MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Gewöhnliche Differentialgleichungen

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The meeting was organized by H. W. Knobloch (Würzburg), J. Mawhin (Louvain-la-Neuve), and K. Schmitt (Salt Lake City).

Whereas in the past the above named conference had been of a general nature, the organizers decided to stress special topics - linear differential equations and equations in the complex field- during the 1987 meeting, in order to review some important developments that have taken place during the past decade.

The theories of linear differential equations and equations in the complex field have experienced a tremendous revival and a surge of research activities in recent years. Many factors account for this: There has been a revived interest in the connections between solution expansions and function and group theoretic methods, the classical theory of majorants has become formalized and is being applied to obtain convergence results of formal solutions, hence finding results of Briot-Bouqet type. In the search for solitary wave solutions of some nonlinear evolution equations (Korteweg - de Vries, Boussinesq, sine Gordon etc.) one is led to the study of nonlinear differential equations with fixed critical points whose solutions exhibit properties of Fuchsian linear equations, thus reviving interest in the representation of solutions, via function theoretic methods, of equations having regular singular points. This being the centenary of O. Haupt, it is of interest to note that some of his research areas are very much alive today and significant new results are being obtained in the spectral theory of differential operators whose coefficients are merely integrable and change sign. There has been a considerable amount of activity in the study of stability properties, exponential dichotomies, asymptotic integration, disconjugacy and oscillation, control theory of linear systems, as well as spectral theory and boundary value problems for such systems.

The study of periodic solutions of nonlinear differential equations is of paramount interest for the mathematician as well as the applied scientist and several new results in this area were presented on Hilbert's 16th problem, on equations with nonlinear restoring terms and related problems. Bifurcation and perturbation phenomena abount in the theory of ordinary differential equations and important new developments were reviewed on persistence of invariant tori, on Hopf bifurcations in quasi-periodic systems, on symmetry breaking, bifurcations for potential operators, on multiplicities of solutions for nonlinear Sturm - Liouville problems and resonance phenomena in nonlinear systems. Applications of differential equations to population dynamics and phase transition also enjoy considerable research activities, as does the study of functional differental equations of delay and neutral type.

The conference was attended by 49 mathematicians who came from 11 different countries. 39 of the participants presented lectures many of which were followed by interesting and lively discussions. Besides the lectures presented many less formal meetings and discussion groups formed and the latest results in the various subspecialties were exchanged. Many old collaborations were continued and some new collaborations were formed.

During the week prior to the conference Prof. Knobloch celebrated his 60th birthday. Prof. Kirchgraber and others edited a special issue of ZAMP for that occasion, this special issue was presented to Prof. Knobloch as a birthday gift during the Monday afternoon session of the conference.

The conference was very much of a success and the institut's administration and staff are to be congratulated for providing a most pleasant atmosphere and environment which contributed to a great extend to this success.

The abstracts of the lectures presented follow.

F. V. Atkinson: *Half-linear second order equations and their perturbations.*This work is concerned with nonlinear ordinary differential equations which have the property that constant multiples of solutions are solutions also. Using this property several qualitative criteria about asymptotic behavior of solutions are derived.

W. Balser: *Transformation of meromorphic differential equations to Birkhoff standard form.* In 1912, G. Birkhoff claimed that every meromorphic system of linear differential equations can, by means of a so-called analytic transformation, be turned into another whose coefficient matrix is a polynomial. He gave a valid proof for a special case only, and in 1959 Gantmacher and Masami independently gave examples of two-dimensional systems for which no such transformation exists. Enlarging the class of transformations to meromorphic ones, one can immediately see from Birkhoff's results that now in fact every equation can be transformed into a polynomial one. Unfortunately, meromorphic equations generally increase the Poincaré rank of a differential equation, so the question was left open whether in general a transformation exists, taking a given equation into polynomial form without increasing the Poincaré rank.

In arbitrary dimensions, this question was answered positively under additional hypotheses by Turrittin and others, and in dimension two (without restrictions) by Jurkat, Lutz, and Peyerimhoff. Using considerably more involved, although elementary, arguments, the author recently was able to treat the case of three dimensional equations also.

P. Bates: *The singular limit in a phase field model.* Necessary conditions on the location of the free boundary for the singular limit of an elliptic problem are derived. Variational as well as classical ODE techniques are used to show convergence of solutions as the coefficient of the leading term approaches zero.

F. Bureau: *Nonlinear systems of ordinary differential equations with fixed critical points - Applications.* Much attention has been given in recent years to some nonlinear evolution equations (Korteweg - deVries, Boussinesq, sine-Gordon, ...). The search for solitary wave like solutions for these equations leads to nonlinear ordinary differential equations with fixed critical points, whose solutions possess some characteristic properties of the solutions of Fuchsian linear equations.

To solve this problem, three methods were introduced. The first, due to Picard and Mittag-Leffler, and the second, due to Painlevé, lead to necessary conditions to have solutions with fixed critical points but have a somewhat heuristic character justified by their conclusions. They also require tedious computations.

The author proposes a third method, much more easy to apply and which provides necessary conditions of stability. Furthermore, this method makes possible a detailed classification of the stable equations, which will be usefull for the applications.

S. Busenberg: *Minimal periods of Lipschitz dynamical systems.* Let U be an open subset of a normed space E, let $f : U \to E$ be Lipschitz continuous with constant L, and consider the differential equation:

$$(1)\ x' = f(x)$$

and the corresponding Euler difference equation:

$$(2)\ x_{i+1} = x_i + f(x_i).$$

If T is the period of an orbit of (1) we show that $L \geq 6/T$, and 6 is the best such constant. When E is an inner product space, then the best such constant is 2π. For orbits of (2) of period $N > 0$, we have the sharp bounds $hL \geq 2sin(\pi/N)$ for inner product spaces, and $hL \geq N/\lambda_N$ for normed spaces, where λ_N is the leading eigenvalue of a positive $N \times N$ integer matrix. In particular, if N is prime,

$$\lambda_N = (N^2 - 1 + \sqrt{N^4 + 22N^2 - 23})/12.$$

We describe some applications of these results to both linear and nonlinear equations.

C. Cosner: *Spectra, generalized spectra, and eigenfunctions for second order systems.* Let

$$L_i u = (p_i(x)u')' + q_i(x),\ p_i > 0,\ q_i \geq 0.$$

We consider problems of the form

$$\begin{pmatrix} L_1 & 0 \\ 0 & L_2 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \lambda & 0 \\ 0 & \mu \end{pmatrix} \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} \ on\ (0,\pi)$$

$$u = v = 0\ for\ x = 0, \pi$$

and describe the set (λ, μ) for which there exist nontrivial solutions. We give some conditions allowing bounds on the ratio u/v if $u, v > 0$ and discuss the possibility of having $u > 0,\ v < 0$ when $m_{ij} > 0$ for all i, j.

W. Eberhard: *On the restrictions of singular differential operators.* We consider singular differential operators on R^+ defined by the linear differential expression

$$l(y) = y^{(n)} + \sum_{k=2}^{n} p_k y^{(n-k)}$$

and by m boundary conditions $u_j(y)$ at 0, where $n/2 \leq m \leq n-1$. In the case $m = n/2$ (n even) they can be interpreted as the restrictions of the selfadjoint or non-selfadjoint differential operator treated by Weyl, Neumark, Glasman, Funtakov a.o.. Using a sequence of irregular eigenvalue problems on the compact interval $[0,b]$ with $n-m$ auxiliary seperated boundary conditions at b and letting $b \to \infty$ it is proved that the resolvent is an integral operator with kernel K. But K is no more exponentially bounded resp. the eigenvalue parameter λ when $\lambda \to \infty$ and $m > n/2$. By the method of L-analytic function theory developed by Fage it can be proved that for a certain class of analytic functions there holds an expansion formula on compact intervals of R^+.

L. Erbe: *Oscillation theory for linear differential systems.* We consider the second order linear differential system

$$(1)\ Y'' + Q(t)Y = 0,$$

where Q and Y are $n \times n$ matrix valued functions with $Q = Q(t)$ symmetric for $t \in [a, \infty)$. We obtain a number of sufficient conditions in order that all prepared solutions $Y(t)$ of (1) are oscillatory (i.e., $detY(t)$ has infinitely many zeros). Two approaches are considered, one based on Riccati techniques and the other on variational techniques, and involve assumptions on the behaviour of the eigenvalues of $Q(t)$ (or of its integral). These results extend some well-known averaging techniques for scalar equations to system (1). In addition we survey some other recent work.

W.N. Everitt: *Linear control theory and quasi-differential equations.* Let I be an interval of the real line, and let A and B be $n \times n$ complex-valued,Lebesgue measurable, matrix functions defined on I such that $A \in L^1_{loc}(I)$ and $B \in L^\infty_{loc}(I)$. If $x = [x_1 x_2 ... x_n]^t$ and $u = [u_1 u_2 ... u_n]^t$ are column vectors defined on I such that $x \in AC_{loc}(I)$ and $u \in L^1_{loc}(I)$ then the linear control problem considered is

$$x'(t) = A(t)x(t) + B(t)u(t),\ \ t \in I\ (*)$$

where x is the response and u is the control.

This paper is concerned with the problem of determining necessary and sufficient conditions on A and B to make $(*)$ fully controllable on I, without departing from the basic requirements $A \in L^1_{loc}(I)$ and $B \in L^\infty_{loc}(I)$.

D. Flockerzi: *Invariant tori for coupled oscillators.* We consider a smooth two-parameter family of ODE's describing the flow of two coupled harmonic oscillators. In particular, we deal with the interaction of two Hopf bifurcations in quasi-periodic systems. In suitable coordinates the system under consideration can be written as

$$(*)\ x' = Ax + X(x,y,\mu),\ \ y' = \omega + Y(x,y,\mu),$$

where X, X_x, Y are of order $O(|\mu|), \mu \to 0, for\ x = 0$. Here (x, μ) is to vary in some neighborhood of the origin in$R^4 \times R^2$ and y is to belong to the torus T^n. The matrix A is supposed to be in real normal form with eigenvalues $\pm i\beta_1, \pm i\beta_2$ $(\beta_1 > \beta_2)$. Under suitable nonresonance conditions on the frequencies $\beta_1, \beta_2, \omega_1, \omega_2, ..., \omega_n$ we prove the existence of invariant tori of dimension $n, n+1, n+2, and\ n+3$ for system $(*)$.

G. Freiling: *Eigenvalue problems depending nonlinearly on the parameter.* We consider the boundary-eigenvalue problems

$$(1)\ l(y,\lambda) = \sum_{j=0}^{n} \lambda^{n-j} (\sum_{k=0}^{n} p_{kj}(x) y^{(k)}) = 0,\ x \in [0,1],$$

$$(2)\ U_\nu(y,\lambda) = \sum_{k=0}^{n-1} (\alpha_{\nu k}(\lambda) y^{(k)}(0) + \beta_{\nu k}(\lambda) y^{(k)}(1)) = 0,\ 1 \le \nu \le n,$$

where p_{kj} is sufficiently smooth, $\alpha_{\nu k}$ and $\beta_{\nu k}$ are polynomials and where the characteristic equation has multiple roots. (1), (2) is called normal if it's Green's function G satisfies $G(x,\xi,\lambda) = O(\lambda^\alpha)$ for $0 \le x, \xi \le 1$ and for at least three rays $\gamma_1, \gamma_2, \gamma_3$ dividing the λ plane into three sectors of opening $< \pi$. The system of derived chains eigenfunctions of (1), (2) is complete and (for $r \ge r_0$) minimal in W^r_{2,U_r}, where the latter is a subspace of finite codimension of the direct sum of Sobolev spaces $W_2^{r+n-1} \oplus W_2^{r+n-2} \oplus ... \oplus W_2^r$.

R. Gerard: *Nonlinear singular operators with regular singularity.* We are constructing a big class of singular nonlinear analytic differential equations having the property that each formal solution is convergent. Our result contains as a particular case all the results of Briot-Bouquet types and concerning Gerrey solutions.

L. Grimm: *Solvability of linear differential systems.* A classical theorem of Perron and Lettenmeyer guarantees the existence of a subspace of solutions of singular differential systems holomorphic at a singular point. Results related to this theorem as well as a theorem of Briot and Bouquet are surveyed.

P. Habets: *Periodic solutions of ordinary differential equations with jumping nonlinearity.* Consider the periodic BVP

$$(1)\ u'' + ku' + g(t,u) = f(t),\ u(0) = u(2\pi),\ u'(0) = u'(2\pi)$$

together with.

PROPERTY P: For any $\mu, v \in L^\infty(0, 2\pi)$ such that

$$a(t) \le \mu(t) \le b(t),\ c(t) \le v(t) \le d(t)$$

the function $u(t) \equiv 0$ is the only solution of the BVP

$$u'' + ku' + \mu u^+ - v u^- = 0,\ u(0) = u(2\pi),\ u'(0) = u'(2\pi)$$

where $u^+ = max(u, 0)$ and $u^- = max(-u, 0)$.

In this context we prove the following:

THEOREM: Let g be Caratheodory and $f \in L^2(0,2\pi)$. Assume there exist $r \in R$, $s \in L^2(0,2\pi)$, $a,b,c,d \in L^\infty(0,2\pi)$ such that $|g(t,x)| \leq r|x| + s(t)$ and

$$a(t) \leq liminf_{u\to\infty} g(t,u)/u \leq limsup_{u\to\infty} g(t,u)/u \leq b(t)$$

$$c(t) \leq liminf_{u\to-\infty} g(t,u)/u \leq limsup_{u\to-\infty} g(t,u)/u \leq d(t),$$

and *property* P holds. Assume further there exists $(\mu_0, v_0) \in R^2_+$ *or* $\in R^2_-$ such that $a(t) \leq \mu_0 \leq b(t), c(t) \leq v_0 \leq d(t)$. Then the problem (1) has a solution.

J. Haddock: *Functional differential equations for which each constant function is a solution.* In this presentation we examine various finite and infinite delay functional differential equations for which each constant function is a solution. The main purpose is to give a short survey regarding conditions which guarantee that each solution tends to a constant at infinity. The techniques include applications of differential and integral inequalities as well as invariance principles. It will be emphasized that one must also be concerned with the choice of the underlying phase space for a given infinite delay equation.

W.A. Harris: *The method of small parameters in the asymptotic integration of linear differential equations.* Floquet theory for the differential system $y' = A(t,\epsilon)y$, $A(t+\omega,\epsilon) = A(t,\epsilon)$; $Y(t,\epsilon) = P(t,\epsilon)e^{tR(\epsilon)}$, $P(t,\epsilon) = P(t+\omega,\epsilon)$ is utilized to effect the asymptotic integration of the system $x' = A(t,h(t))x$ through the transformation $x = P(t,h(t))u$. The relationship $P_t = AP - PR$, yields $u'(t) = \{R(h(t)) - P^{-1}(t,h(t))P_h(t,h(t))h'(t)\}u$, where $h' \in L^1(t_0,\infty)$, allows us to consider $-P^{-1}P_h h'$ a perturbation term in the application of Levinson's theorem. We illustrate this technique with the example

$$y'' + \frac{sin\lambda t}{t^p} y = 0,\ 0 < p < 1.$$

H. Herold: *k-Diskonjugiertheit von* $w^{(n)} + f(z)w = 0$. Der Sturmsche Vergleichssatz für reelle lineare Dgln. 2. Ordnung lässt sich folgendermassen verallgemeinern: Hat die komplexe Dgl. $w^{(n)} = F(x,w)$ $(a \leq x \leq b)$ eine nichttriviale Lösung w mit $|F(x,w(x))| \leq \phi(x)|w(x)|$, $w^{(i)}(a) = 0$, $i = 0,...,n-k-1$, $w^{(j)}(b) = 0$, $j = 0,...,k-1$, so hat die Dgl. $y^{(n)} + (-1)^{k+1}\phi(x)y = 0$ eine nichttriviale Lösung mit einer $(n-k)$-fachen Nullstelle bei $x = a$ und einer k-fachen Nullstelle in $(a,b]$.

Mit diesem Vergleichssatz können für komplexe lineare Dgln. n-ter Ordnung Kriterien für k-Diskonjugiertheit hergeleitet werden. Beispielsweise gilt: Ist p eine für $|z| < 1$ holomorphe Funktion mit

$$|p(z)| \leq \frac{(2m)!}{(1-|z|^2)^m},\ |z| < 1,\ (m \in N),$$

so hat die Dgl. $w^{(2m)} + p(z)w = 0$ keine nichttriviale Lösung mit zwei m-fachen Nullstellen im Einheitskreis.

S. Invernizzi: *The periodic BVP for ODE's with oscillating nonlinearities.* We discuss the existence and multiplicity of periodic solutions for systems of forced coupled pendulum-like ordinary differential equations. As a model example, it is possible also to consider the oscillations of an N-point Josephson junction with external time-dependent disturbances.

F. Kappel: *Size dependent population dynamics.* The lecture discusses the following aspects of the theory of size structured populations:

1.Mechanisms which explain size dispersion for single cohort pulses.

2.Well-posedness of the Sinko-Streifer model in L^1.

3.Numerical approximation.

Concerning size dispersion we assume that the individual growth rate is determined by parameters which are fixed for each individual but considering the total population have to be understood as random variables.

Well-posedness is established by using the Lumer-Phillips theorem of semigroup theory. For simplicity of presentation, numerical approximation is done by step-function approximation for the state. The proof of convergence uses the Trotter-Kato theorem of semigroup theory. Finally some preliminary results on identification of growth and mortality rates are presented. The results reported are are joint with H.T. Banks (Brown) and L. Botsford (Davis).

H. Kielhöfer: *A bifurcation theorem for potential operators and applications to a semilinear wave equation.* We consider the nonlinear wave equation

$$(1)\ u_{tt} - u_{xx} = f(\lambda, x, u),\ \ t \in R,\ \ x \in (0, \pi),\ \ \lambda \in R$$

together with either Dirichlet or Neumann boundary conditions at $x = 0$ and$x = \pi$. We are interested in solutions which are periodic in t and which bifurcate from the trivial solution $u \equiv 0$. A central question is which periods of the linearized problem

$$u_{tt} - u_{xx} = f(\lambda, x, 0)u$$

persist for the nonlinear equation. We assume that $f_u(\lambda, x, 0) = c(\lambda)$ is independent of x and strictly monotone in λ. Then we apply a new bifurcation theorem for potential operators and we can show that for any λ_0 of a dense set in R there exist finitely many periods (depending upon λ_0) which occur as periods of bifurcating solutions of (1). The solutions having the minimal period form a continuous branch emanating at $(0, \lambda_0)$.

U. Kirchgraber: *Stability boundary tracing algorithms and an application to rotor dynamics.* In this paper I report on joint work of F. Meyer, G. Schweitzer and myself on an efficient method for the construction of stability boundaries of two-parameter families of (large) linear periodic Hamiltonian systems. Given a point μ_0 on a stability boundary in the parameter plane we construct first and second order approximations of the stability boundary in a neighborhood of μ_0 using techniques from bifurcation theory like Liapunov-Schmidt reduction and the Newton diagram. Since the construction involves certain information of the monodromy matrix and its derivatives with respect to the parameters at μ_0 we get a numerical predictor-corrector scheme to trace stability boundaries. The algorithm is applied to a five degree of freedom two-blade wind turbine model to illustrate its efficiency.

J. Kurzweil: *On the equation $x' = A(t)x$, A quasiperiodic and fulfilling $A + A^T = 0$ or $A + A^* = 0$.* Let X be the matrix solution of (1) $X' = A(t)X$, $X(0) = I$ ($A(t)$ is an $n \times n$ matrix with real (complex) entries, continuous with respect to $t \in R$). Then

(2) $A(t) + A^*(t) = 0$ (here A^* is the conjugate transpose of A) implies that $X(t)$ is orthonormal (unitary) for $t \in R$.

Hypothesis: Let r be a positive integer. Then there exists a $\kappa = \kappa(r,n)$ that the manifolds $Y = SO(n)$, $n \geq 3$ ($Y = SU(n)$, $n \geq 2$) have the following estimation property: If $V : [0,1]^r \to Y$ is of class $C^{(1)}$, $V(u) = I$ for $u \in \partial[0,1]^r$ and if V is homotopic to the identity map, then there exists such a homotopy $H(s,u)$, $s \in [0,1]$, $u \in [0,1]^r$ that $H(1,u) = V(u)$, $H(0,u) = I$, $||\frac{\partial H}{\partial s}|| \leq \kappa$, $||\frac{\partial H}{\partial u_i}||$, $||\frac{\partial^2 H}{\partial s \partial u_i}|| \leq \kappa(1 + max_{u,j}||\frac{\partial V}{\partial u_j}||)$.

Theorem 1: Assume that the hypothesis holds for $r = 1,2,..,l$ and for some n. If A is quasiperiodic with $l+1$ frequencies at most and $\epsilon > 0$, then there exists a continuous B such that

$$(3)\ ||A(t) - B(t)|| \leq \epsilon,\ B(t) + B^*(t) = 0,\ t \in R$$

(4) the matrix solution Z of $Z' = B(t)Z$, $Z(0) = I$ is quasiperiodic with $l+1$ frequencies at most.

Theorem 2: The hypothesis holds for $r = 1,2$ and all n.

Corollary: Let $l = 1,2$, n being arbitrary. If A is quasiperiodic with $l+1$ frequencies at most, $\epsilon > 0$, then there exists B such that (3) and (4) hold.

The problem, whether the Corollary can be extended to greater l and to almost periodic functions, remains open.

N.G. Lloyd: *Liénard systems with several limit cycles.* Consider the system of the form

$$(*)\ x' = y - F(x),\ y' = -g(x)$$

in which F and g are polynomials and $g(x) sgn x > 0$, $x \neq 0$. Let $k(F) = [\frac{1}{2}(\partial F - 1)]$ where ∂F is the degree of F and [ ] denotes integer part. Lins, de Mels and Pugh proved in 1977 that if $g(x) = x$, then there are systems with $k(F)$ limit cycles. They also conjectured that this is the maximum possible number. The talk describes a number of cases in which this conjecture has been resolved, and it is shown how a large number of systems with this number of limit cycles may be constructed. This is part of a programme to investigate Hilbert's 16th problem, and it will be emphasized that it almost certainly necessary to consider the appropriate complex form of (*) in order to obtain the maximum number of limit cycles.

D.A. Lutz: *On the connection problems for some second order meromorphic differential equations.* The purpose of this talk is to show how some recent results of W. Balser, W. Jurkat and D. Lutz may be applied to obtain solutions in Floquet form for some second order linear differential equations in the neighborhood of an irregular singularity of rank one. Related to this are solutions of the lateral and central connection problems of Birkhoff and Turrittin. In particular, it will be shown how the Floquet coefficients can be expressed as certain convolutions of the formal series solutions with certain explicit formal Laurent series.

P.J. McKenna: *Asymmetric systems.* I will survey recent results on solutions of differential equations where the nonlinear restoring term $f(u)$ has different asymptotically linear behavior at plus and minus infinity. Numerous applications will be given, including the problem of nonlinear oscillations in suspension bridges.

A. B. Mingarelli: *Boundary value problems for second order equations with an indefinite weight-function.* The Sturm-Liouville problem:

$$-(p(x)y')' + q(x)y = \lambda r(x)y, \ y(0) = y(1) = 0, \ (p(x) > 0)$$

is considered. When $q, r \in L(0,1)$ and $r(x)$ changes sign sharp estimates are obtained for the Haupt and Richardson indices (which roughly measure the deviation of the Haupt-Richardson oscillation theorem, from the classical Sturmian type oscillation theorem). If $1/p \in L(0,1)$ and $p(x)$ changes sign , $q = 0$ and $r(x) > 0$ the Neumann problem is considerd, asymptotic estimates for the eigenvalues are derived along with a "positivity-type" result associated with the eigenfunctions of the eigenvalues nearest to $\lambda = 0$. Finally, for certain classes of q, r an " Annulus theorem" is derived relating to the nonexistence of eigenvalues within an annulus whose size depends upon the coefficients only.

J. S. Muldowney: *Generalized Wronskians and disconjugacy.* Let X_n be a real linear space of dimension n and J an interval in R. For $i = 1, ..., k$, let l_i be a linear function from X_n to $C^r(J)$. We are interested in the question of when $x \in X_n$, $l_i x$ has r_i zeros in J $r_1 + r_2 + ... + r_k \geq n$ implies $x = 0$. In the case that $k = 1$, X_n is the nullspace of a real linear differential operator L of order n and $l_1 x = x$, this is disconjugacy of L on J. When $l_i x = x^{(i-1)}$, $i = 1, ..., n$ we have disfocality of L. We obtain a necessary and suffcient condition in terms of determinants which reduces to a criterion of Pólya in the case of disconjugacy.

F. Neuman: *Global theory of ordinary differential equations in the real domain.* Investigations of linear differential equations started in the last century in the papers of E. Kummer, E. Laguerre, A. Forsyth, F. Brioschi, G. Halphen, P. Stäckel, S. Lie, E. Wilczynski, and others. As pointed out by G. Birkhoff in 1910, their results were of local character.

A study of linear differential equations from the global point of view started in the fifties by O. Boruvka, W. Everitt, H. Guggenheimer, G. Gustafson, M. Hanan., P. Hartman, K. Kreith, A. Levin, G. Sansone, C. Swanson, W. Trench and others.

A generalization of these methods and results, together with algebraic toplogical, geometrical tools, and also some methods from the theory of dynamical systems and functional equations, make it possible to develop a general theory of global properties of linear differential equations of an arbitrary order.

K. Palmer: *Exponential dichotomies.* Roughly speaking, an equation $x' = A(t)x$ has an exponential dichotomy (or is "hyperbolic") if its solution space can be split into two subspaces, in one of which the solutions are exponentially decaying and in the other exponentially growing. This property is stable under perturbation in the sense that if $sup|B(t) - A(t)| < \delta$, $\delta > 0$ sufficiently small, then $x' = B(t)x$ also has an exponential dichotomy (proved by Massera and Schäffer). Here we prove the following generalization: Let $T > 0$ be sufficiently large. Then if for any interval J of length T there exists a real number r such that $sup_{r \in J}|B(t) - A(t)| < \delta$, $x' = B(t)x$ still has an exponential dichotomy when $x' = A(t)x$ has. Using similar methods we also prove that if $A(t)$ is almost periodic and $x' = A(t)x$ has an exponential dichotomy on a single sufficiently long finite interval, then it has an exponential dichotomy on the whole real line.

M. Plum: *Eigenvalue inclusions for linear differential operators.* Object of consideration is the eigenvalue problem for linear symmetric differential operators (mainly of second order) with a discrete spectrum $\lambda_1 \leq \lambda_2 \leq ....$ An algorithm and its theoretical background are presented which yield, for given n, guaranteed and close inclusion intervals for the first n eigenvalues. In particular, intervals containing no eigenvalues are computable.

The algorithm is based on Hilbert space analysis and an appropriate homotopy method is needed in order to calculate approximate solutions for linear boundary value problems. Concrete examples are given to illustrate the method.

H. Rüssmann: *Lower dimensional invariant tori in Hamiltonian systems.* An extension of the classical theorem of Kolmogorov-Arnol'd concerning the preservation of invariant tori of integrable Hamiltonian systems under small perturbations is formulated for the general case in which the unperturbed system contains besides of action angle variables an arbitrary number of elliptic and hyperbolic variables near a stationary point. The main tool of the proof is an essential simplification of the measure theoretic part of Arnol'd's original proof.

R. Schaaf: *A class of nonlinear Sturm-Liouville problems with infinitely many solutions.* Consider the nonlinear Sturm-Liouville problem

$$u'' + u + g(u) = h(x),\ u(0) = u(\pi) = 0,$$

where

$$g(u+\omega) = g(u),\ u \in R,\ \int_0^\omega g(s)ds = 0,\ \int_0^\pi h(x)sinxdx = 0.$$

We imbed this problem into a one parameter problem and use techniques from global bifurcation theory (bifurcation from infinity) to conclude that the above boundary value problem has infinitely many solutions. These arguments may be applied to obtain similar results for more general ordinary and semilinear elliptic partial differential equations.

R. Schäfke: *Confluence of several regular singular points into an irregular singular point.* I study a system of differential equations

$$(1)\ (s - B)\frac{dv}{ds} = (\rho - A)v,$$

where s is a complex variable, ρ a parameter, A and B are $n \times n$ matrices and B has n distinct eigenvalues. (1) has regular singular points at $s =$ eigenvalue of B, ∞. If we put $\hat{v}(z) = (-\rho z)^{-\rho} v(-\rho z)$ and pass to the limit $\rho \to \infty$, $\hat{v} \to y$, we obtain

$$\frac{dy}{dz} = (z^{-1}A - z^{-2}B)y.$$

The finite singular points of (1) move to 0, which is an irregular singular point of (2). (1) and (2) were studied recently by Okubo, Kohno, Balser-Jurkat, and RS. Special solutions were defined having particular asymptotic behavior near the singular points. I investigate the behavior of these solutions in the process of confluence and thus give new proofs of theorems on connection formulas for (1) and (2).

Y. Sibuya: *A remark on the relation between Stokes multipliers and central connection matrices (joint with W. Messing).* THEOREM: Let $H(\lambda)$ be an $n \times n$ matrix whose entries are entire in the complex variables $\lambda_1, ..., \lambda_q$, and let L be a linear map : $C^q \to C^q$ such that $L^r = identity$ for a positive integer r. Suppose that H satisfies the condition

$$H(L^{r-1}(\lambda))H(L^{r-2}))...H(L(\lambda))H(\lambda) = I,$$

where I is the $n \times n$ identity matrix. Then there exists an $n \times n$ matrix $E(\lambda)$ such that (i) the entries of $E(\lambda)$ and $E(\lambda)^{-1}$ are entire in λ and (ii) $H(\lambda) = E(L(\lambda))^{-1}H(0)E(\lambda)$.

We shall prove this theorem and explain how this theorem is related to the relation between Stokes multipliers and central connection matrices.

A. Vanderbauwhede: *Period-doublings and orbit-bifurcations in symmetric systems.* We describe a new approach, based on equivariant Liapunov-Schmidt reduction, to discuss the bifurcation of periodic solutions near some nontrivial periodic solution for symmetric autonomous systems depending on a parameter. The approach avoids the usual Poincaré map and has the advantage that the resulting bifurcation problem has the full symmetry of the critical solution at the bifurcation point, and that it keeps track of the symmetry of the bifurcating solutions. We also describe some of the elementary bifurcations that can occur in such systems; these bifurcations include period doublings and orbit pitchforks.

P. Volkmann: *Funktional Differentialgleichungen mit meromorphen Lösungen endlicher Wachstumsordnung.*

SATZ (mit L. Volkmann): Es sei $w = (w_1, ..., w_n) : C \to \hat{C}^n$ eine meromorphe Lösung des Systems $(j = 1, 2, ..., n)$:

$$w_j'(z) = \sum_{k=1}^{n} P_{jk}(z)w_k(z) + R_j(g_1(z), ..., g_l(z), w_1^{(p_1)}(\lambda_1 z + \mu_1), ...$$

$$..., w_1^{(p_m)}(\lambda_m z + \mu_m), ..., w_n^{(p_n)}(\lambda_1 z + \mu_1), ..., w_n^{(p_m)}(\lambda_m z + \mu_m)).$$

Dabei bedeuten: P_{jk} Polynome, $R_1, ..., R_n$ rationale Funktionen, $g_1, ..., g_l$ ganze Funktionen endlicher Wachstumsordnung, λ_k, μ_k komplexe Zahlen, $|\lambda_k| < 1$, $p, ..., p_m$ ganze Zahlen. Dann besitzen $w_1, ..., w_n$ endliche Wachstumsordnung.

E. Wagenführer: *On the evaluation of formal fundamental solutions of linear systems at a singular point.* We consider a system of n linear formal differential equations at $x = 0$,

$$xy'(x) = A(x)y(x), \ A(x) = x^{-s}\sum_{\nu=0}^{\infty} x^\nu B_\nu, \ s \in N_+.$$

This system has a formal matrix solution $Y(x) = H(x)x^J e^{Q(x)}$, where Q is a diagonal matrix containing polynomials in $x^{-1/p}$, J is a constant Jordan matrix commuting with Q and H is a formal meromorphic series in $x^{1/p}$ with a certain $p \in N_+$. The subject of this talk is a new method of evaluating Q, J, H. Certain λ-matrices $A_m^{(r)}(\lambda)$are derived from the leading Laurent series coefficients of $A(x)$: these lead to characterizing polynomials

$\chi^{(r)}(\lambda)$. The coefficients in Q as well as the diagonal elements in J are the zeros of such polynomials. The structure of J as well as the Laurent series coefficients of $H(x)$ can be determined by means of a method developed by the speaker in an earlier paper.

J. Ward Jr.: *Conley index and non-autonomous ordinary differential equations.* We describe a method for applying Conley or homotopy index theories to non-autonomous ordinary differential equations. One is then able to obtain stability-type information by continuation methods. We illustrate this with some applications to time-periodic ordinary differential equations.

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