MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Anwendungen der Infinitesimalmathematik

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This was the sixth conference on Nonstandard Analysis held at Oberwolfach since 1970. It had been organized by S. Albeverio (Bochum), D. Laugwitz (Darmstadt) and W.A.J. Luxemburg (Pasadena), and was attended by participants from 17 countries. The contributions included applications of the method of infinitesimals to many fields, such as Lie groups (Hilbert's Fifth Problem), functional analysis, distributions, partial differential equations, iterations and fractals, perturbations and bifurcations, Brownian motion, measures and integrals, quantum field theory, computer science.

In addition stimulating discussions arose on special subjects as well as on questions of foundations and history, all of them closely related to applications. The meeting showed that the method of infinitesimals furnishes both remarkable progress and intuitive insight in many fields related to present day problems.

Vortragsauszüge

S. ALBEVERIO:

Remarks: Brownian motion, fields, strings

A unified discussion of infinite dimensional "Gibbs measures" and associated random processes and fields is given, with examples and open problems from the theories of Brownian motion, stochastic processes, random fields, gauge fields (stochastic differential geometry) and relativistic strings. In particular the usefulness of hyperfinite discretization methods and the use of hyperfinite dimensional Hilbert spaces is stressed.

L. ARKERYD:

On Loeb Sobolev spaces

Both with a view to applications and as an extension of the corresponding classical theory, it is of interest to study Sobolev spaces of functions with S-integrable derivatives. The talk reports on joint work in this area between J. Bergh and myself. First Loeb L^p-spaces on $^*R^n$ are compared to classical Lebesgue spaces on R^n, and two Fourier multiplier results are given in Loeb L^p, $1 < p < \infty$. They provide the basic tool for the following discussion of embedding and trace theorems. In particular it turns out that the traces of Loeb Sobolev functions are contained in classical Besov spaces. The presentation ends with a simple PDE-example.

I.P. VAN DEN BERG:

Nonstandard asymptotics

Asymptotics is the study of functions depending on a large parameter. Disposing of concrete large numbers, nonstandard analysis provides a natural setting of asymptotics.

The basic classical tools of asymptotic analysis are the, mostly divergent, asymptotic expansions of functions. We present a nonstandard alternative: an expansion of numbers in successive shadows. We treat some fundamental properties of these expansions: uniqueness, existence, relation to asymptotic expansions and its use in pointwise approximation in case of divergence.

Finally, we turn to the problem of changing asymptotic behaviour as a function of the large parameter, and show that the external sets of non-standard analysis play an important part in the study of these phenomena.

N. CUTLAND:

Infinitesimal methods in large deviations

Schilder's Large Deviation Principle (LDP) for Wiener measure is proved in a natural way by representing Wiener measure as an internal 'flat' integral against *Lebesgue measure.

The ideas involved in the proof are developed further to extend the famous result of Ventcel-Freidlin on large deviations for the measures given by solutions to stochastic differential equations. The Ventcel-Freidlin theory deals with homogeneous diffusions, whereas we treat the case of time-dependent functional SDE's.

F. DIENER:

Superstructures versus I.S.T.

During the last ten years, the non-standard mathematicians who use the framework of superstructures, and those who use I.S.T. have developed their theories completely separately. My talk is meant to be a first contribution to an "interface" between both communities. I shall describe an analogue of the Loeb-measure within I.S.T., when this is possible.

Those who are familiar with Loeb-measure should get an idea about the arguments commonly applied by the users of I.S.T. The latter will be informed about the main points of the Loeb-measure.

M. DIENER:

Stream bifurcations

The computer-plottings of solutions of (standard) rational ordinary differential equations of type $dY/dX = Q(X,Y)/P(X,Y)$ have revealed surprising concentrations of solutions forming streams seeming to defy unicity of solutions. A (non-standard) study, using macroscopes, of this phenomenon suggested a mathematical definition of those special solutions. We recall the properties of the streams of the generic case, and give an example due to F. Blais (Paris 7) of a stream bifurcation, for which the generic theory a priori does not apply, but for which it is nevertheless possible to compute asymptotically the behaviour.

M. DRESDEN:

Speculations on the nature and dimensionality of space time

The space time continuum in physics has recently become an item, to be determined by experimental and mathematical considerations. Neither the dimensionality, topological, nor point set characteristics can be assigned a priori. In these investigations a fractal description with an ensuring fractal dimension plays an ubiquitous role. It is through these structures that direct contrast is made with the formalism of non standard analysis. Examples will be presented showing how the notion of "quantum paths" directly leads to fractals and thus to a non standard description. It is conjectured (somewhat hesitantly) that the string picture might also fit most naturally in a fractal, nonstandard picture of a higher dimensional space time.

S. FAJARDO:

Equivalence relations between adapted probability spaces

In this talk I introduce a new way of comparing stochastic processes which live on adapted probability spaces (i.e. structures of the form $(\Omega,(F_t),p)$ where (F_t) is a filtration of σ-algebras). My work is motivated by ideas from model theory (i.e. back and forth games between elementary equivalent models) and the common operation of "localization" of a stochastic process by means of a sequence of stopping times.

I'll first introduce the notion of adapted distribution due to Hoover and Keisler, and recall the main theorems about this relation; namely: the saturation and homogeneity of adapted hyperfinite Loeb spaces.

Then I introduce the new concept and show that every process can be embedded inside the hyperfinite adapted Loeb space under this new relation. Several variants of the new definition will be presented and I'll discuss the relevance of these ideas with respect to an open problem of Keisler in the model theory of probability logic.

E.J. FARKAS:

Parallel computations embedded in star-finite models

Given a programming language PL, closed under while, if ... then ... else, composition, parallel and await statements, and a related language BPL in which while and await statements are bounded by integers $n \in {}^*N$, we develop a representation theory for programs in which programs are represented by formulas of $^*PA$. The representation is based on a system of types for subprograms used to associate with every program a diagram from which the representing formulas can be deduced.

We use this representation to show that every computation tree Tr(P) of $P \in PL$ of BPL is a definable subset of a star-finite set.

Theorem: For every $P \in PL$ there exists a $P' \in BPL$ such that the standard calculations of P are infinitely close to the nonstandard calculations of P'.

Theorem: If Φ is a formula of $^*PA$ representing a computation tree Tr(P') for some $P' \in BPL$, then Φ defines Tr(P').

Corollary: Tr(P') is star-finite.

R. FITTLER:

Nonstandard quantum electrodynamics

Presentation of a nonstandard version of Q.E.D. yielding the known classical numerical results in lowest nontrivial perturbation order but avoiding the classical divergencies.

H. HEJTMANEK:

Asymptotic behaviour of semigroups, nonstandard characterization

The linear Boltzmann equation is a paradigma in the theory of strongly continuous semigroups of positive operators (see Kaper, Lekkerkerker, Hejtmanek [1982] Spectral Methods in Linear Transport Theory, and Nagel editor [1986], One-parameter Semigroups of Positive Operators).
Methods from nonstandard analysis are applied to characterize Gearhart's Theorem (for the Spectral Mapping Theorem of the exponential function) and to analyze the asymptotic behavior, as $t \to \infty$, of linear transport processes.

C.W. HENSON:

Descriptive set theory on hyperfinite sets

Let F be a hyperfinite (but not finite) set. A subset $S \subseteq F$ is Borel if it is in the σ-algebra generated on F by the internal sets; S is analytic if it is obtained from a tree $\{W_\xi \mid \xi \in \mathbb{N}^{<\mathbb{N}}\}$ of internal sets by the Souslin operation:

$$S = \bigcup_{\alpha} \bigcap_{n} W_{\alpha \restriction n}.$$

Equivalently, S is analytic iff there is a hyperfinite set G, an internal function $\pi: G \to F$ and a Borel set $T \subseteq G$ with $\pi(T) = S$.

In close analogy with classical descriptive set theory on Polish spaces, we develop the descriptive set theory of Borel, analytic (and higher projective) subsets of hyperfinite sets. A brief survey of previous work by Henson, Loeb, Kunen, Mills, Ross, Cutland ... was given, as well as several new results and easy proofs of old results. Many interesting problems were posed.

J. HIRSHFELD:

Locally compact topological groups

An infinitesimal α that has some (non standard) power α^n outside of $\mu(e)$ is either "regular" and describes a one parametric subgroup, or "singular" and describes an infinite family of (standard) compact connected subgroups.

Combining ideas similar to this with the main ideas that go into the solution of Hilbert's fifth problem, we get an improved proof of the Gleason-Montgomery - Zippin-Yamabe solution to Hilbert's fifth problem.

a) Every locally Euclidean group has no small subgroups.

b) Every group with no small subgroups is a Lie group.

Indeed our proof holds also for the a-priori larger class of locally connected groups with no small connected subgroups.

D.N. HOOVER:

Topologies for functions with jumps

Stochastic processes whose sample paths are right continuous with left limits appear often in probability theory. For instance, martingales with respect to right continuous filtrations have such paths. To study convergence in distribution, we consider such processes as random variables with values in the space D of right continuous functions with left limits.

In order to use Prokhorov's Theorem, it is desirable to give D a topology which is separable, complete, metrizable, and has many compact sets. The usual topology, due to Skorokhod has the first three properties, but is too fine for some purposes. For instance, addition of functions is not continuous.

In the talk, we discuss the following weaker alternatives to the Skorokhod topology:

(1) The topology of convergence in measure (which has been considered by many authors independently).

(2) A nonstandard convergence notion which does not correspond to any topology (introduced by Hoover and Perkins).

We characterize their compact sets, and show how they facilitate the theory of weak convergence for martingales.

C. IMPENS:

Propagation of microcontinuity for nonstandard polynomials

For hyperreal polynomials, with nonstandard degree and coefficients, the monadic concept of microcontinuity is supplemented with a typically absolute microcontinuity. It is shown that microcontinuity may be confined to isolated monads, but that absolute microcontinuity always propagates itself over non-infinitesimal distances. Examples clarify the limitations to this extension, and its intrinsic difference with extension of convergence of standard power series.

C. KESSLER:

Lattice systems with hyperfinite fiber

We work on a lattice system with a standard discrete lattice but a hyperfinite state space or fiber. We define interactions and specifications both internally and standardly, relating both notions via a standard part map. The processes of generating a specification from an interaction and of taking standard parts commute. Then we characterise pare Gibbs measures for the system (given a standard interaction) in terms of hyperfinite specifications that are generated by a lifting of the given interaction.

The hyperfiniteness of the fiber results in two difficulties that did not exist in [1]. Subsystems with finite parameter sets - still being infinite - now also carry nonatomic measures, and an internal interaction that lifts a standard one cannot be an internal object, so it does not have a unique extension "into the infinite parameters".

[1] C. Kessler, in: Acta Appl. Math. 7 (1986), 252-283.

D. LAUGWITZ:

Poisson and Cauchy: Methods of summation and delta functions

Early uses of delta functions had been associated to the Fourier inversion formula $\int_{x-a}^{x+b} \delta(t-x)f(t)dt \approx f(x)$ where $\delta(t) = \frac{1}{\pi}\int_0^{\mu} \eta(s)e^{its}\, ds$, $\eta(s) \approx 1$ for all finite $s \geqq 0$, $\mu >> 1$. Poisson conjectured that if $\delta(t) \approx g(t)$ in some interval $0 < t_o \leqq t \leqq t_1$ where $g(t)$ is continuous, then $g(t) \equiv 0$; more generally, it can be shown that the continuous function $g(t) = \int_0^{\mu} \eta(s)\phi(s,t)dt$ (on some interval) does not depend on the method of summation η if it exists at all. This is related to nonstandard methods for values of distributions. Summations of divergent series as used by Poisson and his contemporaries appear as special cases.

B.-H. LI:

On the Dedekind completion of ${}^{*}\mathbb{R}$

Denote by ${}^{\#}\mathbb{R}$ the Dedekind completion of ${}^{*}\mathbb{R}$, A the subset of ${}^{\#}\mathbb{R}$ consisting of those u's satisfying $u+u = u$, and $M = f(A)$, where $f: {}^{\#}\mathbb{R} \to {}^{\#}\mathbb{R}$ is given by $f(x) = e^{x}$. It is proved that A is closed in ${}^{\#}\mathbb{R}$ with respect to the order topology, and

$$\{wO/w \in {}^{*}\mathbb{R}\} \quad \text{and} \quad \{wo/w \in {}^{*}\mathbb{R}\}$$

are dense subsets of A, where

$$O = \sup\ \{x/x \in {}^{*}\mathbb{R},\ \text{finite}\}, \quad o = \sup\ \{x/x \in {}^{*}\mathbb{R},\ \text{infinitesimal}\}.$$

The gaps of A are exactly all $[WO, Wo]$ with $0 > w \in {}^{*}\mathbb{R}$ and $[Wo, WO]$ with $0 < w \in {}^{*}\mathbb{R}$.

Let $u \in M$ with $u \geq 0$, and

$${}^{u}\tilde{\mathbb{R}} = \{x \in {}^{*}\mathbb{R}/|x|<u\}, \quad {}^{\frac{1}{u}}\tilde{\mathbb{R}} = \{x \in {}^{*}\mathbb{R}\ /\ |x| < \tfrac{1}{u}\},$$

then

$${}^{u}\mathbb{R} = {}^{u}\tilde{\mathbb{R}} / {}^{\frac{1}{u}}\tilde{\mathbb{R}}$$

is a field and ${}^{u}\mathbb{C} = {}^{u}\mathbb{R} \oplus \sqrt{-1}\ {}^{u}\mathbb{R}$ is an algebraically closed field. ${}^{u}\mathbb{R}$ generalizes Robinson's ${}^{\rho}\mathbb{R}$.

In ${}^{*}\mathbb{R}_{+}$, we define $(\uparrow\downarrow)$-equivalence similar to that given by Puritz for ${}^{*}\mathbb{N}$. For $x \in {}^{*}\mathbb{R}_{+}$, let $\mu(x)$ be the monad of x defined by Wattenberg, and let

$$\bar{\mu}(x) = \sup\ \{|y-z|\ /\ y,z \in \mu(x)\} \in {}^{\#}\mathbb{R}.$$

Then, it is proved that $x \uparrow\downarrow y$ if and only if $\bar{\mu}(x) = \bar{\mu}(y)$. Furthermore, we have

$$\sup\ \{y/\ y\uparrow\downarrow x\} = \bar{\mu}(x)^{-1},$$

and $\bar{\mu}(x)$ is either 0 or an element in M.

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Y. LI:

Multiplication of distributions

For any distribution T on $\mathbb{R}^n$, there exists harmonic function $\tilde{T}$ on $\mathbb{R}^n \times \{y>0\}$ such that

$$\lim_{y\to o} \tilde{T}(\cdot,y) = T(\mathcal{D}').$$

If $\tilde{T}_1$ and $\tilde{T}_2$ are two such harmonic functions, then $\tilde{T}_2 - \tilde{T}_1$ can be extended to a harmonic function h on $\mathbb{R}^n \times \mathbb{R}$ such that $h(\cdot,0) \equiv 0$. For $T,S \in \mathcal{D}'(\mathbb{R}^n)$ and $\phi \in \mathcal{D}(\mathbb{R}^n)$, then

$$\langle {}^*\tilde{T}(x,\rho)\,{}^*\tilde{S}(x,\rho),\ {}^*\phi(x)\rangle \quad \text{mod infinitesimals}$$

is independent of the choice of $\tilde{T}$ and $\tilde{S}$, where $\rho > 0$ is a fixed infinitesimal. Thus the multiplication

$$T\circ S\colon \mathcal{D} \to {}^*\mathbb{R} \quad \text{mod infinitesimals}$$

is well-defined. This multiplication shares lots of properties with the ordinary multiplication of functions.

Example. The finite part of $\delta(x_1,x_2) \circ \delta(x_1,x_2) = \frac{1}{32\pi}\Delta\delta(x_1,x_2)$, where Δ is the Laplacian.

T. LINDSTRÖM:

Brownian motion on fractals

The purpose of this talk is to introduce a construction of Brownian motion on a reasonably wide class of self-similar fractals. These processes are continuous, satisfy the strong Markov property, and have a scaling property reminiscent of classical Brownian motion. The Laplacian Δ_s on a fractal S is defined in the usual way; i.e., such that $-\frac{1}{2}\Delta_s$ becomes the infinitesimal generator of Brownian motion on S. The properties of Δ_s depend crucially on three real parameters; the volume scaling factor μ, the linear scaling factor ν, and the time scaling factor λ. For example, if $n(E)$ denotes the number of eigenvalues of $-\frac{1}{2}\Delta_s$ less than E, it turns out that

$$n(E) = O(E^{\log \mu/\log 2}) \quad \text{as} \quad E \to \infty.$$

This should be compared to a classical theorem by Weyl which says that if

Δ_S is the Laplace-Beltrami operator on a Riemannian manifold S of finite volume, then

$$n(E) = O(E^{d/2}) \quad \text{as} \quad E \to \infty.$$

The relationship between these two results becomes clearer if one first notices that

$$\frac{\log \mu}{\log \lambda} = \frac{\log \mu}{\log \nu} \cdot \frac{\log \nu}{\log \lambda} = d \cdot \frac{\log \nu}{\log \lambda}$$

where $d = \frac{\log \mu}{\log \nu}$ is the Hausdorff dimension of S, and then observes that in the manifold case $\frac{\log \mu}{\log \nu} = \frac{1}{2}$.

W.A.J. LUXEMBURG:

Asymptotic averages

We recall that a bounded linear functional L on the Banach space $l^\infty(\mathbb{N})$ of bounded sequences, x, is called a Banach-Mazur limit if L is positive of norm one and invariant in the sense that $L(\tau x) = L(x)$ for all $x \in l^\infty$, where $(\tau x)(m) = x(m+1)$, $m \in \mathbb{N}$. The family of all Banach-Mazur limits will be denoted by $\mathcal{L}$. $\mathcal{L}$ is a weak*-closed convex subset of the unit ball of the dual of l^∞.

If $x \in l^\infty$ and $L \in \mathcal{L}$, then $L(x)$ will be called an asymptotic average of x. Using the nonstandard sequential representation of the elements of $\mathcal{L}$ it is easy to see that the asymptotic averages can be extended to bounded sequences of elements of a Banach space. Using this notion the following version of the ergodic theorem for contractions can be shown:

Theorem: Let T be a linear contraction on a Banach space E. Then for all $x \in E$ and for all $x' \in E'$, the dual of E, the set of limit points of the sequence $\{\langle \frac{1}{n} \sum_{0}^{n-1} T^k x, x' \rangle\}_{n=1}^{\infty}$ is contained in the closed set $\{\langle x, (\int_N (T')^n dL) x' \rangle, L \in \mathcal{L}\}$. Hence, $x \in E$ is ergodic in the sense of Eberlein if and only if x takes on the same value on every asymptotic average T'-invariant linear functional. In particular, if T' has only one such element every x of E is ergodic under T.

For $l^\infty(\mathbb{N})$ for the shift operator τ, x is τ-ergodic iff x is almost convergent in the sense of Lorentz.

Similar results hold for asymptotic averages of sequences of random variables.

J. MAWHIN:

Non standard analysis and integration

Riemann-type integrals in R^n have been introduced by the author in 1981 to allow a divergence theorem for mere differentiable vector fields in the same way as the Perron integral integrates every derivative in R. Refinements and improvements have been given, among others, by Jarnik-Schwabik-Kurzweil and Pfeffer. We give here a nonstandard characterization of Pfeffer's contribution.

If I is a nondegenerate compact rectangle in R^n, Pfeffer's definition uses, besides the usual notions of generalized Riemann integrals in the Kurzweil-Henstock sense, the concept of regularity $\rho(H,\Pi)$ of a pointed partition Π of I with respect to a finite family H of linear proper submanifolds of R^n parallel to the coordinate axes.

Pfeffer's concept can be characterized as follows in Nelson's IST.

Theorem: Let $f: I \to R^p$ be standard. Then f is integrable if and only if there exists a standard $J \in R^p$ such that the Riemann sum $S(f,\Pi)$ is infinitely close to J whenever Π is a ρ-appreciable pointed micro-partition of I.

Notice that Π is said to be ρ-appreciable if $\rho(H,\Pi)$ is not an infinitesimal when H is standard.

L.C. MOORE:

Invariant means on Z

This is joint work with Gordon Keller. A nonnegative finitely additive set function μ defined on all subsets S of Z is said to be an invariant mean if $\mu(Z) = 1$ and $\mu(S+1) = \mu(S)$ for all S. An invariant mean μ is said to be extremal if there do not exist distinct invariant means μ_1 and μ_2 and $0 < \lambda < 1$ such that $\mu = \lambda\mu_1 + (1-\lambda)\mu_2$. If A is a *-finite subset of $^*Z$ with *-cardinality of A, say ω, infinite and if the set $\beta(A)$ of endpoints of component intervals of A satisfies (*-cardinality of A)$/\omega \approx 0$, then one may define an invariant mean μ_A by $\mu_A(S) = \mathrm{st}\left(\frac{^*\text{-card}(^*S \cap A)}{\omega}\right)$ and every invariant mean has a representation of this form. If μ is extremal, then A may be taken to be an interval in $^*Z$.

S. NAGAMACHI:

Linear canonical transformations on Fermion Fock space with an indefinite metric

On finite dimensional Fermion Fock space we can construct Λ-unitary (unitary with respect to indefinite metric) operators which implement linear canonical transformations. We use nonstandard analysis to apply this result to the infinite dimensional case, that is, we investigate a sufficient condition for a nonstandard extension of the above Λ-unitary operator to have a standard part, where a new concept "almost standard" plays an important role.

J.O. OIKKONEN:

Remarks about two dimensional Brownian motion

We work with a program where we look for a discrete hyperfinite "geometry" which serves as a tool for studying (harmonic) analysis and Brownian motion and their interplay.

In [1] we modified Anderson's well-known construction of a Brownian motion from an internal random walk in such a way that the walker had a hyperfinite uniformly distributed variety of possible steps available. This led to a nonstandard construction of a Brownian motion in the plane. On the other hand, it gave natural hyperfinite representations to plane domains and their boundaries, and furthermore to harmonic measure and harmonic functions. So the old theorem of Kakutani about the connections between harmonic measure and Brownian motion has a proof built in this approach. Also the old result of Levý about the conformal invariance of Brownian motion has a natural and simple proof here.

In [2] we developed this approach further to find out in which sense the conformal invariance of Brownian motion can be generalized to arbitrary C^2 mappings. As a side product, we obtained also an abstract hyperfinite representation theorem for C^2 mappings, K-quasiconformal C^2 mappings, and to conformal mappings of the plane.

[1] Harmonic analysis and nonstandard Brownian motion in the plane, Math. Scand. 57 (1985), 346-358.

[2] The C^2 image of a Brownian motion in the plane, Ann. Acad. Sci. Fenn. Ser. A. I. Math., vol 12 (1987).

H. OSSWALD:

On vector valued Loeb spaces

Two concepts of vector valued Loeb measures are presented.

The first concept: Let $\mathbb{B}$ be a locally convex vector space over $\mathbb{C}$ and let $(\Omega, \mathfrak{A}, \mu, {}^*\mathbb{B})$ be an internal ${}^*\mathbb{B}$ valued finitely additive measure space such that μ is of S-bounded semivariation. Then $\mu(A)$ defines an equivalence class in the nonstandard hull $\hat{\mathbb{B}}$ of $\mathbb{B}$ w.r. to the weak topology on $\mathbb{B}$. Let $\phi \in \mathbb{B}'$. $N \subset \Omega$ is called a $\phi\circ\mu$-nullset, if $\forall^{st}\varepsilon > 0 \ \exists A \in \mathfrak{A} (N \subset A$ and $(\phi\circ\mu)_{\circ}(A) < \varepsilon)$, where $(\phi\circ\mu)_{\circ}$ denotes the total variation of $\phi\circ\mu$. B is called measurable, if there exists $A \in \mathfrak{A}$ such that $A \Delta B$ is a $\phi\circ\mu$-nullset for all $\phi \in \mathbb{B}'$ and we define then $\hat{\mu}(B) := \overline{\mu(A)}$.

Prop.: $(\Omega, L(\mathfrak{A}), \hat{\mu})$ is a σ-additive measure space. Two notions of integrability are shortly discussed.

In the second approach (this is a joined work with P. Loeb), we start with a Riesz-space $\mathbb{B}$ and describe a $\mathbb{B}$-valued version of the functional approach of Peter Loeb to his measure theory in the scalar valued case.

J.P. REVEILLES:

S-topology and applications to iteration theory

A finite modelisation of real numbers analogous to an hyperfinite fixed point arithmetic is presented. Equipped with an external topology, called s-topology, this system can be a frame to develop elementary analysis (s-continuity, s-derivability ...) in a style very close to computational science. As an application we present iteration theory of unimodal functions through functional operators easily defined. The spectral theory of these operators reflects the dynamics of the functions and can be completely described. For a family, depending on a continuous parameter, a s-continuity theorem is given from which we deduce a bifurcation phenomenon of the spectrum analogous to the classical Feigenbaum scheme.

L. ROGGE:

Nonstandard-methods for τ-smooth measures

Let A be an algebra on a set and $\nu: {}^*A \to {}^*[0,\infty)$ be an internal finite content. The set $C(\nu) := C(\nu,{}^*A) = \{A \subset {}^*X: \underline{\nu}(A) = \overline{\nu}(A)\}$ with $\underline{\nu}(A) = \sup\{{}^\circ\nu(B): {}^*A \ni B \subset A\}$, $\overline{\nu}(A) = \inf\{{}^\circ\nu(C): A \subset C \in {}^*A\}$ is according to Loeb a σ-algebra. Define $C_U({}^*A) = \bigcap\{C(\nu,{}^*A): \nu: {}^*A \to {}^*[0,\infty)$ finite internal content$\}$. Let the model be polysaturated. Then for $\zeta \subset {}^*A$:

(i) $\bigcup_{S\in\zeta} \zeta, \bigcap_{S\in\zeta} \zeta \in C_U({}^*A)$

(ii) $\begin{cases} \rho \uparrow T \Rightarrow \nu_L(T) = \sup\{\nu_L(S): S \in \zeta\}, \\ \rho \downarrow T \Rightarrow \nu_L(T) = \inf\{\nu_L(S): S \in \zeta\}. \end{cases}$

Let (X,τ) be a topological Hausdorff space with Borel-field $\mathcal{B}$, then $\mathrm{cpt}\,{}^*X \in C_U({}^*\mathcal{B})$; $\mathrm{pns}\,{}^*X \in C_U({}^*\mathcal{B})$ (for metric); $\mathrm{ns}\,{}^*X \in C_U({}^*\mathcal{B})$ for locally compact, σ-compact and for complete metric spaces; $st^{-1}B \in C_U({}^*\mathcal{B}) \cap \mathrm{ns}\,{}^*X$ for $B \in \mathcal{B}$, if X is regular. If X is regular and $\nu: {}^*\mathcal{B} \to {}^*[0,\infty)$ is an internal finite content, then $\overline{\nu} \circ st^{-1}|\mathcal{B}$ is a τ-smooth and $\underline{\nu} \circ st^{-1}|\mathcal{B}$ is a Radon measure. One can also obtain the extension theorem for τ-smooth Baire-measures, the Theorem of Kakutani, a result of Topsoe and the decomposition theorem for regular Borel-measures by these nonstandard methods.

D. ROSS:

Hyperfinite measures in stochastic optimization

A solution for a stochastic optimization problem is often obtained as a measurable selection from the domain of optimization. Such selections do not in general exist when this domain is not a Polish space, even when the stochastic component is given by a Radon measure.

When the stochastic component is given by a hyperfinite measure, however, selections often exist for very general domain spaces. This means that solutions (often in a weak, or ideal, sense) can be obtained for many otherwise intractable problems. I give examples from Markov decision theory and integration theory for random sets.

T. SARI:

Sur le équations de continuité de la théorie des semiconducteurs

Le système d'équations differentielles

$$\frac{dn}{d\kappa} = -ne + I_n(\kappa), \quad \frac{dp}{d\kappa} = pe - I_p(\kappa), \quad \varepsilon^2 \frac{de}{d\kappa} = p-n + D(\kappa)$$

apparaît dans la description de l'état stationnaire d'une diode (jonction p-n): κ dénote la variable d'espace, $e = e(\kappa)$ le champ électrostatique, $n = n(\kappa)$ la concentration des charges négatives (électrons), $p = p(\kappa)$ la concentration des charges positives (trous), $I_n(\kappa)$ et $I_p(\kappa)$ les courants d'électrons et des trous et $D = D(\kappa)$ la densité de dopage; ε est la longuer de Debye (de l'ordre de 10^{-6} à 10^{-2}). On propose une description des trajectoires de ce système basée sur le fait que ε est infinitésimal. On utilise entre autres techniques la méthode de stroboscopique et le lemme de "l'ombre courbe".

Classiquement le problème est étudié un utilisant des développments asymptotiques des fonctions p, n et e en puissance de ε.

D. SPALT:

Reality and chance - On the philosophical explosive by non-standard analysis

Formalism ('64) - Intuitionism ('84) - what about the position of conventionalism? Non-standard analysis is the very convulsion in mathematics by conventionalism. In the 19th century non-Euclidean geometry already shook the Platonic image of mathematical thinking for the first time, but there were some convenient arguments (internal - simplicity, elegance - as well as external ones - the world of experience -) in favour of the durability of the autocracy of the timehonoured theory (Euclidean geometry). Non-standard analysis now changes the situation completely. Now the convenient scale (simplicity, elegance) produces at least a drawn battle if not some advantage for the new. So the persistence in the ancient cannot be legitimated any more <u>internally</u> but only <u>externally</u> via tradition, compatibility in teaching etc. In other words, what usually is thought to be the very essence of mathematics - its unity - now inevitably turns out to be an object in view the reaching of which has to be gained by hard work, by fighting, and so first of all has to be set up.

M.E. SZABO:

Computational monads

Using idealization, we show that the computations generated by arbitrary algorithms are infinitely close to star-finite computations of algorithms with star-finitely bounded iterations.

We use the progamming languages PL and BPL described by E.J. Farkas in "Parallel computations embedded in star-finite models" to illustrate the fact that if $P \in PL$, $P' \in BPL$, and $m_1,\ldots,m_p$ are finite bounds on while and await statements in P', then it holds for all computations Tr(P) of P that

$$Tr(P) = \lim_{m\to\infty} Tr(P'(m_1,\ldots,m_p))$$
$$= std\ (Tr\ (P'(\mu_1,\ldots,\mu_p))),$$

where $m = \min(m_1,\ldots,m_p)$ are finite and $\mu_1,\ldots,\mu_p$ are infinite integers. This involves the description of a monad μ for which

$$Tr(P) \approx Tr(P'(\mu_1,\ldots,\mu_p)).$$

The basic idea is that two computations of an algorithm are equivalent if and only if no mechanical computing device can distinguish them in finite time at standard inputs.

We illustrate the monad construction in several other situations: finite automata, programs with procedure terms, and computable functionals.

We also show how the method can be used to obtain a model of the λ-calculus without limits and give a star-finite analogue of Herbrand's theorems for partial correctness formulas.

V.M. TOTORELLI:

Γ-limits and infinitesimal analysis

We briefly sketch some kinds of limit-cases in problems in differential equations and calculus of variations, as usually done by showing the elementary example of the one dimensional homogenization.

We introduce the general topological definition of Γ-convergence as a possible abstract setting of these kinds of problems, viewing Γ-limits of sequences as operations of lower-semicontinuous envelope, carried meanwhile the parameter of the sequence goes to infinity.

Then we give an infinitesimal characterization of Γ-limits acting on functions with two variables. They are sup-min° and inf-max° (on the box-monad of standard points) operators. We also remark that similar characterizations for Γ-limits acting on functions depending on more than two variables, have been not proved.

N. VAKIL:

Internal Set Theory with external classes

Edward Nelson has axiomatized nonstandard analysis in what he calls Internal Set Theory (IST). The Nelson IST has been criticized for its inflexibility to external constructions. The remedy suggested by Nelson is to embed IST in one of its models. This sacrifices much of the simplicity of IST, which is the primary purpose of this approach to nonstandard analysis. We provide here for IST a formal framework within which a limited external theory can be developed. This is achieved by adapting Takeuti and Zaring's treatment of ZFC (Zermelo-Frankel set theory with the axiom of Choice) to IST. In their treatise "Introduction to axiomatic set theory", Takeuti and Zaring use the class symbol $\{x \mid \phi(x)\}$ as a defined notion, so that the formulas that contain this symbol are merely abbreviations of the well-formed formulas of ZFC. A class symbol $\{x \mid \phi(x)\}$ will then be called a set if it satisfies the formula $(\exists y)\ [y = \{x \mid \phi(x)\}]$. Class symbols that are not sets are called proper classes. Most of the set theoretic operations that are used to construct new classes from given classes, such as intersection, union, and cartesian product, are defined for all classes. Some operations, such as the formation of power sets and an ordered pair of sets, are restricted to sets only. This approach to the development of set theory reveals its full strength when adapted to IST. The presence of internal and external formulas in the language of IST produce internal and external classes, respectively. While one seldom deals with proper classes in ZFC, they often occur in IST, and as external classes. Thus we develop a restricted external theory within IST which allows us to make certain external constructions. This broadens the applicability of IST while preserving all of its simplicity.

N. VAKIL:

Nearstandard elements in the space of continuous functions with the topology of pointwise convergence

Let f be an internal function from ${}^*R$ to ${}^*R$. Recall that f is called s-continuous at a point $a \in {}^*R$ if

$$(\forall \varepsilon \in R^+)(\exists \delta \in R^+)(\forall x \in {}^*R)\ [|x-a| < \delta \rightarrow |f(x) - f(a)| < \varepsilon].$$

If in this statement, we replace the part $(\forall x \in {}^*R)$ by $(\forall x \in R)$, we obtain a useful property which we have called quasi-s-continuity. In terms of this notion, we can characterize the nearstandard elements of the spaces of continuous functions endowed with the topology of pointwise convergence. As an example of applications of such characterization, we show that a bounded subset H of $C_p(X)$ is relatively compact if and only if every element of ${}^*H$ is quasi-s-continuous on X. Here $C_p(X)$ denotes the space of all real-valued continuous functions on a metric space X with the topology of pointwise convergence.

F. WATTENBERG:

Applications of nonstandard analysis to PDEs

We discuss using a *finite system of ordinary differential equations to model and study phenomena usually studied by means of partial differential equations. These techniques will be illustrated by looking at the wave equation.

M.P.H. WOLFF:

On the dilation of strongly continuous semigroups of contractions

In the following three cases it is well-known how to construct a dilation of a discrete semigroup of contractions (generated by a single operator) and how to interpret this dilation in terms of probability theory or of quantum mechanics: (i) positive contractions of L^1, (ii) contractions of a Hilbert space, (iii) completely positive contractions on C^*-algebras.

With the aid of nonstandard analysis it is possible to generalize these results to the case of strongly continuous one-parameter semigroups of contractions. Applications to ergodic theory as well as to quantum mechanics are presented.

R. ŽIVALJEVIĆ:

An application of infinitesimals in algebraic topology

Our goal is to associate homology type invariants with the Stone-Čech remainders of locally compact metric spaces. It is well known that a natural number $H \in {}^*\mathbb{N}$ determines uniquely an ultrafilter (type) on $\mathbb{N}$ or equivalently an element of $\beta\mathbb{N}$, the Stone-Čech compactification of $\mathbb{N}$. Using this idea a chain complex $\{E_p(I(X)),\ p \geq 0\}$ associated with the increment $I(X) := \beta X \setminus X$ is defined. In particular one has a nice geometric insight in why $h_1(I(R')) \neq \{0\}$, $h_n(I(X)) \neq \{0\}$ for $X = D^n \times C \setminus \{(0,c_o)\}$ where D^n is a n-dimensional disk, C the Cantor set, $c_o \in C$ etc. where $h_p(\cdot)$ are homology groups defined by the complexes above. This together with a proof of triviality of $h_n(I(D^n))$ which is based on a theorem of A. Calder leads to a nonstandard proof of a result of M. Jerison, J. Seigel, S. Weingram about local increments of finite dimensional locally compact topological groups.

P. ZLATOŠ:

A Hamiltonian and a Menger property of connected sets in the alternative set theory

If R is a reflexive and symetric relation on the universe V then for each class X the pair $\langle X,R\rangle$ can be regarded as a graph with the class of edges $\{\{x,y\};\ \langle x,y\rangle \in R \cap X^2 \ \&\ x \neq y\}$.

Theorem: Let R be a π-equivalence and u be a set such that $\langle u,R\rangle$ is connected.

a) For each pair of different elements $x,y \in u$ there is a Hamiltonian path from x to y in $\langle u,R\rangle$.

b) If $u^2 \nsubseteq R$ then for each pair of different elements $x,y \in u$ there is an infinite set s of paths from x to y in $\langle u,R\rangle$ such that each point $z \in u$, $z \neq x,y$, lies exactly on one of the paths from s.

c) If $u^2 \nsubseteq R$ then there is an infinite set c of cycles in $\langle u,R\rangle$ such that each point $x \in u$ lies on exactly one cycle from c.

A bit weaker but similar results can be proved also for σ-equivalences.

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