MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 5/1987

Algorithmen in der kombinatorischen Geometrie

1.2. bis 7.2.1987

Tagungsleiter: A. Dress (Bielefeld), R.L. Graham (Murray Hill)

The Oberwolfach conference "Algorithms in Combinatorial Geometry" brought together mathematicians and computer scientists working on geometric problems. A wide spectrum of problems was touched, clever ad-hoc methods were confronted with first drafts of conceptual machinery. A general consensus emerged that for developing the field intense cooperation with areas from theoretical mathematics like real algebraic geometry could turn out rather helpful.

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

Vortragsauszüge

K. H. BORGWARDT:

Linear programming algorithms admitting a probabalistic analysis

In recent years the behaviour of linear programming algorithms was studied from a probalistic view under different stochastic models. But all the algorithms which admitted such an analysis were modifications of a parametric variant of the Simplex-Method. The talk will explain two such approaches in detail and give some geometrical background. It will be shown that there are geometrical properties and advantages of parametric variants which make them more useful than other variants for the purpose of a probabilistic analysis.

M. SHARIR:

Davenport-Schinzel sequences and their geometric applications

Davenport-Schinzel sequences are sequences composed of n symbols which do not contain any pair of adjacent equal elements, and do not contain any (not necessarily continuous) subsequences of the form $a...b...a...b...a...$, for $a \neq b$, of length $s+2$ (these are (n,s)-Davenport-Schinzel sequences). These sequences arise in the calculation of the lower envelope (i. e. pointwise minimum) of n continuous functions each pair of which intersect in at most s points. The talk reviews recent progress in the analysis and applications of these sequences. We derive sharp upper and lower bounds on the maximal length $\lambda_s(n)$ of an (n,s)-Davenport-Schinzel sequence. In particular $\lambda_3(n) = \Theta(n\alpha(n))$, where $\alpha(n)$ is the (extremely slowly growing) inverse Ackermann's function. For $s > 3$ we have $\lambda_s(n) = O(n\alpha(n)^{O(\alpha(n)^{s-3})})$ and $\Omega(n\,\alpha^{\lfloor \frac{s-1}{2} \rfloor}(n))$. The geometric applications of these sequences include: (i) Efficient $(O(n\alpha(n)\log n))$ preprocessing of a polyhedral terrain to support fast $(O(\log n))$ ray shooting queries from a fixed viewing point in an arbitrary direction. (ii) Planning a purely translational motion separating two (m-sided and n-sided) interlocking simple polygons in time $O(mn\alpha(mn)\log m \log n)$. (iii) An upper bound of $O(kn\lambda_6(kn))$ on the number of free contacts of a k-sided convex polygon B amidst polygonal obstacles having n sides altogether, in which B makes three simultaneous contacts with the obstacles. (iv) Construction of collections of n line segments in the plane, whose lower envelope consists of $\Omega(n\alpha(n))$ subsegments. (v) Tight bounds $\theta(n^2\alpha(n))$ on the combinatorial complexity of the lower envelope of n triangles in 3-D space, and some other sharp bounds on the complexity of the lower envelope of 2-D surfaces.

J. E. GOODMAN

Hadwiger's transversal theorem in higher dimensions (joint with R. Pollack)

A theorem of Hadwiger states that a family of pairwise disjoint compact convex sets in the plane has a line transversal if and only if the family can be ordered so that for any three of the sets there is a line meeting them in the prescribed order. We extend Hadwiger's theorem to hyperplane transversals of compact convex sets in d dimensions. The "ordering" of the sets is replaced by their "order type", and the condition that no two have a common point by the condition that no d meet a common d-2-flat.

F. R. K. CHUNG

The sidewalk problem

Given a finite curve $f = [0,1] \rightarrow \mathbb{R}^n$, we want to determine if we can move two points from one end of the curve to the other end such that the two points are (always) of distance 1 and lie in the curve. In this talk, we will give a short solution to this problem. We will also discuss several other problems on different distances and on polytope graphs.

W. PLESKEN

An algorithm for finding the automorphism group of a lattice in Euclidean space

The motivation for developing such an algorithm was due to a conjecture by J. Thompson on integral laminated lattices with prescribed minimum m. For $m = 3$ and 4 the conjecture was verified thus giving (further) constructions and characterizations for a certain unimodular lattice in 23-space and for the Leech lattice. The algorithm itself consists of a backtrack search of tuples of lattice vectors which have the same scalar products as a given basis of the lattice. Since partial tuples usually cannot be extended to full basis tuples a matrix - called the finger print - is used to guide the search. The finger print records members of vectors (usually of shortest length) having certain scalar products with sequences of basis vectors. It does not only help to construct automorphisms, but can also be used to find isometries to other lattices and to prove or disprove that one has generators for the full automorphism group of the lattice. For integral lattices the algorithm can be improved by using the dual lattice at the same time. (joint work with M. Pohst).

J. M. WILLS

Regular polyhedra with hidden symmetries

The 5 regular compounds can be interpreted as collapsed polyhedral realizations of regular maps. In particular Kepler's Stella Octangula can be interpreted as a collapsed regular torus whose 48 automorphisms all occur as symmetries of the Stella Octangula. This is a surprising and simple example of the general program to investigate various polyhedral realizations of regular maps in E^3. The symmetry group of a polyhedral realization is a subgroup of the automorphism group of the regular map; its index is called the *index* of the polyhedron. The automorphisms which do not occur as symmetries are called *hidden symmetries*.
There are several methods and algorithms to find or enumerate such regular polyhedra or to disprove their existence: By A. Dress for Grünbaum's "new regular polyhedra", by J. Bokowski for regular polyhedra with or without self-intersections, by S. E. Wilson for branched covers and by A. M. Macbeath for unbranched covers. We discuss some results and interesting open problems.

J. O'ROURKE

Moving a ladder in three dimensions

The problem considered is planning the motion of a line segment (a "ladder") in three dimensions in the presence of polyhedral obstacles. An initial and final position are given, together with the obstacles, which have a total of n vertices. We show a lower bound of $\Omega(n^4)$ by exhibiting a collection of obstacles that force a ladder to make that many distinct moves to reach the final position. And we establish an $O(n^6 \log n)$ upper bound by describing an algorithm that requires that much time in the worst case. The algorithm is based on the cell decomposition approach pioneered by Schwartz and Sharir.

A. BJÖRNER

Continuous matroids

The continuous geometries of J. von Neumann are in a precise sense "continuous analogues" of the finite-dimensional projective geometries. In the discrete setting, these have been generalized in the theory of matroids, which are semimodular rather than modular.
We show how to construct "continuous analogues" to other classes of matroids through an embedding procedure. This yields, in particular, a continuous partition lattice. In joint work with L. Lovász we also constructed continuous algebraic matroids (based on field extensions) and continuous transversal matroids.

A. SCHRIJVER

Homotopic Routing

The following problem arises in some of the automatic procedures for the design of integrated circuits. Let $G = (V,t)$ be a planar undirected graph, embedded in the plane $\mathbb{R}^2$, let $I_1,\ldots,I_p$ be some of its faces (including the unbounded face), and let $C_1,\ldots,C_h$ be curves in $\mathbb{R}^2 \setminus (I_1 \cup \ldots \cup I_p)$, each starting and ending in vertices on the boundary of $I_1 \cup \ldots \cup I_p$. Do there exist pairwise edge-disjoint paths $P_1,\ldots,P_h$ in G so that P_i is homotopic to C_i in $\mathbb{R}^2 \setminus (I_1 \cup \ldots \cup I_p)$ $(i = 1,\ldots,h)$? We discuss some special cases where a so-called "cut-condition" is sufficient, and we discuss the result that this cut-condition always is equivalent to the existence of a "fractional" packing of paths.

R. E. JAMISON

Recognizing Angle Orders

Let Σ_k denote the class of convex k-gons in the plane. A partially ordered set $(P,\leq)$ is represented by Σ_k iff it is isomorphic to a subposet of Σ_k under set-inclusion. The class Σ_2 consists of all convex angular regions and the posets represented by Σ_2 were called angle orders by Fishburn and Trotter. There are several minimal posets known which are not representable by Σ_2, but it is unknown whether the number of these is finite or not. The recognition problem for representability by Σ_k (k fixed) is probably NP-hard and seems to be related to the following combinatorial problem:
Let X be a (finite) set and $\mathcal{J}$ a family of subsets. When can X be cyclically ordered so that each set in $\mathcal{J}$ is the union of at most k arcs (intervals) in the cyclic order?

P. ROSENSTIEHL

Jordan Pushing and Pulling

We consider in the plane a set of Jordan arcs monotonic with respect to the Y-axis, called the objects, and a set of Jordan arcs monotonic with respect to the X-axis called the pushers, such that each pusher is incident by one end with the upper side of a first object and incident by the other end with the lower side of a second object; the pusher is said oriented from the first to the second object. Furthermore the objects are pairwise disjoint, and any pusher is disjoint from the objects except by its two incidences; at last whenever two pushers cross, they are both oriented in the positive direction of the Y-axis.
The theorem proved is that the pushing relation between the objects is acyclic. The theorem is closed in the discussion of a system of potential inequality on the Y-coordinates of points belonging to the objects of the type $Y_j - Y_i \geq a_{ij}$ ($a_{ij} < 0$ for a pusher, $a_{ij} > 0$ for a puller). Consequences appear about the existence of Fary-type layout of planar graphs, about tentative of stretching of pseudolines of a pattern, and practically while improving the automatic design of electrical networks. The result generalizes the dominance relation studied by L.J. Guibas and F.F. Yao for the motion in a given direction of segments of any direction.

G. PURDY

Some Extremal Problems in Geometry

We discuss several inequalities and extremal problems in $\mathbb{C}^d$ instead of the more usual $\mathbb{R}^d$, including a conjecture of Dirac, a result of Kelly and Moser and the inequality $n - \ell + p \geq 0$, conjectured by the author, involving the ℓ lines and p planes determined by n points. We use the inequality $t_2 + 3/4\, t_3 \geq n + \sum_{i\geq 3} (2i-1)t_i$ involving the number t_i of lines containing exactly i points from among n given points in $\mathbb{C}^2$, (assuming $t_n = t_{n-1} = t_{n-2} = 0$), proved recently by F. Hirzebruch, using an inequality on Chern numbers.

N.J.A. SLOANE

The Solution to Berlekamp's Switching Game

Four recent results in packing and covering (I) the theta series of diamond. (II) Packing superballs $H = \{(\kappa_1,\ldots,\kappa_n) : \Sigma\, |\kappa_i|^\sigma \leq 1\}$ (with J.A. Rush). (III) Penny-packing (with R.L.Graham). (IV) The solution to Berlekamp's light-bulb game: for the 10×10 game the answer is 34 (with P.C. Fishburn).

B. KORTE

Polynomial Time Algorithms for Convex Shellings

In Combinatorial Optimization very often a polynomial-time (combinatorial) algorithm for a problem and a nice and comprehensive description of the convex hull of its characteristic vectors coincide. There are only few exemptions from this rule: for stable sets in perfect graphs a good polyhedral description but no a good combinatorial algorithm is known. On the other hand the stable set problem in claw-free graphs can be solved in polynomial time, but we do not have a polyhedral characterization. Only very "ugly" facts are known. One aim of this lecture is to contribute another problem to this list of exemptions: We show that the validity problem for the feasibility polytope of semi-convex and convex shelling in the plane can be solved in polynomial time, but we were not able to describe the polytope, since we found also very strange facets.

For some other antimatroids we were able to characterize the convex hull of characteristic vectors nicely. For some other antimatroids we could only characterize the conical hull, while a characterization of the convex hull is NP-hard. For other antimatroids (with long circuits) even the membership problem for the conical hull is NP-complete. - This reports on joint work with L. Lovász.

R.L. GRAHAM

Remarks on some nonstandard packing and covering problems

In this talk we will discuss various results and problems which relate to the following two questions:

1. Given an open convex region $R \subseteq \mathbb{E}^2$, what is the shortest length CL(R) a curve C can have which cannot be covered by R in any position or orientation. For example, if R is a disc D then CL(D) = diam(D). Similarly, if R is a square S, then CL(S) = diam(S). However, as Besicovitch showed in 1965, if R is an equilateral triangle T, then CL(T) < 0,99 diam(T). Which regions (or polygons) R have CL(R) = diam(R)? As R = R(t) continuously changes from a circle (t = 1) to an ellipse of eccentricity t, how does the optimal curve C evolve? In particular, when does it become non-polygonal? Is there a decision procedure for determining CL(R) when R is a polygon? What happens in higher dimensions, e.g., a unit simplex or cube in $\mathbb{E}^3$?

2. What is the densest packing of unit squares in a square of side α? If W(α) denotes the minimum possible area of uncovered space in a packing of unit squares into a large square of side α, it is clear that W(α) = O(α). In 1975, P. Erdös and the author showed that $W(\alpha) = O(\alpha^{7/11})$. This was subsequently improved by Montgomery to $W(\alpha) = O(\alpha^{\frac{3-\sqrt{3}}{2}+\varepsilon})$ for any ε > 0. The best lower bound known is due to Roth and Vaughan. It asserts that for $\alpha(\alpha - [\alpha]) > 1/6$,

$$W(\alpha) > c(\|\alpha\|\ \alpha)^{1/2}$$

for some c > 0 where $\|\alpha\|$ denotes the distance of α to the nearest integer. There are many variants of this question, e.g., packing strips with discs or squares, covering squares by squares, etc.

DAVID DOBKIN

Finding Empty Convex Polygons

Let S be a set of n points in general position in the plane. Let $\Gamma_r(S)$ denote the set of $T \subset S$ such that $|T| = r$ and the points of T form a convex r-gon with no other point of S in it. Let $\gamma_r(S) = |\Gamma_r(S)|$. We show that $\Gamma_r(S)$ can be found in time $O(\gamma_r(S) + \gamma_3(S))$.

Let $g_r(n) = \min\{\gamma_r(S) \mid |S| = n\}$. We show that $\frac{n^2}{2} + cn \leq g_3(n) \leq 2n^2$, $\frac{n^2}{8} + cn \leq g_4(n) \leq 3n^2$, $\frac{n}{6} + c \leq g_5(n) \leq 2n^2$ and $g_6(n) \leq \frac{n^2}{2}$.

DAVID AVIS

Extremal Properties of Fixed Radius Neighbourhood Graphs

For $i = 1,\ldots,n$ let (x_i,r_i) be a sphere in $\mathbb{R}^d$ with centre x_i and radius r_i. A fixed radius neighbourhood graph (F.R.N.G.) is a directed graph on the centres, where (x_i,x_j) is an edge whenever x_j lies on the sphere (x_i,r_i) . Examples are the minimum distance graph, nearest neighbour graph, diameter graph, furthest neighbour graph etc. Bounds are given on the maximum number of edges in F.N.R.G. which are assymptotically tight in even dimensions. It is established that the furthest neighbour graph in $\mathbb{R}^3$ can have at most $\frac{n^2}{4} + \frac{3n}{2} + 255$ edges for sufficiently large n . Examples show that such graphs can have at least $\frac{n^2}{4} + \frac{3n}{2}$ edges.

(Joint work with P. Erdös and J. Pach)

J. J. SEIDEL

Designs and Approximation

Each of the notions: ordinary t-(v,κ,λ) design, spherical t-design, cubature formula of strength t for the unit sphere S , deals with the approximation of the set of all blocks, resp. vectors, by a nice subcollection. We present a general setting for these notions and their generalizations in terms of measures in $V = \mathbb{R}^d$ by defining $d\xi$ to be a measure of strength t whenever $_V\int f_k(x)d\xi(x) = {}_V\int \|x\|^k d\xi(x) \cdot {}_S\int f_k(x)d\tau(x)$, for all $f_k \in Hom_k(V)$, for $k = 1,2,\ldots,t$.

The case of finite support (X,w) with $t = 2$ leads to eutactic stars. Finite support (X,w) with $t = 2e$, and X distributed over enough spheres, yields $|X| \geq \dim Pol_e(V) = \binom{d+e}{e}$.
The case $X \subset S$ leads to cubature formulae and spherical t-designs.
Also lattices fall under the general notion of measure of strength t ; certain results are to be expected.
(Joint work with A. Neumaier)

GÜNTHER M. ZIEGLER

The Face Lattice of Hyperplane Arrangements

Every arrangement H of affine hyperplanes in $\mathbb{R}^d$ determines a partition of $\mathbb{R}^d$ into open topological cells. The face lattice $L(H)$ of this partition was the object of a study by Barnabei and Brini.
We use geometric constructions from the theory of convex polytopes to prove the shellability of $L(H)$ and to determine the topology of its intervals up to homeomorphism.
We will discuss connections to other recent progress in the combinatorics of hyperplane arrangements.

L. J. GUIBAS

On the complexity of many facets in an arrangement of n lines in the plane

What is the largest total number of edges that $\underline{m}$ distinct faces can have in an arrangement of $\underline{n}$ lines on the plane? We prove that this quantity is $\vartheta(m^{\frac{1}{1+\gamma}}n^{\frac{2\gamma}{1+\gamma}} + m + n\log n)$ for any $\gamma > \frac{2}{3}$. Our technique is based on analyzing the space complexity of an algorithm for computing the faces containing m given points $p_1,\ldots,p_m$ in an arrangement of n lines $l_1,l_2,\ldots,l_n$. The algorithm uses a partition-based range searching technique for solving the half-planar range query problem: The search tree is built on the duals of the lines $l_1^*,\ldots,l_n^*$. Then the duals of the points $p_1^*,\ldots,p_m^*$ are sent down this tree. Whenever a line (dual point) reaches a node all of whose points are on the same side, the line is placed on a special bucket associated with that node. The partition subdivision stops when (# of lines at a node) $\geq$ (# of points)^2 . At the bottom we dualize back and compute the full arrangement. We then go back up the tree, combining the cells computed from each of the children.

JÜRGEN BOKOWSKI

Realizable and Nonrealizable Chirotope manifolds of genus 3

Oriented matroids or chirotopes have been proved to be an appropriate structure to study a variety of realizability problems. The talk deals with two cases in which combinatorial complexes are given and the geometric realization is of interest and has to be decided: geometric regular polyhedra and manifolds of genus 3.

The method of the author to decide such cases was discussed and more recent results were given. The shape of polyhedral realizations of these manifolds of genus 3 was shown on slides, video-tapes and a graphic work station.

HENRY CRAPO

Geometric reasoning by computer

1. Computational tools for geometric research

Geometric research has always been impeded by the unavailability of adequate and efficient means for visual representation, and by the unavoidable gap which separates concrete geometric models from their logical and algebraic description. Recent advances in computer-aided design, together with progress in computer-aided geometric reasoning, promise a speedy improvement in the conditions under which geometric research is carried out.

Since computers haven't been told there's anything special about three dimensions, they are perfectly content to work on higher-dimensional problems, when programmed to use the usual techniques of vector representation and linear algebra. Higher dimensional subspaces are easily represented in projective (Grassmann-Plücker) coordinates. Exterior algebra, suitably upgraded to the Rota-Doubilet-Stein double algebra of join and meet, enables one to draw out the consequences of geometric hypotheses for geometric figures of arbitrary dimension. But since computer output is typically no more than 2-dimensional, consisting as it does of essentially 1-dimensional strings of letters, and two-dimensional drawings or screen presentations, some way has to be found adequately to represent and to manipulate higher-dimensional structures in 2-dimensional form.

2. A glimpse at descriptive geometry

Since the time of Gaspard Monge, geometers have successfully developed techniques to bridge the gap between 2 and 3 dimensions. The most substantial effort goes by the name of descriptive geometry. The basic technique in descriptive geometry is to work on plane drawings as if they were already 3-dimensional. The first step is just a question of correct labelling of a plane figure: the visible intersection of two lines in the drawing plane is not taken to be a point unless the two lines are known to be coplanar in the associated spatial realization. As a second step, the descriptive geometry technique of rabattement (rotation of flat polygonal faces down into the plane) can be used to obtain correct Euclidean dimensions for plane faces, so they can be cut from cardboard, and real 3-dimensional models can be built.

There is no obstacle, at least in theory, to extending the methods of descriptive geometry to higher dimensions. Janos Baracs, a colleague of ours and founder of the Structural Topology research group in Montreal, undertook the extension of descriptive geometry to 4 and 5 dimensions, in order to understand the mechanics and statics of bar and joint structures in 3 dimensions.

3. The use of partially defined objects

The natural description of geometric structures, and thus the starting point for any comprehensive geometry software package, is in terms of variable points, and combinatorial statements of incidence and other projective properties. The corresponding calculations can only be accomplished in terms of polynomials in the coordinates of undetermined points. Symbolic computation methods are applicable to this problem, but will need to be extended to cover the gap mentioned in our introductory paragraph. Geometric statements, correctly translated into geometric language, do not necessarily describe single families of geometric models. There is a phenomenon of branching omnipresent in the adjoint situation linking geometric properties and geometric models. Algebraically, this is simply the observation that an radical ideal is an intersection of prime ideals.

4. Outlines of a programming environment

The essential features of a programming environment for automated descriptive geometry would seem to include the following:

- input devices flexible enough to generate exact data of incidence, approximate data of location, without impeding the free use of geometric imagination.
- a data base permitting the stocking of a variety of partially defined geometric figures, either generic, or else with specific projective coordinates, whenever these become available.
- a rich vocabulary of elementary forms
- interactive definition of complex structures, by declarations of incidence, or by other methods of composition starting from simple structures (such as splines). Such structures should include all sorts of configurations of lines and planes, mechanical and architectural structures of bars and joints, of hinged panels, and tensegrity systems of elastic cables and sheets.
- calculation of the effect of geometric operations, principally of projection and intersection, carried out in symbolic form in the double algebra of Doubilet-Rota-Stein.
- automatic construction of generic models (often after taking into account the natural branching into classes of minimal models), and interactive computation of models determined by a series of free choices of heights, bar lengths, dihedral angles, and the like.
- automatic calculation of descriptions of figures with special geometric properties, such as those which lift to higher dimensions, or which admit certain internal motions.
- derivation of the logical consequences of geometric hypotheses, and automatic proof of geometric theorems.
- legible representations of higher-dimensional geometric forms, via screen, plotters, laser printers.

CHEE K. YAP

Moving a robot arm in a partially known environment (or, How to search in the dark)

We report on a joint work with J. Cox. We consider motion planning in a significantly different setting than that used in most current research. This model, first used by Lumelsky, assumes that the environment (obstacles) is not known except that we assume a polygonal environment. The algorithm discovers its environment by making "guarded moves" which consists of prescribing an algebraic motion which is stopped when the arm touches new obstacle points. The arm is assumed to be completely covered by sensors so that all contact points are known. We show that there is a polynomial time algorithm for a 3-link arm in the plane. This case is important because significant new complications arise which are not encountered in Lumelsky's original work on 2-degree of freedom robots. Our method is based on the retraction approach.

LOUIS J. BILLERA

Computing bases for modules of smooth splines

For a simplicial (or general polyhedral) d-dimensioned complex in $\mathbb{R}^d$, we define $C^r(\Delta)$ to be the set of all smooth (of order r) piecewise polynomial functions defined on Δ. We consider the question of computing a free basis for $C^r(\Delta)$ as a module over the polynomial ring $R = \mathbb{R}[x_1,\ldots,x_d]$ and we describe an example of a triangulated 2-disk Δf_n for which $C^1(\Delta)$ is not free. ($C^0(\Delta)$ is free for all triangulated d-manifolds and $C^r(\Delta)$ is free for all 2-manifolds). Finally, we describe a construction of the ring $C^r(\Delta)$ for d-complexes embedded in $\mathbb{R}^N$, $d < N$, which leads, in the case $r = 0$, to a possible generalization of the notion of the face ring to nonsimplicial complexes.

PETER W. SHOR

A Simplified Realization of Davenport-Schinzel Sequences by Segments

We show that the lower envelope of n line segments in the plane can have size $\theta(n\alpha(n))$, where $\alpha(n)$ is the inverse Ackermann function. The lower envelope of the segments forms a Davenport-Schinzel sequence, so this bound is tight. This theorem was first proved by Ady Wiernik. Our construction uses the same combinatorial construction of the sequence as Wiernik, but it simplifies the placing of the line segments by using one real parameter instead of a complicated system of constraints. We hope the same techniques can be used to show non-linearity of other realizations of Davenport-Schinzel sequences.

ALOK AGGARWAL

Geometric Applications of a Matrix-Searching Algorithm

Let i_1 and i_2 be any two rows and let j_1 and j_2 be any two columns of a $(p * q)$-sized, real-valued matrix A . Then, A is called monotone if both $b > a$ and $c > d$ are not simultaneously possible. We show that the maximum values for all rows of a monotone matrix A can be computed with $\theta(p + q)$ questions where every question only asks for the comparison between some two entries of A . We describe some geometric applications of finding the maxima in monotone matrices and also discuss some unresolved problems.

R. POLLACK

Computing the geodesic center of a simple polygon

The geodesic center of a simple polygon is a point inside the polygon which minimizes the maximum internal distance to any point in the polygon. We present an algorithm which calculates the goedesic center of a simple polygon with n vertices in time $O(n \log^2 n)$. This is a generalization of the problem of finding the center of the smallest circle enclosing n given points.
(Joint work with Micha Sharir)

Berichterstatter: A. Dress

MARTIN GRÖTSCHEL

Decomposition and Optimization Algorithms for the Cycle Problem in Binary Matroids

A cycle in a binary matroid is the disjoint union of circuits. The maximum weight cycle problem is the task to find, given a binary matroid M with weights on the elements of the ground set, a cycle of maximum weight. The cycle polytope of M is the convex hull of the incidence vectors of the cycles of M.
For $k = 2$ and 3, we define several k-sums of binary matroids and of cycle polytopes; and we establish interesting relations between these k-sums. We exploit these relationships to construct polynomial time algorithms for the solution of the maximum weight cycle problem for some classes of binary matroids and for the solution of the separation problem of a certain LP-relaxation of the cycle polytope. These algorithms are based on polynomial time matroid decomposition algorithms and on good optimization procedures for certain special cases of the cycle problem. This work is joint with Klaus Truemper (Dallas).

ZOLTAN FÜREDI

The Solution of the Littlewood-Offord Problem in High Dimension

Consider the 2^n partial sums of arbitrary n vectors of length at least one in d-dimensional euclidean space. It is shown that no closed sphere of diameter Δ contains more than $(\lfloor\Delta\rfloor + 1 + o(1))\binom{n}{\lfloor n/2\rfloor}$ out of these sums, and this is best possible. For $\Delta - \lfloor\Delta\rfloor$ small an exact formula is given.
These are joint results with P. Frankl.

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

Tagungsteilnehmer

Dr. A. Aggarwal
IBM Corporation
Thomas J. Watson Research Center
P. O. Box 218

Yorktown Heights , NY 10598
USA

Prof. Dr. J. Bokowski
Fachbereich Mathematik
der TH Darmstadt
Schloßgartenstr. 7

6100 Darmstadt

Prof. Dr. D. Avis
School of Computer Science
McGill University
805 Sherbrooke St. West

Montreal P. Q. H3A 2K6
CANADA

Prof. Dr. K.H. Borgwardt
Mathematisches Institut
der Universität Augsburg
Memminger Str. 6

8900 Augsburg

Prof. Dr. I. Ba'ra'ny
Dept. of Mathematics
University College
Gower Street

GB- London , WC1E 6BT

Dr. F. R. K. Chung
Bell Communications Research Center
435, South Street

Morristown , NJ 07960
USA

Prof. Dr. L. J. Billera
Department of Mathematics
Cornell University
White Hall

Ithaca , NY 14853-7901
USA

Prof. Dr. H. Crapo
INRIA
Domaine de Voluceau - Rocquencourt
B. P. 105

F-78153 Le Chesnay Cedex

Prof. Dr. A. Björner
Dept. of Mathematics
University of Stockholm
Box 6701

S-113 85 Stockholm

Prof. Dr. W. Deuber
Fakultät für Mathematik
der Universität Bielefeld
Postfach 8640

4800 Bielefeld 1

Prof. Dr. D. P. Dobkin
Department of Computer Science
Princeton University

Princeton , NJ 08540
USA

Prof. Dr. R. E. Jamison
Schwarzackerstrasse 30

CH-4411 Seltisberg (BL)

Prof. Dr. A. Dress
Fakultät für Mathematik
der Universität Bielefeld
Postfach 8640

4800 Bielefeld 1

Prof. Dr. B. Korte
Institut für Ökonometrie und
Operations Research
der Universität Bonn
Nassestr. 2

5300 Bonn 1

Prof. Dr. J. E. Goodman
Mathematics Department
The City College
CUNY
137 St. and Convent Ave.,

New York , NY 10031
USA

Prof. Dr. J. O'Rourke
Department of Computer Science
John Hopkins University

Baltimore , MD 21218
USA

Prof. Dr. R. L. Graham
AT & T
Bell Laboratories, 2D - 151
600 Mountain Avenue

Murray Hill , NJ 07974-2070
USA

Dr. W. Plesken
School of Mathematics
Queen Mary College
University of London
Mile End Road

GB- London , E1 4NS

Prof. Dr. M. Grötschel
Institut für
Angewandte Mathematik II
der Universität
Memminger Str. 6

8900 Augsburg

Prof. Dr. R. Pollack
Department of Mathematics
Courant Institute
New York University
251, Mercer Street

New York , N. Y. 10012
USA

Prof. Dr. L. J. Guibas
Digital Equipment Corporation
Systems Research Center
130 Lytton Avenue

Palo Alto , CA 94301
USA

Prof. Dr. G. Purdy
Dept. of Mathematics
Texas A & M University

College Station , TX 77843
USA

Prof. Dr. P. Rosenstiehl
Ecole des Hautes Etudes en Sciences
Sociales, Centre d'Analyse et de
Mathematiques Sociales
54, Boulevard Raspail

F-75270 Paris Cedex 06

Prof. Dr. R. Scharlau
Fakultät für Mathematik
der Universität Bielefeld
Postfach 8640

4800 Bielefeld 1

Prof. Dr. A. Schrijver
Department of Econometrics
Tilburg University
P. O. Box 90153

NL-5000 LE Tilburg

Prof. Dr. J. J. Seidel
Department of Mathematics
Technische Hogeschool Eindhoven
Postbus 513

NL-5600 MB Eindhoven

Prof. Dr. M. Sharir
Department of Mathematics
Courant Institute
New York University
251, Mercer Street

New York , N. Y. 10012
USA

Prof. Dr. P. Shor
AT & T
Bell Laboratories, 2D - 151
600 Mountain Avenue

Murray Hill , NJ 07974-2070
USA

Prof. Dr. N. J. A. Sloane
AT & T
Bell Laboratories, 2D - 151
600 Mountain Avenue

Murray Hill , NJ 07974-2070
USA

Prof. Dr. J.M. Wills
Lehrstuhl für Mathematik II
Universität Siegen
Hölderlinstr. 3

5900 Siegen 21

Prof. Dr. C. K. Yap
Department of Mathematics
Courant Institute
New York University
251, Mercer Street

New York , N. Y. 10012
USA

G. M. Ziegler
Dept. of Mathematics
Massachusetts Institute of
Technology

Cambridge , MA 02139
USA