MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 4/1987

Quantenstochastik

25.1. bis 31.1.1987

The conference was organised by L. Accardi (Rome) and W. v. Waldenfels (Heidelberg).

The main purpose of the meeting was to exchange informations about the most recent results in Quantum Probability and to continue the already existing successful cooperation between the participants (44 mathematicians and physicist from several countries). The lecture programme consisted of topics such as theory of quantum white noise, quantum Poisson processes, quantum stochastic integration, quantum stopping times and further noncommutative (purely algebraic and also analytic) analogs of probabilistic notions and results like entropy, Radon-Nikodym theorem and Dirichlet problem. There were also some physical talks which established the connection between theoretical considerations and applications.

The 32 talks have been followed by lively discussions and a fruitful exchange of ideas has taken place, which to a great extent is due to the stimulating atmosphere of the Oberwolfach Institute.

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Vortragsauszüge

L. Accardi:

A mathematical theory of quantum noise

In classical physics we have learnt that a deterministic system can develop a chaotic behaviour and that there is a whole hierarchy of "chaotic properties". The same question could be asked for a quantum system and since for quantum systems the description must be statistical, the problem is to separate in some sense the deterministic features of a quantum system from the statistical ones. In classical probability theory the mathematical realization of this programme can be achieved to a great extent via the Doob-Meyer theorem which states that, under rather general conditions a stochastic process X_t is described by a stochastic differential equation of the form

$$d X_t = b\, dt + d N \qquad (1)$$

where b is a random function (which may depend on X_t) and N is a "noise" i.e. a martingale (which itself can depend on X_t). It is also known that all the sufficiently regular noises (martingales) can be built up out of the fundamental ones: the brownian motion and the Poisson process.

In the quantum domain a decomposition similar to (1) can be achieved provided the system has the following "chaotic" properties :

1. The algebra of observables A is endowed with a (non trivial) past filtration ($A_{t]}$) : $s \leq t \rightarrow A_{s]} \subseteq A_{t]}$.
2. The past filtration is expected in the sense that there exists a norm 1 conditional expectation $E_{t]}$ with range $A_{t]}$.

Under these conditions an equation similar to (1) is deduced.

D. Applebaum:

Quantum stochastic parallel transport

In A. Connes´ differential geometry, a "non-commutative manifold" is an appriately smooth $*$ - algebra A^{∞} of a C^* - algebra A and a "non-commutative vector bundle" is a finitely generated, projective A^{∞} - module Ξ .If Ξ is Hemitian and possesses a faithful trace, we may define a complex inner product on Ξ and complete to obtain a Hilbert space h_0.

We consider the situation where (A, G, α) is a C^* - dynamical system, G being a Lie group with Lie algebra L which acts as derivations on A^∞ that are annihilated by the trace.
If H is some noise space, such as symmetric Fock space over $L^2(\mathbb{R}^+, \mathbb{C}^d)$, a quantum parallel transport process on $h_0 \otimes H$, if it exists, is a family of adapted unitary operators $U = (U(t), t \geq o)$ safisfying the quantum stochastic differential equation

$$\left. \begin{array}{l} dU = U(\nabla x_j \, dM^j + 1/2 \, d < \nabla x_j \, M^j, \nabla x_k \, M^k >) \\ U(o) = I \end{array} \right\} *$$

where ∇ is a compatible connection on Ξ, $\{x_1, ..., x_n\}$ is an orthonormal basis for L and $\{M^1,..., M^n\}$ are semimartingales in $h_0 \otimes H$.

Examples studied so far are the classical case where * is the Ito form of the usual equation for stochastic parallel transport equation on smooth sections of a vector bundle and the case of Heisenberg modules over the non-commutative torus algebra where $d = 1$ and M_j $(j = 1,2)$ are linear combinations of annihilation and creation processes. This example yields the three canonical forms of quantum diffusion process of Hudson & Parthasarathy when ∇ is chosen to minimize the Yang Mills action of Connes and Rieffel.

A. Barchielli:

Input and output channels in quantum systems and quantum stochastic differential equations

A well known application of quantum stochastic calculus is in the theory of dilations of quantum dynamical semigroups. In physical terms, the quantum Brownian motion, which is the main object in quantum stochastic calculus, plays the role of quantum noise and represents a very idealised bath. However, one can interprete the quantum Brownian motion also as an idealised description of a physical field (the electromagnetic field, for instance), which carries information in and out some system. This change of point of view is very important: now one can use quantum stochastic differential equations for modelling systems such as atoms stimulated by laser fields and which emit light. In the talk I discussed how the field after interaction with the "system" can be described by means of quantum stochastic differential equations and how information on the system can be extracted from this "outer field".

C. Cecchini:
Noncommutative Radon- Nikodym theorems

First, in the framework of a theory of noncommutative L^P - spaces for states on von Neumann algebras developed by the author, a Radon-Nikodym theorem connecting those spaces related to different states on the same von Neumann algebra is given. In the second part a result obtained with D. Petz, giving an explicit Radon-Nikodym formula for ω-conditional expectations is given, when a majorization condition is satisfied between the states. It implies extending canonically a normal faithful state with respect to a given ω-conditional expectation. Finally, this result is generalised with no restrictions.

W. Cegla:
Lattice structure in Minkowski spaces

In Minkowski space we deduce the orthogonality relation from a causal structure. Then we construct the fannily of double orthoclosed sets which form a complete orthomodular lattice.
This lattice is atomistic, with trivial center and does not satisfy the covering law, therefore cannot be represented as the lattice of projections of a von Neumann algebra.

M. Fannes:
An application of De Finetti's theorem

A state on an infinite product of measure spaces on an infinite tensor product of C* - algebras is called symmetric whenever it is invariant under local permutations. De Finetti´ s theorem and, its various extensions identify the extreme symmetric states with the symmetric product states. This theorem is useful to compute the equilibrium states of the discrete mean-field models with permutation invariant Hamiltonians. More general models showed at least allow a non-constant external field q in the Hamilitonian. Therefore the notion of q-symmetric state is introduced for classical lattice systems and an extension of De Finetti´ s theorem is obtained under the condition that q is uniformly bounded and it is conjectured that the result remains true if q has less than logarithmic growth.

W. Ford:

The quantum Langevin equation - the independent oscillator model

The quantum Langevin equation for a Brownian particle in a potential V (x) has the form:

$$m\ddot{x} + \int_{-\infty}^{t} dt' \mu(t-t')\dot{x}(t') + V'(x) = F(t)$$

where the random operator-force F(t) has (symmetric) correlation:

$$\frac{1}{2} < F(t)F(t') + F(t')F(t) > = \frac{1}{\pi}\int_0^\infty d\omega \, \mathrm{Re}\left\{\tilde{\mu}(\omega + io^+)\hbar\omega \coth\frac{\hbar\omega}{2k\pi}\cos\omega(t-t')\right.$$

and commutator

$$[F(t), F(t')] = \frac{2}{i\pi}\int_0^\infty d\omega \, \mathrm{Re}\left\{\tilde{\mu}(\omega + io^+)\right\}\hbar\omega \sin\omega(t-t').$$

Here

$$\tilde{\mu}(z) = \int_0^\infty dt \, e^{izt}\mu(t), \qquad \mathrm{Im}\, z > 0,$$

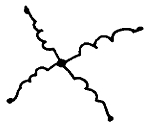
is the Fourier transform of the memory function. This is clearly analytic in Im z > 0. In addition, as a consequence of the second law of thermodynamics, its boundary value on the real axis must have positive real part,

$$\mathrm{Re}\{\tilde{\mu}(\omega + io^+)\} \geq 0.$$

Thus $\tilde{\mu}(z)$ is a positive function, which among other important properties means that it can be represented in the form:

$$\tilde{\mu}(z) = -icz + \frac{i}{\pi}\int_{-\infty}^{\infty} d\omega \frac{\mathrm{Re}\{\tilde{\mu}(\omega + io^+)\}}{z - \omega}$$

All these properties can be derived from the independent oscilator (IO) model, in which the Brownian particle is surrounded by a number of masses, to each of which it is coupled by a spring, as shown in the figure. The Hamiltonian for the S0 model is



$$H_+ = \frac{p^2}{2m} + V(x) + \sum_j \left\{ \frac{1}{2m_j} p_j^2 + \frac{1}{2} m_j \omega_j^2 (q_j - x)^2 \right\}$$

I show how in a few trivial steps one can derive the quantum Langevin equation with

$$\mathrm{Re}\left\{ \tilde{\mu}(\omega + io^+) \right\} = \frac{\pi}{2} \sum_j m_j \omega_j^2 \left[\delta(\omega - \omega_j) + \delta(\omega + \omega_j) \right]$$

Clearly, by an appropriate choice of the distribution of the masses m_j and the frequencies ω_j this can represent the most general positive distribution. Thus this very simple I0 model has remarkable generality!

A. Frigerio:
Quantum Poisson processes and applications

According to conventional wisdom, the reduced time evolution of a system S coupled to a reservoir R displays a Markovian irreversible behaviour when the characteristic relaxation time τ_R of correlation in R is much shorter than the characteristic time τ_S for appreciable effects on S of its interaction with R.Two physical situations in which the condition $\tau_S >> \tau_R$ is satisfied are those of weak coupling and of low density. In the weak coupling limit the system S appears to be driven by a (non-Fock) quantum Brownian motion, and in the low density limit it appears to be driven by a "quantum Poisson process" of some kind. The quantum Poisson process over a von Neumann algebra M with a cyclic and separating vector ξ is defined by $N_t(x; \xi) = W(1_{[0,t]} \otimes \xi)^{-1} \Lambda_t(x) W(1_{[0,t]} \otimes \xi)$ where $W(\xi)$ is a Weyl operator and $\Lambda_t(x)$ is the gauge process (x is an element of M). Quantum stochastic differential equations of the form $dU(t) = d\hat{N}_t(x; \xi)\, U(t)$ are considered, where $\hat{N}_t(a \otimes b; \xi) = a \otimes N_t(b; \xi)$, $a \in A$ (initial algebra), $b \in M$. They are shown to have a unitary solution if and only if $x = u - a$, u being a unitary element of $A \otimes M$. Such unitary operators U (t) may be used to construct dilations of quantum dynamical semigroups on A whose generator L has the form $\langle \Phi, L(a) \Phi' \rangle = \langle \Phi \otimes \xi, [u^* (a \otimes 1) u - a \otimes 1] \Phi' \otimes \xi \rangle$.

G. C. Hegerfeldt:

Noncommutative analogs of probabilistic notions and results

Random variables can be considered as multiplication operators, and mixed moments $E \xi_1 --- \xi_n \equiv m(\xi_1 --- \xi_n)$ can be written as a scalar product $< \phi, \xi_1 ---- \xi_n \phi >$ in $L^2(dP)$, where $\phi(\omega) \equiv 1$. This can be generalized by replacing $\{\xi_i\}$ by operators $A_1, ---, A_n$ in a Hilbert space H, with a common dense invariant domain of definition, and ϕ by some unit vector $\Phi \in H$ so that $m(A_1 ----- A_n) :\doteq < \Phi, A_1 ----- A_n \Phi >$. In quantum field theory these moments correspond to n-point functions. Slightly more general, let M be a set and $\underline{M}$ the free algebra generated by M, let * be an involutive map of M onto itself and extend it to an involution * on $\underline{M}$. We can consider representations π of $\underline{M}$ to possibly unbounded operators in a Hilbert space H with a cyclic vector ϕ_0 and domain $D = \pi(\underline{M})\phi_0$. The notion of random variable is now generalized by $\pi(a)$, $a \in M$, or $a \in M^{(1)}$, the linear subspace of $\underline{M}$ spanned by M, and expectation is replaced by $< \phi_0, \cdot >$, $m \in \underline{M}^*$ is a state if $m(a^* a) \geq 0$, $a \in \underline{M}$, and $m(1) = 1$. Now one can define cumulants in close, but not identical, analogy to the classical case. One can define the analogy of addition of independent random variables, analogs of infinitely divisible random variables. With the noncommutative notions one can prove analogs of results in probability theory, in particular: noncommutative analogs of Marcinkiewicz theorem, of Cramer´s theorem, of the central limit theorem, and of two factorization theorems of Khinchin. There are applications of the results to coherent states in quantum mechanics, to thermal coherent states (by G. G. Emch and the speaker), and to quantum field theory.

A. S. Holevo:

Conditionally positive definite functions and continuous measurement processes in quantum probability

The notions of positive definite and conditionally positive definite functions with values in the space of bounded linear maps of a C*-algebra are introduced. The Schoenberg type theorem, relating the notions , is given. The representation theorems for positive definite and conditionally positive definite functions are established, implying a noncommutative generalization of the Lèvy-Khinchin formula.

The applications to the problem of continuous quantum measurements are discussed. It is shown that this problem is intrinsically related to classical topics of probability theory such as infinite divisibility and functional limit theorems.

R. L. Hudson:

Quantum diffusion and cohomology of algebras

Quantum stochastic calculus as developed by the lecturer and K.R. Parthasarathy is reviewed. It is a noncommutative theory of stochastic integration in which operator -valued adapted processes are integrated against three basic integrators, the gauge, creation and annihilation processes Λ, A and A^+. The quantum Ito product fomula is a multiplication table for the differential of the basic processes. Classical Brownian motion, Poisson processes and even Fermionic fields are all encompaned in the single theory based on Boson Fock space.
The theory is used to construct quantum diffusion. These are quantum stochastic processes based on a * - algebra $\mathcal{A}$ which are governed by systems of stochastic differential equations of form

$$dx = \lambda\ (x)\, d\,\Lambda + \alpha^+ (x)\, d\, A + \alpha\,(x)\, d\, A^+ + \tau\,(x)\, dt,\ x\,(0) = x_0 \in \mathcal{A}\,.$$

Here λ, α^+, α, τ are linear maps from $\mathcal{A}$ to itself called structure maps. The Ito product fomula gives rise to certain identities to be satisfied by the structure map which are analysed from the viewpoint of the Hochschild algebra cohomology theory for $\mathcal{A}$. Examples include the noncommutative torus, where cohomological obstruction to the construction of quantum diffusions are found, and examples involving quantum Poisson processes.

B. Kümmerer:

Noncommutative Poisson processes and continuous Markov dilations

We adapt our standard notation from the theory of (stationary) Markov dilations for completely positive operators on W^*-algebras as they are contained, e.g. in the Proceedings on Quantum Probability I, II, Springer Lecture Notes in Mathematics 1055 and 1136. This definitions form a natural frame for a theory of non-commutative stationary Markov processes. In particular, a Markov dilation is a stationary Markov process for a given semigroup of transition operators.
Given a dynamical system ($\mathcal{A}$, ϕ, T) for discrete time, then ($\mathcal{A}$, ϕ, $e^{(T-Id)\,t}$) is a continuous dynamical system. Observing that

$$e^{(T-Id)t} = \sum_{n=0}^{\infty} e^{-t}\, \frac{t^n}{n!}\, T^n$$

we can use the classical Poisson process for proving the following result:
Theorem.If ($\mathcal{A}$, ϕ, T) has a discrete Markov dilation then ($\mathcal{A}$, ϕ, $e^{(T-Id)\,t}$) has a continuous Markov dilation.

This continuous dilation has a natural interpretation in terms of non-commutative compound Poisson processes. Using earlier results on generalized Bernoulli shifts this construction provides us with the first new examples for white noise beyond the white noises composed from classical Brownian motion and Poisson processes and the non-commutative quasifree shifts on the CCR and CAR algebras.

Applying an approximation theory we proved to the following result.

Theorem. For a given continuous dynamical system $(\mathcal{A}, \phi, T_t)$ the following conditions are equivalent.

(a) $(\mathcal{A}, \phi, T_t)$ has a dilation

(b) For each single t_0 the discrete dynamical system $(\mathcal{A}, \phi, T_{t_0})$ has a dilation.

(c) There exist continuous dynamical systems $(\mathcal{A}, \phi, T_j)_{j\in J}$ which have a dilation and real numbers numbers $(\alpha_j)_{j\in J}$ s.t. T_t = pointwise weak* $\lim_j e^{\alpha_j (T_j - Id) t}$ for all $t > 0$.

If $\mathcal{A}$ is finite dimensional then the same holds for Markov dilations.

In particular, the above result reduces the existence problem for continuous dilations to the existence problems for discrete dilations which is easier to handle with.

J. T. Lewis:

The large deviation principle and models of an interacting Boson gas

We investigate condensation in some models of an interacting Boson gas. The motivation is an old conjecture of F. London:

> Momentum-space condensation is enhanced by a spatial repulsion among particles.

The models we consider are diagonal in the occupation numbers; such models were studied around 1960 by Huang, Yang and Luttinger and by Thouless. These models can be investigated by the methods of classical probability; this enables us to obtain rigorous results on the existence of the pressure and hence on condensation. This is achieved by the use of Varadhan´ s Theorem. We prove that the distributions of various random variables associated with these models in the free-gas grand canonical ensemble satisfy the large deviation principle. This is joint work with M. van den Berg and T. V. Pulè .

G. Lindblad:

Dynamical entropy for quantum systems

I discuss the difficulties of providing a non-commutative generalization of the Kolmogorov-Sinai entropy to any * - automorphism of a W^*-algebra with an invariant normal state. A definition is given which differs from earlier ones by Connes and Størmer and by Emch. This approach is based on operational ideas namely the time-ordered correlation kernels. It takes proper account of the difference between space-time translations for non-commutative systems. The commutative KS entropy is nevertheless included as a special case. Several derived properties of this entropy remain conjectured.

M. Lindsay:

Quantum stochastic calculus with integral kernels

Two illustrations of the kernel calculus were given. The first is the existence and uniqueness problem for a class of quantum stochastic differential equations. There are solved by a recursive procedure which gives the solution in an explicit form. The second is a characterisation of strongly continuous evolutions which are both adapted to the filtration of a non-unit variance ("finite temperature") quantum Brownian motion and are covariant under the group of shifts of the quantum Brownian motion - also called Markovian cocycles. There are determined by a triple of generators via a "kernel differential equation". When the evolution is unitary-valued this leads to a quantum- stochastic Stone´s theorem: any such evolution is the solution of a quantum-stochastic Schrödinger equation

$$dU = U\,[iHdt + LdA^* - L^*dA - 1/2\,(aL^*L + bLL^*)\,dt\,]$$

where a and b are fixed parameters determined by the variance/temperature of the Brownian motion and H is self-adjoint. The presence of the last term is due to the Ito form of the equation. Conversely the solutions of such equations provide unitary Markovian cocycles.

A. Luczak:

Nuclear instruments in von Neumann algebras

Let (Λ, A) be a measurable space and let W be a σ-finite von Neumann algebra. A (dual) insturment ε^* on (Λ, A) acting in W is a σ-additive measure on (Λ, A) with values in the space of linear positive normal mappings of W into itself enjoying the condition $\varepsilon^*_\Lambda(1) = 1$. By the observable of ε^* we mean the semi-spectral measure e defined by $e(E) = \varepsilon^*_E(1)$.
Nuclear instruments are those of the form

$$\varepsilon^*_E(x) = \int P(x, E, A)\, e(d\lambda) \quad , \qquad E \in A,\ x \in W.$$

Weakly repeatable instruments are defined by
$\varepsilon^*_E(\varepsilon^*_F(1)) = \varepsilon^*_{E\cap F}(1)$, E,F ∈ A, and
repeatable by $\varepsilon^*_E\ \varepsilon^*_F = \varepsilon^*_{E\cap F}$.
It turns out that for nuclear instruments weak repeatability and repeatability coincide. If ε^* is a (weakly) repeatable nuclear instrument, than there exists a central projection p in the algebra $\mathfrak{A} = \{ e(E) : E \in A \}$ such that the mapping

$$\phi(x) = p\,\varepsilon^*_\Lambda(x) \quad (x \in W)$$

is a normal conditional expectation from W onto $\mathfrak{A}$. Moreover to each instrument ε^* there corresponds in a canonical way, a number instrument $\hat{\varepsilon}^*$ of the form $\hat{\varepsilon}^*_E(x) = \int_E Q(x,\lambda)\, e(d\lambda)$ which has the same observable as ε^*.

H. Maassen:

A quantum stochastic calculus using integral kernels

A motivation and a construction is given for an explicit quantum stochastic calculus based on several types of noise. Starting from heuristically postulated Ito rules (such as $dB_t^2 = dt$ for Brownian motion or $dN_t^2 = dN_t + \lambda\, dt$ for the compensated Poisson process $N_t = P_t - \lambda t$) a Hilbert algebra is constructed in which these Ito rules are actually valid. This is done starting from the Ansatz

$$N(f) = \int_{\Gamma_I} f(\omega)\, dN_\omega := \sum_{n=0}^{\infty} \int_{t_1 < \dots < t_n} f(\{t_j\})\, dN_{t_1} \dots dN_{t_n}.$$

The Hilbert space in question is the Fock space $L^2(\Gamma_I)$, where $\Gamma_I := \{\omega \subset I : \omega \text{ finite}\}$ and $I \subset R$ is an interval. The algebraic structure in the case $N_t = B_t$ is given by $(f * g)(\sigma) = = \sum_{\alpha \subset \sigma} \int_\Gamma f(\alpha \cup \sigma)\, g(\alpha \cup \sigma)\, d\sigma$. The notions of stochastic integration, stochastic differentiation and the forward derivative are now defined in terms of the kernels f. The fundamental theorem of stochastic calculus is regained and the Ito rules are recovered.

This circle of ideas is repeated for the Bose noise of annihilation and creation operators A^- and A^+, and a stochastic calculus of the same type is obtained with the Ito rules $dA^- dA^+ = c_+ dt$ and $dA^+ dA^- = c_- dt$. Linear quantum stochastic equations can be explicitly solved by recursion. This leads to the construction of cocycles for quantum Markov processes.

K. R. Parthasarathy:

<u>**Local time and fine structure in Fock space calculus**</u>

Define Fourier transform F in $L_2(\mathbb{R})$ by $(Ff)(x) = (2\pi)^{-1/2} \int e^{-ixy} f(y)\, dy$ and extend it to $S(\mathbb{R})'$ the space of tempered distributions by $(F\Lambda)(\phi) = \Lambda(F\phi)$ for all $\phi \in S(\mathbb{R})$. Note that $Fp F^{-1} = q$, $Fq F^{-1} = -p$. The derivative Λ' of Λ satisfies $\Lambda'(\phi) = -\Lambda(\phi')$, $(F\Lambda')(\phi) = i F \Lambda(q\phi)$.

Let H be a Hilbert space, X a selfadjoint operator, $\phi \in S(\mathbb{R})$. Then $< \xi, \phi(X)\eta > = < \xi, F F^{-1} \phi(X) \eta > = (2\pi)^{-1/2} \int (F^{-1}\phi)(y) < \xi, e^{-iyX} \eta > dy$. Taking this as a clue we define for any $\Lambda \in S(\mathbb{R})'$ the form

$$< \xi, \Lambda(X)\eta > = F^{-1} \Lambda\left((2\pi)^{-1/2} < \xi, e^{-ixX} \eta > \right)$$

for all pairs (ξ, η) such that $(2\pi)^{-1/2} < \xi, e^{-ixX} \eta > \in S(\mathbb{R})$ as a function of x.

<u>Theorem 1.</u> Let $H = \Gamma(h)$ be the boson Fock space over h, $f \in h$, $X = P(f) = i(a(f) - a^+(f))$. Then for any two coherent vectors $\psi(u)$, $\psi(v)$ the following holds:

$< \psi(u), \Lambda(P(f))\, \psi(v) > = \Lambda(\phi_1) = F\Lambda(\phi_2)$ where

$\phi_1(x) = (2\pi)^{-1/2} \| f \|^{-1} \exp(< u,v > - 1/2 \| f \|^{-2} (x + i [< u, f > - < f, v >])^2)$

$\phi_2(x) = (2\pi)^{-1/2} \exp(< u, v > - 1/2 \| f \|^2 x^2 + (< u, f > - < f, v >) x)$.

<u>Theorem 2.</u> In Theorem 1 replace f by f_t where $t \to f_t$ is a continuous map from $\mathbb{R}$ into h such that $\| f_t \|^2$ is differentiable in t. Let $u, v \in h$ be such that $< u, f_t > - < f_t, v >$ is differentiable. Then

$$\frac{d}{dt} < \psi(u), \Lambda'(P(f_t))\, \psi(v) >$$

$$= \frac{1}{2}\frac{d}{dt} \|f_t\|^2 < \psi(u), \Lambda''(P(f_t))\, \psi(v) > +$$

$$< \psi(u), \Lambda'(P(f_t))\, \psi(v) > \frac{d}{dt}\, i(< f_t, v > - < u, f_t >).$$

Remark 1. Theorem 2 contains the well known Tannaka formula

$$d\,|w(t)| = [\operatorname{sgn} w(t)]\, dw(t) + \delta(w(t))\, dt$$

2. Theorem 2 implies that for the Ornstein Uhlenbeck process x (t)

$$d\,|x| = \operatorname{sgn} x(t)\, d\,x(t) + \delta(x(t))\, dt.$$

3. Theorem 2 can be interpreted in the generalised sense as

$$d\,\Lambda(P(f_t)) = i\,\{ dA^{+}(f_t)\,\Lambda'(P(f_t)) - \Lambda'(P(f_t))\, dA\,(f_t) \} + \frac{1}{2}\frac{d}{dt}\|f_t\|^2 \Lambda''(P(f_t))$$

D. Petz:

Conditional expectations in quantum probability

After a short review of the work of Moy, Umegaki and Takesaki we treat properties of the ϕ-conditional expectation from a von Neumann algebra M into its subalgebra M_0 (ϕ is a fixed faithful normal state on M).

ϕ is considered as the adjoint of the embedding $M_0 \to M$ when M and M_0 are endowed with an appropriate inner product. As an application of the ϕ-conditional expectation, we say that M_0 is sufficient with respect to a family θ of states on M if the conditional expectation $E_\phi : M \to M_0$ does not depend on ϕ , that is $E_\phi = E_\omega$ for any $\phi, \omega \in \theta$. We characterise sufficient subalgebras in terms of Radon-Nikodym cocycles and transition probabilities.

J. Quaegebeure:

Quadratic variation and Ito 's table in quantum stochastic calculus

Let (T, F) be a measurable space and F_0 a subset of F which is closed for finite unions and intersections. Let x_1,x_2: $F_0 \to \mathfrak{A}$ be finitely additive measures with values in some non-commutative topological algebra $\mathfrak{A}$. We say that x_1 and x_2 have a quadratic variation if

$[\![x_1, x_2]\!] (I) := \lim_{|I_k|} \sum_k x_1 (I_k)\, x_2 (I_k)$ exists in $\mathfrak{A}$ for all $I \in F_o$, where the limit is taken over the net of all finite F_o-partitions I_k of I. We show that the notion of quadratic variation provides the formluation for Ito´s formula in a general quantum stochastic integration theory, namely $dx_1\, dx_2 = d\, [\![x_1, x_2]\!]$.

Next we study the quadratic variation of certain measures taking values in algebras obtained by some representations of the CCR C*-algebra $\mathcal{C}$(H) over a hilbert space H. In the case of quasi-free representations π we construct for a measure x on (T, F_o) with values in B (H) and on $f \in H$, the field measure $B_x^{\ f}$ and the gauge measure Λ_x on (T, F_o) taking values in the operators affiliated with π ($\mathcal{C}$(H)). We prove that $[\![B_{x_1}^{\ f}, B_{x_2}^{\ g}]\!]$, $[\![B_{x_1}^{\ f}, \Lambda_{x_2}]\!]$, $[\![\Lambda_{x_1}, B_{x_2}^{\ g}]\!]$ and $[\![\Lambda_{x_1}, \Lambda_{x_2}]\!]$ exist for the topology of strong convergence on an appropriate domain, provided $[\![x_1, x_2]\!]$ exists in the strong operator topology and x_1 and x_2 are non-atomic. We characterise the Fock representation in terms of the quadratic variation of the field and gauge measures. Finally we prove the existence and give the explicit form of the quadratic variation of the field measures in a non-quasi-free infinitely divisible representation of $\mathcal{C}$(H) . Part of this work was done jointly with L. Accardi.

U. Quasthoff:

Noncommutative Bernoulli flows

The construction of (discrete) Bernoulli shifts using infinite tensor products of finite dimensional matrix algebras is generalized to the continuous case. The underlying von Neumann algebra is constructed with a countable equivalence relation using the procedure of Feldmann and Moore. The Connes-Størmer entropy of such a flow equals the Kolmogorov-Sinai entropy of the corresponding commutative flow. Conditions for conjugacy of non-commutative flows are discussed.

J.-L. Sauvageot:

A Dirichlet problem for C*-algebras

On a C*-algebra A (with unit) consider two given data:

- a quantum Markov semigroup $(\Phi_t)_{t \geq 0}$, norm continuous, whose generator Δ is local
- a bilateral ideal I.

By constructing a suitable dilation of the semigroup, we get a quantum process where the time evolution of the support process of I (in A**) is a commuting family of projections, and thus provides a spectral measure which represents the "first exit time from I."

By stopping the evolution of the process, thanks to this stopping time, we get a completely positive map Λ from $A/_I$ into A^{**} such that:

1. for all $\alpha \in A/_I$ $\Lambda(\alpha)/_{(A/I)^{**}} = \alpha$.
2. for all $\alpha \in A/_I$ $\Lambda(\alpha)/_{I^{**}}$ is an harmonic element.

By assuming the semigroup is a Feller semigroup, plus a standard smoothness condition on the boundary $A/_I$, we show that the previous solution of stochastic Dirichlet problem is also a solution of the topological Dirichlet problem.

M. Schürmann:

Quantum independent increment processes and continuous tensor products

Convolution evolutions of states on involutive bialgebras on the one hand give rise to quantum stochastic processes of independent increments. On the other hand they determine a continuous tensor product with some additional structure consisting in the existence of a shift and of a unit vector which plays the role of the vacuum vector in Fock space.

The special case of the involutive bialgebra $K < d >$ (which we call the quantum anologue of the coefficient algebra of the unitary group) was treated. If the convolution evolution is replaced by a convolution semi-group of states the associated process with independent stationary increments is represented by a unitary evolution on $C^d \otimes H$ (where H is a Hilbert space) satisfying a cocycle identity. The theory yields the result that in special cases (d= 1; Weyl operators; Wigner-Weißkopf atom) the solution of a quantum stochastic differntial equation in the sense of Hudson & Parthasarathy is determined by its generator which is a conditionally positive linear functional on $K < d >$. It is a conjecture that this statement holds in the general case.

G. Sewell:

Entropy, observability and the generalized second law of thermodynamics

Since entropy is a function of the microstate of an observable quantum system, the question arises as to whether thermodynamics extends to processes, demanded by General Relativity, in which the system exchanges matter with unobservable space-time regions, i.e. Black Holes. A widely accepted view, proposed by Bekenstein on phenomenological grounds, is that Black Holes have entropies proportional to their surface areas, and that, consequently, processes in which they participate satisfy a Generalized Second Law (GSL) of thermodynamics, of the form

$\Delta (S + \alpha A) \geq 0$ (*), where S is the total entropy of observable systems, A is the Black Hole area and α is a universal constant. I argue that the hypothesis of a Black Hole entropy is untenable, since observable microstructure is essential to the entropy concept, but that, nevertheless, the GSL (*) is valid. My derivation of this result is based on a treatment of the open system, Σ formed by the matter and fields in the observable region. This treatment rises only on simple basic demands of Quantum Statistical Thermodynamics and Relativistic Mechanics, and the area term in the resultant GSL represents mechanical work done on Σ.

K. B. Sinha:

Stopping times in quantum stochastic calculus

The definition of classical stop time as a $\mathbb{R}$-valued random variable such that the event $\tau \leq t$ is adapted to the filtration $\mathcal{F}_{t]}$ for every $t > 0$ is taken over to define a stop time S in the (boson) fock space as a spectral measure on $\mathbb{R}_+$ such that $S[0,t] \in \mathcal{B}(\mathcal{H}_{t]})$ for every $t > 0$. For such a stop-time, first a stop-time integral of the type $\int_0^\infty W(f_{s]})\, S(ds)\, \psi(g_{s]})\, \xi(s)$ is defined where $\xi(s)$ is a future adapted process satisfying some integrability assumption. With the help of this two Hilbert subspaces $\mathcal{H}_{s]}$ and $\mathcal{H}_{[s}$, the "past" and "future" spaces are defined as follows. $\mathcal{H}_{s]} =$ c.l. sp $\left\{ \int_a^b S(ds)\, \psi(f_{s]}) \mid f, 0 \leq a \leq b \leq \infty \right\}$ and $\mathcal{H}_{[s} =$ range of U^s where $U^s = \int_0^\infty S(ds)\, \Gamma(\theta_s)$ with $(\theta_s f)(t) = 0$ if $t < s$ and $= f(t-s)$ if $t \geq s$. Then there exists an isomorphism from $\mathcal{H}_{s]} \otimes \mathcal{H}_{[s}$ onto $\mathcal{H}$ in a canonical way and $U^s\, W(f)\, U^{s-1}$, the shifted Weyl process is again a Weyl process on $\mathcal{H}_{[s}$. These two results constitute the quantum version of the strong Markov property of the Brownian motion.
One can also define the "past" and "future" algebras $\mathfrak{A}_{s]} = \{ \int_a^b S(ds)\, W(f_{s]}) \mid f \}''$ and $\mathfrak{A}_{[s} = \{ \int_0^\infty S(ds)\, W(f_{[s}) \mid f \}''$. Then one has a surprising result (which is also very non-classical) that $\mathfrak{A}_{s]} = \mathcal{B}(\mathcal{H})$ if the stop time S has every neighbourhood of ∞ in its support.

R. F. Streater:

Non-linear stochastic processes

We want a simple class of models of an autonomous system which moves under stochastic dynamics to an equilibrium state ρ_β, where β is determined by $\rho_\beta(H) = \rho_0(H)$ where ρ_0 is any initial state (of finite energy). H is the Hamiltonian of the system.

Consider a dilute gas; two particles may interact and scatter before meeting any others. Let $\mathcal{H}$, with dim $\mathcal{H} < \infty$, be the Hilbert space of one particle. The scattering gives an automophism of $\mathcal{B}(\mathcal{H})$: $A \to SAS^*$, $A \in \mathcal{B}(\mathcal{H})$, S = scattering matrix. More randomization is achieved if the Hamiltonian of 2 particles is random, giving a doubly stochastic map $A \to TA = E_\omega (S(\omega) A S(\omega)^*)$. Since S commutes with the free 2 body Hamiltonian $h = H \otimes 1 + 1 \otimes H$, T and T* map the spectral projections of h to themselves: $h = \sum EP(E)$, $TP(E) = P(E)$, $T^* P(E) = P(E)$: T* = adjoint as a superoperator on the Hilbert-Schmidt operators. But $T^{*n} \rho$ does not converge to equilibrium, as any function of h is a fixed point - we must add the Stoßzahlansatz: after scattering, particles reenter the population as independent particles: this means we project onto the first Hilbert space of the 2 particle space $\mathcal{H} \otimes \mathcal{H}$. To take into account statistics, we use $\Gamma(\mathcal{H})$ instead of $\mathcal{H} \otimes \mathcal{H}$. The Boltzmann map is $\tau = Q\,T$, T = bistochastic on $\Gamma(\mathcal{H})$, Q = quasi-free projection. Let T be ergodic relative to H, i. e. T T* has only functions of $H = \Sigma\, \omega_k a_k^* a_k$ as fixed points. <u>Theorem</u> $\tau^n \rho \to$ Gibbs state as $n \to \infty$ if ω_j are relatively rational.

G. Süßmann:
<u>Quantum friction</u>

The open system considered is a quantum particle under the influence of some friction force, its motion not necessarily being bound: it may be in a scattering state as well. A pure state is assumed: $W_t = |\psi_t\rangle\langle\psi_t|$ .This irreversible process is different from absorption,where the energy is un - changed (the elastical channel of the optical model), whereas the probability decays. By friction, on the other hand, the probability stays unchanged (= 100 %), whereas the energy decays. In the absorption case a good solution is given by complex potentials U = V - iW, leading to non-hermitean Hamilton operators; in the friction case I have proposed a <u>non-linear</u> potential, leading to a state dependent but somehow hermitean operator. More specifically, the Hamiltonian is bilinear and local, similar to that for the Hartree-Fock approximation for the atomic or nuclear shells.

Y. Suhov:
<u>Classical and quantum degenerate hydrodynamics</u>

The hydrodynamic limit was introduced in the paper of Ch. Morrey (1955) for the purpose to derive rigorously Hydrodynamic Equations (Euler equation) for the Hamiltonian equations of motion. However, Morrey used some assumptions on the solution of the equations of motion -

which are very hard for cheeking in any case of interacting particles. The last years, some model examples were investigated from such a point of view, in particular harmonic oscillators, or, more generally, linear systems (both classical and quantum), and hard rods on the line $\mathbb{R}^1$. The talk contained a review of related results.

K. Urbanik:

Joint distributions and commutability of observables

A Borel probability measure $p_\Gamma^{\mathfrak{A}}$ on the k-dimensional euclidean space R^k is said to be the joint probability distribution of $\mathfrak{A} = (A_1, A_2, \ldots, A_k)$ at the state Γ if for every system $\alpha_1, \alpha_2, \ldots, \alpha_k$ of real numbers the projection of $p_\Gamma^{\mathfrak{A}}$ onto the real line defined by $(x_1, x_2, \ldots, x_k) \to \sum_{j=1}^{k} \alpha_j x_j$ coincides with the one-dimensional distribution

$$p_\Gamma^{\sum_{j=1}^{k} \alpha_j A_j}$$

Let $S(\mathfrak{A})$ be the set of states Γ for which $p_\Gamma^{\mathfrak{A}}$ exists. It is well-known that $S(\mathfrak{A})$ contains all states if and only if $\mathfrak{A}$ consists of commutig operators. A system $\mathfrak{A} = (A_1, A_2, \ldots, A_k)$ is said to fulfil the probabilistic commutation condition if there exists a system $\mathfrak{B} = (B_1, B_2, \ldots, B_k)$ of commuting observables such that $p_T^{\mathfrak{A}} = p_\Gamma^{\mathfrak{B}}$ for all $T \in S(\mathfrak{A})$.

Theorem. If $\mathfrak{A}$ consists of one-sided bounded observables with purely point spectrum, then $\mathfrak{A}$ fulfils the probabilistic commutation condition. Using Weyl transform we can prove that the pair of canonical observables does not fulfil the probabilistic commutation condition.

A. Verbeure:

Detailed balance and critical slowing down

We derive rigorous results about the phenomenon of critical slowing down. For classical lattice systems a stochastic dynamics is constructed satisfying essentially a locality property and the detailed balance property with respect to a state. An upper bound for the energy gap of the evolution is derived in terms of the fluctuation of an observable, choosen in the right way. If one

chooses for the state an equilibrium state of a system chowing a phase transition, the above mentioned upper bound yields the phenomenon. For quantum systems, analogous results are derived for the free Bose gas and for the Dicke-Maser model; for the first model the energy gap tends to zero like $(T - T_c)^2$, for the second model like $(T - T_c)^1$.
All this is based on joint work with various people: M. Fannes, R. Alicki, T. Quaegebeur, D. Gooleris and P. Vets.

W. v. Waldenfels:
The relations of the noncommutative coefficient algebra of the group U(d)

Let H be an infinitedimensional Hilbertspace. Write each element of the unitary group $U(C^d \otimes H)$ in the form $u = (u_{ik})_{i,k=1,\dots,d}$, $u_{ik} \in B(H)$. Define the functions $F_{ik} : U(C^d \otimes H) \rightarrow B(H)$ by $F_{ik}(u) = u_{ik}$ and call $\mathcal{K}_d$ the algebra generated by F_{ik} and F_{ik}^* with respect to pointwise multiplication. Then $\mathcal{K}_d$ is isomorphic to the complex algebra with 1 generated by x_{ik}, x_{ik}^* i,k = 1,...d with relations $(xx^*)_{pq} = \delta_{pq}$ and $(x^*x)_{pq} = \delta_{pq}$. Here x is the matrix $(x_{ik})_{i,k=1,\dots,d}$ and $x^* = (x_{ki}^*)_{i,k=1,\dots,d}$. The algebra $\mathcal{K}_d$ is the noncommutative analoge of the commutative coefficient algebra and is used as a state space for quantum stochastic processes.

J. Wilde:
Stochastic integration

We consider fermion stochastic integration for the Clifford process, and for the creation and annihilation processes in certain non-Fock quasi-free-states and give elementary proofs of fermion analogues of Ito´s theorem (that every Brownian functional is a constant plus a stochastic integral). Thus we obtain very simple proofs of the martingale representation theorems for these theories.

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