MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 22/1987

Elliptische Operatoren auf singulären und nicht kompakten Mannigfaltigkeiten

17.5 bis 23.5.1987

Die Tagung fand unter Leitung von Herrn Brüning (Augsburg) und Herrn Melrose (Cambridge, Mass.) statt. Die Vorträge befaßten sich mit linearen elliptischen Operatoren auf Außenräumen, mit der Geometrie der Enden nicht kompakter Mannigfaligkeiten und mit symplektischer Geometrie. Auch über nicht-lineare Differentialgleichungen geometrischen Ursprungs wurde vorgetragen. Ein Hauptziel der Tagung war es Verbindungen zwischen diesen Themen aufzuzeigen.

Vortragsauszüge:

S. ZUCKER:

L^2-cohomology

L^2 - cohomology is the homological object associated to the L^2-harmonic forms on a Riemannian manifold. Given M, there is an intrinsic notion of L^2 - forms, and the L^2-cohomology is defined by $H^i_{(2)}(M) = Z^i/B^i$, where Z^i is the space of closed L^2-i-forms, and B^i is the subspace consisting of exterior derivatives of of L^2-(i-1)-forms. Here, one can use either C^∞ - forms, or measurable forms with weak derivatives. Taking the latter choice, one sees that $H^i_{(2)}(M) \simeq h^i_2 \oplus R^i$ (general Hodge theorem), where $h^i_2 = Z^i \cap (B^i)^\perp$ is the space of L^2- harmonic forms satisfying the requisite domain condition (Neumann problem), and $R^i = \bar{B}^i/B^i$ is either trivial or of infinite algebraic dimension. As simple examples, we considered a) $M = D^2$ (to show the role of domain conditions) and b) $M = \mathbb{R}^+$ (Euclidean, where $R' \neq 0$). We continued with c) $X \subset \mathbb{CP}^N$ a projective variety (with singularities) and $M = X^{reg}$, with metric induced by the embedding; d) M an arithmetic quotient of a non-compact symmetric space (when M is Hermitian, it has the Baily-Borel Satake compactification X); e) X has metrically conical singularities, M the regular locus. The L^2-cohomology is known to have the topological interpretation <u>on X</u> as the middle intersection homology of Goresky-MacPherson in (d) (Looijenga; Saper-Stern) and (e) (Cheeger). The same is conjectured for (c) by Cheeger-Goresky-MacPherson, and is known only for special classes of X (from Cheeger; Hsiang-Pati). As is well-known, it is a local issue on X, and it has been carried out by explicit calculation.

S. SAPER & M. STERN

Zucker's conjecture

Let D be an hermitian symmetric domain, Γ a neat arithmetic subgroup of the group G of automorphisms of D. With respect to the metric induced from the Bergman metric on D, $\Gamma\backslash D$ is a complete, finite volume, Kähler manifold. Let $\Gamma\backslash D^*$ denote the Bailey-Borel-Satake compactification. This is a normal projective variety which, in general, is highly singular. Let $\mathcal{E}$ be a local coefficient system on $\Gamma\backslash D$ defined by a finite dimensional representation of G. We prove the following theorem conjectured by Zucker.

Theorem. $H^{\cdot}_{(2)}(\Gamma\backslash D, E) \simeq IH^{\cdot}(\Gamma\backslash D^*, E)$ where $H^{\cdot}_{(2)}$ denotes L^2-cohomology and $IH^{\cdot}$ denotes (middle) intersection cohomology.

Let x lie on the codimension k singular stratum of $\Gamma\backslash D^*$. In order to prove this theorem, one needs to show the following vanishing condition:

$$H^{i}_{(2)}(U \cap \Gamma\backslash D, E) = 0, \quad i \geq k,$$

for U belonging to a nice fundamental system of neighborhoods of x in $\Gamma\backslash D^*$. To show this, we establish the estimate

$$\|d\varphi\|^2 + \|d^*\varphi\|^2 \geqq c\|\varphi\|^2 \quad \text{for} \quad \varphi \in \text{dom}\, d^* \cap A^i_c(U \cap \Gamma\backslash D, E).$$

The proof requires extensive use of the structure of Hermitian symmetric spaces.

R. MAZZEO

The Laplacian on asymptotically hyperbolic manifolds

For certain complete manifolds which are asymptotically similar to hyperbolic space (including certain quotients $\mathbb{H}^n/\Gamma$) the Hodge Laplacian may be regarded as a degenerate elliptic operator on a compactification. Operators which degenerate in just this fashion are characterized as polynomials (with C^∞- coefficients) in the vector fields $x\partial_x, x\partial_{y_i}$, where $x = 0$ on the boundary of this compactification, and $y_1, \ldots, y_{n-1}$ are the tangential variables. This class of operators is microlocalized, and a space of pseudo-differential operators is defined which contains parametrices for such degenerate elliptic operators. In particular, in the geometric setting above, this parametrix is used to study the essential spectrum and dimension of the space of L^2-harmonic k-forms on these manifolds. These latter numbers are interpreted topologically in terms of the absolute and relative singular cohomology of the compactification.

S.R. SIMANCA

Spectral properties of the mixed Laplacian

From Fredholm properties of the Laplacian with mixed Dirichlet and Neumann conditions, we construct a suitable inverse for the unbounded mixed Laplacian induced in the space of square integrable functions. Using it, we study the spectrum or this operator and by direct computations show that the counting function for polyhedral domains in the plane has an expansion in which the first and second coefficient can be computed explicitly, showing that for large valued or λ this function is in between the corresponding one with purely Dirichlet and Neumann conditions on the whole boundary.

E. SCHROHE

Complex powers of operators on non-compact manifolds

In order to analyse complex powers of pseudo-differential operators (pdo) on non-compact manifolds, a class of weighted symbols and Sobolev spaces, first investigated by H.O. Cordes on $\mathbb{R}^n$ is transfered to manifolds with a compatible structure. These include manifolds consisting of a compact center and finitely many ends of the form $K \times (1,\infty)$, K compact. In particular, some cases of manifolds with singularities can be treated via coordinate transforms.

For certain elliptic pdo with positive multiplication and differentiation order, a family of complex powers $\{A_s : s \in \mathbb{C}\}$ is constructed. The kernel function $k_s(x,y)$ of A_s, as well as the zeta and eta function of A then shows an analytic behaviour very similar to that in the case of a compact manifold.

R. SEELEY

Extending $\bar\partial$ over limit points in moduli space

One can form a family M_t of Riemann surfaces of genus g, varying with a modular parameter t, by "operating" on a surface M^{g-1} of genus $g-1$: Take two disjoint disks $\{|z| < 1\}$ and $\{|\omega| < 1\}$ in M^{g-1} and, for $|t| < 1$, identify $\{|t| < |z| < 1\}$ with $\{|t| < |\omega| < 1\}$, by the map $zw = t$. Then for $t = 0$,

$$\bar\partial_p : (\Lambda^{1,0})^p(M_t) \longrightarrow (\Lambda^{1,0})^p \otimes \Lambda^{0,1}(M_t)$$

is a holomoprhic family of Fredholm operators. Question (posed by Singer, because of problems in string theory): Can this family be extended to $t = 0$?. Answer (joint with Singer): It can, as a continuous Fredholm family, by using L^2-norms defined in the two annuli by

$$||u(\frac{dz}{z})^p||^2 = \int|u|^2\, r^{-1}dxdy, \qquad ||fd\bar z(\frac{dz}{q})^p||^2 = \int|f|^2 r^{-1}dxdy.$$

Similar results hold for half-integer p. As a consequence, one rederives

$$\text{index}\ \bar\partial_p = (2p-1)(g-1).$$

M. GROMOV

On the singular Hilbert boundary value problem

Consider a subset $R \subset \mathbb{C}^n$ which is "almost everywhere" an n-dimensional totally real manifold. The problem is to describe holomorphic curves $C \subset \mathbb{C}^n$ with boundary $\partial C \subset R$. These basic examples where non-trivial existence theorems are known are as follows:

(a) Immersed Lagrange manifolds $R \subset \mathbb{C}^n$;

(b) Generic surfaces $R \subset S^3 \subset \mathbb{C}^2$.

G. GRUBB

On the heat equation for pseudo-differential boundary problems

Pseudo-differential boundary problems appear in many applications, often arising from manipulations with differential operator problems. We mentioned two new time-evolution cases:

1° In control theory: The "taming" of unbounded solutions (for $t \to \infty$) of the heat equation for a strongly elliptic Dirichlet problem with some negative eigenvalues, by introducing a (non-local) feedback in the boundary condition, $\gamma u = \sum_{1 \leq N} (n, w_j)_\Omega g_i$.

2° In hydrodynamics: Removal of the degeneracy in the parabolicity of the Navier-Stokes initial-boundary value problems, by transformation to problems containing pseudo-differential boundary terms.

The tools for the ps.d.o. heat semigroup construction are given in a recent book (Progress in Mathematics #65, Birkhäuser Boston 1986), and we explained some basic ingredients, comparing with the resolvent construction of R. Seeley for differential problems. Finally, we described the finite asymptotic expansion of the trace for $t \to 0$.

G. UHLMANN

An n-dimensional analogue of the Borg-Levinson theorem

We sketched a proof of the following result generalizing the one dimensional Borg-Levinson theorem:

Let $\Omega \subseteq \mathbb{R}^n$, $n \geq 2$, be a bounded domain with smooth boundary. Let q_i, $i = 1,2$, be smooth, real-valued functions in $\bar{\Omega}$ and let $\mu_j(q_i)$ denote the Dirichlet eigenvalues associated to the Schrödinger equation $-\Delta + q_i$, $i = 1,2$. A corresponding complete set of orthonormal eigenfunctions is denoted by $\varphi_j(q_i)$ $i = 1,2$.

Theorem Assume $\mu_j(q_1) = \mu_j(q_2) \ \forall_j$,

$$\frac{\partial \varphi_j}{\partial \nu}(q_1)\big|_{\partial\Omega} = \frac{\partial \varphi_j}{\partial \nu}(q_2)\big|_{\partial\Omega} \ \forall_j .$$

Then $q_1 = q_2$ in $\overline{\Omega}$.
This is joint work with A. Nachman and J. Sylvester.

M. COSTABEL

Crack singularities in three dimensions

The Neumann problem for the Lamé equations of linear elasticity theory for the exterior of a smooth open surface in $\mathbb{R}^3$ can be reduced to a pseudodifferential equation on the ("crack-") surface. Eskin's factorization technique can be applied to get a precise description of the (local form of the) singular behaviour of the solution near the crack edge. If one splits off the first singular function in form of a tensor product, the resulting a-priori-estimates in terms of Sobolev norms lead to the loss of one order or regularity. Taking this into account, one can use these estimates as starting point for error estimates for certain specially adapted numerical approximation methods (Joint work with E. Stephan).

R. LOCKHART

Fredholm and Liouville properties of Laplacians on noncompact manifolds

Suppose M is a noncompact manifold with ends. Thus outside a compact set M_0 we have $M - M_0 = \partial M_0 \times \mathbb{R}^+$. On such a manifold it is sensible to talk about asymptotically translation invariant metrics. Suppose h is such a metric and $g = e^{2\rho}h$ for $\rho \in C^\infty$ and all $D_h^t\rho$ converging to a translation invariant tensor at infinity. Then the associated Laplacean Δ_g is a bounded operation from $W^p_{s+2,\delta,a}(\Lambda^g M)$ to $W^p_{s,\delta,a+2}$ where the norm for the weighted Sobolev space is $||\sigma|| = (\sum_{t=0}^{s} \int_M ||e^{\delta z+(t+a)\rho} \cdot D_g^t\sigma||_g^p dV_g)^{1/p}$. Furthermore there is a discrete set $\mathcal{D}_\Delta \subset \mathbb{R}$ such that if $\delta \notin \mathcal{D}_\Delta$ then Δ_g is Fredholm.

As a consequence the space of harmonic forms satisfying $||e^{\delta z}\sigma||_g < \infty$ is finite dimensional. Another consequence is that $\exists$ an interpolation space $\tilde{W}^p \to \Delta_g : \tilde{W}^p \to L^p$ is always Fredholm.. Similar results hold for the operator $d + d^*_g$. In particular $L^2(\Lambda^q) = d\tilde{W}^p(\Lambda^{q-1}) \oplus d^*_g\tilde{W}^p(\Lambda^{q+1}) \oplus h^q$. Often one can say much about h^q. For instance if g is g-bounded above then h^q provides unique representatives for the de Rham cohomology classes that have respresentatives in L^2.

P. WERNER

Branch points of the resolvent, standing waves and resonances in some classes of unbounded domains

Let A be the self-adjoint extension of the Laplacian $-\Delta$ in a n-dimensional domain D with respect to the Dirichlet or the Neumann condition. If D is bounded, then the spectrum of A is discrete, the resolvent $(A - z)^{-1}$ is meromorphic with simple poles at the eigenvalues of A, and the solution of the initial and boundary value problem $\partial_t^2 u - \Delta u = fe^{-i\omega t}$ in D, $u = 0$ on ∂D, $u(x,0) = \partial_t u(x,0) = 0$ with $f \in C_0^1(D)$ has resonances of order t if ω is a square-root of an eigenvalue of A. New phenomena arise if D is unbounded. Certain unbounded domains admit resonances of order $t^{1/2}$ and $\ell n\, t$ which are not related to eigenvalues of A. These resonances are connected with branch points of the resolvent on the real axis and discontinuities of the derivative of the spectral family of A. As examples, we discuss the asymptotic behaviour of u in $\mathbb{R}^2$, in two-dimensional exterior domains, in $D_0 = R^n \times (0,1)$, and in local perturbations D of D_0. Resonances occur in these domains at the frequency ω if and only if the corresponding homogeneous boundary value problem for $\Delta U + \omega^2 U = 0$ has certain non-trivial solutions ("standing waves"), which can be characterized by suitable infinity conditions. The resonances are extremely unstable and can be simultaneously removed by small perturbations of the domain.

M.A. SHUBIN

Spectral invariants of elliptic operators on noncompact manifolds

Continuous spectrum of selfadjoint elliptic operators on noncompact manifolds can be studied by means of spectral invariants which are constructed either by some limiting procedures from bounded domains in these manifolds or by von Neumann traces. An example of such an invariant is the integrated density of states $N(\lambda)$ which can be defined e.g. for selfadjoint elliptic operators with almost periodic or random homogeneous coefficients on $\mathbb{R}^n$ and for periodic operators on the universal covering $\tilde{M}$ of a compact manifold M (periodicity means that the operator commutes with deck transformations by the fundamental group $\Gamma = \pi_1(M)$). In the latter case let the operator be the Laplacian Δ_p on p-forms on $\tilde{M}$ constructed by means of some Riemannian metric on M pulled back to $\tilde{M}$, so $N(\lambda)$ becomes

$$N_p(\lambda) = \int_F \operatorname{tr} e_p(\lambda,x,x)\,dx,$$

where $e_p(\lambda,x,y)$ is the spectral function of $(-\Delta_p)$ and F is the fundamental domain of Γ on $\tilde{M}$. It was noticed in a joint paper by S.P. Novikov and M.A. Shubin that if $0 \in \operatorname{spec}(\Delta_p)$ and $N_p(\lambda) \sim c\lambda^{\alpha}$ as $\lambda \to +0$ then $\alpha = \alpha_p$ does not depend on the metric on M and so it has to be a topological invariant of M itself. The same invariant can be constructed by considering the asymptotic behaviour of the heat kernel of p-forms (the Schwartz kernel of $\exp(t\Delta_p)$) as $t \to +\infty$.

R. Mc OWEN

Conformal deformations of Riemannian metrics on noncompact manifolds

I consider the problem of conformally deforming a noncompact Riemannian manifold (M,g) to a complete metric $\tilde{g}$ with constant scalar curvature. One quantity which is useful is the 1^{st} eigenvalue $\lambda_0(L)$ of the "conformal Laplacian" $L = -\Delta + S$ where

S = (scalar curvature)$\cdot\frac{(n-2)}{4(n-1)}$. If (M,g) has a conformal metric $\tilde{g}$ with $\tilde{S} \geq 0$ then $\lambda_0(L) \geq 0$. On the other hand:

Theorem (Aviles, McOwen). If $\lambda_0(L) < 0$ then there is a conformal metric $\tilde{g}$ with $\tilde{S} = 1$. If, moreover, g is complete and $S \leqq -\varepsilon < 0$ outside of a compact set $M_0 \subset M$, then $\tilde{g}$ is complete.

We can use $\lambda_0(L) \geq 0$ to achieve $\tilde{S} = 0$, provided more information is known about ∞, for example, if (M,g) is asymptotically Euclidean.

We can also consider (M,g) compact and Γ a submanifold of dim d and ask for a complete conformal metric $\hat{g}$ on $\hat{M} = M \setminus \Gamma$ with constant $\hat{S}$.

Theorem (Aviles, McOwen). For any compact (M,g) there is a complete g on M with $\hat{S} = -1 \iff d > (n-2)/2$.

S. REMPEL

Regularity results for degenerate operators

We start from elliptic operators in an open smoothly bounded domain which degenerate at the boundary in a special way

$$L(x,D_x) = \sum_{h=0}^{H} \varphi(x)^{k-h} p^{\mu-h}(x,D_x)$$

$H = \min(k,\mu)$, $k,\mu \in \mathbb{N}$, ord $p^{\mu-h} \leq \mu - h$, φ a C^∞- function equivalent to the distance to the boundary, p^μ uniformly elliptic in the closure of the domain. Including suitable boundary conditions the Fredholm property was proved by Bapuendi/Goulaouic, Višik/Grušin, Bolley/Camus ... in different adapted function spaces. Regularity results were shown assuming that a certain strip in the complex plane is free of zeros of a certain characteristic polynomial (depending on the leading symbols of all $p^{\mu-h}$). We give regularity results for solutions of the equation $Lu = f$ independent of the boundary conditions. The zeros of the characteristic polynomial in the sensitive strip and the singular parts of the right hand side give rise to explicitely calculable singular terms of the solution, which are the deviation from the space with better weight. An extension to boundary value problems on manifolds with wedges will be indicated.

G. MENDOZA

Interior elliptic singular problems

A problem proved on a smooth compact manifold M together with a closed submanifold X, both without boundary, is used as a means to describe part of work done jointly with R. Melrose. The problem posed leads to the following situation. Let N be a compact manifold, $\Gamma = \partial N$, $p: \Gamma \to X$ a fibration (with compact fibers). The space V of vector fields on N which are tangent to the fibers of p generates a ring of degenerate differential operators, $Diff_V(N)$. Let $\dot{H}^s$ be the space of distributions on N which extended as 0 to a neighborhood of N and belong to H^s. Let $I_V\dot{H}^s = \{u \in \dot{H}^s \mid Au \in \dot{H}^s, \forall A \in Diff_V\}$. If P is elliptic in the appropriate sense and $u \in [C^\infty(N)]'$ then $Pu \in C^\infty(N) \Rightarrow u = v + w$ with $v \in C^\infty(M)$ and $w \in I_V\dot{H}^s$ for some s. If furthermore $Pu = 0$ to infinite order on Γ then u has an asymtotic expansion at the boundary resembling a Taylor series, a series which in certain significant cases arising from regular elliptic boundary value problems ($p : \Gamma \to X$ is then the identity) is the usual Taylor series. Conditions on the number of terms that vanish together with additional assumptions on P lead to Fredholm properties of the operator.

V. GUILLEMIN

Zollstein manifolds

This talk consisted of three points 1. A review of the Segal approach to scattering for conformally invariant operators on

Minkowski space. In this approach the scattering operator becomes a Floquet operator on the universal cover of the conformal compactification of Minkowski space. 2. A brief discussion of Zollstein manifolds. These are compact Lorentzian manifolds whose null geodesics are periodic. 3. A definition of some conformal invariants for Zollstein manifolds. For those manifolds the scattering operator on the universal cover is of the form $\pm I + K$, K compact, and the spectral invariants of $\pm I + K$ are conformal invaraints of these manifolds.

J. JOST

Harmonic maps between noncompact manifolds

We first discuss the general existence problem for harmonic maps between noncompact manifolds. An important step for getting such a map is to construct a map of finite energy first. We then discuss specific examples in Kähler geometry involving the construction of suitable complete metrics. As an application, we state results about the global rigidity of locally Hermitian symmetric spaces within the class of Kähler manifolds.

C.L. EPSTEIN

Global invaraints of strongly pseudoconvex CR manifolds

We define a global, $\mathbb{R}$-valued invariant of a compact, strictly, pseudo convex, 3-dimensional C.R. manifold M. The invariant arises as the evaluation of a deRham cohomlogy class on the fundamental class of the manifold. To construct the relevant form we start with the structure bundle Y over M. The form is a secondary characteristic class of this structure. If the Euler class of the 2-plane field underlying the holomorphic tangent space of M is trivial then the form can be pulled down to M.

Surprisingly, this form is well defined up to an exact term, and thus its cohomology class is well defined in $H^3(M,\mathbb{R})$. By examples we show that he numerical invariant actually depends on the complex structure.

N. TELEMAN

Index theory on singular spaces

One gives an unifying proof for the index theorems due to Atiyah-Singer, Teleman and Cheeger.

The proof shows also why the ingredients of the topological index appear and to which specific part of the geometry they are due.

S.T. MELO

An index formula for differential operators with semi-periodic coefficients

We consider the differential operator $L = A_1(x)\frac{d}{dx} + A_2(x)$: $(H^1(\mathbb{R}))^N \longrightarrow (L^2(\mathbb{R}))^N$ with continuous, $N \times N$ - matrix valued coefficients $A_i = \chi_+ A_i^+ + \chi_- A_i^- + A_i^0$, $i = 1,2$, where $A_i^{\pm}$ are 2π-periodic, $\lim_{x\to\pm\infty} A_i^0(x) = 0$ and $\chi_+ + \chi_- \equiv 1$, $0 \leq \chi_\pm \leq 1$, $\chi_\pm(x) = 1$ for $\pm x \geq 1$. A necessary condition for L to be Fredholm is that it be uniformly elliptic. We may then assume w.l.o.g. that $A_1 \equiv 1$. Regarding $A_2^{\pm}$ as functions on S^1, we define the operators $L^{\pm} = \frac{1}{i}\frac{d}{d\theta} + A^{\pm}(\theta)$: $(H^1(S^1))^N \longrightarrow (L^2(S^1))^N$ and the set $N^{\pm} = \{\mathrm{Im}\,\zeta; \zeta$ is an eigenvalue of $L^{\pm}\}$.

Theorem (Cordes-Melo) L is Fredholm if and only if $0 \notin N^+ \cup N^-$. In this case, index $L = \#(N^+ \cap \{t; t < 0\}) - \#(N^- \cap \{t; t < 0\})$.

The proof involves the study of a C*-comparison algebra with non-compact commutators where we have defined a complex-valued σ-symbol and a singular-integral-operator-valued γ-symbol.

F. A. MEHMETI

Interaction Problems

The physical problem of vibrations of certain complicated systems leads to nonlinear wave-equations on ramified spaces (notion introduced by G. Lumer, developed further S. Nicaise). We indicate a possibility to construct a self adjoint operator from the elliptic spatial part of the equation by Friedrichs-extension.

The information of the connexion of the media is contained in the choice of a certain closed subspace of the product of function spaces on these media. Much of the formalism works, when we take any closed subspace. This leads to interaction - problems (e.g. identifications in the interior of the media, integral-conditions, mixing of the dimension etc.)

Finally we give an existence-theorem for global solutions of certain nonlinear abstract wave equations with dumping and indicate the applicability to mixed initial-value-interaction - problems. Also the theory of T. Kato and results of J. Shatah can be applied.

Concerning the question of regularity, there are relations to the research on abstract C^{∞}-notions of B. Gramsch.

S. NICAISE

Elliptic operators on ramified spaces

We introudce a class of boundary value problems on two-dimensional polygonal topological networks which are ramified spaces (G. Lumer, C.R. Acad. Sc. Paris, t. 291, A, 1980, p. 627-630) such that each face is homeomorphic to a polygon. This class contains mixed boundary value problems (Komohatier 1967, May' Plamenerskii 1978, Grisrand 1985) or interface problems (Kellogy 1971, Lemrabet 1977) in polygonal domains of the plane. Following

Gisraud (1985), we show that, in a neighborhood of a vertex, the singularities can be calculated using a Laplace operator defined on an associated topological network (one-dimensional ramified space, cf. G. Lumer).

These considerations are related to Felix Ali Mehmeti's results on nonlinear wave equations on ramified spaces.

W. BALLMANN

The Dirichlet problem at infinity for manifolds of nonpositive curvature.

A simply connected, complete Riemannian manifold M with sectional curvatures nonpositive is in a natural way diffeomorphic to the interior of the unit ball B. If M is irreducible and admits a compact quotient, then either M is a symmetric space of higher rank or the Dirichlet problem at $M(\infty) = \partial B$ is solvable on M. This result is related to recent work of Anderson and Sullivan.

A. CHALJUB-SIMON

Ground states of semi-linear elliptic equations in $\mathbb{R}^n$

We consider in $\mathbb{R}^n$ $(n > 2)$ the following equation:

(1) $\Delta u - c^2 u + g(x,u) = 0$

(c is a contant, g is a positive function, such that the growth of $u \mapsto g(x,u)$ is less than u^k, $k < \frac{n+2}{n-2}$). We want to prove the existence of possible solutions of (1), tending to zero at infinity (ground states). For this, we introduce some function spaces, with an exponential weight. First we prove, that : $L = \Delta - c^2$ is an isomorphism in the weighted spaces. Then transforming (1) into a non-linear integral equation:

(2) $u = \underline{G}[\tilde{g}(u)]$

we prove that $\phi : u \mapsto \underline{G}[\tilde{g}(u)]$ is compact, under convenient assumptions on g; then, we prove, there exists an a priori bound for all positive, C^2, bounded solutions of (1). By use of a fixed point theorem in the cone of positive functions in the weighted spaces, we get a solution with the required properties.

L. GUILLOPÉ

On spectral theory of some non compact complete Riemannian manifolds.

Let M be a non-compact complete Riemannian manifold with ends which are either cylindrical or cusps of a locally symmetric space of rational rank one. We reduce the spectral analysis of the Laplacian on M to the study of a simple model: some operator H on a Hilbert space H. The resolvant $(H - \lambda)^{-1}$ has a meromorphic continuation for λ in some cover Σ of $\mathbb{C}$ (with ramifications of order two over some discrete, generally infinite, set σ_H of $\mathbb{R}^+$). We define Eisenstein functions which are meromorphic on Σ and behave nicely under the action of the group of the covering $(\Sigma,\mathbb{C})$, an action which is described by some transfer coefficients $T_{\tilde{\mu},\tilde{\mu}'}$ $(\tilde{\mu},\tilde{\mu}' \in \sigma_H)$. In the case of a locally symmetric space, we recover the Eisenstein series and all the nondiagonal coefficients are zero. However, this is not generically the case, as the differential of these coefficients w.r.t. a conformal variation of the metric shows.

H.O. CORDES

Some Laplace comparison algebras on noncompact spaces

We study C*-algebras of singular integral operators on Ω, a noncompact Riemannian manifold. Specifically, with conical ends: $\tilde{\Omega} = [0,\infty) \times \Theta$, $ds^2 = dr^2 + r^2 d\Theta^2$, Θ compact, and cylindrical ends

$\tilde{\Omega} = [0,\infty) \times B$, $ds^2 = dr^2 + dz^2$. Also a combination of both: $\tilde{\Omega} = [0,\infty) \times \Theta \times B$, $ds^2 = dr^2 + e^2 d\Theta^2 + dz^2$. In each case $\mathbb{A} \hookrightarrow L(\varphi)$. $G = L^2(\Omega, dS)$ is generated by a class $A^{\#}$ of multiplications (by $a \in C^{\infty}(\Omega)$, $a = 0(1)$) and a class of "Riesz-Operators" $D(1-\Delta)^{-1/2}$ with $D = b^j \partial_{x_j} + p \in \mathcal{D}^{\#}$, a class of bounded first order generators.

With suitable choice of $A^{\#}, \mathcal{D}^{\#}$ one obtains an ideal chain $\mathbb{A} \supset \mathfrak{C} \supset K(\varphi)$, $\mathfrak{C} = [\mathbb{A},\mathbb{A}]$ where $\mathbb{A}/\mathfrak{C} = C(\mathbb{M})$, $\mathbb{A}/K(\varphi) = \oplus\, C(\mathbb{E}_j, \mathfrak{S}J(B_j))$ with (locally) composed spaces $\mathbb{M}, \mathbb{E}_j$ (one $\mathbb{E}_j$ for each cylindrical end) and the algebra $\mathfrak{S}J(B)$ of singular-integral operators on B.

A necessary and sufficient condition for $A \in \mathbb{A}$ to be Fredholm is that both induced symbols σ_A and γ_A are invertible.

Details about $\mathbb{M}$, $\mathbb{E}_j$, and criteria for differential generators "within reach" were discussed.

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