MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 9/1987

New Foundations

1.3. bis 7.3.1987

Die Tagung fand unter der Leitung der Herren M. Boffa (Brüssel) und E. Specker (Zürich) statt. Ihr Hauptthema war das System New Foundations, ein Axiomensystem für die Mengenlehre, welches im Jahre 1937 - also vor 50 Jahren - von W.V. Quine aufgestellt wurde. So darf der Vortrag von Quine "The inception of New Foundations" gewiss als Hauptvortrag bezeichnet werden. Auf allgemeinen Wunsch erklärte sich Quine bereit, für die Teilnehmer der beiden parallel stattfindenden Tagungen einen weiteren Vortrag mehr philosophischen Charakters zu halten.

In weiteren Vorträgen wurde der Geschichte von New Foundations nachgegangen und über die aktuellen Probleme berichtet.

Die Teilnehmer waren sich einig im Dank an das Forschungsinstitut für die erfolgreiche Tagung mit ihrer Möglichkeit der Kontaktnahme mit Forschern aus verschiedenen Ländern.

Vortragsauszüge

W. Baur:
Elementary theories of abelian groups with subgroups

Let $L = (0,+,P)$ be the first order language of abelian groups with an additional unary predicate symbol P . Kozlov and Kokorin (1969) showed that the L-theory of all structures (A,B) , A a torsionfree abelian group, B an arbitrary subgroup is decidable. This implies Szmielew's theorem (1955) on the theory of abelian groups. If in the above situation the hypothesis that A be torsionfree is dropped then the resulting theory is undecidable. More precisely: Let p be a prime. Then the theory of all structures (A,B) , A an abelian group satisfying $p^9 A = 0$, B a subgroup is undecidable (1976). The proof proceeds by translating an undecidable word problem into T . - Since T is stable, Peano arithmetic or similar theories are not interpretable in T .

M. Boffa:
The Consistency of $ZF + NF_3$

Let T be a set theory formulated in the first-order language based on the $\in$-relation and containing at least the axiom of extensionality (for example: T = ZF , NF , ...).
Let $T + NF_3$ be the extension of NF_3 obtained as follows:

(i) We add a new constant A , the axiom "A is transitive" and all the axioms of T relativized to A (using new variables running over A);

(ii) We extend the comprehension axiom scheme of NF_3 to the new language, with the rule that only the old variables are submitted to the stratification condition.

By adaption Grišin's method for proving the consistency of NF_3 (Soviet Math. Doklady, 1969), we can show that $T + NF_3$ is a conservative extension of T . In particular, this gives the consistency of $ZF + NF_3$ relatively to ZF . [The consistency of ZF + NFU was previously obtained by S. Feferman, using Jensen's method for proving the consistency of NFU .]

M. Crabbé:
NF with Urelemente

The consistency of NFU is proved by a method which does not depend on Ramsey's theorem (as does Jensen's proof). Combining results of Specker and Grišin we prove that if $\langle M0,M1,M2,M3,\in\rangle$ is a model of TTU 4 and if there is an isomorphism between $\langle M0,PM0,PPM0,\in\rangle$ and $\langle M1,M2,M3,\in\rangle$ then $\langle M0,\in_f\rangle$ is a model of NFU . (PX is the power set of X , $x \in_f y$ means $x \in f(y)$).) Models of TTU as described can be obtained from any model of TT by a compactness argument. - NFB (NF with Begriffe) generalizes NFU by allowing the existence of a set of "concepts" having a common extension B (not necessarily empty, as in NFU). "x is a set" is defined as $\neg\forall y\ (y \in x \leftrightarrow y \in B) \cup x = B$. The consistency of NFU plus one of the following axiomas can be shown: $B = \{x/\ x = x\}$, $B = \{x/\ x$ is a set$\}$, $B = \{x/\ x$ is not a set$\}$.

D. Dzierzgowski:
Elementary Equivalence for Stratified Structures

A theory T in a many-sorted language L can effectively be replaced by a L^*-theory T^*, L^* being the one-sorted language canonically associated to L. On the other hand, some many-sorted notions cannot be directly transposed to the corresponding one-sorted notions (see J.L. Hook, JSL 50,1985, 372-374). Here we study how many-sorted elementary equivalence can be replaced by one-sorted elementary equivalence: If M, N are L-structures, we show when $M \equiv N$ implies, or is implied by $M^* \equiv^* N^*$ (M^*, N^* being the associated L^*-structures). As an example, L is taken to be the language L_{TT} of type theory and four different ways to define L^* are considered depending on the presence of symbols $\in^i$, S^i($x \in y$: x is of type i, y of type $i+1$ and $x \in y$; $S^i(x)$: x is of type i). - Then we characterize the many-sorted languages for which these results can be generalized.

T. Forster:
How to do induction without wellfoundedness

In set theory with $V \in V$ we cannot do $\in$-induction. We need a kind of pseudo-induction principle to use in its stead. To do this we consider games G_x. I picks a member x_1 of x, II picks $x_2 \in x_1$, I picks $x_3 \in x_2$ ... and so on until one player attempts to pick a member of an empty set and thereby looses. The assertion that for every set x, there is a winning strategy in G_x for I or for II is equivalent to the assertion that there exist sets I, II such that $V = \mathrm{I} \,\dot{\cup}\, \mathrm{II}$ and the following rules

$$\frac{\begin{matrix}\forall x \in y\ \exists z \in x\ \varphi(z)\\ \vdots\end{matrix}}{(\forall x \in \mathrm{II})\varphi(x)} \qquad \frac{\begin{matrix}\exists x \in y\ \forall z \in x\ \varphi(z)\\ \vdots\end{matrix}}{(\forall x \in \mathrm{I})\varphi(x)}$$

This "axiom of $\in$-determinacy" can be used to show, e.g. $\forall x \in x \to x \notin \cap x$ by "pseudo-induction": The axiom can be shown to be consistent modulo any subsystem of NF for which term models (in which every element is uniquely identified by a set abstract) exist.

M. Forti:
Ample Models for Frame Theory

Define inductively the equivalence relation $\sim\alpha$ by $x \sim \alpha y$ iff $\forall\beta < \alpha$ ($\forall s \in x\ \exists t \in y\ \ s \sim \beta t$ & $\forall t \in y\ \exists s \in x\ \ s \sim \beta t$). Let X_1 be the free consturction axiom stating: Given $f : A \to P(A)$ there is exactly one function g such that $g(x) = \{g(t)/t \in f(x)\}(\forall x \in A)$. Work in Zermelo-Fraenkel set theory without foundation + choice + X_1.

Theorem 1: There exist unique functions σ_α verifying

(i) $x \sim \alpha\sigma_\alpha(x)$ and $x \sim \alpha y$ iff $\sigma_\alpha(x) = \sigma_\alpha(y)$;

(ii) $\sigma_\alpha(x) \in \sigma_\alpha(y)$ iff $\exists x' \sim \alpha x\ \exists y' \sim \alpha y\ \ x' \in y'$.

The range N_α of σ_α is a transitive set carrying (for α regular) a natural structure of α-metric space.

Theorem 2: If $X \subseteq N_K$ then $\sigma_K X = \bar{X}$. Hence $N_K = cl(N_K)$ and

$$X^Y \cap N_K = C(Y,X) \quad \text{for any} \quad X , Y \in N_K .$$

Theorem 3: (i) N_K is Cauchy-K-complete for any regular K .

(ii) N_K is K-compact iff $K \to (K)^2_2$ ("tree property") .

Corollaries: If N_K is K-compact then

(i) N_K satisfies the positive comprehension scheme (incl. bounded quantifiers), and is closed under parts and sequences of size $< K$.

(ii) Many basic operations and relations on N_K are in N_K : identity, projection, union, powerset operation, membership, inclusion.

(iii) N_K is closed under arbitrary intersection, composition and fibred product, but these operations are not elements of N_K .

M. Fürer:
Learning from History can help
(joint work with E. Specker)

We define a Mazur game to be a game played on subsets of a given infinite set S . Starting with $X(0) = S$ players I and II alternate choosing $X(i)$ (a subset of $X(i-1)$), this choice being restricted by a predesignated condition. A play $X(0),X(1),\ldots$ is a win for I iff the intersection of the sets $X(i)$ is an element of W (a given subset of the power set of S). It is shown that there exist Mazur games where one of the players has a (historical) winning strategy but no positional winning strategy (i.e. no strategy where his choices of $X(i+1)$ only depend on $X(i)$ and not on the previous sets $X(h)$, $h < i$). An example of such a game is defined on the set $\{0,1\} \times \omega_1$. Player I eliminates finitely many elements at each move (i.e. $X(i+1)\setminus X(i)$ is finite), player II eliminates exactly one element at each move. Player II wins a play iff an element $(0,a)$ is eliminated iff $(1,a)$ is. Clearly, II hase a historical winning strategy; not so clearly, he has no positional winning strategy.

R. Hinnion:
Set Theories with a Universal Set

L is the first order language in $\in$, $=$, L_τ the extension of L obtained by allowing terms constructed by using an abstraction operator. If Σ is a class of formulas, Comp(Σ) in L is the usual comprehension scheme for formulas in Σ ; Comp (Σ) in L_τ is the class of formulas $\forall t(t \in \{x/ \varphi(x,\ldots)\} \leftrightarrow \varphi(t, \ldots))$ (φ in $L_\tau \cap \Sigma$) . D is $\exists x \; \exists y \; x \neq y$, Ext the axiom of extensionality. We have:

(1) T = Comp(positive formulas in L_τ) + Ext + D is inconsistent.

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(2) T without Ext has a model in Z_{Δ_o} (Zermelo set theory with Δ_o-separation scheme).

(3) Comp(positive formulas in L) + Ext + D has a model in ZF . (This was proved independently by M. Forti and E. Weydert.)

(4) NF , NFτ , NFPτ have the same theorems in L . (NFτ is the extension of Quine's NF to L_τ , NFPτ is NFτ with comprehension restricted to positive stratified formulas.)

H. Läuchli:
On Counting Countable Order Types

Consider the following equivalence relation on order types: $\varphi \sim \psi$ iff ordered sets X , Y of types $\bar{X} = \varphi$, $\bar{Y} = \psi$ can be partionned into finitely many sets $X = X_1 \dot\cup X_2 \dot\cup \ldots \dot\cup X_m$, $Y = Y_1 \dot\cup Y_2 \cup \ldots \dot\cup Y_m$ such that $\bar{X}_k = \bar{Y}_k$, all k .
Answering a question raised by P.M. Neumann, we prove
Theorem: $|\text{Countable order types}/\sim| = \aleph_1$.
The theorem follows easily from results of R. Laver, On Fraïsse's order type conjecture, Annals of Math.43 (1971).

A. Oberschelp:
A Combination of Set Theory and Stratification

The notion of set is usually understood in a "Zermeloan" way: small classes (e.g. singletons) are sets, any subclass of a set is a set, the complement of a set is not a set, there are no infinite descending chains of sets, etc. NF is not set theory in this sense. However, it seems perfectly possible that the objects of NF can play the role of classes in a "set theory over classes" ZFK admitting classes among its objects.
Problem: Is the theory ZFK + NF consistent if ZFK and NF are?

U. Oswald:
A Survey of the History of "New Foundations"

In the presence of the "Founding Father" of "New Foundations" (NF) , W.V. Quine, a sketch was given of the development of NF from its beginnings in 1937 to the present day: Finite axiomatization, proof of the axiom of infinity, disproof of the axiom of choice, typical ambiguity, fragments of NF (NF_3,NF_2 etc.), the relation of the latter to fragments of type theory, possible reductions of the scheme of ambiguity, strange phenomena like the existence (or inexistence) of Quine atoms, consistency proofs for various fragments, including one for NF_3 + (existence of Nn) , probability of $\aleph_\omega$ - existence in NF (which would imply Con(Z) in NF) . - At the end, W.V. Quine confessed to be amazed at the work that had been done on the subject he had fathered, and at seeing what "Pandora's box" he had invented in 1937.

U. Oswald:
A Decision Method for the Existential sentences of NF

NF_2 is obtained from NF by admitting only 2-stratified comprehension axioms. Let NF_2 be formulated in the language $L = \{=, \in, \Lambda, -, \cup, \{\}\}$ (Λ : empty set; $-$: complement, $\{\}$: singleton set), and let the relation E be defined on N by
(1) $E(i, 2^j)$ iff i is an exponent in the binary expansion of j ;
(2) $E(i, 2^j+1)$ iff i is not an exponent in the binary expansion of j . Then $\mathfrak{M} = \langle N, E, \ldots \rangle$ is a "minimal" model of NF_2, i.e. a model isomorphically embeddable in every model of NF_2. Therefore an existential sentence σ is a theorem of NF_2 iff it holds in $\mathfrak{M}$.
A method is presented which leads either to a contradiction (e.g. if σ is $\exists x \exists y \; (x = -\{x \cup \{y\}\})$, or it produces a sequence of indivuals of $\mathfrak{M}$ satisfying the matrix of σ.

A. Pétry:
A Characterization of Structures which Satisfy the Same Stratified Sentences

Let L be the usual first order language $\{=, \in\}$. We note
$\quad$ Ext $\equiv (\forall x,y)((\forall z)(z \in x \leftrightarrow z \in y) \to x = y)$,
Singl $\equiv (\forall x)(\;y)(\forall z)(z \in y \leftrightarrow z = x)$. Let $\mathfrak{A} = \langle A, \in_{\mathfrak{A}} \rangle$ and $(G_i)_{i\in\omega}$ be a family of permutations of A such that
(1) for any $i \geq 1$, G_i is an automorphism of $\langle A, C_{\mathfrak{A}} \rangle$ (where $C_{\mathfrak{A}}$ is defined by $u \, C_{\mathfrak{A}} v$ iff $\mathfrak{A} \models (\forall z)(z \in u \to z \in v)$)
(2) for any $i \in \omega$ and any $a \in A$, $\mathfrak{A} = G_{i+1}(\{a\}) = \{G_i(a)\}$.

Then we define $\in_{\mathfrak{A}}^{G_o}$ and $\mathfrak{A}^{G_o}$ by

$$u \in_{\mathfrak{A}}^{G_o} v \text{ iff } u \in_{\mathfrak{A}} G_o(v), \quad \mathfrak{A}^{G_o} = \langle A, \in_{\mathfrak{A}}^{G_o} \rangle .$$

The structures $\mathfrak{A}$ and $\mathfrak{A}^{G_o}$ satisfy the same stratified sentences.
We prove the following theorem:
Let $\mathfrak{A}$, $\mathfrak{B}$ be models of Ext + Singl. $\mathfrak{A}$ and $\mathfrak{B}$ satisfy the same stratified sentences if and only if there exist ultrapowers $\pi_D \mathfrak{A}$, $\pi_D \mathfrak{B}$ and a family $(Z_i)_{i\in\omega}$ of permutations of the universe of $\pi_D \mathfrak{A}$ which satisfy for $\pi_D \mathfrak{A}$ the above condition (1) and (2) such that $(\pi_D \mathfrak{A})^{G_o} \cong \pi_D \mathfrak{B}$.

W.V. Quine:
The Inception of "New Foundations"

It may have been to Zermelo that I owed the insight that a meaingful open sentence may or may not determine a class, and that it can be left to axioms to settle which ones do. Having grasped this point, I was able to look to types as a restriction specifically upon classes and not upon language. The purpose of the theory of types was to bar the paradoxes, and this could be done using it only to say which open sentences are to be taken to determine classes. Its efficacy for this purpose is a matter of structure, namely, stratification; and this benefit, I conjectured, can be preserved while abandoning the whole notion of a stratified ontology of classes.- It is evident from "Set Theory and Its Logic" that I do not extoll "New Foundations" over other set theories. Its extension in my book Mathematical Logic has advantages in strength and convenience, notably in affording unrestricted mathematical induction; but "New Foundations" remains a crucial auxiliary in view of Wang's result that the one system is consistent if the other is. "New Foundations" is better than the other for proof-theoretic study because of its greater simplicity.

W.V. Quine:
Holism

The logical positivists of Vienna used to say that the only meaningful sentences were those rooted in sense-experience. Since the truths of mathematics are not derived from experience (and we do wish to retain them!) some other justification had to be found: namely the idea that mathematics is coded in the structure of the language we use. This dichotomy of sentences we recognize as meaningful can be avoided once we recognize that no experiment tests precisely the sentences it was designed to test, but a more inclusive bundle of sentences. Thus, if an experiment turns out negative, we are free to choose which of the bundle to revoke. Among them may be some sentences of applied mathematics; and thus mathematics shares the empirical content of the bundle. But we are disinclined to select a mathematical sentence for rejection, lest we disturb to much of our science; and herein lies the so-called necessity of mathematical truth.

G. Servais:
Grisin's Conditions in Pabion's Models

In 1980 Pabion (Compt. rend. Aca.Sc. Paris 290) gave a method to imbed models of PA_2 into models of TT_3 + AI .
Question: Can this result be improved to get models of NF_3 + AI with the same property? [In general, Pabion's models satisfy Grisin's conditions - Soviet. Math. Dok. 1969 - only if they are countable.] Nevertheless I prove that for models of TT_3 + AI resulting from an application of the Fraenkel-Mostowski method (as Pabion's models do) Grišin's conditions hold at level one if and only if they hold at level two. The "only if" part can be proved by a combinatorial argument: We can split every infinite set of level two into disjoint infinite subsets which are the orbits of some special elements of the given set by some particular infinite sets of finite permutations of level zero (finite in the sense of the model of PA_2). - Boffa (JSL 49, 1984) uses this result to imbed countable models of PA_2 into models of NF_3 + AI .

E. Specker:
NF inconsistent: what remains?

Even if NF should turn out to be inconsistent, there will still be the history of NF just as there is the fascinating history of phlogiston theory. The historical context of two results in NF is sketched: the proof of the axiom of infinity and the relation of NF to typical ambiguity. As a weakening of NF , systems (m)-NF ($m \in \mathbb{N}^+$) are proposed. It was pointed out in the discussion that essentially the same proposal was made by T.E. Forster (in "Quine's New Foundations", Cahiers du Centre de Logique, no.5, Louvain-la-Neuve, 1983) and discussed by M. Crabbé (Typical ambiguity and the axiom of choice, J. Symb. Log. 49, 1984).

J. Truss:
Some Problems about Cardinals in the Absence of the Axiom of Choice - a survey

If in Tarski's notion of cardinal algebra (CA) infinitary addition is replaced by finitary, we obtain "weak CA's" ; many of the properties holding in CA's may be derived in WCA's (e.g. interpolation, distributive laws). Bradford proved undecidability for certain "special" sentences. Sagee and Halpern, Howard independently showed that

$$(\forall x)(x \text{ infinite} \rightarrow x = 2x) \nrightarrow AC .$$

Problem 1: Find a model for FM (or ZF) + $\neg$AC + "every cardinal has a 3-successor" .
(y is a 3-successor of x if $x < y$ & $(\forall z)(z < y \rightarrow z \leq x$.)
Problem 2: Find a model for FM + $\neg$AC +
$\forall x \exists y \exists z(x = y+z$ & y is well ordered & $z^2 = z)$
A surjective cardinal is a $=^*$ - class of cardinals where $|x| =^* |y|$ if there are surjections from x to y and from y to x .
Problem 3: Prove that surjective cardinals form a CA (assuming countable choice).
Problem 4: If $x , y \leq^* z , t$ does there exist u such that $x , y \leq^* u \leq^* z , t$?
Problem 5: Find a model for $(\forall x)(x \text{ infinite} \rightarrow 2x \leq^* x)$ & $(\exists x)$ (x Dedekind finite, not finite).

E. Weydert:
Topological Set Theory

Definition: $\mathfrak{M} = (M,\in_{\mathfrak{M}})$ is an ω-closed set theoretical structure (STS)
(1) $\tau_{\mathfrak{M}} := \{\mathfrak{M}\backslash F_{\mathfrak{M}}(x) \in M)\}$ is a T_2-topology and
(2) $F_{\mathfrak{M}} : (M,\tau_{\mathfrak{M}}) \rightarrow (M^*,\tau_{\mathfrak{M}}^*)$ is a homeomorphism where
$\forall x \in M$ $F_{\mathfrak{M}}(x) := \{z \in M/z \in_{\mathfrak{M}} x\}$, $M^* := \{x \subset M/x \tau_{\mathfrak{M}}\text{-closed}\}$
and $\tau_{\mathfrak{M}}^*$ is the natural set topology on M induced by $\tau_{\mathfrak{M}}$
with subbase $\{\{z \in M^*/z \cap U \neq \emptyset\} , \{z \in M^*/z \subset U\}/U \in \tau_{\mathfrak{M}}\}$.
Because the closure of every class is a set, this is a natural way to approximate the naive comprehension scheme.
Definition: $\mathfrak{M} = (M,\in_{\mathfrak{M}})$ is a strong ω-closed STS iff $\mathfrak{M}$ is a ω-closed STS and $\tau_{\mathfrak{M}}$ is compact.

Definition: A formula of $L(\in)$ is called generalized positive if it is built up by $x \in y$, $\wedge$, $\vee$, $\vee$, $\exists$, $Vz \in x$, $Vz : (\varphi(z) \rightarrow \;)$ (φ of $L(E)$, with at most one free variable).

Theorem. If $\mathfrak{M}$ is an ω-closed STS then ($\mathfrak{M} \models$ generalized positive comprehension) $\leftrightarrow$ $\mathfrak{M}$ is a strong ω-closed STS .

Theorem. In ZF we have a strong ω-closed STS $\mathfrak{M}^A_\omega$ with

$$|M^A_\omega| = 2^{\aleph_0} .$$

E. Weydert:
Classification of Universal Set Theories.

The main aim of unniversal set theory is to approximate the naive comprehension scheme inside of classical logic. To this end, we need an adequate measure for the comprehension theoretic strenght. Let R be the set of syntactical rule schemes over $L^* = L(\in,=,\neq,x,\ldots)$ (x comprehension variable). A regular scale over R is a pair $(\mathcal{X},\mathcal{Y})$ with $\mathcal{H}$, $\mathcal{Y} \subset R$ and $\mathcal{H} \cap \mathcal{Y} = \emptyset$ ($\mathcal{H}$ = {fundamental $(\mathcal{X},\mathcal{Y})$-rules}, $\mathcal{Y}$ = {general $(\mathcal{X},\mathcal{Y})$-rules}). For

$$\Delta(\mathcal{Z}) := \{\varphi \in L^* / \vdash_{\mathcal{Z}} \varphi\}$$

$$\Delta(\mathcal{X}) - KOMP = \{V\vec{y}\ \exists z\ Vx(x \in z \leftrightarrow \varphi(x,\vec{y}))/\varphi \in \Delta(\mathcal{X})\}$$

$Sp(\mathcal{X},\mathcal{X}) := \{\mathcal{X} / \mathcal{Z}$ is maximal with $\mathcal{H} \subset \mathcal{X} \subset \mathcal{H} \cup \mathcal{Y}$ and $Con(\Delta(\mathcal{X})\text{-KOMP+EXT})\}$. Finding the spectres of regular scales is an extremely difficult task. Fortunately, we have at least very good partial results for a very natural small scale, the central scale.

Berichterstatter: E. Specker

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