MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 17/1987

Mathematische Logik

19.4. bis 25.4.1987

Die Tagung fand unter der Leitung von Herrn W.Felscher (Tübingen), Herrn H.Schwichtenberg (München) und Herrn A.S.Troelstra (Amsterdam) statt. An ihr nahmen 22 Wissenschaftler aus 7 Ländern teil, darunter 11 aus Deutschland.

Es wurden 15 Vorträge aus wichtigen Teilgebieten der mathematischen Logik (Beweistheorie, Rekursion-und-Lambdakalkülstheorie, Mengenlehre), Komplexitätstheorie und Verbindungen der Logik zur Informatik gehalten.

Parallel dazu fand ebenfalls eine Tagung der Sprachwissenschaften statt, die zeitlich so gut mit der Logischen koordinierte, dass beiderseits die Möglichkeit bestand, die interresantesten Vorträge dieser Wissenschaften zu besuchen, die auch genutzt wurde.

Vortragsauszüge

Z.ADAMOWICZ: A non-usual universal relation for Δ_o formulas.

We define a Δ_1 universal relation for Δ_o-formulas in the standard model N. The construction of the relation uses König's lemma instead of using the existence of Skolem functions for Δ_o formulas.

K.AMBOS-SPIES: Honest polynomial-time reducibility.

The polynomial-time reducibilities p-many-one and p-Turing are obtained by taking many-one respectively Turing reductions and by requuiring in addition that the reductions can be carried out in polinomial time. A p-reduction is honest if the length of the input and those of the oracle queries are polynomially related. While there are no minimal degrees for the p-reducibilities, the existence of minimal degrees for the honest p-reducibilities might depend on an answer to the P=?NP question. Homer has shown that assuming P=NP minimal degrees for the honest p-reducibilities exist but they cannot be recursive. Homer asked whether there are at least recursively enumerable sets which have minimal hp-degree (assuming P=NP). Here we give an affirmative answer to this question. Our proof requires some results from recursion theory, and it clarifies the role played by the assumption that P=NP. It remains the question, whether this assumption is necessary or not.

G.ASSER: Zur Robinson-Charakterisierung der einstelligen rekursiven Wortfunktionnen.

Es werden mehrere interne induktive Charakterisierungen der Klasse $Prim^1_A$ der einstelligen primitiv rekursiven Wortfunktionen über einem endlichen Alphabet $A = \{a_1,\dots,a_r\}$ $(r \geq 2)$ gegeben. Speziell wird gezeigt, daß $Prim^1_A$ die kleinste Klasse von einstelligen Wortfunktionen über A ist, die die Nachfolgerfunktionen $\sigma_i(w) = wa_i$ $(i = 1,\dots,r)$ und eine geeignete weitere Funktion τ ehthält, und abgeschlossen ist im bezug auf Superposition $*$, Verkettung $\cap$ und Wortiteration it_A :

$(\alpha * \beta)(w)=\alpha(\beta(w))$, $(\alpha \cap \beta)(w)=\alpha(\omega)\beta(w)$,

$it_A(\varphi_1,\dots,\varphi_r) = \psi \Leftrightarrow \psi(\Delta)=\Delta \,\&\, (\forall w) \mathop{\&}_{i=1}^{r} \psi(wa_i)=\varphi_i(\psi(w)))$.

Als Funktion τ kann man u.a. wählen:

$q\hat{}(w)=c^{-1}(q(c(w)))$, wobei $q(x)=(x-[\sqrt{x}]^2)$ und c eine kanonische Bijektion von A^* auf N ist, die Links-bzw. Rechtsfunktion der folgenden injektiven Paarfunktion von $A^* \times A^*$ in A^* :

$\gamma(v,w) = a_1\dots a_1 a_2 v w a_2 a_1 \dots a_1$.

I.BETHKE: Extensional partial combinatory algebras.

We formalize a general method for the construction of non-total λ-models.

Theorem 1: *Let* D *be a* CPo *and* $[D \to_s D]=\{f \in [D \to D] : f(\bot_D)=\bot_D\}$. *Let* $F:D \to [D \to_s D]$ *and* $G:[D \to_s D] \to D$ *be continuous maps such that* $Range(G) \subset (D-\{\bot_D\})$ *and* $F \circ G$ *is identity on* $[D \to_s D]$. *For* $d,d' \in (D-\{\bot_D\})$ *put* $d \cdot d' := F(d)(d')$ if $F(d)(d') \neq \bot_D$ *,undefined otherwise. Take* $\mathfrak{M} := ((D-\{\bot_D\}),\cdot)$. *Then the following are true:*

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(*i*) *$\mathfrak{M}$ can be considered as a non-total λ-model such that the representable functions on $\mathfrak{M}$ are exactly the partial continuous functions on* $D-\{\perp_D\}$.

(*ii*) *Moreover, $\mathfrak{M}$ is extensional, i.e.* $\forall d,d'\in(D-\{\perp_D\})(\forall d''\in(D-\{\perp_D\})(d\cdot d''\simeq d'\cdot d'')\rightarrow d=d')$, *iff* GoF *is identity on* $D-\{\perp_D\}$.

As an example of how a CPo satisfying the domain equation $D-\{\perp_D\}\cong[D\rightarrow_s D]$ can be constructed we describe a modification of the free PSE-algebra (Plotkin-Scott-Engeler) generated by an arbitrary poset A with bottom $\perp_A$. The resulting non-total extensional PCA $\mathfrak{M}$ has the following properties.

Theorem 2: *$\mathfrak{M}$ has neither a completion nor a total submodel.*

Theorem 3: *There are unsolvable λ -terms* $T_0,T_1,\ldots T_\omega$ *such that* $[T_0]^{\mathfrak{M}}\subsetneq[T_1]^{\mathfrak{M}}\subsetneq\ldots\subsetneq[T_\omega]^{\mathfrak{M}}=\sup_n[T_n]^{\mathfrak{M}}$.

E.BÖRGER: The undecidability of the floundering property in MU-PROLOG.

Facing the problem of dealing with negative information in logic programs Clark (1978) has introduced an extension of a Horn clause theorem prover by a special inference rule (known as NAF-rule for Negation As Failure) which allows to include a certain form of negation into Prolog programs. Recently Sheperdson (1984,1985) and Flannagan (1985,1986) have put forwards arguments that there seems to be no simple and useful logical meaning of negation as failure in the general case. Clark's version of NAF whereby negative literals must be closed in order to be evaluated is implemented in MU-PROLOG (Melbourne University Prolog). We show that for the underlying query evaluation process it is undecidable whether it will flounder for a given query with respect to a given program, confirming a conjecture made by Flannagan (1985). Thus we contribute to the impression that this query evaluation process (with negation interpreted as failure) at least in its full generality is not of practical use as a theorem prover. It also puts into perspective the more restricted versions or applications of that query evaluation process proposed recenzly by Lloyd&Topor (1985), Aquiliano et.al.(1986), Barbuti&Martelli (1986).

J.DILLER: Syntax of E-logic.

Stimulated by the observation that the class of Kripke-structured mentioned by D.Scott in LNM 753 is not complete for E-logic, M.Unterhardt, Münster, made a study of the semantics of E - logic (1986) and the following is joint work with him. A general concept of rule in natural deduction is developed. Besides standard redexes and simplifications we take as permutation redex any formula occurrence that is the conclusion of (vE) or (∃E) and major premise of an E-rule. By standard normalization techniques any deduction reduces to a normal (redex-free) deduction in this strict sense. An inductive definition of these normal deductions leads, after applying the crude discharge convention, to an isomorphism between normal deductions and cut-free sequent derivations. In contrast to the situation in logic with global existence M.Unterhardt could show that there are rules that cannot be replaced by axiom schemes, e.g. the Görnemann-rule characterizing Kripke-structures with constant domains.

L.GORDEEV: On modified Kruskal-Friedman combinatorics.

In comparison to the talk given last time in Oberwolfach I present now a more precise estimation of the proof theoretic strength of the modified combinatorial statements M^p_α (p -natural number >0, α -countable ordinal) stating that *for every infinite sequence* (T_1,ℓ_1), (T_2,ℓ_2), $\ldots,(T_n,\ell_n),\ldots$ *of* $(\le p)$*-branching trees* T_n *with labeling functions* $\ell_n:T_n\longrightarrow\alpha$ *there are* $i<j$ *and a hom(e)omorphic embedding* $h:T_i\longrightarrow T_j$ *satisfying:* (1) $\ell_i(x)\le\ell_j(h(x))$ *for every node* x *in* T_i *and* (2) *if* x,x' *are neighbouring nodes in* T_i *then* $\ell_j(z)\ge\min\{\ell_i(x),\ell_i(x')\}$ *for every node* z *placed between* $h(x)$ *and* $h(x')$ *in* T_j . It turns out that from the proof theoretic viewpoint the hierarchy $\{M^1_\alpha\}$ almost exactly imitates the strength of the predicative hierarchy $(\Pi^1_0\text{-CA})_\alpha$ while for p>1 this is true with respect to the impredicative (inductive) hierachy ID_α of subsystems (familiar in the literature) of the second order arithmetic.

E.R.GRIFFOR: Strong Existensional Types and Normalization in the Second Order Typed Lambda Calculus.

The second order typed λ-calculus of Girard is extended with Martin-Löf's dependent type and existensional types with the strong introduction and elimination rules of Howard. It is shown that there are non-normalizable terms in the resulting calculus. This would seem to support the view that impredicativity is incompatible with the identification of types and propositions.

G.JÄGER: Annotations on the consistency of the closed world assumptions.

In the recent years the general theme of *Negative information* has attracted a certain amount of attention especiallly in the context of logic programming, logical data bases, information processing and the like. One of the most important concepts in this connection is Reiter's closed world assumption CWA. In this talk we introduce the notion of *closed world of a first order theory theory T with respect to a sequence of predicates* and give natural conditions for its consistency.

H.LUCKHARDT: Incompleteness and complexity.

Gödel's second incompleteness theorem is equivalent to non-trivial lower bounds (LB) on proof complexity. Therefore usual formal theories T of mathematics cannot prove any non-trivial LB_T on their proof complexity. We set up wide-spread examples of provable formulas F_n (some consisting only of Boolean ones) assuming such LB for elementary or NP≠P -reasons thus constituting a new type of formal proof limit which can only be overcome by making almost all (proved) F_n axioms. So a smaller ideal on the adequacy of formalization is also destroyed, namely that every recognized set of new true statements (like $LB_S(F_n)$ for all consistent $S \supset T$) can be formally derived by adding suitable new (true) formulae as axioms. Our Boolean examples also give indirect hints in favor of Cook's thesis NP≠P and point out some proof-theoretic difficulties a proof has to overcome. Finally we obtain formal undecidability of the following well-known theorems: recursive undecidability, proof-theoretic Π^0_1 -uniformity and proof speed-ups.

A.R.D.MATHIAS: MacLane Set Theory.

Saunders MacLane has drawn attention many times, most recently in his book *Mathematics Form and Function*, to the system of set theory of which the axioms are *extensionality*, *Null Set*, *Pairing*, *Union*, *Infinity*, *Power Set*, *Restricted Comprehension*, *Foundation*, and *Choice*. This system is naturally related to systems derived from topos-theoretic notions concerning the category of sets, and is, as MacLane emphasizes, one that is adequate for much of mathematics. In this paper we give some relative consistency arguments relating MacLane's to other systems of set theory; we show by arguments of Coret and Boffa that MacLane set theory proves all stratified instances of replacement and comprehension; and we exhibit a simple set theoretic assertion - namely that there exists an infinite set of infinite sets, no two of which have the same cardinal - that is unprovable in MacLane system.

D.NORMAN: Inductive definitions in type theory.

We construct a formal type-theory where we include some type-operators given in e.g. Per Martin-Löf type theory together with an operator forming the least fixed-point of certain strictly positive type operators. We may also define a family of types indexed over a type using simultaneous induction. The theory contains a general rule-schema for proofs by induction and construction of types by recursion on the natural ordering of the inductively defined types. Finally it is shown that the strictly positive type-operators are in a sense categorical and that this is provable within the theory.

Z.RATAJCZYK: Functions provably total in Σ_n-induction without parameters.

The following theorem is a generalization to the case for n=1 by Z.Adamowicz and T.Bigorajska. Let $n \geq 2$.

Theorem: *If in the proof of the totality of a recursive function* $f: N \to N$ *in the theory* $I^{-}\Sigma_n$ *we use only* m *different axioms of* Σ_n*-induction then* f *can be bounded (almost everywhere) by a function* $H_{\omega_{n-1}^{\omega^m \cdot k}}$ *in Hardy's hierarchy for a certain* $k \in \omega$*. The estimation of the rapidity of growth for* n=1 *which we obtain is* $H_{\omega^{m+2} \cdot k}$ *for a certain* k*.*

In the proof we use only the usual semantical method and combinatorial consideration. We do not need to analyse proofs but only provability.

A.S.TROELSTRA: An open problem concerning intensional continuous functionals.

Let APP be the arithmetical part of Feferman's theory of operations and classes, i.e. the part without classes. Let IT be the obvious internally defined structure in APP.

Is there a model for APP *(say* $\mathfrak{M}$ *) such that* IT *becomes isomorphic to* ICF, *the structure of the intensional continuous functionals?* More generally,

Is there an $\mathfrak{M}$ *such that* (a) $IT_0 = N$, (b) $IT_{0\to 0} = N\to N$, (c) *type* 2 *objects are continuous* , (d) *the modulus of continuity functional for type* 2 *objects is in* IT *(in the model* $\mathfrak{M}$ *)*?

Unexplained notations in A.S.Troelstra, LNM 344.

S.S.WAINER: Subrecursive Hierarchies.

The Slow-growing (G) , "Hardy" (H) and Fast-growing (F) hierarchies were defined, and their most significant properties outlined:

(1) $G(\alpha)$ represents α as in direct limit

(2) $H(\alpha)$ witnesses the well-foundness of α

(3) $F(\alpha) = H(\omega^{\alpha})$.

These suggest the construction (from below) of a large class of so-called accessible recursive functions. The definition follows standard recursion theoretic principles, using $G(\alpha)$ to "code" α : i.e. generate $F(\alpha)$ only if $\exists\beta<\alpha\ (G(\alpha) = F(\beta))$.

Theorem: *For appropriate "Bachmann-style" collection* $\varphi^n: \Omega_{n+1}\times\Omega_n \longrightarrow \Omega_n$ *we have* $G(\varphi^{n+1}) = \varphi^n$ *where* $\varphi^0 = F$.

Corollary: $G(\text{"}id_{n+1}\text{"}) = F(\text{"}id_n\text{"})$, *so the hierarchy of accessible recursive functions closes off at* "$id_{<\omega}$" .

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