MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 11/1987

Mathematische Stochastik

8.3. bis 14.3.1987

Die diesjährige allgemeine Stochastik Tagung stand unter der Leitung von F. Götze (Bielefeld) und P. Bickel (Berkeley).

Die Arbeitsgebiete der 48 Teilnehmer aus den Vereinigten Staaten und acht europäischen Ländern umspannten weite Gebiete der mathematischen Stochastik. Etwa 40 Vorträge wurden nach verwandten Themen zu Sitzungen zusammengefaßt.

Als Schwerpunkte ergaben sich dabei: Semiparametrische Modelle, asymptotische Entwicklungen, starke Invarianzprinzipien, funktionale Grenzwertsätze, Bootstrap Methoden sowie die Theorie der großen Abweichungen mit ihren vielfältigen Anwendungen auf Probleme der statistischen Physik. Ferner gab es Vorträge zur Verbindung von Informationstheorie und Statistik, zu robusten Schätzproblemen und empirischen Spektralprozessen.

Diese Vielfalt von Vorträgen über zum Teil wesentliche neue Entwicklungen in den verschiedenen Gebieten führte zu einem erfreulich lebhaften Meinungsaustausch auch außerhalb des Vortragsprogrammes.

Vortragsauszüge

R. AHLSWEDE:

On Minimax Estimation in the Presence of Side Information about Remote Data

Model: One person, called "helper" observes an outcome $x^n = (x_1,\ldots,x_n) \in \mathcal{X}^n$ of the sequence $X^n = (X_1,\ldots,X_n)$ of i.i.d. RV's and the statistician gets a sample $y^n = (y_1,\ldots,y_n)$ of the sequence $Y^n(\theta,x^n)$ of RV's with a density $\prod_{t=1}^{n} f(y_t|\theta,x_t)$ ($\theta \in \Theta$ 1-parametric).

The helper can give some (side) information about x^n to the statician via an encoding function $s_n : \mathcal{X}^n \to \mathbb{N}$ with

$$\text{rate}(s_n) \triangleq \frac{1}{n} \log \# \text{range}(s_n) \leq R .$$

Based on the knowledge of $s_n(x^n)$ and y^n the statician tries to estimate θ by an estimator $\hat{\theta}_n$.

Result: For the maximal mean square error

$$\ell_n(R) = \inf_{\hat{\theta}_n} \inf_{s_n:\text{rate}(s_n)\leq R} \sup_{\theta\in\Theta} E_\theta|\hat{\theta} - \theta|^2 .$$

We establish a Cramer-Rao type bound and, in case of a finite $\mathcal{X}$, prove asymptotic achievability of this bound under certain conditions. (Joint work with M.V. Burnasev)

O.E. BARNDORFF-NIELSEN:

A Problem in Fisheries

The talk reported on work in progress, joint with Ian James and

George Leigh, concerning the natural mortality of ocean fish. Let μ denote the rate of natural mortality for fish of a particular species and suppose μ is a constant, i.e., independent of time and age. The basis for inference on μ is the observed recapture times $t_1,\ldots,t_n$ of a total of N fish which have been tagged and then simultaneously released in the ocean, at time 0. The fishing pattern resulting in the recapture of n fish among the N released is considered unknown and is modelled by a distribution function Φ which determine the potential i.e. ignoring natural mortality recapture time of a tagged fish. The maximum likelihood estimator $\hat{\mu}$ of μ is determined as the solution of the equation

$$\sum_{1}^{n} \ell^{\mu t_i} - N = 0 ,$$

while $\hat{\Phi}$ assigns probability $\exp(\hat{\mu} t_i)/N$ to time point t_i, $i = 1,\ldots,n$. In the case where the actual fishing rate is constant and equal to λ, say, the limiting behaviour of the distribution of $\hat{\mu}$ is as follows:

$$\text{For } \lambda > \mu : \qquad N^{\frac{1}{2}}(\hat{\mu}-\mu) \xrightarrow{\sim} N(0,\lambda^2\mu/(\lambda-\mu))$$

$$- \quad \lambda = \mu \quad (N/\log N)^{\frac{1}{2}}(\hat{\mu}-\mu) \xrightarrow{\sim} N(0,1)$$

$$- \quad \lambda < \mu \qquad N^{1-\frac{1}{\alpha}}(\hat{\mu}-\mu) \xrightarrow{\sim} S_\alpha$$

where $\alpha = 1 + \lambda/\mu$ and where S_α indicates a stable law of index α and with mean 0. Normalizing $\hat{\mu} - \mu$ by observed information rather than merely a function of N leads to a somewhat less exotic limiting behaviour.

R. BERAN:

Prepivoting Test Statistics: A Bootstrap View of the Behrens-Fisher Problem, the Bartlett Adjustment, and Nonparametric Analogs

Approximate tests for a composite null hypothesis about a parameter θ may be obtained by referring a test statistic to an estimated quantile of that test statistic's null distribution. Either asymptotic theory or bootstrap methods ban be used to estimate the desired quantile. The bootstrap approach typically leads to more accurate critical values, if the asymptotic null distribution of the test statistic does not depend on unknown paramters. Certain classical refinements to asymptotic test, such as Bartlett's adjustment to likelihood ratio test and Welch's estimated t-distribution solution to the Behrens - Fisher problem, are analytical approximations to natural bootstrap test. Prepivoting is the transformation of a test statistic by the cdf of its bootstrap null distribution. Bootstrap test based on a test statistic prepivoted one or more times have asymptotically smaller error in level than do bootstrap or simple asymptotic theory tests based on the original test statistic. Analytical approximation of the prepivoting transformation is sometimes feasible. Monte Carlo approximation works more generally.

R. BHATTACHARYA:

An L^2 Comparison of Bootstrap & Empirical Edgeworth Methodologies

An empirical Edgeworth expansion of the distribution function of a statistic is obtained by replacing population moments by sample moments in the Edgeworth expansion. The expected squared error of this method of estimation is compared with that of the bootstrap. It is shown that the bootstrap performs better than a two-term empirical Edgeworth for a broad class of statistics under reasonable conditions, if the statistics are studentized.

Markov Processes and Nonlinear Time Series Models

Criteria for ergodicity are obtained for certain classes of Markov processes which are not in general irreducible. These are applied to nonlinear time series models. Classes of functions on the state space for which the central limit theorem holds are found.

P. BICKEL:

Efficient Estimation of ∞ Dimensional Parameters Based on "Maximum Likelihood" Ideas

We discuss several ways of modifying non parametric maximum likelihood estimation to make it applicable in a variety of important situations.In particular, we develop the extension of generalized (M) estimation to ∞ dimensional parameters.Using this notion we construct efficient procedures in the transformation and bivariate censoring models.

E. BOLTHAUSEN:

On Self-Repellent One Dimensional Random Walks

Let $p(x)$, $x \in \mathbb{Z}$, be a probability distribution on $\mathbb{Z}$ satisfying $\Sigma\, x\, p(x) = 0$. The probability distribution of the self-repellent random walk is defined by

$$Q_n^\lambda(\omega) = \prod_{j=1}^{n} p(\omega_j - \omega_{j-1}) \prod_{i,j=1}^{n} (1-\lambda 1_{\omega_i=\omega_j})/Z_n$$

where $\lambda \in (0,1)$, $\omega = (\omega_0 = 0, \omega_1, \ldots, \omega_n)$, $\omega_i \in \mathbb{Z}$ and where Z_n is the appropriate norming factor in order that Q_n^λ becomes a probability measure. Then the following result can be proved:

Theorem If $\Sigma\, \ell^{\varepsilon|x|} p(x) < \infty$ for some $\varepsilon > 0$, then there exists $\lambda_0 > 0$ such that for $\lambda \in (0, \lambda_0]$ there exist $c_1(\lambda)$, $c_2(\lambda) > 0$ with

$$\lim_{n\to\infty} Q_n^\lambda \left(c_1(\lambda) \leq |\omega_n|/n \leq c_2(\lambda)\right) = 1 .$$

J. CUZICK:

Semi-Parametric Regression Models

A class of models of the form $y = \beta(z) + g(\alpha(z) + x) +$ error is considered, where α and β are known parametric functions and g is some unknown smooth funtion. Detailed analysis is carried out in the case when $\alpha(z) \equiv 0$ and $\beta(z) = \beta z$. This special case is tamed semiparametric additive regression. Analysis is based on a local linear predictor $\tilde{\Delta}_i$ of the "pseudo-residuals" $\Delta_i = y_i - \beta z_i$, where $\tilde{\Delta}_i$ has the form $\sum_{\substack{j \neq 0 \\ |j| \leq M}} \alpha_i^j \Delta_{i+j}$.

Simple examples are nearest neighbor, linear interpolation, moving average, and local least sequences. In vector notation $\tilde{\Delta} = A\Delta$ and minimization of $||\tilde{\Delta}\, \Delta||^2$ leads to the estimator $\beta = z'c\, y / z'c\, z$ where $c = (I-A)'(I-A)$.

If (x,z) are drawn from a bivariate distribution the best possible estimator has efficiency $E(Var(z(x))/Var(z)$ compared to the parametric MLE with g unknown. This can be achieved when ε is normal by taking n^{-1} tr $(A'A)^2 \to 0$. Small modifications will also achieve the efficiency for general known errors.

R. DAHLHAUS:

Empirical Spectral Processes

Let $X_1,\dots,X_N$ be a stationary process with mean 0 and spectral distribution function $F(\lambda) = \int_0^\lambda f(\alpha)\, d\alpha$ where $f(\alpha)$ is the spectral density. A common estimate of $F(\lambda) = \int_{-\pi}^{\pi} \chi_{(0,\lambda]}(\alpha) f(\alpha)\, d\alpha$ is,

$$\hat{F}_N(\lambda) = \int_{-\pi}^{\pi} \chi_{(0,\lambda]}(\alpha)\, I_N(\alpha)\, d\alpha$$

where

$$I_N(\alpha) = \frac{1}{2\pi N} \left| \sum_{t=1}^{N} X_t \exp(-i\alpha t) \right|^2$$

is the periodogram. We replace $\chi_{(0,\lambda]}$ by functions $g \in \mathcal{F}$ and prove weak convergence of the resulting empirical spectral process $\sqrt{N}(\hat{F}_N(g) - F(g))_{g\in\mathcal{F}}$. Furthermore, we apply the above result to time series analysis, for example to estimators of the spectral density obtained by minimizing the cross entropy of the process.

H.E. DANIELS:

Local Brownian Motion and the Maximum of Certain Gaussian Processes

Durbin (J.Appl. Prob. 1985) recently used the local Brownian behaviour of a class of not necessarily Markovian Gaussian processes to find approximate first exit time distributions. I use the idea in a different way to extend to Gaussian processes results on the maximum, and the time at which it is attained, of a random walk whose mean path has a maximum (Daniels and Skyrme (Adv. Appl. Prob. 1985), Barbour (JRSS.B. 1975), Groeneboom (Probability Theory and Stochastics, to appear)). The method is applied to the breaking strength and extension of bundles of fibres under general assumptions.

U. EINMAHL:

Strong Invariance Principles for Partial Sums of Independent Random Vectors

A construction method for multidimensional random vectors is prescuted. It is applied to prove strong invariance principles for partial sums of i.i.d. random vectors $\sum_1^n X_k$, $n \in \mathbb{N}$, such that $E[H(|X_1|)] < \infty$, where $H : [0,\infty) \to [0,\infty)$ is a continuous functio[n] satisfying $t^{-2}H(t)$ is non-decreasing and $t^{-4+r}H(t)$ is non-increasing for some $r > 0$.

M. FALK:

Weak Convergence of Some Bootstrap Estimates

Let $X_1,\ldots,X_n$ be iid rv's with common df F. Denote by $T(F)$ the parameter of interest and by F_n the sample df pertaining to $X_1,\ldots,X_n$.

Put

$$Z_n(x) := P_n\{T(F_n^*)-T(F_n) \le x\} - P\{T(F_n)-T(F) \le x\}, \quad x \in \mathbb{R},$$

where $P_n\{T(F_n^*)-T(F_n) \le x\}$ denotes the bootstrap estimate of $P\{T(F_n)-T(F) \le x\}$. Before the sample $X_1,\ldots,X_n$ is drawn, $Z_n(x)$ is unknown, i.e. $(Z_n(x))_{x\in\mathbb{R}}$ defines a stochastic process in $D_{\mathbb{R}}$ which we may call the <u>bootstrap process</u> based on F and T.

In the talk, weak convergence results in $D_{\mathbb{R}}$ for the bootstrap process are presented for several choices of T.

J. FRANKE:

Bootstrapping Kernel Spectral Density Estimates

We consider estimating the spectral density $f(\omega)$ of a linear process $X_t = \sum_{-\infty}^{\infty} b_k \xi_{t-k}$, ξ_t i.i.d., $E\xi_t = 0$, $\operatorname{var} \xi_t < \infty$, using the periodogram $I_T(\omega_j)$, $\omega_j = (2\pi j)/T$, $j=1,\ldots,N$, $N=[T/2]$, from a sample of size T. In view of the asymptotic properties of the periodogram we can reinterpret this task as estimating the regression function $f(\omega)$ in a multiplicative regression model

$$I_T(\omega_j) = f(\omega_j)\varepsilon_j , \quad j = 1,\ldots,N, \varepsilon_j \quad \text{"approximately" i.i.d.}$$

This formal analogy inspires an approach for bootstrapping kernel estimates $\hat{f}(\omega;k)$, where h denotes the bandwidth, of $f(\omega)$: Using an initial estimate $\hat{f}(\omega;g)$ we get empirical residuals $\hat{\varepsilon}_j$. After rescaling them, we draw an i.i.d. sample $\varepsilon_1^*,\ldots,\varepsilon_N^*$ from the empirical distribution of the rescaled $\hat{\varepsilon}_j$, and we define the bootstrap periodogram as $I_T^*(\omega_j) = \hat{f}(\omega_j;g)\, \varepsilon_j^*$. Finally, we get $\hat{f}^*(\omega;h,g)$ as a kernel spectral estimate whith band width h, smoothing the bootstrap periodogram. We prove that this intuitive ad-hoc method works in the sense that, under appropriate conditions, the distribution of $(Th)^{1/2}(\hat{f}(\omega;h)-f(\omega))$ and the conditional distribution, given the original data, of $(Th)^{1/2}(\hat{f}^*(\omega;hg) - \hat{f}(\omega;g))$ converge to each other in Mallows metric d_2 if $h \to 0$ with optimal rate $T^{-1/5}$ and $g \to 0$ a bit slower.

P. GAENSSLER:

On Convergence in Law of Random Elements in Certain Function Spaces

The aim of the present paper is to popularize the applicability of a model for convergence in law of random elements in certain (nonseparable) function spaces being at first especially appropriate for simplifying the presentation of known functional limit theorems for univariate empirical processes (like the uniform one) and which at the same time allows for a straightforward generalization in handling also empirical processes based on multivariate observations up to empirical processes based on random data in arbitrary sample spaces and being indexed by certain classes of sets or functions, respectively.

R. GILL:

Nonparametric Maximum Likelihood Estimation in Semiparametric Models

In many practical situations, estimators are derived in a semiparametric model by appealing to some generalization of the maximum likelihood principle. Centred and scaled, the estimators are asymptotically Gaussian and in fact asymptotically efficient in the sense of achieving the asymptotic infomation bounds of e.g. Begun, Hall, Huang and Wellner [1983] .

Our aim is to understand when and why this happens. We show that (in nice examples) such estimators satisfy an infinite-dimensional version of the score equation: for each of a large family of parametric s models passing through the estimates, we must have that the (parametric) score functions is zero *at* the estimate. So the NPMLE solves an unbiased infinite dimensional estimating equation. Now, assuming that the estimator is $\sqrt{n}$ - consistent in a suitable strong sense, and that the generalized score function is a smooth function of parameter and data, we can identify the limiting distribution of the estimator and show that it is indeed the best possible.

A first version of such a theorem is sketched when the parameter is a cumulative hazard function, considered as an element of $D[0,\tau]$. "Smoothness" is described in terms of compact (=Hademard) differentiability in $D[0,\tau]$, sup norm. The result explains why various versions of the Kaplan-Meier estimator, arising in complety different sampling experiments (e.g. Markov vs. semi-Markov), have an asymptotic distribution of the same form: in all these problems, the generalized score function and hence also its functional derivative are of the same form, and therefore the information bound (achieved by this NPMLE) is the same too.

E. HAEUSLER:

Laws of the Iterated Logarithm For Trimmed Sums

Let F be a distribution function in the domain of attraction of a non-normal stable law with characteristic exponent $\alpha \in (0,2)$ and $F(0-) = 0$. For a sequence $X_1, X_2, \ldots$ of independent random variables with common distribution function F, let $X_{1,n} \leq \ldots \leq X_{n,n}$ denote the order statistics based on $X_1, \ldots, X_n$ for each $n \geq 1$. We discuss the law of the iterated logarithm behavior of trimmed sums of the form $S_n(k_n) \equiv X_{1,n} + \ldots + X_{n-k_n,n}$ where k_n is a sequence of non-negative integers such that $k_n \to \infty$ and $k_n/n \to 0$ as $n \to \infty$. Special emphasis is laid on the sequences $k_n \sim c \log_2 n$ for some $0 < c < \infty$, because they constitute the level which distinguishes between classical and non-classical law of the iterated logarithm behaviour. For these sequences we show that the constant $K(\alpha,c)$ in the law of the iterated logarithm for $S_n(k_n)$ is given by

$$K(1,c) = \frac{1}{2} c^{1/2}[e^M \log M - \int_0^M e^y \log y \, dy - \log M + M],$$

where $M \in (0,\infty)$ is the unique solution of the equation $\frac{1}{c} = M(1-e^{-M})$, and by

$$K(\alpha,c) = \frac{\alpha(2-\alpha)^{1/2}}{2(\alpha-1)} c^{1/2} \left[1 + \min\left\{ \left(\frac{1}{cM} - 1\right) e^{M/\alpha} : M \in (0,\infty) \text{ satisfies } \frac{1}{c} = M^\alpha e^{-M} \int_0^M e^y y^{-\alpha+1} dy \right\} \right]$$

for $\alpha \neq 1$.

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C. HIPP: (joint work with F. Götze)

Local Limit Theorems for m-Dependent Random Fields

Local limit theorems are derived for sums of m-dependent random fields which admit a representation based on an independent random field. The resulting approximations are mixtures of the usual approximations in the iid lattice case on different supporting lattices. The weights of these mixtures are the probabilities that the sum lies in a certain residue class of the integers; these probabilities are nonasymptotic. The same approximations are derived for statistics which are finite range potentials of a) an iid random field or b) a Gibbsian random field with finite and finite range interactions.

K. HORNIK:

Asymtotically Optimal Tests for Markov and Other Related Processes

We first consider a general statistical model and give asymptotic lower bounds for type II errors in fixed alternatives in the case where the (maximal) type I error tends to 0 exponentially fast. As an application we prove a very strong non-local asymptotic optimality property of the likelihood ratio test for the drift parameter of a Wiener process.

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P. JANSSEN:

Strong Uniform Consistency Rates for Estimators of Conditional Functionals

To establish strong uniform consistency rates for kernel type estimators of functionals of the conditional distribution function one can rely on appropriate a.s. properties of randomly weighted d.f. of the form

$$G_{tn}(x) = n^{-1} \sum_{i=1}^{n} \gamma_t(Y_i) I(X_i \leq x) , \quad x \in \mathbb{R}$$

where (X_i, Y_i) , $i=1,\ldots,n$ are bivariate observations (say) and where, for some interval I , $\{\gamma_t, t \in I\}$ is a family of real-valued measurable functions on $\mathbb{R}$. These properties are discussed and for nonparametric curve estimation specific applications, including M- and L-smoothers, are given.

J.L. JENSEN:

Asymptotic Expansions for Strongly Mixing Harris Recurrent Markov Chains

Asymptotic expansions are derived for sums on the form $\sum_{0}^{N} f(x_i)$ and $\sum_{1}^{N} f(x_i, x_{i-1})$, respectively, where x_i is a Harris recurrent Markov chain and the distribution of $f(x_i)$ has a continuous component. Contrary to previous results it is only necessary to assume that the Markov chain is strongly mixing with a polynomially decreasing mixing coefficient. This is achieved by introducing an atom into the state space. The work is based on the papers by Bolthausen (1982) and Hipp (1985).

References

Bolthausen, E. (1982): The Berry-Esseen theorem for strongly mixing Harris recurrent Markov chains. Z. Wahrscheinlichkeitstheor. Verw. Geb. 60, 283-289.

Hipp,C. (1985): Asymptotic expansions in the central limit theorem for compound and Markov processes. Z. Wahrscheinlichkeitstheor. Verw. Geb. 69, 361-385.

C.A.J. KLAASSEN:

Adaptive Estimation in Partially Irregular Models

Consider estimators which behave locally asymptotically like an average of some function taken at the observations. This function is called the influence function and one calls such estimators locally asymptotically linear. It can be shown that the influence function of a locally asymtotically linear estimator can be estimated consistently and conversely, that, given a consistent estimator of the influence function, estimators can be constructed which are locally asymptotically linear in that influence function. With the help of these results an adaptive estimator may be constructed for a partially irregular model.

J.P. KREISS:

On Stochastic Adaptive Estimation

We deal with the estimation problem of the parameter ν in first order autoregression. Since for such models, under some regularity conditions, local asymptotic normality holds true, we are interested in locally asymptotically minimax estimators for ν which does not depend on the error-distribution.

We give a method which should work quite well, also for small sample sizes. In a first step we estimate the score-statistic by using a Fourier expansion for the score-function with estimated coefficients. Then we use a local random search to find a parameter value which minimizes the absolute value of the estimated score-statistic. This estimator appears to be locally asymptotically minimax. To show the practical relevance of the proposed procedure we add some simulation results.

H.R. LERCHE:

On M. Kac's "Can One Hear the Shape of a Drum?"

A classical result of M. Kac on the number of holes of a drum is derived by using a second-order approximation of the exit probabilities of Brownian Bridge from a circle. A somewhat more general result is this: Let f denote a smooth function with $f(0)=f'(0)=0$ and $f''(0)\neq 0$. Let $W=(W_1,W_2)$ denote the two dimensional Bronwnian Bridge with a starting- and endpoint $\xi=(0,y)$ with $y<0$.

Let

$$T_\varepsilon = \inf\{t>0 \mid W_2(t) \geq f(\sqrt{\varepsilon}W_1(t))/\sqrt{\varepsilon}\}$$

Then as $\varepsilon \to 0$

$$P_{\xi,\xi}\{T_\varepsilon < 1\} = \exp(-2y^2)\{1-\sqrt{\varepsilon}f''(0)y^2 \frac{\Phi(2y)}{\varphi(2y)} + o(\sqrt{\varepsilon})\}.$$

The lecture described is joint work with D. Siegmund.

B. LINDSAY:

Rank Pseudo-Likelihood (Preliminary Report)

Pseudo-likelihood is an inferential mechanism developed for use in certain spatial models on lattices, where the true likelihood is computationally horrendous. Consideration will be given here to construction of rank pseudo-likelhoods which seem to provide a range of solutions from the fully efficient to the partially efficient but robust. A goodness-of-fit statistic is a natural auxiliary of the analysis.

P. MAJOR:

Limit Theorems in Statistical Physics on the Role of Continuous Symmetry

We have discussed limit theorems which appear in statistical physics in a natural way, and compared them with the problems of classical probability. They are very similar, but because of some instability properties of the operators appearing in statistical physics problems of a much more sophisticated picture arises there. This instability is the deeper cause of such phenomena as phase transities, critical phenomena e.t.c.

D. MASON:

Some Remarks about Strong Approximation and the Darling-Erdös Theor

Let $\{X_n, F_n, n \geq 0\}$ be a martingale difference sequence and for each integer $n \geq 1$ set $s_n^2 = \sum_{i=1}^{n} E(X_i^2 | F_{i-1})$ and $S_n = X_1 + \cdots + X_n$, with $S_0 = s_0^2 = 0$. Define the partial sum process on $[0,\infty)$ based on these random variables to be $S(t) = S_n$ whenever $s_n^2 \leq t \leq s_{n+1}^2$. A number of sufficient conditions are presented under which the Darling-Erdös theorem holds for such a process $S(t)$, meaning that

$$a(T) \sup_{1 \leq t \leq T} S(t)/t^{1/2} - b(T) \xrightarrow{\mathcal{D}} E \text{ as } T \to \infty ,$$

where $a(T) = (2 \log \log T)^{1/2}$, $b(T) = 2 \log \log T + 2^{-1} \log \log \log T$

$-2^{-1}\log(4\pi)$ and E is an extreme value random variable with distribution function $\exp(-\exp(-t))$, $-\infty < t < \infty$.

One such set of conditions is

(I) $s_n^2 \to \infty$ a.s. as $n \to \infty$.

There exists a sequence of positve constants $\varepsilon_n \downarrow 0$ such that with probability one

(II) $$\sum_{n=1}^{\infty} P(|X_n| > \varepsilon_n s_n/(\log\log s_n)^{1/2} \mid F_{n-1}) < \infty$$

(III) $$\frac{\log\log s_n}{s_n^2} \sum_{i=1}^{n} E(X_i^2 \, I(|X_i| > \varepsilon_i s_i/(\log\log s_i)^{1/2} \mid F_{i-1}) \to 0,$$

(IV) $$\sum_{i=1}^{\infty} \frac{(\log\log s_i)^2}{s_i^2} E(X_i^2 \, I(|X_i| \leq \varepsilon_i s_i/(\log\log s_i)^{1/2}) \mid F_{i-1}) < \infty.$$

When $X_1, X_2, \ldots$, form an i.i.d. sequence, these conditions hold if and only if $E X_1^2 \log\log(|X_1| \vee 3) < \infty$, which is the best sufficient moment condition on X, for the Darling-Erdös theorem to hold in this case attainable by strong approximation techniques or by a refinement of the original methods of Darling-Erdös (1956).

H. MILBRODT:

On the Asymptotic Power of the Two-Sided Kolmogorov-Smirnov Test

The asymptotic power of the two-sided one-sample Kolmogorov-Smirnov test is investigated. Contrary to the common belief, it is found that this test behaves very much like a test against a onedimensional alternative, and not like a well-balanced procedure for higher-dimensional alternatives: There is essentially only one direction of deviations from the hypothesis for which it has reasonable asymptotic power! This optimal direction is determined. Its knowledge provides a quick and easy check of the performance of the Kolmogorov-Smirnov test for any other direction of alternatives.

Methods are as follows: Employing Abstract Wiener space techniques, a representation of the asymptotic power function near the hypothesis is derived. This representation is in terms of an orthogonal series in the "tangent space" of directions of alternatives. The beginning of this series is evaluated numerically, thus computing the curvatures of the asymptotic power function for a system of 50 orthonormal directions. This yields "local efficiencies" of the KS test which are high for one direction only, and then rapidly dcreas to zero.

D.W. MÜLLER:

Estimating the Shape of the Error Distribution in a Linear Model

Let $y_i = x_i^T \theta + \varepsilon_i (i=1,\ldots,n)$ be a linear model with ε_i iid $\sim F$ and $x_i, \theta \in \mathbb{R}^p$. The design x_i is deterministic, the parameter dimension p is allowed to tend to infinity with $n \to \infty$. Let $\hat{\theta}_n$ be an estimator of θ. Let $\hat{\varepsilon}_i = y_i - x_i^T \hat{\theta}_n$ be the ith residual. The following question (*) is considered: does a $(1-\alpha)$ Kolmogorov-Smirnov-band around the empirical distribution of residuals contain a translate of F with probability $\geq 1-\alpha$? $\hat{\theta}_n$ is said to have rate q_n, if $(\hat{\theta}_n - \theta)^T X_n^T X_n (\hat{\theta}_n - \theta) = O_p(q_n)$. Which rates of consistency of $\hat{\theta}_n$ yield a positive answer to (*) ?

Proposition (by E. Ioannidis)

Let f be the density of F s.t. $\sup_t f(t) < +\infty$, $\sup_t |f'(t)| < +\infty$. For estimators with rate q_n the answer to (*) is positive provided that

$$\text{(i)} \quad q_n n^{-1/2} \to 0 ;$$

$$\text{(ii)} \quad p\, q_n^{1/2} n^{-1/2} \log(p^3 q_n) \to 0 .$$

For the good rate of $q_n = p$ (see Yohai & Maronna 1979) one obtains

$p^{3/2}\ n^{-1/2}\ \log(p) \to 0$ as a sufficient condition.

J. PFANZAGL:

Fixed Versus Random Nuisance Parameters

Let $P_{\theta,\eta} | A(\theta,\eta) \in \Theta \times H$ be a family of p-measures, the problem is to estimate θ. Suitable is a sample $(x_1,\dots,x_n) \sim \overset{n}{\underset{i}{X}} P_{\theta,\eta_\gamma}$, with $(\eta_1,\dots,\eta_n)$ unknown. For the case of random nuisance parameters $(\eta_1,\dots\eta_n) \sim \Gamma^n$, methods for determining a lower bound for the asymptotic variance of regular estimates for θ are known, as well as methods for the construction of estimator-sequences attaining this asymptotic bound. The question is discussed whether these bounds also refer to the case of unknown (non-random) nuisance parameters. No final answer to this problem is obtained so far.

P. REVESZ (E. WILLEKENS):

Strong Theorems for Renewal Process

Let $X_t, X_2, \dots$ be a sequence of positive i.i.d. r.v.'s with $S_0=0$, $S_n=X_1+X_2+\dots+X_n (n \geq 1)$. Denote by $\tau_t = \sup\{n : S_n \leq t\}$. Some almost sure upper and lower bounds for $M_t = \max\{X_1, X_2, \dots, X_{\tau_t}, t-S_{\tau_t}\}$ are proved. A typical result is the following:

Let $\mathbb{P}\{X_1 > x\} = \frac{1}{x^\alpha L(x)}$ $(0<\alpha<1)$ where $L(x)$ is a slowly varying function then

$$\liminf_{t\to\infty} \frac{M_t}{t} \log\log t = \beta(\alpha) \quad \text{a.s.}$$

where $\beta(\alpha)$ is the solution of the equation $\sum_{k=1}^{\infty} \frac{\beta^k}{k!} \frac{\alpha}{k-\alpha} = 1$.

H. RIEDER:

Robust Regression Estimators and their Least Favorable Contamination Curves

In the multiparameter linear regression model $y = x'\theta+u$, the ideal model distribution may be subjected to infinitesimal perturbations of the following types: (c) ε-contamination, (h) Hellinger; errors-in-variables, error-free-variables with fixed contamination curve $\varepsilon(x)$, or, given $p \in [1,\infty]$, with any $\varepsilon(x)$ satisfying an L^p-norm constraint (p) $\|\varepsilon\|_p \leq 1$. For the resulting variety of contamination models, robust estimators and corresponding least favourable contamination curves $\varepsilon^*(x)$ are determined. For example, the Hampel-Krasker estimator, which is minimax in model (c,1), has an $\varepsilon^*(x)$ with essentially $\varepsilon^*(x)/|x|$ increasing from 0 to its finite maximum as $|x|$ tends from 0 to ∞. The Huber estimator turns out to be minimax in model (c,2) with $\varepsilon^*(x)$ roughly proportional to $|x|$. The least favorable contaminations look similar in the general models (c,p) and (h,p), though the respective minimax estimators are rather different, since residuals are bounded in case (c) but only downweighted in case (h).

J. RITOV (P.J. BICKEL):

Large Sample Theory of Estimation in IID Biased Sampling Regression Model

Consider the semiparametric linear regression model where (x,y) are observed with $y = \beta^T x + \varepsilon$, x and ε independently with distribution functions H and G respectively. Suppose that we don't sample from this distribution but there are s strata and stratum j is sampled with probability λ_j (known or unknown). Given that stratum j was sampled, the pair (x,y) is sampled according to the conditional probability density $w(j,x,y)\ g(y-\beta^T x)\ h(x)/W(j,G,H)$ where $w(.,.,.)$ are known weight functions. We discussed the estimation of β, G and H when H has a known finite support. We described an M-estimator which is LAN under any G,H and asymptotically efficient at a particular distribution G_0.

U. RÖSLER:

Quicksort and a Fixed Point Theorem for Distributions

Quicksort is an algorithm to sort a random list of numbers. Pick by random a number out of the unordered list. Compare all other numbers with the chosen one and build up a list of smaller and a list of larger numbers. Then apply the same procedure to the new lists until we end up with all numbers in their natural order.

Let X_n be the number of comparisons to sort a list of n numbers. The random variable is thought to be proportional to the time used by this algorithm.

The appropriate normalization of X_n is $Y_n = \frac{X_n - E(X_n)}{n}$. In distribution Y_n converges to a r.v. Y being a fixed point of the transformation S: distr. → distr. given by

$$S(F) = \mathcal{L}(\tau X + (1-\tau)\overline{X} + c(\tau))$$

$X, \overline{X}, \tau$ independent, $\mathcal{L}(X) = F = \mathcal{L}(\overline{X})$, τ uniformly on $[0,1]$ distributed, $C : [0,1] \to \mathbb{R}$ explicitely known.

Our main result is: S is a contraction on the space of distribution functions F, $\int x\,dF(x) = 0$, $\int x^2\,dF(x) < \infty$, with respect to the D_2 Wasserstein metric. Furthermore the unique fixed point $\mathcal{L}(Y)$ has all exponential moments.

Another good sorting algorithm is Heap sort. This algorithm has the advantage, that X_n for Heap sort is bounded by $n \ln n$ and $4 n \ln n$. X_n for Quicksort is bounded by $n \ln n$ and n^2 with expectation $E(X_n) = 2 n \ln n$. In order to compare both one is interested in the probability of the event X_n Quicksort $\geq 4 n \ln n$. This probability is small, less to $\text{const}(\gamma) n^{-\gamma}$ for every $\gamma > 0$.

M. ROSENBLATT:

Remarks on Limit Theorems for Nonlinear Functionals of Gaussian Sequences

Limit theorems for sums of nonlinear functionals of Gaussian sequences typically obtain as limit distribution that of a single term in an expansion given by Dobrushin for a process subordinate to a Gaussian process. One shows how one can obtain limit theorems of this type where the limit distribution is that of a full expansion of Dubrushin's type.

A. SCHICK:

On Efficient Estimation of Functionals

The question of efficient estimation of functionals is addressed in the case when finite-dimensional submodels satisfy the LAMN condition. A conditional convolution theorem for regular estimates and a lower bound on the local asymptotic risk of an estimate are presented. A sufficient condition for an estimate to be efficient in the sense of these results is given. An application to branching processes is discussed.

A.W. VAN DER VAART:

On the Asymtotic Information Bound

For locally asymptotically normal models we discuss asymptotic bounds on the performance of estimators with values in a vector space. For estimators with values in $\mathbb{R}^k$ we give a generalized convolution theorem and discuss the situation that the tangent cone is not a linear space. In particular we show that the convolution and LAM theorem remain true if the assumption of linearity of the tangent cone is relaxed to convexity. Next we give a convolution and LAM theorem for estimators of functionals with values in (more) general vector spaces. For this we extend the notion of differentiable functionals as given in Pfanzagl (1982).

W.R. VAN ZWET:

Why Do Edgeworth Espansions Work?

Edgeworth expansions were originally developed to provide corrections for skewness, kurtorsis etc. to the normal approximation for more or less arbitrary distributions. Theoretical work, however, has long concentrated on distributions of sums of independent random variables and vectors. Now recent work (van Zwet, ZW 1984), (Friedrich, Ann. Statist. 1988?), (Bickel, Götze, van Zwet, Ann Statist. 1986) is leading up to a point where it can be shown that the distributions of functions of independent random variables possess Edgeworth expansions under mild conditions. These results will finally provide a theoretical justification for the extensive use of Edgeworth expansions in practice. They also indicate that it will generally be possible to construct so-called empirical Edgeworth expansions which may provide an alternative to bootstrap resampling in certain situations.

Y. VARDI:

Asymptotics for Emprirical Distributions in Selection Bias Models

The empirical distribution function in selction bias models is an important data analytic tool often needed in the analysis of observational studies and encountered (as opposed to sampled) data. In this paper we discuss the large sample behauviour of such empirical distributions, and give informal proofs for the stated asymptotic properties.

W. WEFELMEYER:

Estimation of Functionals in the Independent, Not Identically Distributed Case

In the i.i.d. case, the tangent cone introduced by Chernoff and LeCam is useful to describe bias bounds for the asymptotic risk of estimators. The concept of a tangent cone can be carried over to models for independent, not identically distributed observations. This makes it feasible to define canonical gradients for vector-valued functionals on such models. We obtain a convolution theorem and an asymptotic minimax bound in terms of the canonical gradient. The asymptotic minimax bound is attained on contiguous neighborhoods for all bounded bowlshaped loss functions if and only if the estimator is asymptotically linear with optimal score function.

J.A. WELLNER:

Semiparametric Models and ACE

Models incorporating both "parametric" (finite - dimensional) and "nonparametric" (infinite - dimensional) components are called semi-paramteric models. In this talk I will introduce several examples of such models in which the Method of Alternating Projections (MAP) arises naturally in the construction of efficient estimates. Some of the history of the method of alternating projections will be reviewed, and connections made with recent work on alternating conditional expectation (ACE) methods in regression theory.

M. WOODROOFE:

Corrected Confidence Levels for Adaptively Designed Experiments

Consider a non-linear model $y_k = g(x_k;\omega)+e_k$, $k = 1,2,...,$ in which the design variables, $x_k = x_k(y_1,...,y_{k-1})$, may be functions of the previous responses. A very weak asymptotic expansion for the distribution of the maximum likelihood estimator is presented, from which the effect of the adaptive nature of the design may be seen.

C.F.J. WU, J. SHAO:

Some General Theory for the Jackknife

It is well known that the delte-one jackknife gives inconsistent variance estimators for nonsmooth estimators such as the sample quantiles. Consistency can be restored by using a more general jackknife with d, the number of observations deleted, depending on a smoothness measure of the point estimator. Our general theory explains why jackknife works or fails. It also shows that, (i) for "sufficiently smooth" estimators, the jackknife variance estimators with bounded d are consistent and asymptotically unbiased; (ii) for "nonsmooth" estimators, d has to go to infinity at a rate explicitly determined by a smoothness measure to ensure consistency and anymtotic unbiasedness. Improved results are obtained for several classes of estimators. In particular, for the sample p-quantiles, the jackknife variance estimators with d satisfying $n^{1/2}/d \to 0$ are consistent and asymptotically unbiased.

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