MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Darstellungstheorie endlicher Gruppen

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This meeting was organized by Prof. B. Huppert (Mainz) and Prof. G.O. Michler (Essen). Since the first conference on representation theory of finite groups at the Oberwolfach Institute in 1983, the subject has seen a lot of progress in various directions.

A number of talks were devoted to new results in classical character theory, in particular to degree problems in ordinary and modular representation theory, and to representations of Chevalley groups. Several reports were given on a new view on Clifford theory and on the structure of permutation modules, which are also a new tool in coding theory. Furthermore, recent progress on Galois groups and on the isomorphism problem for integral group rings was presented. The participants also got a survey of cohomological methods in group representation theory and of computer algebra, which both have become important new topics in the last years.

The conference was attended by 51 participants from Australia, Danmark, England, France, Ireland, Switzerland, USA, USSR, and West Germany. Apart from the lectures, the discussions during the breaks and in the evening were an invaluable part of the meeting; in particular, the participants from western countries had a chance to get some information on the recent research in the USSR from their Russian colleagues.

J. ALPERIN: Loewy structure of permutation modules for p-groups

The theorems of Jennings and Hill on the structure of $k[P]$, P a finite p-group, k a field of characteristic p, are generalized to the $k[P]$-module $k\Omega$ where Ω is a set on which P acts transitively, so answering, in the case of Hill's theorem, a question of Peter Neumann.

D. BENSON: Specht modules and cohomology of mapping class groups

Let $M^n_{g,k}$ denote an oriented 2-manifold of genus g with n punctures and k boundary components, and $\Gamma^n_{g,k} = \pi_0 \operatorname{Top}^+(M^n_{g,k})$ denote the group of connected components of the group of orientation preserving self homeomorphisms of $M^n_{g,k}$ (the <u>mapping class group</u>). Using recently developed diagrammatic methods in modular representation theory (due to myself and Jon Carlson), and the theory of Specht modules, I obtain the cohomology ring of $\Gamma^0_{2,0}$ with coefficients in any field. The interesting characteristics are 2,3 and 5. For example, we have

$$H^*(\Gamma^0_{2,0}, \mathbb{F}_2) = k[\alpha,\beta,\delta,\gamma]/(\beta\delta,\alpha\beta,\beta^2,\alpha^2+\delta^4,\alpha\delta^3+\delta^5) .$$

As intermediate results we obtain information about the cohomology of $\Gamma^n_{0,0}$; the case of interest is $n = 6$ because there is a short exact sequence

$$1 \longrightarrow \mathbb{Z}/2 \longrightarrow \Gamma^6_{0,0} \longrightarrow \Gamma^0_{2,0} \longrightarrow 1 .$$

Since the cohomology of $\Gamma^n_{0,0}$ is expressed in terms of Specht modules for Σ_n (the symmetric group), the diagrammatic methods are applied for modules for Σ_6 .

F. BERNHARDT: Groups with only few p-modular character degrees

If G is a finite solvable group, then it is conjectured that the derived length of G is bounded by the number of character degrees of G over the complex field $\mathbb{C}$. This conjecture was proved if the group has only at most four character degrees or if $|G|$ is odd.

Looking at a p-modular analogue it is true that, assuming $O_p(G)$ trivial, the derived length of G is bounded by four if the number of p-modular Brauer characters is less or equal to two and also there are such examples.

H. BLAU: Table algebras

A <u>table algebra</u> is defined as a commutative algebra over the complex numbers with a specified basis which satisfies certain properties. It generalizes both the character ring and class algebra of a finite group, and permits a unified proof of various analogous theorems concerning both of these objects. Joint work with Z. Arad on the general theory of table algebras and its applications to finite groups will be discussed.

J.F. CARLSON: Exponents of modules and maps

Let G be a finite group and let R be a P.I.D. of characteristic 0. Let L and M be RG-lattices. If $\alpha: L \to M$ then $\exp(\alpha)$ is a generator for the ideal of all $r \in R$ such that $r\alpha$ factors through a projective. Also $\exp(M) = \exp(Id_M)$. Suppose that $\zeta_1, \ldots, \zeta_r$ are homogeneous elements in $H^*(G,R)$ such that the radical of the ideal that their reductions modulo P generate is the annihilator of the cohomology of M/PM for every prime ideal $P \subsetneq R$. Such a set can be found with $r = c_G(M)$. Then

$$\prod_i \exp(\zeta_i) \geq \exp(M).$$ In case $M = R$ we have that

$$\prod_i \exp(\zeta_i) \geq |G|.$$

Suppose that R is a complete d.v.r. with prime element π. If $\exp(M) = \pi^a$ then M has property E provided $\pi^{a-1} \cdot \hat{\mathrm{Ext}}^0_{RG}(M,M) = \mathrm{Soc}\ \hat{\mathrm{Ext}}^0_{RG}(M,M)$ (Tate cohomology). Joint work with A. Jones shows that property E is preserved under the Green correspondence for absolutely indecomposable lattices. The work suggests that the height-zero conjecture might be provable by purely local methods.

E.C. DADE: Compounding Clifford theory

We present Clifford theory as a single, thoroughly natural equivalence between the subcategory $\mathrm{Mod}(FG|V)$ of $\mathrm{Mod}(FG)$ generated by the module V^G induced by a simple module V over FN, where $N \trianglelefteq G$, and the category $\mathrm{Mod}((FG)')$ of all modules over the endomorphism ring $(FG)' = \mathrm{End}_{FG}(V^G)$. If, in addition, $N \leq M \trianglelefteq G$ and W is a simple FM-module lying over V, then we easily obtain a commutative diagram of equivalences of categories defined by Clifford theory

$$\begin{array}{ccc} \mathrm{Mod}(FG|W) & \approx & \mathrm{Mod}((FG)'_W) \\ \wr\wr & & \wr\wr \\ \mathrm{Mod}((FG)'|W') & \approx & \mathrm{Mod}((FG')'_{W'}) \end{array},$$

where $(FG)'_W = \mathrm{End}_{FG}(W^G)$, etc. Since the two vertical equivalences here are induced by Clifford theory for V and FG, while the top and bottom equivalences are Clifford theory for W and FG and for W' and $(FG)'$, this says that "Clifford theory preserves Clifford theories".

P. FERGUSON: Applications of prime characters

If χ is a quasi-primitive irreducible character of G, let $Z(\chi)$, $F^*(\chi)$, and $M^*(\chi)$ be defined by $Z(\chi)/\ker(\chi) = Z(G/\ker(\chi))$, $F^*(\chi)/\ker(\chi) = F^*(G/\ker(\chi))$, and $M^*(\chi) = F^*(\chi)/Z(\chi)$. χ is a prime character if χ is a quasi-primitive irreducible character, $\chi_{F^*(\chi)}$ is irreducible and $M^*(\chi)$ is homogeneous. I discuss the following:

Theorem 1 If χ is a quasi-primitive irreducible character of G, there is an extension $(\hat{G},\pi)$ of G such that $\ker\pi \subseteq \hat{G}' \cap Z(\hat{G})$ and χ factors uniquely (up to associates) as $\chi = \prod_{i=1}^{n} \rho_i$ where $\{\rho_1,\rho_2,\ldots,\rho_n\}$ is an admissible set of prime characters.

As an application, I indicate a proof of the following theorem:

Theorem 2 Suppose χ is a quasi-primitive irreducible character of G of odd degree. If $\Psi \in \mathrm{Irr}(S)$ where S is a universal covering group of a non-Abelian composition factor of G and $\Psi(1)\,|\,\chi(1)$, assume $\frac{\chi(1)}{\Psi(1)}\Psi$ is not induced from a proper subgroup of S and if $\Psi(1) = p^r$, then $p \nmid \left|\frac{Z(S)}{\ker\Psi}\right|$, then χ is a primitive character of G.

B. FISCHER: The character-table of $E_6(2)$

$E_6(2)$ contains a parabolic subgroup NK where $N \trianglelefteq NK$, $|N| = 2^{16}$, $K \simeq D_5(2)$. The character-table of NK was computed for a table of a certain subgroup of the monster. Since it was known it could be used for the computation of the character-table of $E_6(2)$. I have to thank S. Black and J. Janisczcak for their help.

P. FLEISCHMANN: Periodic simple modules for Chevalley groups in the describing characteristic

Let k be an algebraically closed field of char $p>0$, $q = p^n$, G a finite group and M a kG-module. If $\Omega^i(M) = \Omega(M)$, for some $i>1$, then M is called periodic (Ω is the Heller operator). ST:= Steinberg module. Now the following holds for the non-twisted Chevalley groups:

$A_1(q)$: Periodic simple modules $\neq$ St exist and are classified (Jeyakumar 1979). $A_n(q)$, $n\geq 2$, $D_n(q)$, $E_6(q)$, $E_7(q)$, $E_8(q)$, $F_4(q)$: no periodic simple modules $\neq$ ST exist! (Janiszczak 1985) Same result holds for $B_2(q)$, $G_2(q)$ (Janiszczak and Jantzen 1987).

And for twisted Chevalley groups: ${}^2A_2(q^2)$, periodic simple modules $\neq$ ST exist and are classified (Fleischmann 1986). ${}^2A_n(q^2)$, $n\geq 3$, ${}^2D_l(q^2)$, $l\geq 4$, ${}^2E_6(q^2)$, ${}^3D_4(q^3)$, no simple periodic modules exist ($\neq$ ST). ${}^2B_2(2^{2m+1})$, Suzuki groups: periodic simple modules $\neq$ ST exist and are classified. ${}^2G_2(3^{2m+1})$, Ree groups: no periodic simple modules $\neq$ ST exist.

D. GLUCK: Prime factors of character degrees of solvable groups

Let $\sigma(G)$ be the maximum number of primes dividing any one character degree of G and let $\rho(G)$ be the set of primes which divide some character degree of G. For G solvable, Huppert has conjectured that $|\rho(G)|\leq 2\sigma(G)$. In this joint work with O. Manz, we show that $|\rho|\leq 3\sigma+32$ for every solvable group, considerably improving earlier results of Isaacs and of Gluck. We also obtain $|\rho|\leq 2\sigma+32$ when G is solvable with no normal nonabelian Sylow subgroups.

R. GOW: Reduced K-degrees of irreducible characters

This is a report of joint work with B. Huppert. Let G be a finite group and K a field of characteristic 0. Let L be a splitting field for G containing K and let $\chi \in \mathrm{Irr}_L(G)$. We define the reduced K-degree of χ to be $t_K(\chi) = \chi(1)/m_K(\chi)$, where $m_K(\chi)$ is the Schur index of χ over K. We propose to study how information about the set of numbers $t_K(\chi)$ can lead to group-theoretic information about G. When $K = L$, our problem is just the classical character degrees problem investigated by several researchers in the past twenty years.

We give two examples of the sort of results we have proved.

Problem 1 Suppose that $t_K(\chi) = 1$ or $k>1$ for all $\chi \in Irr_L(G)$. What can be said about G? We investigate this problem using the following result:

Thompson-type theorem Suppose that there exists a prime p such that $p | t_K(\chi)$ for all $\chi \in Irr_L(G)$ with $t_K(\chi) \neq 1$. Then G has a normal p-complement.

Using this theorem, we prove that if G is a group described in Problem 1 and π is the set of prime divisors of k, G has a normal π-complement N, N is certainly metabelian and G/N is nilpotent.

Problem 2 Suppose that $t_K(\chi) = 1$ or a prime number for all $\chi \in Irr_L(G)$ (not always the same prime). What can be said about G? Using the classification of simple groups, we show that G is solvable, and has derived length at most 4, a bound that can be attained by certain groups of even order.

G. HISS: Modular representations of Chevalley groups in special characteristic

Let G be a group with a split BN-pair, i.e. $B,N \leq G = \langle B,N \rangle$, $B = UH$, $U \trianglelefteq B, U \cap H = 1$; $N = N_G(H)$, $B \cap N = H$, $N/H = W$, the Weyl group of G.

Let r be a prime and D a Sylow r-subgroup of H. Suppose the following conditions are satisfied:

1. D is a Sylow subgroup of G
2. $C_G(D) = H$.

In this situation we determine the decomposition matrix of the principal r-block of G. Two corollaries should be mentioned (k denotes a splitting field of characteristic r):

Corollary 1. The permutation module over k on the cosets of B is completely reducible. Its constituents are exactly the simple kG-modules in the principal block. The unipotent characters in the principal series are irreducible modulo r .

Corollary 2. The Green correspondents of the simple kG-modules in the principal block are exactly the simple kN-modules which have H in their kernel, i.e. the simple kW-modules.

The proof is a straightforward application of Green correspondence. L. Puig has obtained these results independently by using his theory of source algebras.

I.M. ISAACS: Characters and solvable groups

Because of the fairly extensive theory of characters of solvable groups which has been developing in the past fifteen or so years, it has become possible to answer for solvable groups certain questions which are (as yet) intractible for arbitrary groups. (For example, the McKay conjecture and the Brauer height conjecture have been proved for solvable groups.)

Recently, in the work of P. Ferguson, the following question has arisen: If $H \subseteq G$ and $\theta \in \mathrm{Char}(H)$ where $\theta^G = a\chi$ for some primitive $\chi \in \mathrm{Irr}(G)$, does this imply that $H = G$?

For solvable groups, it turns out that it is relatively easy to answer this question affirmatively. (This was done independently by Ferguson and myself.) To demonstrate some of the techniques of solvable character theory, and in particular factorization theory, this lecture presents (in some detail) a proof of this result.

A.V. IVANOV: Non rank 3 graph with 5-vertex condition

Ordinary graph with t-vertex condition is defined. The graph with t-vertex condition is also the graph with t'-vertex condition for every $2 \leq t' \leq t$. The examples of such graphs are regular graphs $(t = 2)$, strongly regular graphs $(t = 3)$, rank 3 graphs $(\forall t)$.

The graph G with 5-vertex condition is constructed. Its parameters are $(v, k, \lambda, \mu, \alpha, \beta) = (256, 120, 56, 56, 784, 672)$ and $|Aut(G)| = 2^{20} \cdot 3^2 \cdot 5 \cdot 7$. Two subgraphs G_1 and G_2 of graph G are the graphs with 4-vertex condition. Their parameters are $(120, 56, 28, 24, 216, 144)$, $(135, 64, 28, 32, 168, 192)$, respectively and $|Aut(G_1)| = |Aut(G_2)| = 2^{12} \cdot 3^2 \cdot 5 \cdot 7$. All these graphs are not rank 3 graphs.

Up to now all examples of non rank 3 graphs with t-vertex condition were known for $t \leq 3$ only.

W. KIMMERLE: Sylow subgroups and isomorphic integral group rings

Theorem 1. Assume $\mathbb{Z}G \cong \mathbb{Z}G^*$. Let $P \in Syl_p(G)$ and $P^* \in Syl_p(G^*)$. Suppose that P is abelian. Then $P \cong P^*$.

The result can be extended to Hamiltonian Sylow subgroups. The proof uses for $p > 2$ the classification of the finite simple groups. One crucial point of the proof is that the integral group ring of a finite group determines the chief factors.

Theorem 2. Assume $\mathbb{Z}G \cong \mathbb{Z}G^*$. Let

$$1 = K_0 \triangleleft K_1 \ldots \triangleleft K_{n-1} \triangleleft K_n = G$$

be a chief series of G. Then G^* has a chief series

$$1 = L_0 \triangleleft L_1 \ldots \triangleleft L_{n-1} \triangleleft L_n = G^*$$

such that $L_{i+1}/L_i \cong K_{i+1}/K_i$, $0 \leq i \leq n-1$.

Theorem 1 was proved in joint work with S. Sandling (Manchester). Theorem 2 was proved by R. Lyons (Rutgers University) and R. Sandling, and independently by me. It is written up in a joint paper.

E.A. KOMISSARTSCHIK: Intersections of maximal subgroups in simple groups of order less than 10^6 and associated amalgams

Let G be a finite group and H and K be subgroups of G.
Problem 1. Describe $H^g \cap K$ for all $g \in G$.
Result 1. List of intersections of all pairs of maximal subgroups in simple groups of order less then 10^6 excluding $PSL(2,q)$.
The natural generalization of the Problem 1 is
Problem 2. Let $G_1,\ldots,G_n$, $n \geq 3$, be subgroups of a group G. Describe all residually connected amalgams $(G_1^{g_1},\ldots G_n^{g_n})$ for all g_i from G, such that G is generated by G_i, $1 \leq i \leq n$.
Result 2. List of all residually connected amalgams for
(1) $G \cong J_1$, and
(2) $G \cong U_4(2)$ with the additional assumptions that all G_i are maximal subgroups of G and $\bigcap_1^n G_i$ is nontrivial.

L.G. KOVACS: The Grothendieck ring of $\mathbb{F}_q SL_3(q)$

Consider the homomorphism of the commutative polynomial ring $\mathbb{Z}[x,y]$ to the Grothendieck ring of $\mathbb{F}_p SL_3(p)$ which maps x to the natural module and y to its dual. This homomorphism is surjective and its kernel is the ideal generated by the two polynomials

$$x^p - x + pR(x,y) \quad \text{and} \quad y^p - y + pR(y,x)$$

where

$$R(x,y) = \Sigma \frac{(i+j+k-1)!}{i!\ j!\ k!} x^i(-y)^j \in \mathbb{Z}[x,y]$$

with summation over all non-negative i,j,k such that $i + 2j + 3k = p \neq i$. A similar result holds for $\mathbb{F}_q SL_3(q)$ whenever q is a power of p.

B. KÜLSHAMMER: Morita equivalent blocks and Clifford theory

Clifford theory is concerned with the relationship between representations of groups K,H,G occurring in a finite group extension

$$1 \longrightarrow K \longrightarrow H \longrightarrow G \longrightarrow 1 .$$

Let B be a (G-stable) block of K, and let A be a block of H covering B. J. Alperin has proved recently that A and B are isomorphic if and only if their Brauer correspondents are. We generalize his result, replacing isomorphism by a Morita equivalence satisfying a certain natural condition. When A and B are Morita equivalent, questions about A can often be reduced to questions about B. Thus our reduction process complements other tools in Clifford theory such as the Fong-Reynolds correspondence.

P. LANDROCK: Ideals and codes in group algebras

Let C be a right ideal in a group algebra F[G], F of char p, and define the divisor d of C as the greatest common divisor prime to p of all the cardinalities of subsets of G which form the support of some non-zero element of C. Then there exists a subgroup H of G with d dividing the order of H, such that furthermore C is contained in the permutation ideal $(\sum_{h \in H} h)\,F[G]$. H is called the induction kernel of C and the result above is inspired from work by H.N. Ward in coding theory. This may also be used to explain how to determine the set of subgroups {X} of G such that $S^H \neq 0$ for some simple module S. In particular, we get an improved version of the Nakayama Relations for ideals.

O. MANZ: Brauer characters of q'-degree

We denote by IBr(G) the set of Brauer characters w.r.t. the prime p and let q be a prime different from p . As a modular analogue of Ito's theorem we prove

Theorem 1. Suppose that G is p-solvable and $q \nmid \beta(1)$ for all $\beta \in IBr(G)$.

a) Then the q-length $l_q(G)$ is at most 2 and the q-factors in every q-series are abelian. (This is equivalent with $Q \in Syl_q(G)$ being at most metabelian.)

b) If furthermore $q \nmid p-1$ and $(p,q) \neq (2,3)$, then $l_q(G) \leq 1$.

Question. Is it in general true that $Q \in Syl_q(G)$ is at most metabelian if $q \nmid \beta(1)$ for all $\beta \in IBr(G)$?

A "local" version of Theorem 1 above is the following:

Theorem 2. Let $N \trianglelefteq G$, $\alpha \in IBr(N)$, G/N solvable and $q \nmid \beta(1)/\alpha(1)$ for all $\beta \in IBr(G|\alpha)$. Then for $Q \in Syl_q(G/N)$, we have $dl(Q) \leq \begin{cases} 2, & q \geq 5 \\ 3, & q \leq 3 \end{cases}$.

All results mentioned above are joint work with Tom Wolf (Athens, Ohio).

B.H. MATZAT: Zöpfe und Galoissche Gruppen

Die Fundamentalgruppe des s-fachen Produkts der Riemannschen Zahlenkugel, aus dem die schwache Diagonale (bestehend aus den Elementen mit zwei oder mehr gleichen Komponenten) herausgeschnitten ist, heißt die reine Hurwitzsche Zopfgruppe. Durch das Studium dieser Zopfgruppe erhält man Rationalitätskriterien für Galoiserweiterungen $N/\mathbb{C}(t_1,\ldots,t_s)$, die außerhalb $t_i = t_j$ für $i \neq j$ (geometrisch) unverzweigt sind.

Unter Verwendung dieser Kriterien konnte unter anderem erstmalig bewiesen werden, daß auch die größte der Mathieugruppen, M_{24} , als Galoisgruppe über $\mathbb{Q}$ realisierbar ist.

W. PLESKEN: Applying representation theory: a soluble quotient algorithm

To design feasible algorithms to compute the biggest finite soluble quotient group of a finitely presented group in case it exists has been an open problem in computational group theory ever after the spectacular success of the nilpotent quotient algorithm.

Given a finitely presented group G and an epimorphism ϵ of G onto a "known" group H. To test whether ϵ can be lifted to an epimorphism onto an extension of a simple H-module by H amounts to solving linear equations. Is H soluble one can find the simple modules and the cocycles describing the extensions algorithmically. This way one obtains a soluble π-quotient algorithm, where π is a finite set of primes. The proper choice of π can be made if the irreducible H-modules are known for the various quotient groups H of G which occur.

L. PUIG: Generalization of Brauer's second main theorem to virtual modules

Let G be a finite group, A an interior G-algebra, M an A-module and χ the character of M. For any pointed group H_β on A, denote by χ^β the character of the OH-module i.M where $i \in \beta$ and if K_γ is a pointed subgroup of H_β denote by φ_γ^β the Brauer character of the $kC_H(K)$-module $s_\gamma(i)A(K_\gamma)s_\gamma(j)$ where $j \in \gamma$. In "Pointed groups and construction of characters" we prove $(^*)$ $\chi^\alpha(us) = \sum_\epsilon \varphi_\epsilon^\alpha(s)\chi^\epsilon(u)$ where $\alpha \in P_A(G)$, $u \in G_p$, $s \in C_G(u)_{p'}$, and ϵ runs over $LP_A(u)$. In particular, for any local pointed group Q_δ on A such that $Q_\delta \subset G_\alpha$, we get $(^{**})$ $\chi^\delta(u) = \sum_\epsilon m_\epsilon^\delta \chi^\epsilon(u)$ where $u \in Q$, $m_\epsilon^\delta = \varphi_\epsilon^\delta(1)$ and ϵ runs over $LP_A(u)$. Conversely, if for any local pointed subgroup Q_δ of G_α we choose a virtual character λ^δ of Q in such a way that (**) holds, then (*) defines a virtual character of G. Our main purpose here is to prove analogous statements replacing the ring of $\mathbb{C}$-valued cen-tral functions on G by the Green ring of G over Q, virtual characters by virtual modules, and values of virtual characters by "residues" of virtual modules on the subgroups H of G such that $H/O_p(H)$ is cyclic.

G.R. ROBINSON: On permutation modules

In this talk, G denotes a finite group, p a prime, k an algebraically closed field of characteristic p, S a Sylow p-subgroup of G. We discuss various connections between the structure of permutation modules (and their endomorphism rings) of G and the group-theoretic structure of G.

In section 1, on fusion, I discuss the following result (and related ones).

Theorem: The vertices of the non-projective summands (in the principal p-block) of Ind_S^G (K) constitute a conjugation family for S in G.

In section 2, on simplicial complexes and related topics, I discuss (among other things) the following result related to a conjecture of Quillen.

Theorem: Let Δ_p be the simplicial complex associated to the poset of non-trivial p-subgroups of G. Suppose that $p \geq 5$, that $O(G) = 1$, and that the components of G are of characteristic 2-type. Then Δ_p is not contractible.

I also discuss the following result (proved jointly with R. Knörr).

Theorem: Let B be a block of kG, and for a simplex $C \in \Delta_p$, let B_C be the Brauer correspondents (i.e. blocks of kG_C) (which are defined). Let G_C act on B_C by conjugation. Then $\sum_{C\in\Delta_p/G} (-1)^{|C|}\mathrm{Ind}_{G_C}^G(B_C)$ is a virtual projective module.

This result relates to a conjecture of Alperin, and has applications to groups for which Alperin's conjecture has been verified.

K.W. ROGGENKAMP: News on the isomorphism problem

This is a report on joint work with L.L. Scott. The following result and its consequences are discussed:

Theorem: Let R be an unramified extension of the p-adic integers. G is a p-constrained group with $O_{p'}(G) = 1$. Let $\alpha: RG \longrightarrow RG$ be an augmented automorphism with $\alpha(I(O_p(G))\uparrow^G) \subset I(O_p(G))\uparrow^G$ - $I(X)$ is the augmentation ideal of the subgroup X of G. Then there is a unit u in RG such that $\alpha(G) = uGu^{-1}$.

Corollary 1: Let G be as above. Then the Zassenhaus conjecture is true for $\mathbb{Z}G$. (In particular, $\mathbb{Z}G \simeq \mathbb{Z}H$ implies $G \simeq H$.)

Corollary 2: Let G be a finite group with a normal Sylow p-subgroup. Then the defect group of the principal block B_0 of RG is uniquely determined - up to conjugacy - by B_0.

Corollary 3: Let G be a solvable group. Then for every prime p, the group $G/O_{p'}(G)$ is determined by $\mathbb{Z}G$; in particular, the Sylow p-subgroups are determined.

A.V. ROMANOVSKII: On the finite linear groups with Frobenius section

Related to the results of Brauer, Leonard and Sibley on finite linear groups is the following theorem.

Theorem 1. Let a finite group G have a faithful complex character φ and $C_G(x) \subseteq PC_G(P)$ for each non-trivial element of some non-normal Sylow p-subgroup P of G. Then

1) If every irreducible constituent of character φ is of degree less than $(|P|-1)/2$, then $G \simeq Sz(q)$.
2) If $\varphi(1) \leq (|P|-1)/2$, then either $G \simeq Sz(q)$ or $G/Z(G) \simeq PSL(2,|P|)$.

This result has been obtained together with N.A. Romanovskaja. The proof is based on the theory of exceptional characters and not the classification of simple finite groups.

Following result has been obtained by the author and A.A. Jadchenko.

Theorem 2. Let G be a p-solvable group with an abelian Sylow 2-subgroup. Let G have a faithful representation of degree $2p - 2$ over the complex number field. Then G has a normal Sylow p-subgroup.

G. SCHNEIDER: Dixon's character table algorithm revisited

Let C_i, $1 \leq i \leq k$ denote the classes of a finite group and χ_i the characters. The algorithm given by J. Dixon in 1967 for the automatic computation of group characters can be significantly improved by using the equation

$$\frac{|C_r| \cdot \overline{\chi_i(x_r)}}{\chi_i(1)} \cdot \chi_i(x_t) = \sum_{s=1}^{k} \chi_i(x_s) \cdot c_{rst} .$$

The c_{rst} are the class multiplication constants, i.e. the number of solutions in G to the equation $x_r \cdot x_s = x_t$, $x_r \in C_r$, $x_s \in C_s$ and fixed $x_t \in C_t$. The characters are therefore row eigenvectors of the class matrix $M_r = (c_{rst})_{s,t}$ and can be obtained by successive computation of eigenspaces of various class matrices.

The new approach allows to predict whether a class matrix will split an existing space into smaller subspaces without having to determine the matrix. In addition, not all columns of a matrix have to be computed. An algorithm was presented that finds the characters that span a 2-dimensional space without the need of a class matrix.

The performance of the new algorithm was demonstrated by giving the CPU-time requirements of various test cases. An implementation is available to the users of the CAYLEY system.

G. SEITZ: Restrictions of irreducible representations to subgroups

Let G be a finite classical group over a field of r^c elements. In view of a recent result of Aschbacher, the problem of determining the maximal subgroups of G reduce to finding triples (X,Y,V), where X,Y are quasisimple groups, $X < Y$, and both X,Y are absolutely irreducible subgroups of G. Here V is the natural module for G.

Let $X = X(p^a)$ be of Lie type in characteristic p. For $p = r$ (the <u>generic</u> case) a great deal of work has been done and this case is now in reasonably good shape. Here we consider the cross-characteristic case, $p \neq r$.

The most likely candidate for Y is $Y = Y(p^b)$, also of Lie type in char p, and we assume this to be the case. Hence, we have the embedding $X(p^a) < Y(p^b) < G = G(r^c)$. We have the following

<u>Theorem</u> Assume $p^a > 3$ and Y is of classical type. Then there is a subgroup $C < Y$ such that $X \leq C$ and the pair $(C/Z(C), Y/Z(Y))$ is one of the following:

1) $(PSp_{2n}(q), PSL^{\varepsilon}_{2n}(q))$ $\quad (q = p^b)$

2) $(P\Omega_{n-1}(q), P\Omega_n(q))$

3) $(G_2(q), B_3(q))$

4) $(PSp_{2n}(q^s), PSp_{2ns}(q))$

In particular, if Y is minimal, then $(X/Z(X), Y/Z(Y))$ is one of the pairs 1) - 4).

For most of the types 1) - 4) a suitable V does exist, but we have been unable to determine all possible V's.

U. STAMMBACH: Resolutions as multiple complexes

(PhD Thesis of R. Schmid)

Over a finite abelian group the (minimal) resolution of the field k can be described as the total complex of a multiple complex. Schmid has shown that this result generalizes to the case of a p-solvable group with abelian p-Sylow subgroup. In principle the resulting complex can be used to calculate the cohomology of such a group.

Some time ago J. Alperin raised the question whether there always is a resolution that can be written as total complex of a multiple complex. Recently Benson and Carlson have shown that this is indeed the case. Their proof however is not constructive. Regarding their multiple complex as iterated double complex, Schmid has proved that the associated spectral sequences have the property that $E_2 = E_\infty$. As a consequence the cohomology groups can be calculated as iterated homology. This shows that the cohomology groups obey very strong periodicity rules.

R. STASZEWSKI: On the Loewy structure of the modular group algebra over a finite p-group

Let P be a finite p-group and K an arbitrary field of characteristic p, p a prime. In 1941, S. Jennings proved the following about the dimensions of the Loewy-factors $J(KP)^i/J(KP)^{i+1}$. Let $\kappa_1 := P$ and $\kappa_n := [\kappa_{n-1}, P]\kappa_m^p$, where m is the least integer with $pm \geq n$. Then $\kappa_1 \geq \kappa_2 \geq \ldots \geq \kappa_l > \kappa_{l+1} = 1$ is a central series with elementary abelian factors. If $|\kappa_n/\kappa_{n+1}| =: p^{d_n}$ and $\dim_K(J(KP)^i/J(KP)^{i+1}) =: c_i$, we have $\prod_{n=1}^{l} (1+t^n+\ldots+t^{n(p-1)})^{d_n} = \sum_{i=0}^{s} c_i t^i$ (in $\mathbb{Z}[t]$). From this we easily have $c_i = c_{s-i}$, and for quite some time, the question circulated whether the Loewy-series is even monotonic, which means $c_{i-1} \leq c_i$ $(1 \leq i \leq s/2)$. In 1986, Stammbach-Stricker and Manz-

Staszewski independently found counterexamples, but in all of them, the prime p was either 2, or 3, or 5. Searching for counterexamples in a systematic manner, C. Leedham Green and the author first proved some conditions which the sequence $d_1, d_2, \ldots$ has to fulfill whenever this sequence arises from a group and afterwards wrote a computer program which constructs all d_i-sequences which might arise from a group and calculates the c_i's (up to a certain order of the group). By this method, it was shown that there is no counterexample P with $|P| \mid 7^{15}$, $|P| \mid 11^{17}$ or $|P| \mid 17^{19}$, whereas the smallest counterexamples are of order 2^5 , 3^5 or 5^5 , respectively.

A. TURULL: Central extensions as Galois groups

Report of joint work with Núria Vila

A_n can be realized as the Galois group of some regular extension E of $Q(x)$ for every "rigid" triple of conjugacy classes of S_n . For triples that contain the class of n-cycles we calculate the obstruction to the embedding of E into some extension with Galois group $\hat{A}_n$, the non-split central extension of A_n by Z_2 .

T.R. WOLF: Variations on McKay's conjecture

Let P be a Sylow p-subgroup of a finite group G . McKay's conjecture proposes that G and $N_G(P)$ have the same number of irreducible (ordinary) characters of p'-degree. Okuyama and Wajima have proved this (and the more refined block-by-block AlperinMcKay conjecture) for p-solvable G .

Let q be a prime (equal to or different from p) and let Q be a Sylow q-subgroup of G . We show that if G is $\{p,q\}$-separable, then G and $N_G(Q)$ have the same number of Brauer characters (for the prime p) of q'-degree. Furthermore, p and q may be replaced by sets of primes π and ρ, provided G is π-separable and ρ-separable. From this follows the Alperin-McKay conjecture for π-blocks of π-separable groups as well as an unpublished result of Isaacs regarding the number of π-special characters of a π-separable group.

A.E. ZALESSKII: Eigenvalues of matrices of complex representations of finite Chevalley groups

Let $G = G(p^a)$ be a Chevalley group and let F be a field of characteristic $f \neq p$. If φ is a representation of G then $\deg \varphi(G)$ denotes the degree of the minimal polynomial of $\varphi(g)$.

<u>Theorem 1</u>. Let G be indecomposable in a non-trivial direct product. Assume that $g \in G$, $|g| = p$ and $\deg \varphi(g) < p$ where $\dim \varphi > 1$. Then $G \in \{A_1(p^2), {}^2A_2(p), C_n(p)\ (n \geq 1)\}$. Furthermore, if 1 is not an eigenvalue of $\varphi(g)$ then $G \in \{A_1(p^2), {}^2A_2(p), C_1(p), C_2(p)\}$.

<u>Theorem 2</u>. Let $F = \mathbb{C}$, $G \in \{G_2(p^a), p > 2, {}^2G_2(3^a), F_4(p^a), {}^2F_4(2^a), E_8(p^a)\}$ and let $g \in G$ be semisimple. If φ is an arbitrary representation of G over $\mathbb{C}$ with $\dim \varphi > 1$ then $\varphi(g)$ has eigenvalue 1.

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