MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 25/1986

Gambling and Optimal Stopping

8.6. bis 14.6.1986

Die Tagung fand unter der Leitung von Herrn Prof. Dr. L. Dubins (Berkeley) und Herrn Prof. Dr. U. Krengel (Göttingen) statt. Es nahmen 35 Wissenschaftler aus insgesamt 10 Ländern teil.

Es wurden 27 Vorträge gehalten. Der Schwerpunkt lag bei den wahrscheinlichkeitstheoretischen Strategie-Problemen mit diskreter Zeit. Zum Thema "Gambling" gab es u.a. Beiträge über die Potentialtheorie von "gambling houses", Ziel-Probleme, Existenz stationärer Strategien und einen Vortrag über Strategien beim Roulette. Ein wiederkehrendes Thema waren "Propheten-Ungleichungen" (bei negativer Abhängigkeit, iteriertem Stoppen, freier Wahl der Beobachtungsrechenfolge, Transformierte von Prozessen, etc.). Andere Fragestellungen aus der Theorie des optimalen Stoppens betrafen nichtlineare Beobachtungskosten, mehrparametrige Prozesse, Bayes Schätzer und sequentielle Bestimmung des Zeitpunkts eines Wechsels. Ferner gab es interessante Vorträge über Banditenprobleme (sequentielle Wahl von Experimenten), Markov'sche Entscheidungsprobleme und Ungleichungen für Martingal-Transformierte.

Den Mitarbeitern des Instituts gebührt besonderer Dank für die hervorragende Organisation, Unterbringung und Verpflegung.

Teilnehmer

Ahlsmeyer, G., Kiel
Berti, P., Florenz
Berry, D.A., Minneapolis
Bruss, F.Th., Namur
Burkholder, D. Urbana
Dellacherie, C., Bois-Guillaume
Dieter, U., Graz
Dubins, L., Berkeley
Gilat, D., Tel Aviv
Gittins , J.C., Oxford
Heath, D., Ithaca
Hill, T.P., Atlanta
Hordijk, A., Leiden
Irle, A., Kiel
Jensen, U., Stuttgart
Kadelka, D., Karlsruhe
Kennedy, D.P. Cambridge
Kertz, R.P., Atlanta
Krengel, U., Göttingen
Kohlmann, M., Konstanz
Mazziotto, G., Issy les Moulineaux
Millet, A., Angers
Pestien, V.C., Coral Gables
Ramakrishnan, S., Coral Gables
Rieder, U., Ulm
Rinott, Y., Jerusalem
Rösler, U., Göttingen
Schäl, M., Bonn
Schmitz, N., Münster
Schulte-Geers, E., Osnabrück
Siegmund, D. Stanford
Sonin, I.M., Moskau
Stadje,W., Osnabrück
Sucheston,L., Columbus
Sudderth, W., Minneapolis

Vortragsauszüge

D.A. BERRY : Bandit Problems and Optimal Stopping

Consider two Bernoulli sequences $X_1, X_2, \ldots, Y_1, Y_2, \ldots$ where $P(X_i=1) = \Theta$ and $P(Y_i=1) = \lambda$; given (Θ,λ), all X's and Y's are independent. A strategy indicates at each stage i whether to observe X_i or Y_i (call the resulting observation Z_i), depending on all previous selections and observations. Suppose λ is known, and Θ is unknown with known prior distribution measure F. The objective is to maximize $E[\sum_1^\infty \alpha_i Z_i | F,\lambda]$ where $A = (\alpha_1, \alpha_2, \ldots)$ with $\alpha_i \geq 0$ and $\sum_1^\infty \alpha_i < \infty$ is a discount sequence. Necessary and sufficient conditions on A are given for the bandit problem to be one of optimal stopping for all (F,λ): When is the problem such that the decision maker need only decide when to stop observing the X's and switch permanently to the Y's?

"Index strategies" are examined when there are k processes with unknown characteristics.

F.Th. BRUSS : Stationary decision strategies

Suppose that one or more decisions have to be made on a given time interval $[0,t]$ and that neither the number nor the qualities of options are known in advance. Decisions have to be spontaneous, and they are irrevocable. The aim is to find a strategy which maximizes the expected payoff as a function of a certain number of top options.

We shall investigate the possibilities of modelizing this type of decision problems and briefly discuss the advantages and disadvantages of existing Best Choice models. A "waiting time" model will be selected for its simplicity and for the performance of the corresponding optimal strategies, and we shall describe the methods to prove the existence of optimal solutions and to compute them explicitly. It is interesting to see that in many cases optimal behaviour does not depend on sequential observation of the arrival process and this class of (stationary) strategies will be used to propose a simple model of self-teaching decision strategies. Several concrete problems will be given to motivate our approach.

D.L. BURKHOLDER : Sharp inequalities for martingale transforms and the optimal control of martingales

The best constants in some old and some new martingale inequalities can be obtained by using biconvex and biconcave functions u and zigzag martingales Z . These results and methods have applications to the study of the optimal control of martingales and to problems in functional and harmonic analysis. They lead to some strage inequalities for stochastic integrals, the unconditional constant of any monotone basis of $L^p(o,1)$, and to new results about singular integral operators.

C. DELLACHERIE : Potential theory for gambling houses

We show how nice are the analytic gambling houses through an exposition of the most inportant results included in the third volume of "Probabilitês et Potentiel" (joint work with P.A. Meyer): Mokobodyki's theorem on analytic sublinear functionals, analyticity of the balayage order, extension of Strassen's theorem,...

U. DIETER : Roulette as a ruin game: Optimal strategies

Roulette is considered as a ruin game: the gambler starts with the amount z and plays until he is either ruined (loss z) or has reached a pre-fixed capital a (gain a-z). If he is only allowed to bet on even chances, this model is treated in various textbooks. However, if he is allowed to bet on different combinations of numbers, the model can only be discussed by theory combined with computer calculations.

If the strategy is fixed for every capital z , the ruin probabilities, the mean duration of the game and his mean stakes can be calculated recursively by a kind of Gauß-Seidel iteration.

Furthermore, optimal strategies, which minimize the ruin probabilities, can be calculated if the set of possible strategies is finite. The numerical values are close to the ones which are given by the Dubins-Savage theory for an idealized roulette game. In European casinos, where the *prison rule* is usually valid, gambling on even chances (red, black,...) is preferred if $z \geqq a/3$; at American casinos betting on single numbers has more advantages. This empirical observation was confirmed by computer calculations.

D. GILAT : Prophet inequality with order selection

A complete determination is made of the possible values of $m = E(X_1 \vee X_2)$ and $w = \max\{E(X_1 \vee EX_2), E(X_2 \vee EX_1)\}$ for X_1, X_2 bounded and independent random variables. It turns out, for example, that for X_1, X_2 ranging over an interval of length L, the maximum of the difference $m - w$ is $0.09L$, while the least upper bound of the ratio m/w, for nonnegative independent X_1, X_2, is $5/4$. These bounds are strictly bigger than the corresponding ones for i.i.d. X_1, X_2 which have been obtained by Hill & Kertz (1982). The problem was implicitly suggested by T.P. Hill (1983) and has yet to be resolved for three or more random variables for which a plausible conjecture is formulated.

J.C. GITTINS : Queues of competing projects

A policy which gives priority on the basis of dynamic allocation indices is known to be optimal for a family of alternative bandit processes, which includes the use of a set of jobs which yield discounted rewards on completion. The result extends to the situation of jobs subject to precedence constraints in the form of an out-tree, which includes the possibility of a random arrival process for further jobs. Only, however, in the limiting case as the discount rate tends to 1 is it possible to derive a tractable explicit form for the dynamic allocation indices. The paper discusses what can be done in the practically relevant situation of a general discount rate, and jobs with a completion (hazard) rate which has a unique local maximum.

D. HEATH : Betting to leave an interval

In treating a problem in combinatorial optimization, Joel Spencer conjectured that a gambler betting on the outcome of a coin toss, restricted to bets of size 1 or smaller and attempting to win or lose at least G (an integer) in N bets, should always bet 1 until winning +/-G. We shall discuss Spencer's application and, using the framework of Dubins and Savage gambling theory, prove the conjecture.

Th.P. HILL : Prophet inequalities: some applications and open problems

Several generalizations of the original prophet inequalities will be briefly discussed, followed by a series of applications of prophet inequalities to problems involving order selection, non-measurable stop rules, look-ahead stop rules, iterated maps, and a double-process inequality. Some general open problems concerning prophet inequalities will be mentioned.

A. HORDIJK : Sensitive optimal policies in denumerable Markov decision chains

In this talk we consider a discrete-time Markov decision chain with a denumerable state space and compact action sets and we assume that for all states the rewards and transition probabilities depend continuously on the actions.

An analysis for average and more sensitive optimality without assuming a special Markov chain structure is presented. In doing so, new conditions which include completely the finite state and action model are given.

A. IRLE : On optimal stopping with concave costs of observation

For optimality results in sequential analysis which contain explicit description at optimal strategies the assumption of linear costs of observation has been of major importance. In this talk we investigate the question how the shape of nonlinear costs of observation influences the shape of optimal stopping boundaries. It is found that costs of observation of the form $ct^{a+\frac{1}{2}}$, $o < a < \frac{1}{2}$, lead to optimal stopping boundaries which grow in the order of $t^{\frac{1}{2}-a}$ a $t \to \infty$.

U. JENSEN : Optimal stopping rules for processes in semimartingale representation

Let $(\Omega, \mathcal{F}, P)$ be a probability space with a filtration $(\mathcal{F}_t), t \in \mathbb{R}_+$ and $Z = (Z_t), t \in \mathbb{R}_+$ a stochastic process adapted to this filtration. The problem of optimal stopping in continuous time is then to maximize $EZ_\tau, \tau \in C$ in a given class C of $(\mathcal{F}_t)$-stopping times. This problem is considered for processes Z which admit the following semimartingale representation

$Z_t = Z_o + \int_o^t fs\, ds + M_t$, where $E|Z_o| < \infty$, (f_t), $t \in \mathbb{R}_+$ is a progressively measurable process with $E(\int_o^t |fs| ds) < \infty$ and M is a martingale.

The monotone case in continuous time is introduced similar to that one considered by A. Irle. Conditions are given under which the so called infinitesimal-look-ahead stopping rule is optimal. Furthermore it is shown that the reduction of the semimartingale representation to discrete time leads to the well known discrete monotone case. But the integrability conditions which are necessary to prove the optimlity of the one-step-look-ahead stopping rule differ slightly from the classical ones.

D. KADELKA : On existence of optimal policies in stochastic scheduling

A general model of stochastic scheduling for a project with n jobs (activities) is presented for the preemptive case (running jobs may be interrupted at some cost) and the nonpreemptive case. A particular realization of the jobs duration vector with joint probability measure becomes increasingly known during execution of the project. The problem is to find a schedule plan which minimizes the expected costs, where some continuous cost function is given. Constraints, such as precedence, are allowed. As it turns out the main point in proving the existence of an optimal schedule plan is to show the existence of an optimal stopping rule for some particular stopping problem.

D.P. KENNEDY : Prophet-type inequalities for multi-choice optimal stopping

Let $\{X_n, n \geq 1\}$ denote a sequence of independent, non-negative random variables and let T denote the set of finite-valued stopping-times with respect to the natural filtration of the sequence. The now well-known prophet inequality due to Krengel, Sucheston and Garling (cf. [2],[3]) gives

$$E \sup_n X_n \leq 2 \sup_{T \in T} EX_T .$$

This is referred to as a "phrophet" inequality because the left-hand side is the expected gain to a prophet with complete foresight from choosing one of the sequence while $v_1 = \sup_{T \in T} EX_T$ is the maximal expected reward that a gambler may achieve using non-anticipative stopping rules.

Let $v_r = \sup E(X_{\tau_1}+\dots+X_{\tau_r})$, where the supremum extends over all stopping times $\tau_1<\dots<\tau_r$. It is shown that for each r there exists a (best) universal constant C_r, $1 < C_r \le 2$ with $E \sup_n X_n \le C_r v_r$. These constants C_r may be computed recursively with $C_1 = 2$, $C_2 = 2(2-\sqrt{2})$, and $C_r \to 1$ as $r \to \infty$.
In addition, if the random variables take values in $[0,1]$ it may be shown that $E \sup_n X_n \le F_r(v_r)$, where the functions F_r are defined recursively by

$$F_r(x) = \sup_{\frac{x}{r} \le y \le \min(x,1)} [y+y(1-y)F_{r-1}((x-y)/y)], \quad 0 \le x \le r,$$

for $r \ge 1$ with $F_0(x) \equiv 1$. This extends a result of Hill and Kertz [1].

R e f e r e n c e s

1. Hill T.P., Kertz R.P. Additive comparisons of stop rule and supremum expectations of uniformly bounded independent random variables. - Proc. Amer. Math. Soc. 1981, 83, p. 582-585.

2. Krengel U., Sucheston L. Semiamarts and finite values. - Bull. Amer. Math. Soc. 1977, 83, p. 745-747.

3. Krengel U., Sucheston L. On semiamarts and amarts, and processes with finite value. - In: Probability on Banach Spaces (ed. J. Kuelbs) - New York: Marcel Dekker.

<u>R.P. KERTZ</u> : <u>Prophet Problems in optimal stopping and stochastic Control</u>

For independent r.v.'s $X_1,X_2,\dots$ taking values in $[0,1]$, exact comparisons of $V(X_1,X_2,\dots)$ and $E(\sup_{j\ge1}X_j)$ have been given by Krengel and Sucheston, Hill, and others. First, an extension is given in which $V(X_1,X_2,\dots)$ is compared to $E(k^{-1}\sum_{i=1}^{k} M_i(X_1,X_2,\dots))$, where $M_i(X_1,X_2,\dots)$ is the i^{th} largest order statistic of the sequence $X_1,X_2,\dots$. Second, Krengel and Sucheston's variation of the original comparison results, in the setting of transforms of sequences of independent r.v.'s , is discussed. Throughout, reduction techniques are emphasized, and extremal distributions are given. This extension and variation provide insights into the original prophet comparison.

U. KRENGEL : Macroscopic models for processes with interaction (joint work with M. Akcoglu)

Assume that a large number N of particles are distributed among d states, such that f_iN particles are in state i , $(1 \le i \le d)$. The particles move independently of each other, but the probability of a transition of state i to state j may depend on $f = (f_i)$. E.g., i leads to $i+1$ with probability $(2-f_{i+1})/4$ and to i with probability $(2+f_{i+1})/4$. (Addition mod d). If N is large, the distribution Tf of the particles after one time unit is described by the frequencies

$$(Tf)_i = f_i(2+f_{i+1})/4 + f_{i-1}(2-f_i)/4 .$$

This is an example of a system in which the movement of the particles is affected by the distribution of the remaining particles. In contrast to the linear Markovian case, T is nonlinear. But T can be extended to an operator in $L = (\mathbb{R}^+)^d$ with the following properties: $T0 = 0$; $\int Tf = \int f$, where $\int f = \sum f_i$; and $f \le g \Rightarrow Tf \le Tg$. Under an aperiodicity condition fulfilled above, $T^n f$ converges to the unique fixed point $\tilde{f}$ with $\int f = \int \tilde{f}$. Under even more general conditions T becomes asymptotically periodic.

G. MAZZIOTTO : Stochastic control of two-parameter processes application: the two-armed bandit problem

(with A. Millet)

This paper studies a control problem for two-parameter stochastic processes which generalizes the classical two-armed bandit problem Given an upper semi-continuous process X indexed on $\Pi^2 = \mathbb{N}^2$ or $\mathbb{R}_+^2$, say $(X_{z};z\in\overline{\Pi}^2)$ such that $E(\sup_z|X_z|) < \infty$, we associate to an arbitrary optional increasing path $Z = (Z_u;u\in\Pi)$ in Π^2 the average pay-off

$$C_{X,V}(Z) = E\left(\int_0^\infty X_{Z_u}\, dV_u\right)$$

where (dV_u) is a fixed random measure on Π.

By developing a compactification method, which extends these from optimal stopping, we prove the existence of an optimal increasing path Z^* such that

$$C_{x,v}(Z^*) = \sup\{C_{x,v}(Z);\ Z \text{ optimal increasing path}\}$$

In the discrete case $(\Pi = \mathbb{N})$, an explicit construction of the optimal solution is obtained.

A. MILLET : A new probabilistic approach of the reduite
(with N. El Karoui and J.R. Lepeltier)

We use randomized stopping times to study the reduite $R^{\alpha}f(x) = \sup \mathbb{E}_x(e^{-\alpha\tau}f(X_\tau))$ of a function f for a strong Markov process (X_t). We obtain a unified approach of several known results: independence of the realization, continuity, connection with Snell's envelope.

V. PESTIEN : Minimizing the expected time to reach a goal

An object moves on the negative half line according to an Ito process, where the infinitesimal mean and standard deviation at each point are chosen from a given control set. The problem of minimizing the expected time to reach zero is formulated as a continuous-time gambling problem, and the standard Bellman-equation approach from optimal control theory is seen to be inadequate. Necessary and sufficient conditions are established on the control set for zero to be attainable in arbitrarily small expected time. A new "verification lemma" is presented as a tool for characterizing the optimal return function. Examples are discussed, and the theory is extended to cover certain situations where the set of available controls depends on the position of the object. The talk is based largely on joint work with D. Heath, S. Orey, and W. Sudderth.

S. RAMAKRISHNAN : Non-existence of uniformly adequate stationary plans on a fortune space of cardinality c

Under the axiom of choice, there exists a fortune space of cardinality c, the gambling house consisting of discrete gambles with almost a twopoint support, each fortune having almost three gambles available at it (the gambling problem is in the sense of Dubins and Savage (1965)), and for this gambling house, stationary plans are not uniformly adequate. This result improves a theorem of Ornstein (1969).

Y. RINOTT : Optimal stopping values and prophet inequalities for negatively dependent random variables
(joint work with Ester Samuel-Cahn)

Let $\underset{\sim}{X} = (X_1,\dots,X_n)$ be a vector of random variables and $V(\underset{\sim}{X}) = \sup_t\{EX_t : t$ is a stoprule in $X_1,\dots,X_n\}$. For certain negatively dependent random variables $\underset{\sim}{Y} = (Y_1,\dots,Y_n)$ (to be defined and discussed in the talk) we obtain the inequality $V(\underset{\sim}{X}) \leq V(\underset{\sim}{Y})$, where X_i are independent, having the same marginal distributions as $Y_i, i=1,\dots,n$. This was motivated by a result of O'Brien comparing stopping values for sampling with and without replacement.

We obtain prophet inequalities of the type given by Krengel and Sucheston and Kennedy (and others) for negatively dependent variables. Threshold rules turn out to be useful in these comparisons. The results are extended to infinite sequences.

We also consider partial replacement schemes (to be defined) and indicate some results and open problems.

M. SCHÄL : On the chance to visit a goal set infinitely often

The probability of visiting a goal set infinitely often is a typical criterion in the theory of gambling founded by Dubins and Savage in their book "How to gamble if you must". This criterion is more difficult to handle than the usual criteria in dynamic programming (total return and average return per unit time). So the existence of optimal strategies was known only for a model with finite state space and finite action space. In the present paper that existence result will be extended to the case of a compact action space under the continuity assumptions known from the average return criterion. Also the methods of proof are borrowed from dynamic programming with the average return criterion.

D. SIEGMUND : Sequential detection of a change-point

The problem of sequentially detecting a change of distribution is introduced. The procedures of Page (1954) and Shiryayev (1963) are described; and for the case of two completely specified distributions the optimality properties obtained by Shiryayev (1963), Lorden (1971), and Pollak (1985) are reviewed. For detecting a one-sided change in the drift of Brownian motion, approximations to the average run length are given and used to compare the Page and Shiryayev procedures numerically. Some results which indicte how one can make similar comparisons for discrete time processes are given. Extensions to more complex situations are briefly discussed.

I.M. SONIN : The theorem on separation of jets and some properties of random sequences

Three related topics are treated in the talk. A mathematical model regarding asymptotic behaviour of the nonhomogeneous solution in a system of vessels (discrete stream) is considered. The theorem (on separation of jets) states that every stream with a bounded number of vessels in every moment can be decomposed into such jets that stabilization of volume and concentration takes place and every jet and overflow between different jets are finite on an infinite time interval. This theorem may be reformulated also in terms of nonhomogeneous Markov chains. The above problem is connected with classical gambling - to maximize the probability to hit some particular set infinitely often. The result of T. Hill on existence of good Markovian strategies is generalized. The proof of both theorems uses essentially the theorem about the existence of nonrandom sequences (barriers) such that the expected number of intersections between barrier and trajectories of martingale type random sequences is finite on infinite time interval.

W. STADJE : A sequential estimation procedure for the parameter of an exponential distribution

A Bayesian sequential estimation problem for the parameter θ of n i.i.d. exponential variables $\xi_1,\dots,\xi_n$ is considered. If observation is stopped at time t, a $\sigma(\xi_1\wedge t,\dots,\xi_n\wedge t)$-measurable estimator $\hat{\Theta}_x$ has to be used, and the loss is given by $(\hat{\Theta}_t-\theta)^2 + aN_o(t) + bt$, where $a,b > o$ and $N_o(t)$ is the number of ξ_j which are $\leq t$. One can interprete the ξ_i as the unknown lifetimes of n "units" so that at time t it is known, how many of them failed up to t and when these failures happened. For a gamma prior distribution of Θ the optimal stopping time and the minimal Bayes risk are found.

L. SUCHESTON : Prophet compared to gambler: case of transforms

Let r be an arbitrary integer, X_i, $i = 0,\dots,r$ are random variables, $EX_i = e_i$, $E(X_i|X_{i-1})=e_i$. The maximal gain G of the gambler is defined as

$$G = \sup_U \sum_{o\leq i<r} U_{i+1}(X_{i+1}-X_i)$$

where $U_{i+1}\in\sigma(X_i)$, $0 \leq U_i \leq 1$. The maximal gain P of the prophet differs only in that U_i is arbitrary, $0 \leq U_i \leq 1$. The corresponding "signed" expressions are G_s and P_s, obtained by allowing $-1 \leq U_i \leq 1$.

Theorem 1: Assume $\mu(X_o) \leq \mu(X_r)$, where $\mu(X) = EX - E(X-EX)^+$. (If the random variables X_i are positive, this is not a loss of generality since one may add infront $X_o = 0$.) Then $P \leq 3G$. If $e_o = e_1 = \ldots = e_r$, then $P \leq 2G$.

Theorem 2: Assume $\mu(X_o) \leq \mu(X_r)$ and $e_o \geq e_r$. (Both conditions hold if $e_o = X_r$.) Then $P_s \leq 3G_s$. The constant 3 is optimal in both theorems.

(Joint work with Ulrich Krengel).

W. SUDDERTH : Continuous-time gambling

After certain results from descrete-time, measurable gambling have been recalled, two approaches to the continuous-time theory will be considered. The first approach is global in time while the second in local and defines the gambling problem by specifying infinitesimal parameters for Ito processes. The theory is illustrated by a continuous-time version of red-and-black, a pathological example is given to point out measurability difficulties.

Berichterstatter: U. Krengel

Tagungsteilnehmer

Dr. G. Alsmeyer
Mathematisches Seminar
Universität Kiel
Olshausenstr. 40
2300 Kiel 1

Prof. Dr. D. A. Berry
Dept. of Theoretical Statistics
University of Minnesota
270 Vincent Hall
Minneapolis, Minnesota 55455
U S A

Prof. Dr. Patrizia Berti
Istituto di Matematica
Applicata "G.Sansone"
Via S. Marta, 3
I-50139 Firenze
Italien

Dr. F. Th. Bruss
Department of Mathematics
University of Namur
8, Rempart de la Vierge
B-5000 Namur
Belgien

Prof. Dr. D. L. Burkholder
Department of Mathematics
University of Illinois
1409 West Green Street
Urbana, IL 61801
U S A

Prof. Dr. C. Dellacherie
169 rue Danguy
F-76230 Bois Guillaume
Frankreich

Prof. Dr. U. Dieter
Institut für Statistik
Technische Universität Graz
Lessingstr. 27
A-8010 Graz
Österreich

Prof. Dr. L. E. Dubins
Department of Mathematics
University of California
Berkeley, Calif. 94720
U S A

Prof. Dr. D. Gilat
School of Math. Sciences
Tel-Aviv University
Ramat-Aviv 69395
Israel

Dr. J. C. Gittins
Mathematical Institute
24-29 St. Giles
Oxford OX1 3LB
Grossbritannien

Prof. Dr. D. Heath
School of Operations Research
Cornell University
Upson Hall
Ithaca, N.Y. 14853
U S A

Dr. Th. P. Hill
School of Mathematics
Georgia Institute of Technology
Atlanta, Georgia 30332
U S A

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

Prof. Dr. A. Hordijk
Department of Mathematics
and Computer Science
University of Leiden
Postbox 9512

NL-2300 RA L e i d e n

Prof. Dr. M. Kohlmann

Fakultät für Wirtschaftswis-
senschaften und Statistik
Universität Konstanz
Postfach 5560

7750 K o n s t a n z 1

Prof. Dr. A. Irle

Mathematisches Seminar
Universität Kiel
Olshausenstr. 40

2300 K i e l 1

Prof. Dr. U. Krengel

Institut für Math. Stochastik
Universität Göttingen
Lotzestr. 13

3400 G ö t t i n g e n

Dr. U. Jensen

Institut für Angew. Mathematik
und Statistik der
Universität Hohenheim
Postfach 700562/110

7000 S t u t t g a r t 70

Prof. Dr. G. Mazziotto

PAA/TIM/DRI/PB - Centre National
d'Etudes des Télécommunications
38-40, Rue du Général Leclerc

F-92131 Issy les Moulineaux
Frankreich

Dr. D. Kadelka

Institut für Math. Statistik
Universität Karlsruhe
Postfach 6980

7500 K a r l s r u h e 1

Prof. Dr. Annie Millet

Faculté des Sciences
Université d'Angers
2, boulevard Lavoisier

F-49045 A n g e r s - Cedex
Frankreich

Prof. Dr. D. P. Kennedy

Statistical Laboratory
University of Cambridge
16 Mill Lane

C a m b r i d g e CB2 1SB
Grossbritannien

Prof. Dr. V. C. Pestien
Dept. of Math. & Computer Sc.
University of Miami
P.O.Box 249085

Coral Gables, Florida 33124

U S A

Prof. Dr. R. P. Kertz

School of Mathematics
Georgia Institute of Technology

A t l a n t a , Georgia 30332

U S A

Prof. Dr. S. Ramakrishnan
Department of Mathematics
University of Miami
P.O.Box 249085

Coral Gables, Florida 33124

U S A

Prof. Dr. U. Rieder

Abteilung Mathematik VII
Universität Ulm
Oberer Eselsberg

7900 U l m

Prof. Dr. Y. Rinott

The Hebrew University
Faculty of Social Sciences
Department of Statistics

J e r u s a l e m 91905
Israel

Prof. Dr. U. Rösler

Institut für Math. Stochastik
und Wirtschaftsmathematik
Lotzestr. 13

3400 G ö t t i n g e n

Prof. Dr. M. Schäl

Institut f. Angew. Mathematik
Universität Bonn
Wegelerstr. 6

5300 B o n n

Prof. Dr. N. Schmitz

Institut für Math. Statistik
der WWU Münster
Einsteinstr. 62

4400 M ü n s t e r

Dipl.-Math. E. Schulte-Geers
Fachbereich Mathematik/Inf.
Universität Osnabrück
Albrechtstr. 28
Postfach 4469

4500 O s n a b r ü c k

Prof. Dr. D. Siegmund

Department of Statistics
Stanford University

S t a n f o r d , CA 94305

U S A

Prof. Dr. I. M. Sonin

Economic Mathematical Institu
USSR Acad. of Sciences

M o s k a u

Sowjetunion

Prof. Dr. W. Stadje

Fachbereich Mathematik/Inf.
Universität Osnabrück
Albrechtstr. 28

4500 O s n a b r ü c k

Prof. Dr. L. Sucheston

Department of Mathematics
Ohio State University

C o l u m b u s , Ohio 43210

U S A

Prof. Dr. W. Sudderth
Dept. of Math. & Computer Sc.
University of Miami
P.O.Box 249085

Coral Gables, Florida 33124

U S A