n be the subalgebra of the Steenrod algebra generated by

$Sq^1, Sq^2, \dots, Sq^{2^n}$.

Then

$$Ext^*_{A_1}(\mathbb{F}_2, \mathbb{F}_2) \cong Ext^*_{SD_{16}}(\mathbb{F}_2, \mathbb{F}_2)$$

(where SD_{16} is the semidihedral group of order 16)

$$\cong \mathbb{F}_2[x,y,z,w] / (x^3, xy, xz, z^2+wy^2)$$

Thus we have an Adams spectral sequence which collapses ($E_2 = E_\infty$)

and which horizontally depicts real Bott periodicity, and vertically depicts cohomology of the semidihedral group.

We conjecture that in some sense the same thing happens for A_n, $n > 1$. In particular we conjecture that there is a finite 2-group with the same cohomology as A_n. The order of this group looks as though it should be $2^{\binom{n+2}{3}}$.

A.J. BERRICK:

Acyclic and abelian groups: McLain groups and Eilenberg-MacLane spaces

We exhibit some natural constructions of discrete groups having properties indicated by the work of Baumslag, Dyer & Heller (1980) and Kan & Thurston (1976). Let G be an arbitrary abelian group. We give a natural embedding of G as the centre of an acyclic group. The quotient Q is a perfect group having G as its Schur multiplier. Moreover, application of Quillen's plus-construction to the classifying space K(Q,1) yields a "group-theoretic" model for the Eilenberg-MacLane space K(G,2).

R. BIERI:

Higher geometric invariants of groups

Joint work with Burkhardt Renz.

A group G is said to be of type $(FP)_n$ if the trivial G-module $\mathbb{Z}$ admits a free resolution $\underline{\underline{F}} \longrightarrow \mathbb{Z}$ which is finitely generated in all dimensions $\leq n$. Assuming that G is of type $(FP)_n$ we define, for every $k \leq n$, a subset Σ^k of the sphere

$$S(G) = (\mathrm{Hom}(G, \mathbb{R}) - \{0\}) / \mathbb{R}^*_+ .$$

The definition uses the resolution $\underline{\underline{F}}$ but is independent of all choices.

Main results are:

(a) Σ^1 is the set Σ of Walter Neumann's talk (see p.18)

(b) Σ^k is open in S(G).

(c) A normal subgroup $N \lhd G$ containing the commutator subgroup G' is of type $(FP)_k$ if and only if Σ^k contains all points of S(G) represented by homomorphisms $\chi : G \longrightarrow \mathbb{R}$ with $\chi(N) = 0$.

K.S. BROWN:

Valuations on groups

I introduce a notion of valuation on a group in order to compute the Bieri-Neumann-Strebel invariant Σ associated to a finitely generated group. Valuations are closely related to abelian tree actions. As an application I compute the finitely generated normal subgroups with abelian quotient for an arbitrary one-relator group.

J.F. CARLSON:

Projective resolutions by multiple complexes

Let G be a finite group and let K be a field of characteristic p. A recent result of Dave Benson and the speaker is that a KG-projective resolution of the trivial module K can be obtained as a tensor product $Y = X_1 \otimes \ldots \otimes X_n$ where each X_i is a periodic complex of KG-modules which are not all projective. Each X_i is constructed directly from the cohomology element $\zeta_i \in \mathrm{Ext}^*_{KG}(K,K)$. The varieties of the elements $\zeta_1, \ldots, \zeta_n$ must intersect trivially. The question arises as to when such a resolution is multiplicative. That is, do there exist homomorphisms $X_i \longrightarrow X_i \otimes X_i$ such that the product map $Y \longrightarrow Y \otimes Y$ is a chain map on projective resolutions that lifts the identity on K ? With some modifications, such homomorphisms exist provided that ζ_i annihilates $\mathrm{Ext}^*_{KG}(L_{\zeta_i}, L_{\zeta_i})$ where L_{ζ_i} is the kernel of the cocycle $\zeta_i : \Omega^m(K) \longrightarrow K$, $m = \deg(\zeta_i)$.

For $p > 2$ we can give a complete answer. Specifically, ζ annihilates the cohomology of L if and only if the degree of ζ is even. If $p = 2$ then the problem is far more difficult.

R. CHARNEY:

Torsion in the homology of the mapping class groups

Joint work with R. Lee.

Let $S_{g,r}$ be an oriented surface of genus g with r boundary components, and let $\Gamma_{g,r}$ be the mapping class group of $S_{g,r}$.

$\Gamma_{g,r}$ = isotopy classes of orientation preserving homeomorphisms $S_{g,r} \circlearrowright$ pointwise fixing $\partial S_{g,r}$

One can define a limit group $\Gamma = \varinjlim_g \Gamma_{g,1}$ which by a theorem of J. Harer satisfies $H_*(\Gamma) \cong H_*(\Gamma_{g,r})$ for all $r \geq 0$ providing $g \gg *$

There is a natural homomorphism $\Gamma_{g,1} \longrightarrow GL_{2g}(\mathbb{F}_p)$ which takes a homeomorphism of $S_{g,1}$ to the induced map on $H_1(S_{g,1}; \mathbb{F}_p) \cong \mathbb{F}_p^{2g}$. Passing to the limit groups and applying Quillen's + - construction gives rise to a map $f_p : B\Gamma^+ \longrightarrow BGL(\mathbb{F}_p)^+$. We prove

Theorem: If l and p are odd primes sucht that p generates $(\mathbb{Z}/_{l^2})^*$, then f_p induces split surjections on the l-primary torsion in homology and homotopy. Hence $H_*(\Gamma; \mathbb{Z})_l$ (resp $\pi_*(B\Gamma^+)_l$) contains a direct summand isomorphic to $H_*(GL(\mathbb{F}_p); \mathbb{Z})_l$ (resp. $K_*(\mathbb{F}_p)_l$).

The latter groups have been completely computed by Huebschmann (resp. Quillen).

D.J. COLLINS:

Automorphism groups of free products

We discuss the structure of the automorphism group of a free product and evidence for the following conjectures

1) If $G = \ast_{i=1}^{n} G_i$ with each G_i finite then

   a) Aut G is torsion-free by finite

   b) vcd (Aut G) = n-1

2) If $G = \ast_{i=1}^{n} G_i$ with G_i and Aut G_i torsion-free for all i, then Aut G is torsion-free by finite.

M. CULLER:

IA-outer automorphisms of free groups

The IA-outer automorphism group of a group G is the subgroup IAOut(G) of Out(G) consisting of those outer automorphisms which act trivially on the abelianization G/G' .

This talk is a report on joint work with Karen Vogtmann which indicates that IAOut(F_n), where F_n is free of rank n, is not finitely presented. (IAOut(F_n) is known to be finitely generated.) This has interesting connections to known results about IAOut(G) for G either a closed surface group or a free metabelian group.

M.J. DUNWOODY:

Accessibility of finitely presented groups and related topics

A group is almost finitely presented if it acts freely on a

2-complex K so that $G\backslash K = L$ is finite and $H^1(K; \mathbb{Z}_2) = 0$.

<u>Theorem</u>: Let G be almost finitely presented. Let T be a G-tree. There exists a G-tree T' such that

(i) all edge stabilizers of T' are finitely generated,

(ii) there is an upper bound n(L) (independent of T) depending on L, on the number of G-orbits of vertices of T' of valency > 2,

(iii) there exists a G-morphism $\alpha : T' \longrightarrow T$.

It is possible to deduce from this theorem that almost finitely presented groups are accessible. Also the knot $S^n \longrightarrow S^{n+2}$ ($n = 1$, $n \geq 3$) can be written as a sum of indecomposable knots.

M.N. DYER:

Some applications of Rosset's localization of group rings

Let G be a group with a non-trivial, abelian, torsion free subgroup A. Call such groups the class $\mathcal{R}$. Let R be any subring of $\mathbb{Q}$, the rationals, and $S = RA \setminus \{0\}$. Then S is multiplicatively closed and one may form the left ring of fractions $RG \rightarrowtail S^{-1}RG = U$. This talk will apply the properties of this localization [S. Rosset, Math.Z., <u>185</u> (1984) 211 - 215] to

(1) extending the Kaplansky rank to a rank function $t : U \longrightarrow \mathcal{F}$ taking values in the field of fractions $\mathcal{F}$ of $\mathbb{Q}A$,

(2) extending the Gottlieb-Stallings theorem on the Euler characteristic of groups of type FP over $\mathbb{Q}$, and

(3) determining the deficiency of finitely presented groups G with quotient G/L in $\mathcal{R}$ and subgroup L having $S^{-1}(H_1 L) = 0$ and $wt_G L < \infty$ (L is normally generated in G by finitely many elements).

B. ECKMANN:

Cyclic homology of groups and the Bass conjecture

The "cyclic homology" (Connes) $H_i(\mathbb{Q}G)$ of the group algebra $\mathbb{Q}G$ decomposes into a direct sum $\bigoplus_{[x]}$ over the conjugacy classes [x] in G (Burghelea 1985). For x of infinite order the [x]-summand is $H_i(C_x/\langle x\rangle;\mathbb{Q})$, C_x = centralizer of x in G, $\langle x\rangle$ = subgroup generated by x.

The character maps $Ch^l: K_0(\mathbb{Q}G) \longrightarrow HC_{2l}(\mathbb{Q}G)$ decompose accordingly; they are compatible with $S: HC_{2l}(\mathbb{Q}G) \longrightarrow HC_{2l-2}(\mathbb{Q}G)$ of the Connes-Gysin sequence, $l \geq 1$. In particular, the map $Ch^0: K_0(\mathbb{Q}G) \longrightarrow HC_0(\mathbb{Q}G) = \bigoplus_{[x]} \mathbb{Q}$ is easily seen to be the Hattori-Stallings rank r_P of the finitely generated projective modules P representing elements of $K_0(\mathbb{Q}G)$.

If, for a group G, $H_i(C_x/\langle x\rangle;\mathbb{Q}) = 0$ for all large i then it follows that $r_P = \Sigma\, r_P(x)[x]$ vanishes on elements x of infinite order ("Bass conjecture"). We show, using different techniques, that this is the case for the following classes of groups with $hd_{\mathbb{Q}}G = n < \infty$:

(a) solvable groups

(b) linear groups $G \subseteq GL_n(F)$, char F = 0 ,

(c) groups with $vcd_{\mathbb{Q}}G \leq 2$

J.-C. HAUSMANN:

A Kan-Thurston theorem for duality groups

Definition: A group G is in the class $\mathbb{D}_n$ if there exists a $\mathbb{Z}G$-module D and $e \in H_n(G;D)$ so that

$$- \cap e : H^k(G;B) \longrightarrow H_{n-k}(B \otimes_{\mathbb{Z}} D)$$

is an isomorphism for all $\mathbb{Z}G$-module B, and if BG = K(G,1) is homotopy equivalent to a finite complex of dimension n.

Let $\mathbb{D} = \bigcup_n \mathbb{D}_n$.

Theorem: For any finite complex X, there exists $G_X \in \mathbb{D}$ and a map $f: BG \longrightarrow X$ such that $f_*: H^*(BG;B) \longrightarrow H_*(X;B)$ is an isomorphism for all $\mathbb{Z}\pi_1(X)$-modules B.

Problem: Can we find $G_X \in \mathbb{D}_n$ as above with n = homotopy dimension of X ?

There exists also a group G_X as above with $G_X \in \mathbb{D}_\infty$, where $\mathbb{D}_\infty$ is is the class of groups G such that $H^*(G;\mathbb{Z}G) = 0$ (but such a G_X is not finitely generated).

J. HOWIE:

Equations over groups and pictures

Given a system $p_1(\underset{\sim}{X}) = \ldots = p_m(\underset{\sim}{X}) = 1$ of equations in a set $\underset{\sim}{X} = \{ X_1, \ldots, X_n \}$ of unknowns over a group G , we construct a pair $K \subset L$ of 2-complexes, such that the system is solvable in an overgroup of G iff $\pi_1 K \longrightarrow \pi_1 L$ is injective.

This problem, and others, can be studied using spherical diagrams over the quotient complex L/K , or their dual - spherical pictures. This method is applied, in particular, to the one-relator product

$$G = \frac{A * B}{\langle r^m \rangle}, \text{ where } m \geq 4.$$

Versions of the Freiheitssatz and Lyndon's identity theorem for this object are proved, among other results.

J. HUEBSCHMANN:

The cohomology of nilpotent groups

Let G be a nilpotent group and let

$$G = G_0 \supset G_1 \supset \ldots \supset G_n = e$$

be a central series so that the associated graded group (say) A is abelian. Then G may be viewed as A together with a perturbation of its multiplication law. Accordingly, an appropriate perturbation applied to a free resolution for A yields a free resolution for G. We shall offer an explicit perturbation of this kind which applies to the standard small (Koszul type) free resolution for A. Among others this yields explicit free resolutions for arbitrary nilpotent groups. For small nilpotency class and few generators, the resolution is so explicit that computations can be done by hand.

S. JACKOWSKI:

Group automorphisms inducing the identity map on cohomology

Report on a joint paper with Z. Marciniak.

Let G be a finite group. The automorphisms of G which induce the identity map on integral cohomology of G form a subgroup $Aut^*(G)$ of the group of all automorphisms Aut(G). The Atiyah spectral sequence which relates cohomology of a group to its complex representation ring suggests that $Aut^*(G)$ is related to the subgroup $Aut_c(G) \subset Aut(G)$ consisting of automorphisms preserving conjugacy classes. Unfortunately $Aut^*(G) \not\subset Aut_c(G)$. However we prove that any $\varphi \in Aut^*(G)$ preserves conjugacy classes of elements of <u>prime</u> order. Moreover, assuming that the order of φ is prime to p we prove that φ preserves conjugacy classes of elements whose order is a power of p. For soluble groups we deduce that $|Aut^*(G)|$ is divisible only by primes dividing $|G|$.

Clearly the group of inner automorphisms $Inn(G) \subset Aut^*(G)$. Trying to find non-inner automorphisms in $Aut^*(G)$ we defined a subgroup $Aut_v(G)$

consisting of the automorphisms of G which are restrictions of inner automorphisms of the group ring $\mathbb{Z}G$ which normalize the subset $G \subset \mathbb{Z}G$. For every group G, $Aut_v(G) / Inn(G)$ turns out to be an elementary abelian group and for groups with normal Sylow 2-subgroup $Aut_v(G) = Inn(G)$.

P.H. KROPHOLLER:

Cohomological dimension of soluble groups

Theorem: If G is a soluble group then the following are equivalent

(i) $cd(G) = hd(G) < \infty$

(ii) G is of type (FP)

(iii) G is a duality group

(iv) G is torsion-free and constructible

Then implication (i) $\Longrightarrow$ (iv) will be discussed: It depends on computing cohomology groups $H^1(A;M)$ where A is a period abelian group and M is a direct sum of finite dimensional $\mathbb{C}A$-modules. The other implications follow from work of Bieri, Gildenhuys and Strebel.

R. LEE:

Cohomology of the moduli space of K-3 surfaces of degree two

Joint work with F. Kirwan.

The cohomology of the moduli space of K-3 surfaces is related to the cohomology of discrete subgroups in SO(2,19). Using the method of F. Kirwan in equivariant cohomology theory, we studied these moduli spaces, and obtained a complete answer of its intersection cohomology and its usual cohomology at the situation of K-3 surfaces of degree two.

M. LUSTIG:

Preferred points on hyperbolic surfaces

Finitely many preferred points on an orientable surface of small genus (i.e. $M_{g,r}$ with $(g,r) = (2,0)$ or $(1,1)$), provided with a hyperbolic structure, are exhibited as the only possible intersection points of simple geodesics with geometric intersection number 1. For $M_{2,0}$ they coincide with the six Weierstraß points ($M_{2,0}$ understood as Riemannian surface), but no analogous statement is true for surfaces of higher genus.

As algebraic consequence one obtains the splitting of a subgroup of finite index in Aut $(\pi_1 M_{g,r})$ (g,r as above) into a semidirect product $R \rtimes S$, where R is the normal subgroup of inner automorphisms.

Z. MARCINIAK:

Cyclic homology and idempotents in group rings

We study the Idempotent Conjecture:

Let G be a torsion-free group and let k be a field of char. 0. Then kG has no idempotents except 0 and 1.

We use Burghelea's Theorem on cyclic homology:

$$HC_*(kG) \approx \bigoplus_{c \in T_0 G} H_*(G_c;k) \otimes HC_*(k) \oplus \bigoplus_{c \in T_\infty G} H_*(G_c;k)$$

Here $T_0 G$ denotes the set of conjugacy classes of torsion elements in G, $T_\infty G$ the remaining classes; $G_c = C_G(z)/\langle z\rangle$ for some $z \in c \subseteq G$.
For each $n >> 0$ we construct functions $t_c^n : HC_{2n}(kG) \longrightarrow k$ which coincide with the Hattori traces when $n = 0$. We prove that they factor through $H_{2n}(G_c;k)$ for $c \in T_\infty G$.
Moreover, for each $e = e^2 \in kG$ we define elements $e^{(n)} \in HC_{2n}(kG)$ so that for all c holds: $t_c^n(e^{(n)}) = t_c^{(n+1)}(e^{(n+1)})$.
We prove

Theorem: Let G be a torsion-free group.

If for each $c \in T_\infty G$ there is $n(c) \gg 0$ with $H_{2n(c)}(G_c;k) = 0$

then kG has no idempotents except 0 and 1.

It applies immediately e.g. to polycyclic-by-finite groups.

We also give a sketch of an algebraic proof of Burghelea's Theorem.

W. METZLER:

Deficiency of free products and homotopy type of 2-complexes

Joint work with Cynthia Hog & Martin Lustig.

Generators of a free product can be transformed to lie in the factors (Grushko). A corresponding question for 2-complexes is, whether $K_\pi \simeq K_{\pi_1} \vee K_{\pi_2}$ if $\pi = \pi_1 * \pi_2$.

Counterexamples to this splitting arise by a construction which also yields that the deficiency in general is not additive under the operation of forming the free product of groups. The factors may even be chosen to be finite abelian. There exist many presentations for the same examples which may contribute to the homotopy theory of 2-complexes, for instance whether homotopy type and simple homotopy type always coincide.

M.L. MIHALIK:

End invariants of finitely presented groups

Two end invariants of finitely presented groups and their relationships to group cohomology will be discussed. They are simple connectivity at ∞ and semistability at ∞. If a finitely presented group G is simply connected at ∞, then $H^2(G; \mathbb{Z}G)$ is trivial. If G is semistable at ∞, then $H^2(G; \mathbb{Z}G)$ is free abelian.

The two results to be discussed are:

<u>Theorem</u> A: Assume G is a finitely presented infinite solvable group with commutator series

$$G \rhd G^{(1)} \rhd \ \dots \ G^{(n)} \rhd \ G^{(n+1)} = 1$$

If $G^{(n)}$ contains an element of infinite order, then either G is simply connected at ∞, or G contains a subgroup A, of finite index, and A contains a finite normal subgroup N, such that A / N is one of the groups $\langle x,y : x^{-1} y x = y^p \rangle$, for some integer p.

<u>Theorem</u> B: If all 1-ended finitely presented groups are semistable at ∞ , then all finitely presented groups are semistable at ∞ .

J. MOODY:

An induction theorem for polycyclic-by-finite groups

Starting with the group algebra $k\Gamma$ of a poly-$\mathbb{Z}$-by-finite group Γ over a noetherian domain k we first stabilize a little, replacing k by some $M_n(k)$ and Γ by some $\mathbb{Z}^d \rtimes \Gamma$, suitably chosen.

The result is an Ore localization of a graded noetherian ring $X = X_0 \oplus X_1 \oplus \dots$, such that X and X_0 have finite dimension over each other, and X_0 is a product of matrix algebras over k-group algebras of certain subgroups of Γ of strictly smaller rank.

It follows that

$$K_0'(X) \longrightarrow\!\!\!\!> K_0'(k[\mathbb{Z}^d \rtimes \Gamma]) \longrightarrow\!\!\!\!> K_0'(k\Gamma)$$

and for $i \geq 0$ $\quad K_i'(X_0) \cong K_i'(X)$

and the group $K_i'(X_0)$ splits into the direct sum of K_i' groups of k-group algebras of these subgroups.

Repeating the process for each of these smaller groups, and so on, we arrive at finite groups and the surjectivity of the induction map $\bigoplus_{\substack{H \leq \Gamma \\ \text{finite}}} K_0(kH) \longrightarrow K_0(k\Gamma)$ generalizing Brauer-Swan induction theorem to the case of poly-$\mathbb{Z}$-by-finite groups.

W.D. NEUMANN:

A geometric invariant of discrete groups

Joint work with Robert Bieri and Ralph Strebel.

If G is a finitely generated group and

$$S(G) = (\mathrm{Hom}(G, \mathbb{R}) \setminus \{0\}) / \mathbb{R}^*_+$$

we define a subset $\Sigma \subseteq S(G)$ which captures the information about finite generation of normal subgroups of G with abelian quotients. More generally, if A is a finitely generated G-operator group on which G' acts by inner automorphisms, we define a subset $\Sigma_A(G) \subseteq S(G)$ which contains the information about finite generation of A over subgroups of G. These subsets are open and functorial in a suitable sense. We describe various properties and computations of these sets.

J. RATCLIFFE:

A uniqueness theorem for amalgamated product decompositions of groups

A uniqueness theorem for amalgamated product decompositions

$$G = A *_C B$$

of a group will be discussed.

A splitting theorem for a 3-manifold along a properly embedded annulus will be given as an application.

B. RENZ:

Endlichkeitseigenschaften von Normalteilern in Gruppen vom Typ F_n

Let G be a group of type F_n, i.e. G has a K(G,1)-complex with finite n-skeleton. The character sphere S(G) if G is defined as

$S(G) = (\mathrm{Hom}(G, \mathbb{R}) - \{0\})$ modulo multiplication by positive reals;

$S(G) \approx S^{d-1}$, where d is the $\mathbb{Z}$-rang of the abelianization G/G' of G. For $1 \leq k \leq n$ one can define open subsets ${}^*\Sigma^k \subseteq S(G)$. These geometric invariants of the group G generalize the invariant Σ introduced by Bieri, Neumann and Strebel. In particular they contain complete information about the finiteness properties F_k of normal subgroups N of G with abelian quotient.

Let $S(G,N) = \{ [\chi] \in S(G) \mid \chi(N) = 0 \}$. The criterion then is the following

Theorem: Let G be a group of type F_n and N a normal subgroup of G with $G' \leq N \leq G$. Then

N is of type F_k for $1 \leq k \leq n$ if and only if $S(G,N) \subseteq {}^*\Sigma^k$

The openess of ${}^*\Sigma^k$ implies that the finiteness property F_k for normal subgroups with quotient $\cong \mathbb{Z}^j$ in a group of type F_n is an "open condition". This was proved for finitely presented normal subgroups by D. Fried and R. Lee.

D.J. ROBINSON:

Cohomology of locally supersoluble groups

There has been much work on the (co)homology of nilpotent groups and more recently of locally nilpotent groups. We announce some results which show that there is a similar theory for the cohomology of a supersoluble and locally supersoluble group G. The results take the form of vanishing theorems for modules which have no non-zero cyclic G-submodules or G-quotients.

U. STAMMBACH:

On the non-vanishing of Ext between simple modules over a finite group

Joint work with P. Linnell.

Let G be a finite group and let k be a field of characteristic p, where p / |G|.

Theorem: (a) G p-constrained; M, N simple kG-modules in the principal block.

Then there exists $r \geq 0$ with $\operatorname{Ext}^r_{kG}(M,N) \neq 0$

(b) G p-solvable; M,N simple kG-modules in the same block.

Then there exists $r \geq 0$ with $\operatorname{Ext}^r_{kG}(M,N) \neq 0$

An example shows that it is not possible to replace in part (a) the principal block by an arbitrary block.

O. TALELLI:

On groups with property $\mathcal{P}_1$

Definition: A group G is said to have $\mathcal{P}_1$ if there exists a $\mathbb{Z}G$-module A such that

(i) $\text{p.d}_{\mathbb{Z}G} A \leq 1$

(ii) $H^0(G;A) \neq 0$, and

(iii) A is torsion-free as $\mathbb{Z}$-module.

We prove

A: Let $G = \varinjlim_{i \in I} G_i$ with G_i finitely generated accessible subgroups of G and $|I| = \aleph_n$.

If G has $\mathcal{P}_1$ then G is the fundamental group of a graph of finite groups.

It follows that if G is torsion-free with $|G| = \aleph_n$ and has $\mathcal{P}_1$ then G is free.

B: If G is torsion and has $\mathcal{P}_1$ then G is a countable locally finite group.

We then show

C: Countable locally finite groups have $\mathcal{P}_1$

D: Certain groups of period q after 1-step have $\mathcal{P}_1$

Note that B + C imply that a torsion group G has $\mathcal{P}_1$ iff it is a countable locally finite group.

K. VOGTMANN:

The Euler characteristic of the group of automorphisms of a free group

In joint work with M. Culler, we produce a finite dimensional contractible simplicial complex on which the group $Out(F_n)$ of outer automorphisms of a free group acts with finite stabilizers.

This talk is a report on joint work with J. Smillie, in which we use this complex to obtain a generating function for the Euler characteristic $\chi(Out(F_n))$. Computations for small values of n show apparent pattern in the primes appearing in the denominators. By studying automorphisms of finite graphs we find an upper bound, which is often exact, for the power of a prime which appears in this denominator.

P.J. WEBB:

Sequences of cohomology groups

A simplicial action of a finite group G on a simplicial complex Δ will be called admissible if for every simplex $\sigma \in \Delta$ the isotropy group G_σ fixes σ pointwise. We fix a prime p and let $\mathcal{C}$ be the class of subgroups $\{ H \leq G \mid H$ has a normal p-subgroup $H_p \triangleleft H$ with H/H_p cyclic $\}$ Let $\mathfrak{X}$ be a class of subgroups of G closed under subconjugation.

Theorem: Let G act admissibly on a finite simplicial complex Δ and suppose that for all subgroups $H \in \mathcal{C}$ with $O_p(H) \notin \mathfrak{X}$ the fixed points Δ^H are mod p acyclic. Then

(a) For every $\mathbb{Z}G$-module M and integer $n \geq 1$ there are split exact sequences

$$0 \longrightarrow H^n(G,\mathfrak{X};M)_p \longrightarrow \bigoplus_{\sigma \in \Delta/G} H^n(G_\sigma,\mathfrak{X}_{G_\sigma};M)_p \longrightarrow \cdots$$

$$\longrightarrow \bigoplus_{\sigma \in \Delta/G} H^n(G_\sigma,\mathfrak{X}_{G_\sigma};M)_p \longrightarrow 0$$

and another sequence with the same groups and the arrows in the reverse direction. The notation indicates relative cohomology, and $\mathfrak{X}_{G_\sigma} = \{ H \in \mathfrak{X} \mid H \leq G_\sigma \}$.

(b) In the case $\mathfrak{X} = \{ 1 \}$ the chain complex $C_\bullet(\Delta) \otimes_{\mathbb{Z}} \mathbb{Z}_p$ has a split acyclic subcomplex $D_\bullet$ so that in each dimension $C_r(\Delta) \otimes \mathbb{Z}_p = D_r \oplus P_r$ where P_r is a projective $\mathbb{Z}_pG$-module.

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