MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 6/1987

Flächen in der geometrischen Datenverarbeitung

8.2. bis 14.2.1987

Die dritte internationale Tagung über "Flächen in der geometrischen Datenverarbeitung" im Mathematischen Forschungsinstitut Oberwolfach stand unter der Leitung von R. E. Barnhill (Arizona State Univ., Tempe), W. Böhm (z. Zt. Rensselaer Polytechnic Inst., Troy, N.Y.) und J. Hoschek (Techn. Hochschule Darmstadt).

Die 45 Teilnehmer (darunter 9 aus dem europäischen Ausland, 13 aus den USA, 1 aus China) kamen nicht nur von Universitäten und Hochschulen, sondern auch von Forschungsinstituten und aus der Industrie. Sowohl die regen Diskussionen nach den einzelnen Vorträgen wie auch die zahlreichen Gespräche im Verlauf der Tagung zeigten, daß die Teilnehmer eine gelungene Mischung aus Theoretikern und Anwendern darstellten.

Zu folgenden Themenkreisen wurden u.a. die neuesten Forschungsergebnisse vorgestellt:

Geometrische Stetigkeit, Glätten von Kurven und Flächen, Powell-Sabin Interpolation im $\mathbb{R}^3$, de Boor Algorithmus, diskrete Daten und Kurven, Streichen von Knotenlinien bei Tensorprodukt-Splines, quartische C^1-Splines über Dreiecken, Approximation von Offset-Kurven, Ermittlung von Schnittkurven, Einflüsse von Ungenauigkeiten bei der Berechnung geometrischer Objekte, Interpolation über Dreiecken in Abhängigkeit von der Triangulation, algebraische Flächen, u.v.m..

Vorträge von in der Industrie tätigen Teilnehmern rundeten diese Problemfelder ab und gaben vielseitige Anregungen zu aktuellen Fragen der Praxis.

Vortragsauszüge

J. A. Gregory

Geometric Continuity and Polygonal Patches

The problem of filling a polygonal hole within a parametric C^2 rectangular patch complex is discussed. The theory of geometric continuity between patches is used in the construction of a polygonal interpolation patch which has a curvature continuous join with its rectangular neighbours (GC^2). The general GC^k solution to the problem is also discussed.

H. Nowacki, H. Meier

Jerk Minimized Curves

In the past the minimization of flexural strain energy has played an important part in the fairing of curves. This energy is proportional to the curvature or 2nd derivative.
Recently investigations have been carried out in which the so-called jerk (or jerkiness), i.e., the change of curvature or the third derivative, is used as a norm and is minimized over the range of the curve taking boundary conditions into account. This procedure appears to better simulate certain properties of manual fairing. - Combinations of the two approaches are also conceivable.
The talk will discuss first experiences with the new criterion.

A. Worsey

A Three-Dimensional Powell-Sabin Interpolant

We consider the problem of producing a piecewise quadratic, C^1 interpolant to function and gradient values at the vertices of a tessellation in $\mathbb{R}^3$ of tetrahedra. By splitting each tetrahedron in an appropriate way we show how the problem may be solved. The splitting process is a generalization of that for the bivariate Powell-Sabin element.

W. Böhm

On the Structure of the de Boor Algorithm

It is wellknown that some modifications of the de Boor algorithm permit de Boor-like evaluations of a Bernstein polynomial (de Casteljau '59), Lagrange interpolants (Beatty/Farin '85) and Polya polynomials (Goldman '86). In this paper a generalization is given, that shows the simple underlying common structure of these algorithms. In particular it allows simple changes of bases, and e.g. the calculation of the Bézier points of any segment of a Lagrange interpolant or a spline.

H. W. McLaughlin

Shape of Discrete Data

For a finite set of ordered points in the plane, consideration is given to defining, in computer understandable terms, a notion of shape. Armed with a definition of shape of discrete data, one can look for algorithms designed to insert, in a shape preserving manner, additional points into a, relatively sparse, given discrete point set. In addition, such a definition of shape, permits one to look for algorithms designed to delete, in a shape preserving manner, most of the points in a relatively dense discrete point set. Discussion of the boundaries of the study is provided.

H. Prautzsch

Uniform Refinement

Curves and surfaces (triangular and rectangular) are considered which can be uniformely subdivided. For example such curves and surfaces can be generated by de Casteljau's construction, de Rham's algorithm, and box spline subdivision algorithm. Generally uniform refinement can be described in terms of some matrices. Given p ($p \in \mathbb{N}$) arbitrary matrices it is shown when there is a curve (surface) refineable by these matrices and how higher differentiability and existence of polynomial components of the curve (surface) depend on the p refinement matrices.

T. Lyche

Knot Removal for Parametric B-Spline Surfaces

We will present an algorithm for removing knotlines from a tensor product parametric B-spline surface. Given a tolerance, the algorithm computes a B-spline surface with fewer knot lines and which in norm differ from the original surface by no more than the tolerance. The reduced surface also tends to be smoother than the original surface. Thus the algorithm is at the same time a data reduction and a smoothing method.

B. Piper

An Explicit Basis for C^1 Quartic Bivariate Splines

The dimension of the space of C^1 bivariate piecewise polynomials defined on a triangulation of a polygonal domain is established. Our approach is to construct minimal determining sets and associated explicit bases for the space.

S. C. S. Cohen

A New Three-Point Interpolation Yielding Triangular Patches with Local Shape Parameter

This paper presents a new 3-point interpolation yielding triangular patches with shape parameters.
Local shape modifications of composite surfaces become possible, at individual patches or over a subset of such patches.

G. Chang

The Convexity for Triangular Bernstein-Bézier Polynomials of Functions in $C^2(t)$

Let T be a given domain triangle. $B^n(f;p)$ denotes the nth Bernstein-Bézier polynomial of f defined on T.
Among other things, two main results are presented:

i) for $f \in C^2(T)$, the uniform limit of

$$n^2 \left[B^n(f;p) - B^{n+1}(f;p) \right] \text{ as } n \to \infty \text{ is determined;}$$

ii) for $f \in C^2(T)$, the strict convexity of f implies the strict convexity of $B^n(f;p)$ for sufficiently large n.

J. M. Hahn

Geometric Continuous Patch Complexes

A theory of geometric continuity of arbitrary order of continuity is presented. Conditions of geometric continuity at vertices where a number of patches meet are investigated. Geometric continuous patch complexes are introduced as the appropriate setting for the representation of C^k-surfaces in CAGD. The theory is applied to the modelling of closed surfaces and to filling triangular patches into a geometric continuous patch complex.

J. Hoschek

Spline Approximation of Offset-Curves

The offset-curves of a spline-curve are non spline-curves. Therefore the question was imposed how to approximate the offset-curves by spline curves with a given error tolerance. The introduced technique uses geometric continuity as boundary conditions, parameter optimization, linear least square methods and non linear optimization methods. The technique doesn't need any restrictions for the degree of the spline-curve and for the degree of the approximation of the offset-curve.

H. Hagen

Automatic Smoothing with Geometric Surface Patches

The generation of smooth surfaces from a mesh of three-dimensional data points is an important problem in the field of Computer Aided Geometric Design. A new method based upon generalized Gordon-Coons patches and calculus of variation is presented.

F. N. Fritsch

BIMOND3: An Improved Monotone Piecewise Bicubic Interpolation Algorithm

This is a contribution to shape-preserving interpolation of gridded data in two independent variables. Improvements to the monotonicity-preserving piecewise bicubic interpolation algorithm since the previous Oberwolfach meeting will be described. This is a joint work with Dr. Ralph E. Carlson.

M. Ahlers

Practical Problems in Data Exchange

The topic is to point out some problems arising in the field of exchanging data between different CAD/CAM-systems. The main problem is to look for the future representation form for free-form curves and surfaces. Therefore the rational B-spline is in discussion. Other problems are mentioned when in an application the designer has to accept or to reject incoming data or to work with accepted data.

The purpose of the talk was to have feedback for solving problems and future decisions.

R. E. Barnhill

Surface/Surface Intersections

Finding the intersection of two surfaces is important for many Computer Aided design tasks concerned with surface modeling. An adaptive algorithm is developed for finding the intersection curve(s) of pairs of rectangular parametric patches which are continuously differentiable. The balance between robustness and efficiency of the algorithm is controlled by a set of tolerances. A suite of examples concludes the paper.

B. Piper

Continuous Triangulations

We consider triangulation based interpolation schemes for arbitrarily spaced data. If a triangulation of just the data sites is used, the resulting surfaces changes discontinuously with respect to the data sites. This difficulty may be overcome by using a triangulation of the Dirichlet tessellation. A 3 minute move will be shown to illustrate these concepts.

R. T. Farouki

Imprecise Geometric Computation

We describe preliminary results in a theory of imprecise geometric computation, which attempts to incorporate awareness of the consequences of finite precision arithmetic in geometric algorithms. The focus is on algebraic geometry representations and the systems of polynomial equations arising in their intersection. Running error analyses are formulated for several basic polynomial procedures, and theoretical arguments are used to demonstrate the enhanced conditioning of the Bernstein basis for polynomial root determination.

M. Goldapp

Surface Representation by Reflection Lines

This talk describes how reflection lines can be used to visualize local surface geometry.
Reflection lines are the curves that a viewer can see as the reflection of a set of parallel lines (light sources) on a surface. They are suitable for surface display because they are of lower differentiability order than the surface itself. A method of numerical computation is also given.

H.-J. Hochfeld

Methods of Diagnosis and Visualization of Sculptured Surfaces in the CAD System VWSURF of Volkswagen

The CAD system VWSURF is operational at Volkswagen and Audi for

- generation, design, manipulation and
- visualization, diagnosis

of car body sculptured surfaces.
It is based on piecewise polynomial representation by Bézier-Bernstein methods. The generated surfaces must match special requirements that are sometimes contradictory, i.e.

- approximation accuracy
- surface smoothness
- stylistical appearance

The possibilities and methods of diagnosis and visualization which are parts of the CAD system are explained and illustrated by examples.

W. Degen

Algebraic Surface Segments as Interpolants

It was pointad out that parametric representation of curve and surface segments by polynomials define these objects at same time being algebraic varieties. Taking this fact into account, one can gain principal information about their behaviour, even in approximation and interpolation context. As examples of this point of view were treated: CG^2-continuity of Bézier curve segments, approximation by quadrics, questions of order determining, rational nets on algebraic surfaces, especially with conics as parameter lines lying on generalized cyclids.

F. F. Little

Computing Surfaces

Rational polynomials provide constructive models for complicated surfaces. Adaptive techniques render them into controls for fabricating, displaying and analizing these surfaces. Rendering requires the solution of geodesic and arc length problems. The rational Bézier form facilitates the solution of many rendering problems.

K. Höllig

Unusual Approximation Orders

We discuss two results which are in apparent contrast to the standard error estimates for spline approximation:
Theorem A: A smooth planar curve with nonvanishing curvature can be approximated by C^2 piecewise cubic splines with order $O(h^6)$ where h is the maximal distance of adjacent knots.

Theorem B: Denote by S the space of C^1 piecewies cubics on the triangulation of $\mathbb{R}^2$ generated by the three directions (h,0), (0,h), (h,h). We show that dist(f,S) $\neq O(h^3)$ for smooth function f.

W. Böhm

On the Geometry of the de Boor Algorithm

The wellknown de Boor algorithm was viewed and proved from the view point of the geometricians of mid 19th century.

P. Brunet

Including Shape Handles in Recursive Subdivision Surfaces

In this talk, the problem of the generation of an interpolating surface for a given, general polyhedron is studied. The surface must interpolate the set of vertices of the initial polyhedron, and allow a certain shape control. The Nasri work on the Doo/Sabin procedure is used, and shape handles associated to the initial vertices are discussed. They allow to model and increase the quality of the shape of the surface, without effecting the interpolating properties.

W. Schempp

Ambiguity Surfaces

The purpose of this lecture is to point out the rôle played by the concept of ambiguity surface in the field of Computer Aided Design of radar signals and in the geometry of opto-couplers used in laser opto-electronical technology. As a consequence we establish a series of remarkable identities for theta-null values.

S. Steiner

A Generalization of the Shift Operator Method and some Applications of this Method

The shift operator method is known to be a very elegant way for proving or deriving formulas for Bernstein-Bézier curves and surfaces.
The main purpose of this contribution is to show that this algorithm can be considered a special case of a more general substitution algorithm of points or vectors into polynomials or formal power series of (one or) several variables. In this setup several well-known formulas and schemes of CAGD are easy consequences of the basic algebraic operations for polynomials, e.g. the de Casteljau algorithm (which corresponds to multiplication of polynomials) or the transformation formulas from Bézier to Taylor and vice versa (which correspond to substitution of polynomials into polynomials).

L. Piegl

Coons-Type Surface Patches

The talk looks at the problem of defining Coons-type surface patches expressed in terms of geometric data: the four corner points, eight tangent vectors and four direction vectors used to replace the twists in the original definition of Coons patches. The creation of such surface patches goes back to the definition of rational Bézier- and B-spline surfaces having control points at infinity. The problem of a reliable shape control and continuity conditions between adjacent patches are discussed in details.

T. Varady

Overlap Patches, a Method for Interpolating Topologically Irregular Networks

After presenting a classification of free-form curve networks from topological point of view, a new method for interpolating so-called PFP-type networks by patches with C^1 and VC^1 continuity is given. The overlap patches are composed of individual vertex-patches, which may overlap each other in many different ways. The basic benefits and deficiences of overlap patches in comparison with previous methods are also discussed.

M. Daehlen

Bivariate Interpolation with Quadratic Box-Splines

This paper is concerned with bivariate interpolation using translates of box-splines on a three-direction mesh. Mainly we will concentrate on quadratic box-splines. The interpolation is over polygonal finite convex regions in $\mathbb{R}^3$. We show uniqueness of the interpolation problem for several configurations of the interpolating points.

T. W. Sederberg

Root Isolation of Bernstein-Form Polynomials

An algorithm is presented for isolating the leftmost root of a polynomial in Bernstein form. The algorithm determines a step which can never cross more than one root.

Parametric Surface Patch Intersections

A sufficient condition is presented for two surface patches to not intersect in any closed loops. This means that all branches of the intersection curve must pass through a boundary curve.

Tensor Product Piecewise Algebraic Surfaces

The present discussion pertains to a tensor product algebraic surface over a parallelepiped lattice of control points. This provides easier continuity constraints, along with greater design flexibility.

H. Müller

Realistic Computer Graphics and Free Form Surfaces

Crucial for the efficiency of realistic image synthesis by computers, following the ray tracing approach, is to find quickly an intersection point closest to a ray's origin in a given spatial scene. For parametric free form surfaces, this requires to restrict the candidate patches as well as to find efficiently the intersection of rays with these patches. Besides a survey we introduce a new method that may be seen as a generalization of the classical depth buffer algorithm of computer graphics.

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