MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 1/1987

Fraktale und ihre Bedeutung in Mathematik und Naturwissenschaften

4.1. bis 10.1.1987

Die Tagung wurde geleitet von den Herren B.B. Mandelbrot, Yorktown Heights und Harvard, H.-O. Peitgen, Santa Cruz und Bremen, und P.H. Richter, Bremen.

Ziel der Tagung war es, einen Überblick über das Gebiet der fraktalen Geometrie zu geben, das sich insbesondere seit der Entdeckung der Mandelbrot-Menge 1980 stürmisch entwickelt hat. Innerhalb sehr kurzer Zeit haben sich bereits so viele Teilgebiete herausgebildet, daß nicht alle auf der Tagung berücksichtigt werden konnten. Dennoch wurde im Gespräch zwischen Mathematikern und Physikern die fachübergreifende Bedeutung der Konzepte der fraktalen Geometrie sehr deutlich. Gemeinsame Grundlage der Diskussionen ist das Interesse an iterierten Abbildungen bzw. dynamischen Systemen. Während allerdings Mathematiker das Studium konformer Abbildungen bevorzugen, sind Physiker eher an reellen dissipativen und konservativen Prozessen interessiert.

The meeting attempted to give a survey on the field of fractal geometry which received a particularly strong impetus from the discovery of the Mandelbrot set in 1980. Within a rather short time, a number of special disciplines have developed, not all of which could be represented at the meeting. It became, however, apparent in the discussion between mathematicians and physicists that fractal geometry provides strong concepts with obvious applicability in many disciplines. The common denominator is the interest in iterated maps, i.e. in dynamical systems. While mathematicians prefer to exploit the analytic properties of conformal mappings, physicists are more immediately interested in real dissipative or conservative processes.

Monday, Jan. 5, 1987:

Morning session; chairman: B.B.MANDELBROT

R.L. DEVANEY: **Fractal images generated by families of transcendental functions**

The dynamics of entire transcendental functions such as the sine, cosine, and exponential functions provide interesting examples of bifurcation in complex dynamics. We prove that the Julia sets of these maps may explode as a parameter is varied, and that they may also pulse intermittently. These phenomena are illustrated by computer graphics images and several short computer generated films.

S. GROSSMANN: **How do Misters Navier and Stokes manage to produce a self-similar eddy field in turbulent flows ?**

The eddy energy distribution D(r) in a turbulent flow field is evaluated from the underlying hydrodynamic Navier-Stokes (NS) equation without introducing scaling arguments or an ad hoc ansatz. $D(r)=\langle\langle|\vec{u}(\vec{x}+\vec{r})-\vec{u}(\vec{x})|^2\rangle\rangle$ is regular, $(\varepsilon/3\nu)r^2$, for small r and of scaling form $b(\varepsilon r)^{2/3}$, $b=8.4\pm 20\%$, for the self-similar range. All this is derived from the NS-eq. by introducing a variable range -(r)- decomposition of the flow field, integrating out the small scales in a Galilean invariant (Lagrangian) manner, factorizing 4th order moments (mean field), and restricting to a Markovian approximation for the large scales. A closed non-linear diff.- int. equation for D(r) is obtained, Galilean invariant, free of infrared divergencies, without any adjustable parameter. Its solution reproduces quite well the experimental turbulent eddy distribution through all ranges r. The key to this mean field solution of the NS-eq. is the evaluation of the Lagrangian time correlation function that represents the eddy viscosity.

Afternoon session; chairman: P.H. RICHTER

F.M. DEKKING: **On the size of projections of random Sierpinski carpets**

This talk might be considered as a mathematical comment on the following quote from Mandelbrot's treatise: "In the limit, however, the projections of two points almost never coincide. The dust is so sparse as to leave space essentially transparent" (FGN, caption to plate 219 - Implementation of Hoyle's model using random curdling in a grid). The results are based on recent work in the theory of branching processes in a random environment.

T. BEDFORD: **Hölder exponents of self-affine functions**

A self-affine function f (possibly generated by non-linear affinities) has the property that there exists h>0 such that for Lebesgue almost all x the Hölder exponent of f at x equals h. In simple cases a formula can be given and there is an inequality involving the box dimension of graph (f) and h. A corollary is that for certain expanding maps of the cylinder, the unique repelling fractal circle has Hölder exponent $\chi_2(\mu)/\chi_1(\mu)$ at μ almost all points, where μ corresponds to the Bowen-Ruelle measure.

M. SERNETZ/H.R. BITTNER: **Organisms as bioreactors: Fractal structure and heterogeneous catalysis**

Organisms and bioreactors with immobilized enzymes correspond closely as open dissipative systems with respect to structure and function. Turnover in these multiphase systems is governed by reaction and transport, i.e. by heterogeneous catalysis. The fractal folding of tissue and branching of vascular transport systems is the structural equivalent to turbulent mixing in bioreactors. Models of vascular systems are achieved in recursive construction rules (growth and branching), using characteristic parameters, and assessed by geometrical and dynamical properties, such as fractal dimensions and the residence time distribution. The understanding of the organism as a fractal volume-area-hybrid with surface dimension $D_F = 2.25$ explains the so-called reduction law of metabolism and is of essential importance for the standardization of physiological quantities.

Tuesday, Jan. 6, 1987:

Morning session; chairman: S.J. PATTERSON

P.CVITANOVIĆ: **Renormalization description of strange attractors**

We apply renormalization techniques to description of strange attractors of Hénon type. In contrast to all of the previous applications of renormalization methods to this subject, which describe transitions from orderly motions to weak forms of turbulence, such as aperiodic motions of length 2^{∞}, we describe truly chaotic strange attractors. We use the unstable periodic orbits of increasing length to resolve the attractor. The corresponding cycle points lie on a pair of binary trees; the self-similarity of these binary trees, and the simple representation of the forbidden cycles in the binary representation make feasible the renormalization description.

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G. MAYER-KRESS: **Numerical studies on the reliability of dimension algorithms**

The concept of fractal dimensions has been proved to be an important tool for the analysis of complex time series data, such as the ones generated by non-linear, chaotic dynamical systems.

The classical method of spectral analysis is in these cases only of limited use, since typically the spectra of chaotic systems contain a continuum of frequencies. With the help of fractal dimensions it is in principle possible to distinguish between chaotic time series with identical spectra and quantify the number of non-linear modes involved in the generation of the data.

Today, there exist several numerical algorithms for estimating variants of the fractal dimension. For sets which are defined recursively with no limitations to resolution, many interesting and rigorous results have been obtained.

The measuring process for realistic chaotic systems, however, gives rise to new questions as to how to obtain reliable information about these dimensional observables. We simulated various algorithms for situations as they occur in realistic experiments, where measuring time and resolution are limited. Furthermore in typical experimental situations the scaling behavior, which is assumed in the definition of dimensions is not always given. This can lead to logarithmic corrections. We discuss numerical results from representative model systems and compare them to experimental data from human electro-encephalograms (EEG).

O.E. RÖSSLER: **Cloud attractors**

They were discovered by Julia (1918) as the inverses of a repelling Julia boundary (with coin tossing deciding the alternative pre-images) in a rational noninvertible map. A first real example was given by Yorke et al. (1979). Barnsley and Harrington (1985) and Barnsley (1985) found arbitrary self-similar attracting fractal patterns to be possible in piecewise continuous (even piecewise linear) real 2-D maps. The transition to the continuous complex-analytic case is uniform (see Kahlert's abstract for this meeting).

In invertible systems, they were not seen so far. Barnsley (1985) proposed a modified Smale solenoid (3-D) in which such shapes can arise as a projection of the attractor; an explicit noodle map may possess the same property (Rössler, Hudson & Yorke, 1986).

It is shown that the fractal patterns found in 2-D noninvertible maps can be re-tained, as a section through a higher-dimensional attractor, in simple 4-D diffeomorphisms. They therefore should arise in simple 5-variable ODE's like chemical reaction systems.

The result is first shown for a lower-dimensional analogue, in two steps.
1) A Cantor boundary in a 1-D noninvertible map becomes a "frontière floue"(fuzzy boundary) in a "corresponding" 2-D diffeomorphism like Hénon's map (Mira, 1979). 2) Time-inversion of the latter map does not yield an attractor as the map is expanding (in a direction orthogonal to the former noninvertible map). It is therefore necessary to add a third dimension diffeomorphically, to make the map (a) contracting and (b) to "fold back up" the protrusion into the attracting region. This can be done easily, resulting in a flat-noodle map. The very same steps can be done starting from a 2-D noninvertible map (like that of Rössler et al., 1986), resulting in a hyper noodle map. Hence the result.
All references of the text are found in the following two papers: Rössler et al., Z.Naturforsch. 41 a, 810-822 (1986), and Rössler, O.E., J.L.Hudson and J.A.Yorke, ibidem, pp.979-980.
Coauthors: C.Kahlert, J.Parisi, J.Peinke, B.Röhricht, J.L.Hudson.

Afternoon session; chairman: C.POMMERENKE

S. USHIKI: **Configuration of Herman rings and dynamics on trees**
A report on M.Shishikura's paper "Configuration of Herman Rings and Dynamical Systems on Trees". - The configuration of Herman rings of rational functions are represented in terms of trees and "piecewise linear" maps of them. Their properties are investigated. A sufficient condition for trees to be a configuration of some rational function is obtained by means of holomorphic surgery. Shishikura's theorem on the number of cycles of attractive basins, parabolic basins, Siegel disks, and Herman rings is also reported.

I.N. BAKER: **A problem on Julia sets**

Let $\mathcal{J}(f)$ denote the Julia set of a rational function f. Does $\mathcal{J}(f) = \mathcal{J}(g)$ imply $f(g) = g(f)$? For f, g polynomial the answer is yes, except when $\mathcal{J}(f)$ has a rotational symmetry. The exceptions are discussed.
For f, g rational a weaker result is proved: if $\mathcal{J}(f)$ has an infinite set of cusps, then $\{ g \mid \mathcal{J}(f) = \mathcal{J}(g) \}$ is countable.

C. KAHLERT : **Julia-like boundaries from non-holomorphic maps**

An autocatalytic (resp. damping) term is added to the complex logistic map, yielding: $F(z) = z^2 + c + a\mathrm{Re}z$. The basins of attraction of cycles and fixed points (including infinity) are investigated numerically (for $c = 0.32+0.043i$) and analytically. For $a \neq 0$ the map F is not holomorphic. Nevertheless, for values of a not too far from zero, Julia-like boundaries are found. Decreasing the parameter beyond -0.390. . . yields smooth segments of the boundary of a basin shaped like a wild goose. The latter object eventually disappears at $a \approx -1.5708...$ after a sort of blue sky catastrophy.
The question of an analogue to the Mandelbrot set for non-holomorphic maps with fixed a is raised. We conjecture that the initial value -a/4 takes over the role of the critical point in the analytic case. A set $M'_a := \{ c \mid F^n(-a/4)$ remains finite for all $n \}$ is constructed. Numerical investigations suggest that for $c \in M'_a$ at least one attractor ($\neq \infty$) possessing a basin of finite measure appears; while otherwise almost all points of $\mathbb{C}$ seem to go to infinity.

S. THOMAE : **Final state sensitivity in one-hump maps**

The sensitive dependence on the final state can globally be described by the uncertainty exponent α which is related to the capacity dimension d of the basin boundary via $\alpha = D-d$. D is the topological dimension of the embedding space. Numerical simulations indicate that in windows of period $p \geq 3$ of maps $f_\mu(x) = 1-\mu |x|^\zeta$, $\zeta > 1$ the leading order μ-dependence of α is always given by $\alpha(\mu_0 + \delta\mu) = \alpha(\mu_0) + \tilde{\alpha}(\mu_0) \cdot \delta\mu^{1/2}$. This result can be explained by representing α in the form $\alpha = 1 - \ln\lambda / \ln s(\mu)$. λ is the largest eigenvalue of a matrix encoding the topological structure of the underlying p-cycle and thus constant throughout a given window. $s(\mu)$ is the average slope of the branches of the p-th iterate of f_μ, excluding neighborhoods of local extrema. From $s(\mu_0 + \delta\mu) = s(\mu_0) + \tilde{s}(\mu_0)\, \delta\mu^{1/2}$ follows the numerically observed behavior. for all $\zeta > 1$.

Wednesday, Jan. 7, 1987:

Morning session; chairman: P. BLANCHARD

P.J. RIPPON: **Iteration of exponential functions**

A report on joint work with Noel Baker on the iteration properties of the family

$$f_c(z) = e^{cz},$$

which attempts to explain the "Mandelbrot set" for this family, and make comparisons with the Mandelbrot set for quadratic maps. For example, the regions in parameter space corresponding to attractive basins are (with the exception of the cardioid corresponding to an attractive fixed point) unbounded and simply connected. The complement of these regions appears to form a Cantor set of curves, but it is not clear how to prove that the union of the regions is dense.

J. PEYRIERE: **Frequency of patterns in random or automatic graphs**

Mandelbrot's squigs as well as Penrose's tilings involve graphs invariant under an operation of substitution which may be deterministic or not. It is shown that any pattern appearing in such a graph appears in fact with a uniform density.

B. BRANNER : **New knowledge of the Mandelbrot set through iteration of higher degree polynomials**

Using holomorphic surgery it is possible to construct a homeomorphism from the limb $M_{1/2}$ of the Mandelbrot set M to a certain subset $F_{1/2}$ of cubic polynomials where both critical points have bounded orbits. The set $F_{1/2}$ can also be obtained as $F_{1/2} = \lim_{r \downarrow 1} E_r(0)$, where $E_r(0)$ consists of a certain subset of cubic polynomials where one critical point excapes to ∞ at the rate log r. The set $E_r(0)$ have infinitely many components, among which infinitely many are copies of M. The turning operator $\Upsilon : E_r(0) \longrightarrow E_r(0)$ does in general permute the copies of M. Therefore from the study of cubic polynomials we can extract some symmetry properties in $M_{1/2}$. The limb $M_{1/3}$ of M is related to fourth degree polynomials with only 2 critical points.

This reports on joint work with A. Douady and John H. Hubbard.

Thursday, Jan. 8,1987:

Morning session; chairman: K. KIRCHGÄSSNER

M. BARNSLEY: **The application of fractal geometry to image compression**

A new class of iterated function systems is introduced, which allows for the computation of non-compactly supported invariant measures, which may represent, for example, grey-tone images of infinite extent. Conditions for the existence and attractiveness of invariant measures for this new class of randomly iterated maps, which are not necessarily contractions, in metric spaces such as $\mathbb{R}^n$, are established. Estimates for moments of these measures are obtained.

Special conditions are given for existence of the invariant measure in the interesting case of affine maps on $\mathbb{R}^n$. For non-singular affine maps on $\mathbb{R}^1$, the support of the measure is shown to be an infinite interval, but Fourier transform analysis shows that the measure can be purely singular even though its distribution function is strictly increasing.

I. PROCACCIA: **Thermodynamics and statistical mechanics of multifractals**

Nonlinear physics provides examples of fractal measures, like e.g. strange attractors of dynamical systems. The analysis of such measures is performed on two levels. On the phenomenological level one characterizes these measures by their generalized dimensions D_q or by the distribution of scaling exponents $f(\alpha)$. This description is analogous to the thermodynamics of macroscopic systems. A calculational scheme is obtained by mapping the process of refinement of the covering of such measures onto transfer matrices of appropriate 1-dimensional spin models. The largest eigenvalue of this transfer matrix provides the thermodynamic functions of the phenomenological levels. This approach has been applied to sets at the borderline of chaos and to some sets off this borderline, and it was found to yield very efficient characterization and rapid convergent computations.

J. HARRISON: **Continued fractals**

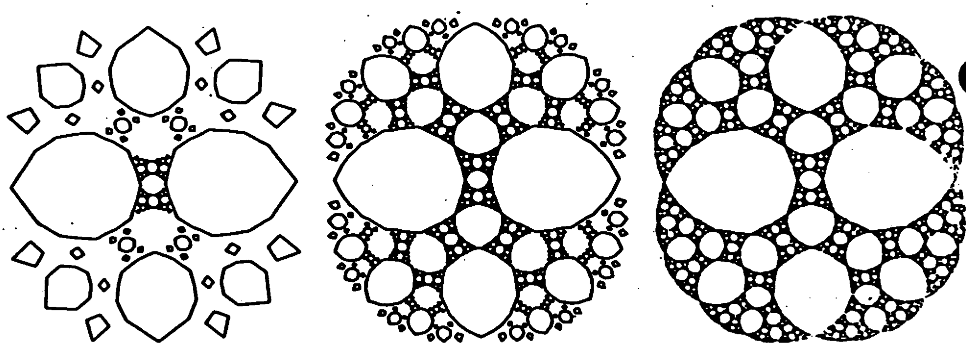
A sequence of real numbers generates a Jordan curve in the plane which geometrically exhibits the asymptotic behavior of the sequence. For example, let $X_n = n\alpha \pmod 1$ where $\alpha \in \mathbb{R} - \mathbb{Q}$. The curve Q_α generated by X_n is embedded, self-similar and has dimension >1, if α is algebraic.
For Diophantine α, the shape of the curve is simply related to the Diophantine type of α. For any α it is possible to read off the integers of the continued fraction expansion of α.
If α is algebraic, there exists a smooth ($C^{3-\varepsilon}$) diffeomorphism of S^2 for which Q_α is an attractor. Q_α has rotation number α. This curve Q_α is an example of a Julia set for a smooth diffeomorphism.

Afternoon session; chairman: D. MAYER

M. HORTMANN: **Partitioning Julia-sets**

The computation of Hausdorff-dimension for Julia sets makes it necessary to consider partitions of these sets.
The Julia sets of the rational functions $f(z) = z^2$ and $g(z) = 1/z^2$ are circles, for which one finds arbitrarily fine partitions or polygonal approximations by iterating backwards the repulsive fixed-point $z_0 = 1$, and systematically putting the preimages "in their right place".
In a similar way one treats the family $f_c(z) = z^2+c$, where c lies in a hyperbolic component of the Mandelbrot set. If c is complex, one can no longer use the symmetry with respect to the real axis in ordering the preimages, but has to use the distances to the points already constructed. Also, for c not in the main cardioid, one has to use as starting points of the iteration all repulsive cycles, which have become indifferent at some root on the way to the hyperbolic component of c, and whose external angles are therefore fixed.
The family $R_a(z) = ((z+a) / (z-1))^2$ consists of renormalization mappings of a hierarchical Potts-model for magnetism. To construct partitions one first finds an analytic conjugacy to the family $S_b(z) = (bz+1)/z^2$, which for b=0 contains the preconsidered simple case, and that algorithm works in the whole hyperbolic component of 0. Modifying the combinatorics one can find par-

titions for the other components, and also recursive graphs describing the Julia sets.



I. KAN: **Shadowing properties of non-uniformly hyperbolic maps**

Typically an orbit $\{x_k\}$ of a dynamical system $f: M \to M$ produced in a numerical experiment is in fact a δ-pseudo-orbit. That is, $d(f(x_k), x_{k+1}) < \delta$. If the map f has positive Liapunov exponent then the true orbit of x_o diverges from the pseudo-orbit at an exponential rate. f is said to have the shadowing property if for any $\varepsilon > o$ there exists a $\delta > o$ such that for any δ-pseudo-orbit $\{x_k\}$ there exists an x so that $d(f^k(x), x_k) < \varepsilon$ for all k.

A complete understanding of the shadowing properties of the one parameter family of maps $f(x) = 1-s|x|$ is presented. As are some partial results on the real quadratic and Lozi families of maps.

K.J. FALCONER: **Iterated Venetian blinds and digital sundials**

The "Iterated Venetian blind" construction originated in Besicovitch's work of the 1930s when it was used to construct a subset of the plane of positive 1-dimensional Hausdorff measure that had projections of zero length in all directions. The basic idea has been used more recently in situations where it has been required to control the projections of a set. These include constructions of sets with prescribed projections (in 2 or more dimensions) and Mattila's counter-example to Vitushkin's conjecture in complete variable theory.

T. GEISEL : **1/f -noise due to Cantori in Hamiltonian systems**
We report on a new mechanism for 1/f-noise in nonlinear dynamical systems. It shows up in the velocity fluctuations of a particle moving in a 2-dimensional periodic potential. It is caused by trapping of the orbit in a hierarchy of Cantori (broken KAM-tori). A statistical description is given in terms of a random walk in an ultrametric space.

Friday, Jan. 9, 1987:

Morning session; chairman: G. CAGLIOTI

R. GRAHAM : **Quantization of maps with fractal attractors**
Dissipative dynamical systems are quantized in terms of quantum Markovian semigroups. For discrete maps these take the form of linear integral equations mapping the Wigner function. For the Kaplan Yorke map an explicit expression for the invariant Wigner distribution is obtained, which reduces to the classical phase-space density in the classical limit and shows how the classical strange attractor is blurred on small scales quantum mechanically. For the Hénon map similar results are obtained numerically. The quantum noise is found to consist of two components, one associated to dissipation by the fluctuation dissipation theorem, one being a consequence of the nonlinearity of the map. The two components are found to scale in cases where the classical map scales, but with two separate exponents.

R.T. BUMBY : **Number theory related to the Apollonian gasket**
The Apollonian gasket is formed by iterating the removal of a circular disk tangent to all sides of a triangle formed from three pairwise tangent circular arcs. A continued fraction for producing best approximant to complex numbers by elements of the Gaussian field using this construction and a dual decomposition of circles with tree marked points was given by A.L. Schmidt in Acta

Mathematica 134 (1975). This suggests that methods which had been applied to compute Hausdorff dimension of sets defined by properties of the ordinary continued fraction (R.T. Bumby: Springer Lecture Notes 1135) might apply here to sharpen the results of David Boyd who approximated the dimension of the Apollonian gasket in a series of papers in the early 1970's.

B.B. MANDELBROT : **Concluding remarks**

see his three books:

Les objets fractals, Flammarion, Paris 1975

Fractals: Form, Chance, and Dimension, Freeman, San Francisco 1977

The Fractal Geometry of Nature, Freeman, San Francisco 1982

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