MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 2/1987

Mathematical Economics

11.1. bis 17.1.1987

Tagungsleiter: G. Debreu (Berkeley)
W. Hildenbrand (Bonn)
D. Sondermann (Bonn)

This was the fourth meeting on Mathematical Economics in Oberwolfach. As for the previous meetings, the conference was limited to those areas of Economic Theory and related fields which are challanging both from an economic and a mathematical point of view. Two research areas — theory of incomplete markets, and equilibrium theory with increasing returns to scale — where important research has come to light during the last four years, were the main topics of the conference. The survey presentation of the theory of incomplete markets was concerned with such problems as the existence of equilibrium, indeterminacy of equilibrium allocations, behaviour of producers, continuous time trading, etc. Some special topics related to the theory of incomplete markets were presented in short communications. The survey presentation of the equilibrium theory with increasing returns to scale was concerned with general pricing rules for firms and existence and optimality of equilibrium. There were also some short presentations related to this topic.

The excellent facilities provided by the Mathematisches Forschungsinstitut created a stimulating atmosphere which was appreciated by all the participants.

Teilnehmer

Beth Allen, Philadelphia
P. Artzner Strasbourg
M. Beckmann, München
H. Bester, Bonn
T.F. Bewley, New Haven
V. Böhm, Mannheim
J.S. Chipman, Minneapolis
B. Cornet, Louvain-la-Neuve
E. van Damme, Bonn
P. Dehez, San Domenico
E. Dierker, Wien
Hildegard Dierker, Wien
R. Dos Santos Ferreira, Strasbourg
D. Duffie, Stanford
U. Ebert, Bonn
W. Eichhorn, Karlsruhe
H. Föllmer, Zürich
Birgit Grodal, Kopenhagen
W. Härdle, Bonn
S. Hart, Tel-Aviv
M. Hellwig, Bonn
R. Henn, Karlsruhe
W. Hildenbrand, Bonn
M. Jerison, Bonn
Y. Kannai, Rehovot
A. Kirman, Marseille
K.J. Koch, Bonn
H. König, Saarbrücken
W. Krelle, Bonn
W. Leininger, Bonn
M. Maschler, Jerusalem
Sigrid Müller, Mannheim
M. Nermuth, Bielefeld
W. Neuefeind, St. Louis
J.M. Ostroy, Los Angeles
J. Rosenmüller, Bielefeld
A. Rubinstein, London
D. Schmeidler, Tel Aviv
K. Schürger, Bonn
W. Shafer, Bonn
D. Sondermann, Bonn
H. Sonnenschein, Princeton
T. Stoker, Bonn
H.G. Tillmann, Münster
W. Trockel, Bielefeld
K. Vind, Kopenhagen
S. Weber, Ontario
J. Werner, Bonn
H. Wiesmeth, London

Vortragsauszüge

B. Allen: **Smooth Preferences and the Approximate Expected Utility Hypothesis**

Mild smoothness conditions on continous complete preorders over lotteries imply that various local versions of the expected utility hypothesis are satisfied — i.e., for small deviations, there is a utility (representing the individual's preferences) that is linear in probabilities. For these results, notions of smooth preferences over an infinite dimensional set of probability measures are developed.

Ph. Arzner: **Pricing the Policy Loan Option by the Harrison–Kreps Martingale Method**

(joint work with F. Delbaen)

We present a description of price evaluation of bonds under the assumption that interest rates are stochastic. Such a model was previously introduced by Vasicek (JFE '77). We obtain the same expressions using the general appraoch offered by Harrison and Kreps (JET, 1979). Our approach also yields solutions of the following problems:

a) the valuation of call options on bonds.

b) Callable bonds.

c) Early reimbursement of loans.

d) Policy loans in life insurance. The same methods will eventually be applied to stock options under the assumption of stochastic interest rates.

H. Bester: **Non-Cooperative Bargaining and Spatial Competition**

A bargaining approach to spatial competition is considered. Sellers compete by choosing locations in a market region. Consumers face a cost to moving from one place to another. The price of the good is determined as the perfect equilibrium of a bargaining game between seller and buyer. In this game, the consumer has the outside option to move to another seller so that prices at all stores are independent. Existence of a location — price equilibrium is established. The outcome approaches the perfectly competitive one if the consumer's cost of travelling become negligible or if the number of sellers tends to infinity.

T. Bewley: **Knightian Decision Theory**

A theory of choice under uncertainty is proposed which removes the completeness assumption from the Anscombe-Aumann formulation of Savage's theory and introduces an inertia assumption. The inertia assumption is that there is a status quo and an alternative is accepted only if it is preferred to the status quo. The theory is one way to give rigorous expression to Frank Knight's distinction between attitudes toward risk and uncertainty.

J.S. Chipman: **A Multi–Commodity Intertemporal Model of Consumer Demand**

(joint work with Guoqiang Tian)

The consumer's intertemporal optimization problem is formulated as that of maximizing

$$\sum_{t=0}^{\infty} \beta^t V\big(p(t), y(t) + R(t-1)b(t-1) - b(t)\big)$$

with respect to $b(t)$, where $0 < \beta < 1$ and $V\big(p(t), c(t)\big)$ is the period indirect utility function, $p(t+j)$, $y(t+j)$, $R(t+j) = 1 + r(t+j)$ are expected values — conditional on information at time t — of the price vector, nonproperty income, and interest factor, $b(t)$ is the holding of one-period bonds at the end of period t, and $c(t) = p(t) \cdot x(t)$ is the value of consumer expenditure at time t. The initial condition $b(-1) = 0$ and terminal condition $lim_{t \to \infty} \prod_{\tau=0}^{j-1} R(t+\tau)^{-1} b(t+j) = 0$ (to rule out indefinite borrowing) are imposed. Once $b(t)$ is obtained, the demand vector $x(t)$ is obtained from the Antonelli-Allen-Roy PDE.

The problem is solved by considering the analogous finite-horizon problem and differentiating the maximand with respect to $b(t)$ and equating to zero. In the case of the "linear expenditure system" this leads to a second-order difference equation which is solved for $b(t)$.

B. Cornet: **Nonconvexities and Incresing Returns in Equilibrium Theory**

We survey the field. A general equilibrium model of an economy with increasing returns to scale or more general types of nonconvexities in production is presented. The firms are asked to set their prices according to general pricing rules which include marginal cost pricing, average cost pricing, profit maximization .... The status of the existence problem is presented together with the link with Pareto optimality (of marginal cost pricing equilibria).

E. v. Damme: **Renegotiation–proof Equilibria**

The folk theorem for repeated games states that (under full dimensionality condition) any feasible and individually rational payoff of a one–shot game can be approximated by subgame perfect equilibrium payoffs of the δ–discounted supergame as the discount factor δ tends to 1. "Nonmyopic" equilibria usually involve threats of mutual punishment after deviations from an agreed upon path, i.e. both the punishers and those who are punished are hurt by carrying out the threat. If players cannot suppress the temptation to renegotiate such equilibrium are not viable since a player will deviate on the expectation that she can convince her opponents not to punish since expost it is not in their interest to do so.
I introduce renegotiation–proof equilibria, i.e. equilibria in which the punisher always profits by carrying out the threat. Two results are demonstrated:
(i) In prisoners' dilemma, the renegotiation–proofness requirement does not bile, i.e. all feasible, i.e. points can be obtained for δ large.
(ii) In symmetric Cournot oligopoly models (with complete information) only prices in the neighbourhood of the static Cournot–Nash price can be sustained by means of a renegotiation–proof equilibrium when the number of players is large.

P. Dehez: **Distributive Production Sets and Equilibria with Increasing Returns**

(joint work with J. Drèze)

Consider a *production set* Y (a closed and comprehensive subset of $I\!R^l$ containing 0) on the boundary of which we define the correspondence φ by $\varphi(y) =: \{p \in I\!R^l_+ \mid py \geq py', \forall y' \in Y, y' \leq y^+\}$, the set of prices at which it is profitable to produce the output vector y^+, $y^+_h =: Max(0, y_h)$, *instead* of *producing less* (voluntary trading). W then consider the set of prices in $\varphi(y)$ for which profit is minimized *at given input prices*. This defines a correspondence φ^* on ∂Y which is then enlarged using a technique developed in a previous paper to construct a proper *pricing rule* (a correspondence with closed graph whose values are non-degenerated convex cones with vertex zero). It is shown to coincide with the normal cone when Y is a *convex* set, i.e. voluntary trading and minimal profit characterize profit maximization in the convex case. It is also shown to select *average cost* prices when Y is *output-distributive*. Output-distributivity is defined by applying to outputs (instead of inputs) the definition introduced by Scarf (1963, 1986). More precisely, a production set Y (in which inputs are a priori distinguished from outputs) is output-distributive if for any finite set (y^j) of points in Y, Y contains the intersection of the convex cone (with vertex 0) generated by these points with the set of vectors whose output components are not smaller than the output components of all the y^j's. Such production sets have convex isoquants and display increasing returns to scale. It is then shown that Y is output-distributive iff for all $y \in \partial Y$, $\varphi(y)$ contains a non-zero element, i.e. output-distributivity is a *necessary and sufficient* condition under which average cost pricing is compatible with voluntary trading.

H. Dierker: **Existence of Nash Equilibrium in Pure Strategies in an Oligopoly with Price Setting Firms**

Consider an ologopoly model with differentiated producers and with a fixed, finite number of price setting firms. I assume that cost functions are convex and demand is derived from a consumption sector with a continuum of consumers. I study existence of Nash equilibrium in pure strategies by imposing assumptions on the distribution of consumer's wealth and tastes rather than postulating a concave profit function. I employ a result of W. Hildenbrand's saying that demand for a commodity is a decreasing function, if the wealth distribution has a falling density. The necessary second order conditions on aggregate demand are derived from assumptions on the distribution of consumer's tastes which imply that preferences are price–dispersed.

R. Dos Santos Ferreira: **On Monopolistic Competition and Involuntary Unemployment**

(joint work with C. d'Aspremont and L.–A. Gérard–Varet)

In a recent C.O.R.E. Discussion Paper (1986), with this title, the authors address the question of the possibility of "involuntary" unemployment, as a consequence neither of intrinsic price or wage rigidity nor of unions' power, but of the oligopolistic structure of output markets. In a simple general equilibrium model with two output goods, produced by price–setting firms, one non produced good and labour they establish conditions for the existence, at any given wage, of a Nash–equilibrium in prices, when these are restricted to a feasible set where excess demand for labour is nonpositive. Then they show that for a sufficiently low level of per capita endowments in the non produced good, and under additional conditions, all equilibria will be interior, i.e. unemployment ones, even at a wage arbitrarily close to zero. In further work, which is still in progress, they elaborate the general equilibrium structure of the model, having aside the initial use as a microeconomic foundation for unemployment theory. The model is generalized for n produced goods and the simplifying assumption of identical homothetic consumers' preferences is abandoned. A particular feature of this model is that producers are assumed to know the true demand for their own products and to take account of all the feedback effects of their own actions through consumers' incomes (the so–called Ford effects).

D. Duffie: **Stationary Markov Equilibria**

(joint work with Geanakoplos, Mas–Colell, and McLennon

Consider an economy with Polish state space S, with $P(S)$ the probability measures on S. Let $s \mapsto G(s) \subset P(S)$ be a closed graph (possibly empty valued) correspondence characterizing intertemporal consistency. A $J \subset S$ is

self-justified if $G(s) \cap P(J) \neq \emptyset \quad \forall s \in J$. A *stationary Markov equilibrium* for G is a self-justified set J and a measurable selection $\pi : J \to P(J)$ from $G|_J$.
Theorem: *If J is compact self-justified, there exists a stationary Markov equilibrium with an invariant measure.* This follows from a Theorem of L. Blume, and is used to prove existence of ∞-horizon equilibria (stationary) in three settings: (i) extension of the Lucas '78 model of asset pricing to heterogeneous agents (ii) stochastic games, and (iii) a stochastic overlapping generations equilibria.

U. Ebert: **On the Optimal Income Tax**

The paper deals with optimal income taxation within the framework of Mirrlees' model. This model takes into account the individuals' optimizing behaviour by adding the corresponding first-order conditions as restriction to the planner's problem. An example is presented which demonstrates that this first-order approach is not generally correct. The implications of this example and an extension of the model in which the inplementability of the tax system is guaranteed are discussed.

W. Eichhorn: **On a Class of Inequality Measures**

A (statistical) inequality measure is a Schur-convex function $F : \mathbb{R}^n_+ \to \mathbb{R}$ which satisfies

(1) $\qquad F(\xi e) = 0 \qquad$ for all $\xi \in \mathbb{R}_+ \quad (e := (1, \ldots, 1) \in \mathbb{R}^n)$.

A function $F : \mathbb{R}^n_+ \to \mathbb{R}$ is Schur-convex if and only if

(2) $$F(xP) \leq F(x)$$

for all $x \in \mathbb{R}^n_+$ and all doubly stochastic (n,n)-matrices P, i.e., if and only if it satisfies Dalton's "principle of transfers" and is symmetric. Pfingsten calls a function $F : \mathbb{R}^n_+ \setminus \underline{0} \to \mathbb{R}$, $\underline{0} := (0, \ldots, 0) \in \mathbb{R}^n$, μ-invariant if it satisfies, for a fixed $\mu \in [0,1]$, the functional equation

(3) $$F(x + \tau(\mu x + (1-\mu)e)) = F(x)$$

for all $x \in \mathbb{R}^n_+ \setminus \underline{0}$ and all $\tau \in \mathbb{R}$ such that

$$(x + \tau(\mu x + (1-\mu)e)) \in \mathbb{R}^n_+ \setminus \underline{0}.$$

For $\mu = 0$ and $\mu = 1$ one gets $F(x + \tau e) = F(x)$ and $F(\lambda x) = F(x)$, i.e., leftists' and rightists' equation, respectively, in Kolm's terminology. It is shown, among others, how all functions F which satisfy equations (1) and (2) or which satisfy equations (1), (2), and (3) simultaniously, look like. Some properties of the μ-invariant inequality measures are also derived.

B. Grodal: **Equilibria in Economies with incomplete Markets as Walras equilibria with coordination (joint work with Karl Vind)**

We consider an Economy $\mathcal{E} = (L, C, (X_c, \succ_c)_{c \in C}, (M_i)_{i \in I}$ with L commodities, C consumers with describtion $(X_c, \succ_c)$ where $X_c \subset I\!R^{LI}$, and a system of markets $(M_i)_{i \in I}$ where $M_i \subset I\!R^L$ is a linear subspace.
We prove that this economy can be transformed into a related economy $\hat{\mathcal{E}}$ with externalities, but with *one* market for each agent, in such a way that the set of equilibria in $\hat{\mathcal{E}}$ is identical to the set of Walras equilibria with coordination in $\hat{\mathcal{E}}$.
Using the existence theorem for equilibria in Social Systems with coordination the following existence *theorem* is proved:
If X_c is compact, convex, $0 \in intX_c$ and $\succ_c$ can be extended to a local nonsadiated, convex, continuous preference relation on an open set containing X_c, then there exists an equilibrium in $\mathcal{E}$.
Since $X_c \subset I\!R^{LI}$ the compactness of X_c implies short sales restrictions. However since preferences are also defined on X_c we can avoid the compactness assumption by assuming indifference (preference only depend on the sum of the net trades) and transaction costs.

W. Härdle: **A nonparametric framework for estimating Engel Curves**

Let $(X_i, Y_{il})_{i=1}^{n}, {}_{l=1}^{d}$ denote a sample from family budget data, where $X =$ income, $Y_l =$ expenditure for commodity l and $i = 1, \ldots, n$ the individual index. The problem of finding the Engel Curve $m_i(X) = E(Y_i | X - x)$ is considered in a nonparametric fashion. I propose kernel estimations to approximate the function $m_i(x)$. Let

$$\hat{m}_h(X) = n^{-1} \sum_{i=1}^{h} K_h(x - X_i) Y_i / n^{-1} \sum_{i=1}^{h} K_h(x - X_i)$$

denote the kernel smoother with bandwidth h. How to determine the smoothing parameter ? I propose to select $\hat{h}$ by crossvalidation which is asymptotiacally optimal since the ratio

$$\frac{d_A(\hat{h})}{inf_h\, d_A(h)} \xrightarrow{a.s.} 1$$

where $d_A = n^{-1} \sum_{i=1}^{h} [\hat{m}_h(X_i) - m(X_i)]^2 w(X_i)$ is the average square error. An important quantity to estimate in demand theory is the socalled income–effect–matrix:

$$\Theta_{lk} = \int m_l m_k' f dx,$$

where f denotes the density of the income distribution. I construct an estimate of Θ_k that is root–n consistent and assumptotically normal. In this sense there is no loss against parametric methods when using the more widely applicable and flexible nonparametric framework.

S. Hart: **Potential and Consistency**

(joint work with A. Mas-Colell)

Let P be a real-valued function defined on the space of games, satisfying the following condition: in every game, the marginal contributions of all players (according to P) are efficient. It is proved that there exists just one such function P — called the potential — and moreover that the resulting payoff vector coincides with the Shapley value (in the TU-case) and with the egalitarian solution (in the NTU-case).

"Consistency" is the following property of a solution function: eliminating some of the participants, after paying them according to the solution, does not change the outcome for any of the remaining ones. It is shown that consistency provides a characterization of the above mentioned solutions.

M. Hellwig: **On the Existence of Radner Equilibria**

The paper discusses the existence of Radner's "Equilibrium of Plans, Prices and Price Expectations" in sequential incomplete markets under the assumption of an uncountable state space. Because the price random variables must be measurable, one cannot use Radner's approach of treating the price space as a (finite) product of simplices, one for each date and state of nature. Under a short sales condition, the paper provides a new approach that is based on support prices of first-period asset positions as integrals of state-dependent support prices. On the atomless part of the state space, Lyapunov's theorem yields convexity and upper hemi-continuity of the support price integrals so that an existence proof through Kakutani's fixed-point theorem becomes available.

M. Jerison: **Consumer Preference Aggregation, Stability and Social Welfare Analysis**

Restrictions on consumer Engel curves are presented ensuring that aggregate demand satisfies Slytzky symmetry or negative semidefiniteness given a particular distribution of income. The relevence of these restrictions for social welfare analysis and for stability of Walrasian tâtonnement is discussed. The possibility of testing the restrictions with data on consumer expenditures is examined. In the case of collinear endowments, the restrictions imply that the variance of consumers' excess demands for any good does not decrease when the consumers' endowments rise proportionately. These restrictions are necessary for local acyclicity of Kaldor's compensation criterion and for the income distribution to be optimal with respect to some social welfare function. The restrictions are also related to bounds on the dimension of the subspace spanned by individual or aggregate Engel curves, as implied by exact income aggregation.

Y. Kannai: **The Utility of Special Coordinates in Demand Theory**

An n-dimensional vector $a = (a_1, \ldots, a_n)$ can be thought of as a (unit) package of goods, to be traded at fixed proportions $(\lambda a_1, \ldots, \lambda a_n)$, with unit price $p_a = \langle p, a \rangle$. One then investigates the demand for a when p_a changes. Usefull packages are then normal to the indifference surface and the principal directions of that surface.

K.-J. Koch: **Consumer Demand and Aggregation**

We consider consumption sectors of finitely many individuals who satisfy the weak axiom of revealed preference and hold a fixed share of the total income. We compute a condition the mean demand function of such a consumption sector will fulfill independently of the distribution of individual characteristics. This condition does not impose any local restriction on the mean demand. In order to isolate the local impact of the weak axiom of revealed preference on the aggregate we then allow individual demand functions to have negative components. On any compact set of strictly positive prices and incomes this setup does not imply any restrictions on the class of mean demand functions other than the budget constraint.

W. Krelle: **Keynes after 50 Years: A New Interpretation**

Assume that the economy can be modeled as a system of nonlinear difference equations. The Keynesian results can be interpreted as a solution of this system for the next period, given the initial conditions and the exogenous variables. If the system converges to an equilibrium growth path, this "long-term"-solution conforms to the results of the neoclassical theory. Thus the two approaches can be reconciled.

W. Leininger: **The Sealed-Bid Mechanism for Bargaining with Incomplete Information**

(joint work with P.B. Linhard and R. Radner)

We analyse the "Sealed-Bid Mechanism" as a bargaining procedure in the presence of two-sided incomplete information. To decide whether (and at what price) trade should take place between a (potential) buyer and a seller of a product both of them simultaneously submit sealed bids. If the buyer's bid is at least as high as the seller's bid, then the transaction takes place at a price equal to the average of the two bids; otherwise there is no trade.
We show that, for the uniform prior case, the sealed-bid game has a very large set of equilibria. In fact, not only is there a continuum of equilibria, but the expected gains from trade for the set of equilibria range from second-best to zero.
We also consider the case of general independent priors. Here we pose the question whether strategies with certain structural properties that sustain "good" equilibria in the uniform prior case also yield "good" equilibria over the family of non-uniform priors. Specifically, we show that for all priors in the family considered there exists a *unique* equilibrium in linear strategies and those equilibria

are good as measured by the ratio between (total gains from trade realized) and (second–best total gains from trade realizable).

M. Maschler: Formation of Coalition Structures

Two games — the seller and two buyers, and the quota game [10, 20, 30, 40] — are analyzed. It is assumed that the players believe in the nucleolus, provided that they agree on the game in which they are playing. It is also assumed that the players are allowed to leave the game (if someone pays them to do so) or enter the game only after other coalitions were formed (if they find it convenient, or if one pays them to do so). The result of the analysis are two characteristic function games — called the power games — whose nucleoluses yield highly plausible outcomes.

W. Neuefeind: Quantity Guided Price Setting

We consider an economy with two sectors. The first sector consists of competitively behaving consumers and producers; the second, non competitive, sector, the *P*–sector, consists of firms (*P*–firms) producing commodities (*P*–goods) that are not produced in the competitive sector. The *P*–firms receive their gross output levels and the market prices of their inputs as decision parameters. They minimize costs and set prices for their outputs according to a specific pricing rule. There is also a planning agency that ensures that a certain net production (gross production minus the intra–consumption in the *P*–sector) of the *P*–goods is achieved.
We give assumptions assuming the existence of equilibrium which requires market clearing, meeting the production aspirations of the planning agency, and setting prices for the *P*–goods which are compatible with market prices in the sense that the market prices cannot be higher than the prices to be charged by the *P*–firms, and if the target for the *P*–goods is exceeded, the price charged by the *P*–firms equals the market price.

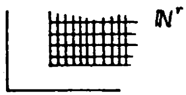
J.M. Ostroy: **Efficient Dominant Strategy Mechanisms in Large NTU Economies**

(joint work with L. Makowski)

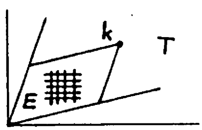
The family of all efficient dominant strategy mechanisms is characterized. It consists of mechanisms that reward agents with their "marginal products" minus perhaps a lump sum.
The existence of such a mechanism is shown for large private goods ecomonies: it consists exactly of Walrasian allocations. It is also shown that no such mechanisms exists in large public goods economies except in the trivial case when the collective good is costless to produce.

J. Rosenmüller: **Nondegenerate Game and Walrasian equilibrium**

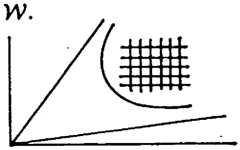
Call $(k,m) \in \mathbb{N}^r \times \mathbb{N}^r$ nondegenerate w.r.t. $\lambda \in \mathbb{N}$ if the system of "minimal winning profiles" $\underline{Q}_\lambda = \{s \in \mathbb{N}_0^r \mid s \leq k, < m,s >= \lambda\}$ determines m uniquely. If we find vectors $s^1, \ldots, s^r \in \mathbb{N}^r$ such that (1) : $0 \leq s^\rho \leq k$ $(\forall \rho)$; (2) : $(s\ :)$ nonsingular and (3) : $\sum_i s_i^\rho m_i = \lambda$ $(\forall \rho)$ then (k,m) hom λ. A Theorem of H.G. Weidner and the author establishes that, given m, a full intervall, of the ideal spanned by $m_1, \ldots, m_r$ yields λ's with (k,m) hom λ provided k is an element at a suitable region of $\mathbb{N}^r$.



The procedure involving non: degeneracy also applies to an equivalence theorem for the core $\mathcal{C}$ and the Walrasian equilibrium $\mathcal{W}$ in a T.U.–exchange economy with piecewise linear utility function. Given some gradient of a utility, let T denote the cone of profiles of economies having this gradient as $\mathcal{W}$–price.



Pick $k \in T$. If one can find vectors $s^1, \ldots, s^r \in \mathbb{N}^r$ such that (1) : ... (2) : ... *and* (3) : $s^\rho \in E$ $(\forall \rho)$ then $\mathcal{C} = \mathcal{W}$.



Using Minkowski's ("2nd") Theorem, we device areas in $\mathbb{N}^r$ such that the that the corresponding profiles yield economies with $\mathcal{C} = \mathcal{W}$.

A. Rubinstein: **The Structure of Nash Equilibrium in Repeated Games with Finite Automata**

(joint work with D. Abreu)

In the standard formulation of a repeated game, players are assumed to be able to costlessly implement strategies of arbitrary complexity. We relax this assumption, pursuing a line of research initiated in Rubinstein [1984]. We assume that strategies vary in their implementation costs. As a consequence, players' strategic choices balance the twin objectives of maximizing repeated game payoffs and minimizing implementation costs.

Our main results concern the structure of machines used in an N.E.. They provide *necessary* conditions on the structure of equilibrium *strategies* and *plays*.

D. Schmeidler: **Maxmin Expected Utility with a Non-Unique Prior**

(joint work with I. Gilboa)

Acts are functions from states of nature into finite-support distributions over a set of "deterministic outcomes". We characterize preference relations over acts which have numerical representation by the functional $J(f) = min\{\int u \circ f dP \mid P \in C\}$ where f is an act, u is a von-Neumann-Morgenstern utility over outcomes, and C is a closed and convex set of finitely additive probability measures on the states of nature.

In addition to the usual assumptions on the preference relation as transitivity, completeness, continuity and monotonicity, we assume uncertainty aversion and certainty-independence. The last condition is a new one and is a weakening of the classical independence axiom.

W. Shafer: **A Survey of Incomplete Markets**

(joint work with D. Duffie)

We examine the results obtained in the last few years on integrating financial markets into a general equilibrium framework, following the lead of Radner [1972]. Without imposing short sales restrictions on security transactions, equilibrium has been shown to exist by Cass, Werner, and Duffie in the case of pure financial securities. Geanakoplos & Mas–Colell, Cass, Balasko and Cass, have shown that such a model, when markets are imcomplete, has an infinite number of solutions. In the case of real financial assets and complete markets, existence has been shown by Repullo, McManus, and Magill & Shafer with incomplete markets, existence has been shown by Duffie–Shafer, Husseini, Lasry & Magill and Geanakoplos–Shafer. Duffie–Shafer add production to the model and show an infinite number of equilibria under the assumption of market value maximization on the part of firms.

D. Sondermann: Reinsurance in Arbitrage–free Markets

Consider the following stochastic processes:

$$S(t) = \text{aggregated claims up to } t$$
$$P(t) = \text{aggregated premiums to } t$$
$$r(t) = \text{return of 1 \$ invested at } t = 0 \text{ at } t$$

Ass.: S, P, r adapted to $(\Omega, \Pi_{[0,1]}, P)$.
Def.: A dynamic reinsurance policy (r.i.p.) is a predictable stochastic process. To any r.i.p. ϕ is associated the (discounted) capital flow

$$C_u(\phi) = \int_0^u \phi(t,w)d\hat{S}(t,w) - \int_0^u \phi(t,w)d\hat{P}(t,w) = \int_0^u \phi_t d\hat{L}_t \qquad 0 \leq u \leq T$$

where $\hat{L}_t = \hat{S}_t - \hat{P}_t = \frac{S_t - P_t}{r_t}$. A reinsurance contract X (r.i.c.) is an $\mathcal{F}_T$–measurable random variable. X is attainable iff $X = c_0 + C_T(\phi)$ for some r.i.p. ϕ. Let $\mathbb{P} = \mathbb{P}[\hat{L}] = \mathbb{P}(S,P,r)$ be the set of P–equivalent probability measure which make $\hat{L}$ a martingale. Then Harrison–Kreps techniques lead to a unique arbitrage value of all attainable r.i.c., which is the (unique) expected value of $\hat{X}$ under any $Q \in \mathbb{P}$.

H. Sonnenschein: Sequential Bargaining, Simple Markets, and Perfect Competition

(joint work with A. McLennan)

Douglas Gale has provided an attractive noncooperative foundation for competitive equilibrium in large economies. In each period the agents in the market are randomly paired with each other, with one agent in the pair proposing a net trade (possibly 0) and the other accepting or rejecting the proposal. There is a constant flow of new agents into the market, and each agent must leave the market in finite time or suffer a utility lower than that provided by his or her initial endowment, but there is no penalty for staying in a long time.

We provide a characterization of Walrasian allocations that is closely related to one given by Schmeidler and Vind and leads to a short proof of a result similar to the one proved by Gale. Roughly, this result characterizes the allocations generated by perfect steady state Nash equilibria of the above trading game as Walrasian.

T. Stoker: **Nonparametric Tests of Derivative Constraints**

Many basic tenets of economics, as well as simplifying restrictions, can be written as constraints on the derivatives of economic relationships. This paper proposes a nonparametric statistical approach for studying such derivative constraints, including tests of their validity. The approach relies on no prior assumptions on the functional form of the relationships.

H. G. Tillmann: **Myopic Topologies on $L^\infty(I\!N, X)$.**

Concepts and some results of Brown–Lewis, Econometrica 49 ('81), 359–368, are generalized from $X = I\!R^1$ to general commodity spaces $X(\tau_0)$.

Prop. 1: If X is an arbitrary vector space with locally convex topology τ_0, on $L := L^\infty(I\!N, X(\tau_0))$ exists a finest strongly–myopic topology, τ_{SM} and a finest weakly myopic topology, τ_{WM}.

Prop. 2: If $l : L \to I\!R$ is a linear functional,

(i) l is τ_{SM}–continuous $\Leftrightarrow l(\hat{x}_n) \to 0 \quad \forall\, \overline{x} \in L$

(ii) l is τ_{WM}–continuous $\Leftrightarrow L(\hat{c}_n) \to 0 \quad \forall$ constant $\overline{c} \in L$,

here $\overline{x} = \{x_1, \ldots, x_n, \ldots,\} \in L^\infty, \hat{x}_n\{0, \ldots, 0, x_{n+1}, x_{n+2}, \ldots,\}$

Prop. 3:

(i) $(L^\infty(I\!N, X(\tau_0))_{\tau_{SM}})' = L^1_b(I\!N, X') = L'$, where summability in L' is defined with respect to seminorms $p_B(u_n) = sup_{x \in B} |u_n(x)|$, B bounded in $X(\overline{c}_0).L' = \{\overline{u} = \{u_n\} \mid \overline{p}_B(\overline{u}) = \sum_i^\infty p_B(u_n) < \infty, \forall B\} = L'(\tau_1)$

(ii) If X is the dual of a barrallel space $Y = X(\tau_o)'_b$ and τ_0 admissable for the duality $< Y, X >$, then τ_{SM} and τ_1 are admissable on L resp. L' for the duality $< L, L' >$.

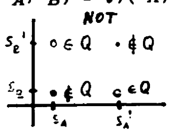
(iii) If $X = I\!R^N$ or $X = l_\infty, Y = l_1$, then τ_{SM} is the Mackey–topology $\tau_k(L, L')$.

W. Trockel: Classification of Price-Invariant Preferences

A classification of price-invariant preference relations on $I\!R^l_{++}$ is given. Price-invariant preferences correspond via a group isomorphism to translation invariant preferences on $I\!R^l$. It is shown that non-trivial continuous preferences are representable by a Cobb-Douglas utility function if and only if they are monotone and price-invariant. The only "reasonable" price-invariant preferences are the ones which are Cobb-Douglas representable.

K. Vind: Independent Preferences (MSRIo7917–86)

Let $Q \subset S = \Pi_{j \in N} S_j$.$Q$ is *essential* if $\forall j \in N, \forall s_j \in S_j \exists (s_i)_{i \neq j}, (s'_i)_{i \neq j}$ such that $(s_j, (s_i)_{i \neq j}) \in Q$ and $(s_j, (s'_i)_{i \neq j}) \notin Q$.$Q$ is *independent* if for all partitians (A, B) of N and all $s_A, s'_A \in S_A = \Pi_{j \in A} S_j, s_B, s'_B \in S_B$; $(s'_A, s'_B) \in Q, (s_A, s'_B) \notin Q, (s'_A, s_B) \notin Q$ and $(s_A, s_B) \in Q$ is excluded (fig.)



For Q independent a total preorder can be defined an S_A for all $A \subset N$. Q is *open* and *connected* if $\forall j \in N S_j$ is open and connected in the relative order topology from S_A for $j \in A$.

Theorem. *Let $\#S_j > 1; \#N > 3$ and assume Q essential, independent, open, and connected. Then there exists $u_j : S_j \to I\!R$ such that $s \in Q \Leftrightarrow \sum u_j(s_j) > 0$.*
One of the many applications of this result is to $Q = \text{graph}P$, where $P : X \to 2^X$ is a preference correspondence. The theorem then yields for example $y \in P(x) \Leftrightarrow \sum u_i(x_i, y_i) > 0$.
This means that most of economic theory based on additive utilities – expected utility, discounted utility – has been generalized to preferences which are not total or transitive.

S. Weber: **On Oligopolistic Model of Product Differentiation (joint work with V. Ginsburgh and B. McLeod)**

The data on European car industry demonstrate that Belgium is the most competitive country, while the U.K. is the least competitive one. Moreover the difference in the prices in two countries is a decreasing function of quality.

In this paper we develop a formal model of an oligopolistic market by introducing an additional parameter-"style" which affects consumers' choice. Thus we propose the following scheme. The firms choose their price schedules, the consumers maximize their utilities given the firms' prices and styles and thus a profit of a firm is determined. In the case of symmetric firms we prove an existence of a unique symmetric Nash Equilibrium, at which a profit of the firms is a decreasing function of quality of produced cars. This theoretical result is consistent, therefore, with our data.

J. Werner: **Asset Prices and Real Indeterminacy in Equilibrium with Financial Markets**

We analyze a general equilibrium in an exchange economy with spot commodity markets and incomplete futures markets for financial assets. We show that every asset price system that admits no arbitrage opportunity is an equilibrium asset price system for a given economy. Generically, distinct asset price systems correspond to distinct allocations of commodities in equilibrium. The indeterminacy of asset prices is, however, not the only reason for the indeterminacy of allocations in equilibrium. Even with a fixed asset price system, there is a "large" set of equilibrium allocations. We provide a description of the degree of this indeterminacy.

H. Wiesmeth: **Endogenously Generated Price Fluctuation in Incomplete Markets**

This model shows the possibility of endogenously generated price fluctuations in asset models, in constrast to many papers, which explain excessive price variability alone. It is given by a sequence of exchange economies, each extending over two periods of time. In each period there are spot markets for commodities and markets for financial assets with uncertain state–contingent return next period. The dynamic link between the various periods of time is provided by the specification of the information structure alone. Agents receive information signals in each period of time and derive additional information from last periods' price funtions. An "equilibrium price sequence" is then given by a closed, finite sequence of price functions, mapping vectors of signals to equilibrium price vectors. The price functions are linked by the above assumption on the information structure. Existence of equilibrium depends on various assumptions on the information structure, and on certain assumptions on the structure of the equilibrium manifolds. Indeterminacy of Radner–equilibrium plays an important role, too. The conclusion of the model is that there is, in general, no chance to explain high price variability by exogenous factors alone. In addition to that, the regular structure of the closed equilibirum price sequence behind the irregularly fluctuating prices allows a discussion of the problem of "learning from price fluctuations".

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