MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 23/1987

Kommutative Algebra und algebraische Geometrie

24.5. bis 30.5.1987

Die Tagung wurde von E.Kunz (Regensburg), H.-J. Nastold (Münster) und L.Szpiro (Paris) organisiert.
Die behandelten Themen erstreckten sich über ein breites Spektrum aus der algebraischen Geometrie und der kommutativen Algebra. Fragen der klassischen "projektiven Geometrie" (Kurven im P^3, Flächen im P^4, vollständige Durchschnitte im P^3, Hilbertschema für Kurven im P^3) waren ebenso von Interesse wie neueste Ergebnisse aus der "transzendenten Theorie": Beispiele von Flächen vom allgemeinen Typ, die homöomorph und nicht diffeomorph sind, mit Methoden von Donaldson, R.Friedman und J.Morgan konstruiert. In der kommutativen Algebra wurden u.a. neuere Ergebnisse über Cohen-Macaulay-Ringe von "endlichem maximalen Cohen-Macaulay-Typ" vorgestellt, die im Zusammenhang stehen mit "einfachen Singularitäten", über Fortschritte in Richtung der "homologischen Vermutungen" von M.Hochster und anderen berichtet, die z.B. einen überraschend einfachen Beweis des Satzes von Hochster-Roberts über die Cohen-Macaulay-Eigenschaft des Invariantenringes eines regulären Ringes erlauben, sowie über den Beweis des "Neuen Schnittsatzes" von P.Roberts im charakteristikungleichen Fall. In der K-Theorie wurde über neue Kürzungssätze bei projektiven Moduln referiert. Auch Beziehungen zur Computer-Algebra kamen zur Sprache. Vermutungen über elliptische Kurven, welche den großen Satz von Fermat zur Folge haben, wurden in dem Bericht von L.Szpiro behandelt.

Von den Teilnehmern kamen 38 aus Deutschland und anderen europäischen Ländern, 17 aus Nordamerika, und je einer aus Japan, Indien und Israel.

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

Vortragsauszüge

M.AUSLANDER

Introduction to Almost Split Sequences in the Category of Maximal Cohen-Macaulay Modules

The purpose of this talk was to give the basic definitions and existence theorems for almost split sequences as developed by I.Reiten and myself. Also applications to studying Cohen-Macaulay rings of finite Cohen-Macaulay type were given including the fact that such rings are isolated singularities, that their Grothendieck group can be described using almost split sequences and Cohen-Macaulay modules as well giving a criterion for describing when a Cohen-Macaulay isolated ring is of finite Cohen-Macaulay type.

H.BASS

The Jacobian Conjecture and Differential Operators

Let $A=\mathbb{C}[x_1,\ldots,x_n]\subset B$ be an étale extension of polynomial algebras. The Jacobian Conjecture says that A=B. The $\delta_i=\delta/\delta x_i$ act on B, so $A\subset B$ are modules over the Weyl algebra $W=\mathbb{C}[x_1,\ldots,x_n,\delta_1,\ldots,\delta_n]$. In fact B is a holonomic W-module, hence cyclic of finite length. The linear derivations $\varepsilon_{ij}=x_i\delta_j$ span $gl_n\subset W$. Let $\mathfrak{g}\subset gl_n$ be a Lie subalgebra, and $U=U(\mathfrak{g})$. If $\dim_{\mathbb{C}}(\mathfrak{g})>n$ then B must be a torsion U-module. One can thus try to prove the JC by choosing $\mathfrak{g}$ of dim $>n$ and showing that B/A is a torsion free U-module.

We attempt this for n=2, A=C[x,y], with $\mathfrak{g}$ spanned by $\varepsilon_x = x\delta/\delta x$, $\varepsilon_y = y\delta/\delta y$, and $\delta = x\delta/\delta y$. It can be shown that B/A is torsion free over $C[\varepsilon,\delta]$, $\varepsilon=\varepsilon_x+\varepsilon_y$, and $C[\varepsilon_x,\varepsilon_y]$. Proof of the later case invokes Siegel's theorem on algebraic curves with infinitely many Z-points, and Fabry's Theorem (1896) stating that a lacunary series $f=\sum_n a_n z^{r_n}$ with $r_n/n \to \infty$ is singular on the entire circle of convergence. Results for the full algebra $U(\mathfrak{g})=C[\varepsilon_x,\varepsilon_y,\delta]$ are still partial.

M.BRODMANN

<u>Asymptotic depth and connectedness of fibers</u>

Let $(R,\mathfrak{m})$ be a local noetherian ring, and let $I\subseteq\mathfrak{m}$ be an ideal. Let t denote the asymptotic depth of the higher conormal modules I^n/I^{n+1} : $t=\text{depth}(I^n/I^{n+1})$, h>>0.

Moreover consider the blowing up-morphism $\pi:\text{Proj}(\bigoplus_{n\geq 0} I^n)\to$ Spec(R) and the exceptional fiber $E:=\text{Proj}(\bigoplus_{n\geq 0} I^n/I^{n+1})$ of π as well as the special fiber $S:=\text{Proj}(R/\mathfrak{m}\otimes(\bigoplus_{n\geq 0} I^n))$ of π.

One knows the inequality dim S<dim R-t. We improve this estimate by the following result:

Assume that t>1 and assume that at least one of the following conditions holds:

(i) R is excellent and normal, and I is $\neq 0$

(ii) $\text{depth}_I(R)>1$

(iii) $\text{Spec}(\hat{R})-V(I\hat{R})$ is $\neq\emptyset$ and connected.

Then E-S is connected and satisfies the inequality

(*) $\underline{c}(\hat{O}_E|E-S)\geq t-2$.

Thereby $\hat{\cdot}$ stands for the m-adic completion, and -for an arbitrary coherent sheaf $\mathcal{F}$ over a noetherian scheme X- $\underline{c}(\mathcal{F})$ is defined by $\underline{c}(\mathcal{F})=\min(\dim_x(\overline{T}_1\cap\overline{T}_2|x\in\overline{T}_1\cap\overline{T}_2$ closed; $T_1,T_2\subseteq Ass(\mathcal{F})$, $T_i\neq\emptyset$, $T_1\cup T_2=Ass(\mathcal{F})\}$. So (*) implies in particular that E-S is connected in dimension t-2.

S.P.DUTTA

On the canonical element conjecture

We mainly study the three equivalent conjectures: Direct summand, canonical element and improved new intersection conjecture. (We prove the equivalence of the last two in the course of this talk.) These conjectures are open in the mixed characteristics case. First we study the effect of the Frobenius map on free complexes with finite length homologies in characteristic p>0. We prove the following theorem:

Theorem. Let A be a complete local equidimensional ring without any embedded components in char.p>0. Let $F_\cdot$ be a free complex with finite length homologies and let N be a finitely generated module. Let $\omega_{j,n}$ be the j'th homology of $Hom(F_n, {}^{f^n}N)$ where $f^n: A\to A$ is given by $f^n(x) = x^{p^n}$. Then

i) if dim N < dim A, $\lim \ell(\omega_{j,n})/p^{nd}=0$;

ii) if dim N = dim A, a) j<dim A, $\lim \ell(\omega_{j,n})/p^{nd}=0$;

b) j≥dim A, $\underline{\lim}\ \ell(\omega_{j,n})/p^{nd}=\underline{\lim}\ \ell(H_{j-d}(f^n(F_\cdot))\otimes N^*)/p^{nd}$, where

$d = \dim A$, $N^* = \mathrm{Hom}(H^d_{\mathfrak{m}}(N),E)$, E = injective hull of the residue class field.

We deduce the improved new intersection conjecture in char.p>0 and a special case of positivity of Serre's conjecture on intersection multiplicity from the above theorem. We then discuss some cases of the canonical element conjecture in the mixed characteristics and reduce it to a question of understanding the limits of lengths of some special cyclic modules of finite length under the Frobenius map.

H.FLENNER

Almost factorial singularities

Let X be a projective manifold over the complex numbers and NS(X) its Neron-Severi group. It is well known from classical Hodge theory that the map induced by logarithmic derivation $\mathrm{dlog}:NS(X)_{\mathbb{C}} \to H^1(\Omega^1_X)$ is injective. In this lecture we gave a generalization to singularities. Let $A = \mathbb{C}[\![X]\!]_n/\mathfrak{a}$ be of pure dimension ≥ 3 satisfying (S_2). By a result of Boutot, the Picard group of the punctured spectrum U of A has an algebraic structure and so $NS(U) = \mathrm{Pic}\ U/\mathrm{Pic}^o U$ makes sense. Again there is a map induced by logarithmic derivation $\mathrm{dlog}:NS(U)_{\mathbb{C}} \to H^1(\Omega^1_{U/\mathbb{C}})/dH^1(\mathcal{O}_U)$, and the main result is, that for algebraic singularities this map is injective. The proof heavily depends on the vanishing theorems of Grauert-Riemenschneider and Steenbrink. As an application one gets under suitable depth assumptions for A and Ω_A^1 criteria for A to be almost factorial.

M.FLEXOR

<u>Algorithms for finding roots of a polynomial</u>

Newton givês an algorithm for finding the roots of a polynomial $P(X))=a_d X^d+\ldots+a_o, a\in\mathbf{C}$. Namely, the algorithm is: $N(z)=z-\frac{p(z)}{p'(z)}$, it is really working when all the roots are real and for z in a neighbourhood of a complex simple root.

Euler generalizes Newton algorithm: $N_h(z)=z-h\frac{p(z)}{p'(z)}$ $0<h\leq 1$. For $h=1$, $N_h=N$ this algorithm works for z in a neighbourhood of any complex or real root. But in general, this algorithm is not generically convergent. We prove (joint work with A.Douady and P.Sentenac) that there exist a lot of $u=he^{id}$, with $|1-u|<1$ such that, if we define $N_u(z)=z-u\frac{p(z)}{p'(z)}$, $z\in\mathbf{C}$, N_u is an algorithm generically convergent (here $d\in]-\pi/2,\pi/2[$).

H.-B.FOXBY

<u>Fibres of morphisms of local rings</u>, work with Avramov (Sofia)

Let A and B denote local rings, and let $\varphi:A\to B$ be a morphism ($\varphi(\mathfrak{m}_A)\subseteq\mathfrak{m}_B$). Assume that the flat dimension $fd_A B$ is finite. The fibre of φ is $F(\varphi) = k_A\otimes_A G$, where $k_A = A/\mathfrak{m}_A$ and G is a bounded DG-A-algebra resolution of B. (DG = differential graded.)

Let D be a DG-ring with H(D) bounded and Noetherian, and let M be a DG-D-module with H(M) bounded and Noetherian. The Bass series is $I_D^M(t) = \sum_{i\geq 0}[Ext^i(k_D,M):k_D]t^i \in Z[t]$, when D maps onto the field k_D.

Thm.1. $I_B^{M\otimes_A B}(t) = I_A^M(t) I_{F(\varphi)}^{F(\varphi)}(t)$, when M is a f.g. A-module

Thm.2. A and $F(\varphi)$ are Gorenstein if and only if B is Gorenstein.

Here $F(\varphi)$ is said to be Gorenstein, if $I_{F(\varphi)}^{F(\varphi)}(t)$ is a monomial. Let D_1, D_2, D_3 be DG-rings like D above and $D_1 \to D_2$ and $D_2 \to D_3$. Assume $fd_{D_1}D_2 < \infty$ and $fd_{D_2}D_3 < \infty$. Fibres of morphism of augmented DG-rings are defined as for local rings.

Thm.3. $F(D_1 \to D_2)$ and $F(D_2 \to D_3)$ are Gorenstein if and only if $F(D_1 \to D_3)$ is Gorenstein.

Thm.4. If $F(A \to B)$ is Gorenstein and A has Gorenstein formal fibres, then $F(\hat{A} \to \hat{B})$ is Gorenstein and B has Gorenstein formal fibres.

W.FULTON

On the Space of Plane Triangles (with Alberto Collino).

Schubert's space X of (ordered) plane triangles is a closed subvariety of $P^2 \times P^2 \times P^2 \times \check{P}^2 \times \check{P}^2 \times \check{P}^2 \times G_2(\mathbb{P}^5)$. Using the natural forms action on X and known relations among divisors, one can compute the intersection ring $A^{\cdot}(X) = H^{\cdot}(X) = Z[a,b,c,\alpha,\beta,\gamma,d]/I$, where I is generated by relations (1) $a^3,\ldots,\gamma^3$; (2) $a\beta - a^2 - \beta^2,\ldots$, (3) $(b-c)(b+c+\beta+\gamma-d)$, $(\beta-\gamma)(b+c+\beta+\gamma-d),\ldots$, (4) $(d-a-b-c)(d-\alpha-\beta-\gamma)$. (The last relation does not follow from Schubert's equations). As an application one can calculate the number of triangles inscribed in a given curve, circumscribed about another. One must allow ordinary nodes and cusps in the first curve, so the dual argument applies to the second; this corrects the formula given by Schubert.

A.V.GERAMITA

<u>The Ideal of an Arithmetically Buchsbaum Curve in</u> $\underline{P^3}$ (joint work with Juan Migliare)

Let $\mathcal{C}$ be a curve in P^3, (closed, pure 1-dimensional, locally C.-M.) and $I_{\mathcal{C}} \subseteq S=k[x_0,x_1,x_2,x_3]$ its defining ideal, $\mathcal{J}_{\mathcal{C}}$ its ideal sheaf. The <u>Hartshorne-Res</u> module $\mathcal{C}$ is the graded S-module, $M(\mathcal{C})=\oplus M(\mathcal{C})_n = \oplus H'(\mathcal{J}_{\mathcal{C}}(n))$. $M(\mathcal{C})$ is an S-module of finite length.

<u>Def</u>: $\mathcal{C}$ is arithmetically Buchsbaum (a.B.) if S_1 acts trivially on $M(\mathcal{C})$.

If $m(\mathcal{C})_n = \dim M(\mathcal{C})_n$, then $N = \Sigma m(\mathcal{C})_n$ is called the Buchsbaum invariant of $\mathcal{C}$.

Let $\alpha(\mathcal{C})$ = least integer t such that $(I_{\mathcal{C}})_t \neq 0$;

$\beta(\mathcal{C})$ = least integer t such that $(I_{\mathcal{C}})_t$ contains a regular sequence of length 2, then

<u>Prop.1</u>: If $\mathcal{C}$ is a.B., $\alpha=\alpha(\mathcal{C})$ and $\mathcal{H}$ is a hyperplane not containing a component of $\mathcal{C}$. Then

i) $\alpha-1 \leq \alpha(\mathcal{C}\cap\mathcal{H}) \leq \alpha$

ii) If $\alpha(\mathcal{C}\cap\mathcal{H}) = \alpha-1$ then $h^0(\mathcal{J}_{\mathcal{C}\cap\mathcal{H}}(\alpha-1)) = m(\mathcal{C})_{\alpha-2}$.

iii) $M(\mathcal{C})_i=0$ for all $i\leq\alpha-3$.

Prop.2: $\mathcal{C}$ a.B., $\mathcal{H}$ a hyperplane not containing a component of $\mathcal{C}$, t = least integer such that $h^1(\mathcal{J}_{\mathcal{C}\cap\mathcal{H}}(t)) = 0$. Then $\mathcal{J}_{\mathcal{C}}$ is generated in degrees $\leq t+1$.

Prop.3: Let $\mathcal{C}$ be reduced and irreducible a.B. curve, $\alpha=\alpha(\mathcal{C})$, $\beta=\beta(\mathcal{C})$, N = Buchsbaum invariant of $\mathcal{C}$. then $I_{\mathcal{C}}$ can be generated in degrees $\leq\alpha+\beta-d2N$. Moreover, if $\alpha(\mathcal{C}\cap\mathcal{H}) = \alpha-1$ then $\alpha=\beta$ and so $I_{\mathcal{C}}$ can be generated in degrees $\leq 2(\alpha-N)$.

R.HARTSHORNE

Set-theoretic Complete Intersections on Cones

This is a report of the work of David Jaffe in his PhD thesis (Berkeley 1987).

It has been known for some time that certain rational curves in P^3, such as the nonsingular quartic curve given by $x=u^4$, $y=tu^3$, $z=t^3u$, $w=t^4$ are set-theoretic complete intersections in characteristic p>0. For example, if p=7 one may use the equations

$$y^4-x^3w=0 \text{ and } z^7-xyw^5=0.$$

On the other hand, it is not known whether this curve is a complete intersection in characteristic 0, and some authors had verified already that at least on that cone $y^4-x^3w=0$, it is not a set-theoretic complete intersection in characteristic 0.

Hence the objective of Jaffe's thesis is to study curves on cones, and to decide when they are set-theoretically the intersection of that cone with some other surface.

The general situation is this. Let $D \subseteq \mathbf{P}^2$ be an irreducible plane curve. Let $S \subseteq \mathbf{P}^3$ be the cone over D. Let $C \subseteq S$ be an irreducible nonsingular curve lying on S. Let $v \in S$ be the vertex.

When D is nonsingular the situation is easily understood:

Proposition. Suppose D is nonsingular. If $v \notin C$, then C is a (strict) complete intersection on S. If $v \in C$, the tangent line to C at v determines a point $P \in D$. Then C is a set-theoretic complete intersection on S if and only if the class of P in $\mathrm{Pic}D/\mathbf{Z}\cdot\mathcal{O}_D(1)$ is torsion.

To state the main result, we need some definitions. Let $\underline{\mathrm{Pic}}D$ be the Picard scheme of D, and let $\underline{\mathrm{Pic}}^{\mathrm{O}}D$ be the connected component. There is an exact sequence of group schemes

$$0 \to (\mathrm{Pic}^{\mathrm{O}}D)_{\mathrm{mult}} \times (\mathrm{Pic}^{\mathrm{O}}D)_{\mathrm{unip}} \to \mathrm{Pic}^{\mathrm{O}}D \to (\mathrm{Pic}^{\mathrm{O}}D)_{\mathrm{ab}} \to 0$$

where ab denotes the abelian variety which is the Jacobian of the normalization of D; mult denotes the multiplicative part, which is a product of $\mathbf{G}_m$'s, and unip denotes the unipotent part, which is a successive extension of $\mathbf{G}_a$'s. We say that D is of cuspidal type if $(\mathrm{Pic}^{\mathrm{O}}D)_{\mathrm{mult}}=0$ and $(\mathrm{Pic}^{\mathrm{O}}D)_{\mathrm{unip}}\neq 0$. We say D is of nodal type if $(\mathrm{Pic}^{\mathrm{O}}D)_{\mathrm{mult}}\neq 0$ and $(\mathrm{Pic}^{\mathrm{O}}D)_{\mathrm{unip}}=0$.

Theorem (Jaffe). Assume that D is singular.

a) If C is a set-theoretic complete intersection on S, then

1) $\mathrm{char}.k=p>0$, and

2) D is of cuspidal type.

b) Conversely, suppose that a1) and a2) are satisfied, and assume furthermore either (i)$v \notin C$, or (ii) D is rational. Then C is a set-theoretic complete intersection on S.

Cor1 [Hartshorne]. For each $d \geq 4$ and for each char.$k=p>0$, the rational curve $x=u^d$, $y=u^{d-1}t$, $z=ut^{d-1}$, $w=t^d$ is a set-theoretic complete intersection in $\mathbf{P}_k^3$.

Cor2 [Ferrand]. If C is a nonsingular curve in $\mathbf{P}^3$ over a field of char.$p>0$,and if there is a $0 \notin C$ such that the projection from 0 sends C birationally onto a plane curve $\bar{C} \subseteq \mathbf{P}^2$ having only cusps for singuarities, then C is a set-theoretic complete intersection.

Ex. The rational quartic curve mentioned above is not a complete intersection of a cone with any other surface in characteristic 0.

The proof depends on a careful study of the groups $\mathrm{Picloc}(s) = \mathrm{Pic}(\mathrm{Spec}\, \mathcal{O}_{S,s} - \{s\})$ for the singular points $s \in S$.

J.HERZOG

Graded maximal Cohen-Macaulay modules (a survey)

This lecture is a report on joint papers with Eisenbud, Buchweitz, Backelin and Sanders. Jointly with Eisenbud we showed that the already known list of homogeneous Cohen-Macaulay rings of finite representation is complete, as long as the Cohen-Macaulay ring is defined over an algebraically closed field of char.0.

Next we discussed the representation theory of quadric hypersurface rings over an arbitrary field, which is completely understood and described in a joint paper with Buchweitz and Eisenbud.

Finally we reported on a paper with Backelin and Sanders in which we extend the method and results obtained for quadratic forms to forms of higher degree.

A.HIRSCHOWITZ

Principal generic space curves

Generic (smooth connected) space curves are curves parametrized by generic points of irreducible components of the Hilbert scheme of P^3. Principal ones are these with generic moduli, which exist and are uniqe for given degree d and genus g.

We prove that the family of plane sections of these curves enjoyes any general position property one would expect, for instance no flex (as known of Eisenbud-Harris), no quintisecant line, no quadritangent plane. All these properties flow from similar original properties of the Hilbert scheme of points in the plane thanks to the condition $H^1(N(-1))=0$, where N is the normal bundle of the curve. This condition is proven using reducible curves (unions of a lower degree generic curve and a cubic curve meeting in five points) and the smoothing result of Hartshorne-H., in the same way as other results on the normal bundle were obtained previously by Ellingsrud-H.

M.HOCHSTER and C.HUNEKE

Tight closures of ideals I,II.

We introduce the notion of tight closure for ideals and submodules in certain cases, and then use this notion to give new proofs that rings which are direct summands of regular rings are Cohen-Macaulay and of the Briancon-Skoda Theorem, as well as to obtain new constraints on the behaviour of systems of parameters. In characteristic p>0 we define the tight closure I^* of $I \subseteq R$ as follows: $x \in I^*$ if there exists $c \in R \setminus \cup\{\text{minimal primes of } R\}$ such that for all $e \geq 0$, $cx^{p^e} \in I^{[p^e]}$, where $I^{[p^e]} = \{i^{p^e} : i \in I\}$. A key point is that under mild conditions, $(x_1,\ldots,x_n):x_{n+1} \subseteq (x_1,\ldots,x_n)^*$, when the x_i are parameters. In a regular ring, every ideal is tightly closed, and rings with this property are called F-regular. If $I \subseteq R$, an algebra finitely generated over a field K of characteristic 0, we say that $x \in I^*$, if there exists a finitely generated Z-algebra $D \subseteq R$, a finitely generated D-subalgebra R_D of R, an ideal $I_D \subseteq R_D$ and an element $c \in R_D$, not in any minimal prime of R, such that $R = K \otimes_D R_D$, $I = K \otimes_D I_D$ and for all $u \in U$ in a certain open subset U of Max-Spec D, if $\mathfrak{m}$ is the corresponding maximal ideal, $\mathfrak{k} = D/\mathfrak{m}$ and $p = \text{char}.\mathfrak{k}$ then for all $e \geq 0$, $1 \otimes cx^{p^e} \in I_{\mathfrak{k}}^{[p^e]} \subseteq R_{\mathfrak{k}}$ where $R_{\mathfrak{k}} = \mathfrak{k} \otimes_D R_D$ and $I_{\mathfrak{k}} = IR_{\mathfrak{k}}$.

In general $I \subseteq I^* \subseteq \bar{I}$, the integral closure; I^* is usually much smaller than $\bar{I}$. F-regularity implies rational singularities if the ring has isolated singularities or is graded and has rational singularities except possibly at the irrelevant ideal.

G.HORROCKS

Algebraic equivalence of vector bundles

A is a regular local ring with coefficient field k. Bundles over the punctured spectrum of A are said to be algebraically equivalent if they can be joined by a sequence of local algebraic deformations. The main result is that algebraically equivalent bundles determine isomorphic bundles over $\bar{k}\otimes_A S$, $S=A[y_1,...,y_n]/(\Sigma x_o y_i -1)$, $y_1,...,y_n$ indeterminates, $x_1,...,x_n$ a base for the maximal ideal. Thus a class of algebraically equivalent bundles corresponds to a set of descent data on a projective module up to equivalence of data. This gives a standard model with which to approach the construction of moduli spaces.

F.ISCHEBECK

Homology and rational equivalence on real varieties

Let X be smooth, projective over **R** and

$$cl_k : Zyc_k X \rightarrow H_k(X(\mathbf{R}),Z/2)$$

the canonical homomorphism.

Theorem: Ker $cl_k = Zyc_k^{th}(X)+P_k(X)$, where $Zyc_k^{th}(X)$ is the group of "thin" cycles, which is generated by k-dimensional subvarieties $V \subset X$ with $\dim_{top} V(\mathbb{R}) < \dim V$, and P_k is the group of cycles, which are rational equivalent to 0.

H.LINDEL

Unimodular elements and cancellation

Let $A=R[\underline{X},\underline{Y}^{\pm 1}]$ be a Laurent extension over a noetherian ring R, dim R = d < ∞, and let P be a projective A-module of rank(P) ⩾ d+1. Generalizing a well known conclusion from a theorem of Eisenbud and Evans in case A=R, we show that for every ideal $\mathfrak{a} \subset R$ the canonical map Um P → Um P/$\mathfrak{a}$P is surjectiv.

Moreover, we give a new proof of the following result of R.A.Rao:

Theorem: Under the assumptions and notations above the elementary group of A⊕P acts transitively on Um(A⊕P), if rank P+d⩾2.

This implies that projective A-modules P with rank P⩾d+1 are cancellative, i.e. $P \oplus A \simeq P' \oplus A$ implies $P \simeq P'$.

H.MATSUMURA

Some Problems on dimension of fibres of ring homomorphisms

Let f:A → B be a morphism of noetherian rings and assume that the going-down theorem holds for f (e.g. f is flat). For a prime ideal $\mathfrak{p}$ of A, define $\alpha(f,\mathfrak{p}) := \dim B \otimes_A k(\mathfrak{p})$, the dimension of the fibre at $\mathfrak{p}$ ($k(\mathfrak{p}) := A_{\mathfrak{p}}/\mathfrak{p}A_{\mathfrak{p}}$).

Theorem. If $\mathfrak{p}\subset\mathfrak{q}$ then $\alpha(f,\mathfrak{p})\geq\alpha(f,\mathfrak{q})$.

Problem 1. Suppose that A,B,f are local. If $\mathfrak{p}\subset\mathfrak{q}\neq\mathfrak{m}_A$ and $\alpha(f,\mathfrak{p}) = \dim(B/\mathfrak{p}B)-1$, does it follow that $\alpha(f,\mathfrak{q}) = \dim(B/\mathfrak{q}B)-1$?

When A is local and f is the natural map $A \to \hat{A}$, set $\alpha(A) := \max\{\alpha(f,\mathfrak{p}):\mathfrak{p}\in\operatorname{Spec} A\}$. By the above theorem we have $\alpha(A/\mathfrak{p})= \alpha(f,\mathfrak{p})$. If A is essentially of finite type over a field then $\alpha(A) = \dim A-1$ (if $\dim A>0$). If A dominates a non-trivial complete local ring or if A is I-adically complete for some ideal I with ht $I>0$, then $\alpha(A)<\dim A-1$, and usually we get $\alpha(A) = \dim A-2$. Put $N(A):=\{\mathfrak{p}\in\operatorname{Spec} A:\alpha(A/\mathfrak{p}) = \dim A/\mathfrak{p}-1$ and $\dim A/\mathfrak{p}\geq 2\}$.

Problem 1′. If $\mathfrak{p},\mathfrak{q}\in\operatorname{Spec} A$ with $\dim A/\mathfrak{p}\geq 2$, $\dim A/\mathfrak{q}\geq 2$, and $\mathfrak{p}\subset\mathfrak{q}$, $\mathfrak{q}\subset N(A)$ does it follow that $\mathfrak{p}\in N(A)$?

Problem 2. Are there local rings A with $0<\alpha(A)<\dim A-2$?

Remark Huneke says that Heinzer and he constructed examples of A with $\alpha(A)=1$, dim A arbitrary. Someone says that Abhyankar constructed A with $\alpha(A)=m$, where m is any integer between 0 and dim A-1.

N.MOHAN KUMAR

Set-theoretic Complete Intersections

We prove the following "addition" theorem about set-theoretic complete intersections of curves.

Theorem (Local version)

Let R be a (Cohen-Macaulay) local ring. $I,J\subset R$ two set-theoretic complete intersection "curves" such that $I+J$ is primary to the maximal ideal. Then $I\cap J$ is also a set-theoretic complete intersection.

Theorem (Projective version)

Let $C \to P^3$ be a set-theoretic complete intersection curve; $L\subset P^3$ any line such that $C\cup L$ is connected. Then $C\cup L$ is a set-theoretic complete intersection. In particular any connected union of lines is a set-theoretic complete intersection.

B.MOISHEZON

Some new results about surfaces of general type

Using a new invariant of Donaldson recently R.Friedman and J.Morgan proved that under some conditions the canonical class of an algebraic surface is a diffeomorphism invariant.

This allows to construct first examples of surfaces of general type which are homeomorphic and not diffeomorphic.

Let $X = CP^1\times CP^1$, $(x_1,\ldots,x_m)$ be a sequence of positive numbers. Define inductively finite morphisms $g_k(x_1,\ldots,x_k) : X(x_1,\ldots,x_k) \to X$ as follows. Assume $g_{k-1}(x_1,\ldots,x_{k-1})$ is constructed. Let $f_k:X(x_1,\ldots,x_k) \to X(x_1,\ldots,x_{k-1})$ be a simple cyclic covering ramified at a non-singular curve $B_k \in |3x_k(g_{k-1}(x_1,\ldots,x_{k-1}))^*E|$, where $E=\ell_1+\ell_2\subset CP^1\times CP^1$, $\ell_1=CP^1\times pt$, $\ell_2=pt\times CP^1$. Then define

$$g_k(x_1,\ldots,x_k) = g_{k-1}(x_1,\ldots,x_{k-1})\cdot f_k.$$

Examples of homeomorphic not diffeomorphic simply-connected minimal surfaces of general type are the following pairs of surfaces.

$\{X(1,1,1,1,6,z_1,\ldots,z_m),\ X(2,10,16,3z_1,\ldots,3z_m)\}$ where $\{z_1,\ldots,z_m\}$ is an empty set or any sequence of positive numbers with $\sum_{i=1}^{m} z_i \equiv 0 (\mathrm{mod}\ 2)$.

M.P.MURTHY

Projective modules over finitely generated rings

We report on a work done jointly with Mohan Kumar and Amit Roy on cancellation of projective modules over finitely generated commutative rings over **Z**.

Theorem 1. Let A be a finitely generated ring of dimension $d \geq 2$ over **Z**. Let P, P' be projective A-modules of rank $\geq d$. Then $P \oplus A \simeq P' \oplus A$ imply $P \simeq P'$.

We say that a ring A has projective stable rank $\leq r$ ($psr(A) \leq r$) if for any projective A-module P of rank $\geq r$ and $(x,a) \in P \oplus A$ unimodular, there is a $y \in P$ such that $x+ay$ is unimodular.

Theorem 2. Let A be a finitely generated ring of dimension $d \geq 2$ over **Z**. Suppose all projectives of rank d have unimodular elements. Then:

1) If $d \geq 3$, $psr(A) \leq d$.

2) If $d=2$ and there is an $n>1$, $n \in$ **Z** such that $1/n \in A$, $psr \leq 2$.

The above theorem generalize work of Vaserstein.

Theorem 3. Let A be a finitely generated ring of dimension $d\geq 2$ over F_p. If $d\geq 3$, $psr(A)\leq d$. If $d=2$ and A is regular, then $psr(A)=2$.

C.PESKINE

Smooth surfaces in P_4.

Which smooth surfaces can be embedded in $P_4(C)$? and how?

Implicit problem in Severi's theorem (except for Veronese surface smooth surfaces in P_4 are linearly complete).

Hartshorne conjectured that there is only a finite number of families of smooth rational surfaces (in P_4).

With G.Ellingsrud we prove this fact, and the same about K_3, Abelian and birationally ruled surfaces. More precisely:

Let $a<6$. There is only a finite number of families of smooth surfaces in P_4 verifying $K^2\leq a\chi(O_S)$.

By an easy numerical argument, the proof of the theorem is reduced to the proof of the following technical lemma:

Let σ be a positive integer. There exists a polynomial P_σ of degree σ with positive leading coefficient such that for every smooth surface S of degree d lying in a degree σ hypersurface of P_4 one has $P_\sigma(\sqrt{d})\leq\chi(O_S)$.

Conjecture: If $S\subset P_n$, S smooth then $\chi(O_S)\geq 0$?

J.RATHMANN.

Double structures on Bordiga surfaces

A Bordiga surface S is a rational surface of degree 6 in P^4. As an abstract surface, S is isomorphic to a blowing up $\tilde{P}^2(x_1,\dots,x_{10})$ and embedded in P^4 by the complete linear system $|4L - \sum_{i=1}^{10} E_i|$.

Some of them admit double structures $\tilde{S}$ with $\omega_{\tilde{S}} \simeq O_{\tilde{S}}(-2)$ which implies that there exists (by a result of Serre) a rank 2 vector bundle E on P^4 and a section $s \in H^0E$ with $\tilde{S}=(s=0)$. For Bordiga surfaces, E splits as $E \simeq O(3) \oplus O(4)$, i.e., $\tilde{S}$ is a complete intersection. These special Bordiga surfaces can be characterized in two different ways:

1. They contain a certain nondegenerate curve C and lie on its second variety where C is one of the following:

a) a rational normal curve of degree 4 in P^4,

b) a union of two conics which intersect in one point.

2. The 10 points $x_1,\dots,x_1$ are in special position, namely

in a) fix a smooth quadric surface $F \subset P^3$ and a 2:1-projection $F \to P^2$. Then $x_1,\dots,x_{10}$ are the 10 double points of a rational sextic (which is projection of a curve of bidegree (1,5) on F).

in b) $x_1,\dots,x_{10}$ are the two ordinary double points x_1,x_2 and 8 of the 9 intersection points of two rational cubics. Furthermore, the nine'th intersection point lies on the line through x_1 and x_2.

M.RAYNAUD

Automorphisms of order p of semi-stable curves

Let R be a complete discrete valuation ring of mixed characteristics, K its quotient field, k its residual field of char. $p>0$. Suppose the maximal ideal is generated by p.

Let X be a smooth, proper R-curve and u an automorphism of X of order p.

Then, for $p\geq 5$, u acts freely on X.

Question: Suppose X is smooth and proper over R of dimension d. Let u be an automorphism of X of order p - does u acts freely on X when $d<p-2$?

L.ROBBIANO

Interaction between Computer and Commutative Algebra: Some New Aspects

I report on a joint paper of myself and Teo Mose, which deals with the description of the maximal monomial ideals associated to an ideal I in the polynomial ring $A=k[x_1,...,x_n]$ over a field k.

First we associate to every ordering of the monoid T of terms of A its "half line of first vectors".

This enables to associate to every set of orderings a suitable cone, and a first result is that for every ideal I in A there is a partition of $(\mathbf{R}^n)^+$ into a fan of polyhedral cones; over each of them the reduced Gröbner basis and the maximal monomial ideals are constant. The fan can be got constructively.

Another aspect of our work is to show that these cones may extend outside $(R^4)^+$, giving rise to the so called <u>Gröbner region</u> of I.

The main feature of it is that every ordering "inside" it behaves, with respect to I, like a term-ordering.

P.ROBERTS

<u>The New Intersection Theorem</u>

Let A be a local ring. The Intersection theorem states that if $F_\bullet = 0 \to f_n \to \ldots \to F_0 \to 0$ is a complex of free A-modules such that the homology is of finite length, and if $F_\bullet$ is not exact, then $n \geq$ the dimension of A. This theorem was proven by Peskine and Szpiro in positive characteristic and for rings of finite type over a field and extended to all rings containing a field by Hochster. We present here a proof in mixed characteristics. The proof uses the theory of local Chern characters of Fulton-MacPherson- the idea is to reduce modulo p to a complex $\overline{F}_\bullet$ of lenght exactly equal to the dimension of A/pA and show that if $F_\bullet$ were a counterexample to the theorem, we would have, first, that the $d^{\underline{th}}$ chern character, where d = dim A, is zero since $\overline{F}_\bullet$ is the reduction of a complex over A. Secondly, we show that the fact that length $(\overline{F}_\bullet)$ = dim A/pA implies that this number is positive, that a complex violating the theorem could not have existed.

L.SZPIRO

Elliptic curves and diophantine equations

We conjectured in 1982 the following: Let k be a number field and $\varepsilon>0$. There exists a constant $C(k,\varepsilon)$ such that for any semistable elliptic curve on k its minimal discriminant satisfies $|\Delta|\leq C(k,\varepsilon)(\prod_{v\in S} N(v))^{3+\varepsilon}$ where S is the set of places where the curve has bad reduction.

We gave in the talk evidence of the conjecture over function fields (any characteristic) and we also explained how this is linked to the Fermat conjecture via the work of G.Frey. Thanks to him one can read the conjecture: Let a+b=c natural numbers with no common factor then for all $\varepsilon>0$ exists $C(\varepsilon)$ such that

$$|abc|\leq C(\varepsilon)N^{3+\varepsilon} \text{ where } N = \prod_{p|abc} p$$

We there explained the latest conjectures of Nasser-Osterlé, Vojta and Parshin which implied this conjecture.

B.ULRICH

Residual intersections (with C.Huneke)

Let R be a local Gorenstein ring, let I be an R-ideal of grade g, and let $s\geq g$. An R-ideal J is called an s-residual intersection of I if grade $J\geq s$, and there exists an ideal $K\subset I$, $\mu(K)\leq s$, such that $J=K:I$. We prove:

Theorem Suppose I is in the even linkage class of an ideal which is strongly Cohen-Macaulay and (G_∞), and let $J=K:I$ be an s-residual intersection of I. Then

a) J is a Cohen-Macaulay ideal of grade s

b) depth R/J = dim R-s

c) $\omega_{R/J} = S_{s-g+1}(I/K)$. In particular, J is Gorenstein if and only if I/K is cyclic.

V.VASCONCELOS

<u>Symmetric algebras and factoriality</u>

Let R be a regular local ring -or a polynomial ring- and let E be a f.g. R-module. There are two puzzling questions regarding the symmetric algebra S(E) of E.

i) Question 1: If S(E) is factorial, must it be a complete intersection? This is equivalent to saying $pd_R E \leq 1$. It is known that $pd_R E \neq 2$.

ii) Denote by $B = \oplus S_t(E)^{**}$ = graded bidual of S(E). B is a factorial domain.

Question 2: Is B Cohen-Macaulay?

A major aspect here is whether B is Noetherian, at least when R is a polynomial ring.

We report on a computer-assisted approach to Question 2. Several classes of modules have B Cohen-Macaulay through the examination of the defining equations of the subalgebras B(r), generated by the forms of B of degree $\leq r$.

Berichterstatter: R.Waldi

Tagungsteilnehmer

Prof.Dr. M. Auslander
Dept. of Mathematics
Virginia Polytechnic Institute and
State University

Blacksburg , VA 24061
USA

Prof.Dr. M. Brodmann
Mathematisches Institut
der Universität Zürich
Rämistr. 74

CH-8001 Zürich

Prof.Dr. H. Bass
Dept. of Mathematics
Columbia University

New York , NY 10027
USA

Prof.Dr. W. Bruns
FB Naturwissenschaften/Mathematik
Fach Mathematik
Abt.Vechta der Univ. Osnabrück
PF 1349, Driverstr. 22

2848 Vechta

Prof.Dr. R. Berger
Fachbereich 9 - Mathematik
Fachrichtung Mathematik
Universität des Saarlandes
Bau 27

6600 Saarbrücken

Prof. Dr. R.-O. Buchweitz
Institut für Mathematik
der Universität Hannover
Welfengarten 1

3000 Hannover 1

Prof.Dr. J. Bingener
Fachbereich Mathematik
der Universität Regensburg
Universitätsstr. 31
Postfach 397

8400 Regensburg

Prof.Dr. E. D. Davis
Dept. of Mathematics
State University of New York
at Albany

Albany , NY 12222
USA

Prof.Dr. J. F. Boutot
Institut de Mathematiques
Universite Lous Pasteur
7, rue Rene Descartes

F-67084 Strasbourg Cedex

Prof.Dr. S. P. Dutta
Mathematical Science Reserach
Institute
2223 Fulton Street
1000 Centennial Drive

Berkeley , CA 94720
USA

Prof.Dr. E. G. Evans
Dept. of Mathematics, University of
Illinois at Urbana Champaign
273 Altgeld Hall
1409 West Green Street

Urbana , IL 61801
USA

Prof.Dr. H. B. Foxby
Matematisk Institut
Kobenhavns Universitet
Universitetsparken 5

DK-2100 Kobenhavn

Prof.Dr. D. Ferrand
Departement de Mathematiques
Universite de Rennes I
Campus de Beaulieu

F-35042 Rennes Cedex

Prof.Dr. W. Fulton
Dept. of Mathematics
Brown University
Box 1917

Providence , RI 02912
USA

Prof.Dr. H. Flenner
Mathematisches Institut
der Universität Göttingen
Bunsenstr. 3-5

3400 Göttingen

Dr. A. Garcia
c/o Professor Dr. P. Roquette
Mathematisches Institut
Universität Heidelberg
Im Neuenheimer Feld 288

6900 Heidelberg

Prof.Dr. M. Flexor
Mathematiques
Universite de Paris Sud (Paris XI)
Centre d'Orsay, Bat. 425

F-91405 Orsay Cedex

Prof.Dr. A. V. Geramita
Department of Mathematics and
Statistics
Queen's University

Kingston, Ontario K7L 3N6
CANADA

Prof.Dr. R. Fossum
Dept. of Mathematics, University of
Illinois at Urbana Champaign
273 Altgeld Hall
1409 West Green Street

Urbana , IL 61801
USA

Dr. G. Gotzmann
Isselstraße 34

4440 Rheine

Prof.Dr. G.-M. Greuel
Fachbereich Mathematik
der Universität Kaiserslautern
Postfach 3049

6750 Kaiserslautern

Prof.Dr. M. Hochster
Dept. of Mathematics
University of Michigan
3220 Angell Hall

Ann Arbor , MI 48109
USA

Prof.Dr. R. Hartshorne
Dept. of Mathematics
University of California

Berkeley , CA 94720
USA

Prof.Dr. G. Horrocks
Dept. of Mathematics
The University

GB- Newcastle-upon-Tyne , NE1 7RU

Prof.Dr. M. Herrmann
Mathematisches Institut
der Universität Köln
Weyertal 86-90

5000 Köln 41

R. Hübl
Fachbereich Mathematik
der Universität Regensburg
Universitätsstr. 31
Postfach 397

8400 Regensburg

Prof.Dr. J. Herzog
FB 6 - Mathematik
der GHS Essen
Universitätsstr. 3
Postfach 103 764

4300 Essen 1

Prof.Dr. C. Huneke
Dept. of Mathematics
Purdue University

West Lafayette , IN 47907
USA

Prof.Dr. A. Hirschowitz
Mathematiques et Physiques
Universite de Nice
Parc Valrose

F-06034 Nice Cedex

Prof.Dr. F. Ischebeck
Mathematisches Institut
der Universität Münster
Einstein-Str. 62

4400 Münster

Dr. M. Kersken
Fakultät und Institut f. Mathematik
der Ruhr-Universität Bochum
Gebäude NA, Universitätsstr. 150
Postfach 10 21 48

4630 Bochum 1

Prof.Dr. K. H. Kiyek
Fachbereich Mathematik/Informatik
der Universität Paderborn
Warburger Str. 100

4790 Paderborn

M. Kreuzer
Fachbereich Mathematik
der Universität Regensburg
Universitätsstr. 31
Postfach 397

8400 Regensburg

Prof.Dr. E. Kunz
Fachbereich Mathematik
der Universität Regensburg
Universitätsstr. 31
Postfach 397

8400 Regensburg

Prof.Dr. H. Lindel
Mathematisches Institut
der Universität Münster
Einstein-Str. 62

4400 Münster

Prof.Dr. J. Lipman
Dept. of Mathematics
Purdue University

West Lafayette , IN 47907
USA

Prof.Dr. W. Lütkebohmert
Mathematisches Institut
der Universität Münster
Einstein-Str. 62

4400 Münster

Prof.Dr. H. Matsumura
Dept. of Mathematics
College of General Education
Nagoya University
Chikusa-Ku

Nagoya 464
JAPAN

Prof.Dr. N. Mohan-Kumar
Nation.Cent.of the Governm.of India
f. Nuclear Science + Mathematic
Tata Inst. of Fundamental Resea
Homi Bhabha Road

Bombay 400 005
INDIA

Prof.Dr. B. Moishezon
Dept. of Mathematics
Columbia University

New York , NY 10027
USA

Prof.Dr. M. P. Murthy
Dept. of Mathematics
Eckhart Hall
University of Chicago
5734 University Ave.

Chicago , IL 60637
USA

Prof.Dr. O. Riemenschneider
Mathematisches Seminar
der Universität Hamburg
Bundesstr. 55

2000 Hamburg 13

Prof.Dr. H.-J. Nastold
Mathematisches Institut
der Universität Münster
Einstein-Str. 62

4400 Münster

Prof.Dr. L. Robbiano
Dep. di Matematica
Universita di Genova
Via L. B. Alberti, 4

I-16132 Genova

Prof.Dr. C. Peskine
Institut de Mathematiques Pures et
Appliquees, UER 47
Universite de Paris VI
4, Place Jussieu

F-75252 Paris Cedex 05

Prof.Dr. P. C. Roberts
Dept. of Mathematics
University of Utah

Salt Lake City , UT 84112
USA

Dr. J. Rathmann
Sonderforschungsbereich 170
"Geometrie und Analysis"
der Universität Göttingen
Bunsenstr. 3-5

3400 Göttingen

Prof.Dr. P. Samuel
Mathematiques
Universite de Paris Sud (Paris XI)
Centre d'Orsay, Bat. 425

F-91405 Orsay Cedex

Prof.Dr. M. Raynaud
Mathematiques
Universite de Paris Sud (Paris XI)
Centre d'Orsay, Bat. 425

F-91405 Orsay Cedex

Prof.Dr. G. Scheja
Mathematisches Institut
der Universität Tübingen
Auf der Morgenstelle 10

7400 Tübingen 1

Prof.Dr. R. Y. Sharp
Dept. of Mathematics
University of Sheffield
Hicks Building
Hounsfield Road

GB- Sheffield , S3 7RH

Prof.Dr. W. V. Vasconcelos
Dept. of Mathematics
Rutgers University

New Brunswick , NJ 08903
USA

Prof.Dr. U. Storch
Fakultät und Institut f. Mathematik
der Ruhr-Universität Bochum
Gebäude NA, Universitätsstr. 150
Postfach 10 21 48

4630 Bochum 1

Prof.Dr. U. Vetter
FB Naturwissenschaften/Mathematik
Fach Mathematik
Abt.Vechta der Univ. Osnabrück
PF 1349, Driverstr. 22

2848 Vechta

Prof.Dr. L Szpiro
29, rue de Vaugirard

F-75006 Paris

Prof.Dr. B. Wajnryb
Dept. of Mathematics
Columbia University

New York , NY 10027
USA

Prof.Dr. B. Ulrich
Dept. of Mathematics
Michigan State University

East Lansing , MI 48824-1027
USA

Dr. R. Waldi
Kurt-Schumacher-Str. 19a

8400 Regensburg