MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 25/1987

Diskrete Geometrie

7.6. bis 13.6.1987

Die Tagung wurde von A. Florian (Salzburg) und G. Fejes Tóth (Budapest) geleitet. Sie wurde von 36 Teilnehmern besucht, von denen 33 Vorträge hielten.

In den Vorträgen wurden verschiedene metrische und kombinatorische Eigenschaften von diskreten Mannigfaltigkeiten geometrischer Objekte untersucht. Die Themen der Vorträge gliederten sich in drei größere Gruppen: Packungen und Überdeckungen, Theorie der konvexen Polytope und kombinatorische Geometrie. Eine kleinere Gruppe von Beiträgen beschäftigte sich mit Raumzerlegungen. Eine erfreuliche Entwicklung ist darin zu sehen, daß Probleme aus der Theorie der linearen Programmierung und aus "computational geometry" behandelt wurden.

Die Anzahl der Teilnehmer war ideal, und daher gab es genügend Zeit zur Arbeit in kleineren Gruppen. Dabei sind einige Probleme in internazionaler Zusammenarbeit gelöst worden. Es ist jedoch zu bedauern, daß mehrere der Eingeladenen nicht kommen konnten. So warwen die sovjetische und indische Schulen der diskreten Geometrie, die besonders die Beziehungen zur Geometrie der Zahlen pflegen, überhaupt nicht vertreten.

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Vortragsauszüge

I. BÁRÁNY:

Convex bodies, coverings by cups, random polytopes

Let $K \subset R^d$ be a convex compact set with $\operatorname{int} K \neq \emptyset$. Denote by K_n the convex hull of n points choosen from K randomly and independently and according to the uniform distribution. Define, further, $K(\varepsilon)$ for $\varepsilon > 0$ as the set of points $x \in K$ for which there exists a halfspace H with $x \in H$ and $\operatorname{vol}(H \cap K) \leq \varepsilon$. The main result is that there are constants c_1 and c_2 depending on d only such that

$$c_1 \operatorname{vol} K(\varepsilon) < \operatorname{Exp} \operatorname{vol}(K - K_n) < c_2 \operatorname{vol} K(\varepsilon).$$

For the proof we use an economic cap-covering of $K(\varepsilon)$. The results are joint with David Larman.

U. BETKE:

Geometric aspects of a new linear programming algorithm

To solve LP: min $c^T x$, $Bx=b$, $x \geq 0$ we consider the affine plane $E(t)=\{x \mid Bx=b,\ c^T x=t\}$ and the positive ortant $S=\{x \mid x \geq 0\}$. We define $d(t)=d(E(t),S)$ where $d(\ ,\ )$ denotes euclidean distance. Apparently the smallest zero t_0 of $d(t)$ gives the solution of LP. Here we show that the exact t_0 can be computed in finitely many steps by Newton's algorithm and obtain thus an algorithm for LP.

A. BEZDEK:

On common transversals

A finite family A of convex sets in the plane is said to have property T if the family admits a common transversal, that is, if there is a straight line which intersects every member of A. The family A has property $T(m)$ if every m-membered subfamily of A has property T. $L(k)$ will mean that the family splits into k subclasses such that each has property T. With K. Bezdek we proved the followings.

Th. 1. If a finite family of congruent, disjoint circles satisfies T(3), then there exists a line which intersects every circle except at most 18.

Th. 2. For any N≥b there is an arrangement of N congruent, disjoint circles having T(3) but not T(N-1).

Th. 3. In case of a finite family of homothetic convex sets T(3) implies L(4) and T(4) implies L(3).

Th. 4. If a finite family of translates of a convex set has T(3) then it has L(2) too.

K. BEZDEK:

Covering curves by translates of a convex set

In this joint paper with R. Connelly we investigated among others the following problem: What conditions will insure that one convex set can be translated into the other? For instance Wetzel showed that for a given acute triangle, if a closed curve has length equal to or less than the pedal triangle, then the closed curve can be translated into the given triangle. Another example is when the covering set is a circular disk of diameter 1/2. Then any closed (planar) curve of length one or less can be translated into the disk. With the help of the technique of the so called billiard triangles we proved the following.

Theorem: Let X be any compact convex set of constant width 1/2 in the plane. Then any closed curve of length one or less in the plane can be covered by a translate of X. Furthermore, if Y is any compact convex set such that any closed curve of length one or less can be covered by translates of Y, then the length of perimeter of Y is equal to or larger than $\pi/2$ with equality if and only if Y has constant width 1/2.

We looked at other related problems and generalizations regarding translation covers, and mentioned several more results that we could obtain with our technique.

ST. BILINSKI:

Windschiefe archimedische Polyeder höheren Geschlechtes

Die klassische Theorie der Platonischen Körper wurde in einigen der letzten Jahrzehnten schon mehrere Male und von mehreren Authoren im Sinne der topologisch-kombinatorischen und der affinen Geometrie verallgemeinert. Es scheint aber, dass die elementare Theorie der klassischen Archimedischen Polyeder in diesem Sinne noch nicht erweitert wurde.
Es wird versucht diese Theorie auf die topologische und affine Archimedische Polyeder zu verallgemeinern mit dem Hauptziel, die hinreichende Existenzbedingung für das Polyeder $\{C;p\}$ zu finden. Um diese Idee zu realisieren wäre aber notwendig über etwas reicheren Anschauungsmaterial von mehreren verallgemeinerten Archimedischen Polyeder zu verfügen. Bis jetzt waren zu diesem Zweck schon einige besondere Polyederfamilien, z.B. die Familie der quasiregulärer Polyeder aus der Menge aller verallgemeinerten Archimedischen Polyeder untersucht.
Jetzt wird die Familie der windschiefen Polyedern $\{(3,3,3,3,n);p\}$; $n=4,5,6,\ldots$ betrachtet, welche für $n=4$ und $n=5$ die zwei bekannten elementaren windschiefen Archimedischen Polyeder enthällt.

T. BISZTRICZKY:

On the convex hulls of convex sets

Let S be a set of points in the plane, no three collinear and $n\geqq 4$. Then there exists a smallest integer $f(n)$ such that if $|S|\geqq f(n)$, then S contains the vertices of a convex n-gon. It is known that $f(n) \geqq 2^{n-2}$ and conjectured that $f(n)=2^{n-2}$. The conjecture has been proved for $n=4$ and 5.
This problem is generalized as follows. Let F be a family of pairwise disjoint <u>ovals</u> (compact convex sets). Three ovals are <u>collinear</u> if one is in the convex hull of the other two. Let $D=\operatorname{conv}(\bigcup_{A\in F} A)$. Then $X\in F$ is a <u>vertex</u> of D if $X\not\subset \operatorname{conv}(\bigcup_{A\in F\setminus\{X\}} A)$

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and D is an <u>n-gon</u> if it has n vertices.

<u>Theorem</u>: Let F be a family of mutually disjoint ovals no three collinear and $n \geq 4$. Then there exists a smallest integer $g(n)$ such that if $|F| > g(n)$, then F contains the vertices of an n-gon.

We mention that $g(n) \geq f(n)$, $g(4)=5$, $g(5)=8$ and conjecture that $g(n)=f(n)$.

G. BLIND:

<u>Ein Kreisüberdeckungsproblem auf der Sphäre</u>

Untersucht wird, welcher Teil der Sphäre S^2 von n kongruenten Kreisen $\{K_1,\dots,K_n\}$ <u>einfach</u> bedeckt werden kann; dabei sind bei gegebenem $n \in \mathbb{N}$ die Mittelpunkte der Kreise und der Kreisradius variabel. Vorausgesetzt wird, daß kein Kreismittelpunkt im Inneren eines anderen Kreises liegt.
Gezeigt wird: Ist M die von $\{K_1,\dots,K_n\}$ einfach bedeckte Teilmenge der S^2, so gilt für ihren Flächeninhalt

$|M| \leq 12(n-2)F(\frac{\pi}{6}\frac{n}{n-2})$ mit $F(\alpha) := \alpha - \pi + 2\arccos\frac{1}{4\cos\frac{\alpha}{2}}$. Diese Schranke ist für $n \to \infty$ und für $n=3, 4, 6, 12$ scharf; in den Fällen $n=4, 6, 12$ liegen die Kreismittelpunkte in den Ecken eines regulären tetraeders bzw. Oktaeders bzw. Ikosaeders, und jeder Kreis wird von den 3 bzw. 4 bzw. 5 benachbarten Kreisen in den Ecken eines regulären 6-Ecks bzw. 8-Ecks bzw. 10-Ecks geschnitten.

M.N. BLEICHER:

<u>Tightly packing a convex set with similar sets</u>

Let K be a d-dimensional convex body, $n \geq 2$, an integer and $\ell \geq 0$.

Let $K_n^{d,\ell} = \sup\{\frac{\Sigma d(K_i)^\ell}{d(K)^\ell} : K \supset \bigcup_i^n K_i,\ K_i \cap K_j = \emptyset,\ i \neq j, \text{ and } K \sim K_i\}$.

Let $\alpha_n^{d,\ell} = \inf\{K_n^{d,\ell}\}$, $\beta_n^{d,\ell} = \sup\{K_n^{d,\ell}\}$. We wish to determine the values of α and β, and the cases, if any, of equality.
$K_1, K_2,\dots,K_n$ are said to be packed tightly in K iff

$K_n^{d,\ell} = (\sum_i^n d(K_i)^\ell)/d(K)^\ell$. In 1971 Beck and Bleicher (Acta Math. Hung. 22) proved $\alpha_2^{2,1} = 1$ and $\beta_2^{2,1} = 2$ with equality for β iff K is an isoceles right trianle or a parallelogram with side ratio $\sqrt{2}$, and equality for α iff K has constant width or is a regular polygon. Here we show that for the non-trivial case, $0 < \ell < d$, $\beta_n^{d,\ell} = n^{(d-\ell)/2}$, with equality iff K is an n-rep-tile (n-similarity tile), and thus K is a polytope. For $d = 2$, K has at most five edges, with five edges possible only if $n \geq 6$. It is conjectured that pentagons are not possible. It is also shown that $5/4 \leq \alpha_3^{2,1} \leq 6\sqrt{3} - 9$ with equality only for the circular disc.

KÁROLY BÖRÖCZKY:

Packing problems

Let P be a packing of the space A^n with unit balls. Let $\{O_i\}$ and $\{DV_i\}$ be systems of the centres of balls and of Dirichlet-Voronoi cells, respectivily. The centre of a supporting sphere is denoted by C^n.
A k-dimensional sphere $(1 \leq k \leq n)$ is called a k-supporting sphere if it lies on the boundary of a supporting sphere and contains k+1 point of $\{O_i\}$ which do not lie in a (k-2)-dimensional subspace. C^k denotes the centre of a k-supporting sphere. C^k affiliated to DV_j if O_j lies on the previous k-supporting sphere $(C^k \in DV_j)$.
The k-dimensional closeness of the point-system $\{O_i\}$ is defined by

$$\sup_i \max_{C^k \in DV_i} O_i C^k = \rho(n,k).$$

Theorem: The 2-dimensional closeness of a packing of unit balls in 3-dimensional Enclidean space is at least $\sqrt{\frac{3}{2}}$, and equality holds only for the space-centred cubic lattice where the edge-length of the cube is $\frac{4}{\sqrt{3}}$.

J. CHALK:

Exponential sums and volumes of sum-sets

The classical inequalities (asymptotic formula) of Vinogradov, Mordell and Tietävänem are dependent upon upper bounds for

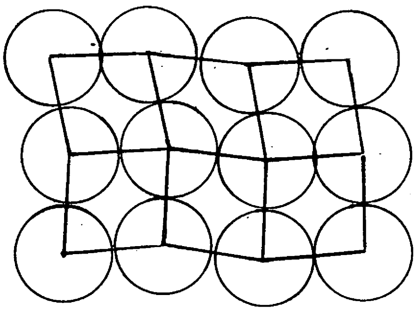
$$V(S+C)-V(S),$$

where V denotes the Jordan contant of a convex set in R^N and $S+C$ denote the vector sum of S and C.

R. CONNELLY:

Uniformly stable circle packings

A packing is finitely stable if every finite number of packing elements is held fixed by the rest. For example the square lattice packing of equal circles in the plane is finitely stable. The following zig zag packing is not finitely stable:



Related to this notion is the following definition.
A packing is ε-uniformly finitely stable if there is no other rearrangement moving a finite number of packing elements less than ε. Thm (Bárány, Dolbilin): The close packing of equal circles in the plane with disjoint holes is uniformly finitely stable. Theorem (with A.Bezdek): The square lattice packing of equal circles in the plane is not uniformly finitely stable.

The idea is to use the zig zag packing to replace a piece of the square packing.

H.E. DEBRUNNER:

Dissection-theoretical analogoues of Schläfli's formula for the volume of orthoschemes

Several geometrical dissections of regular simplexes and cross-polytopes in spherical, euclidean and hyperbolic n-space are considred. They exhibit in a purely combinatorial way volume relations previously deduced by L. Schläfli (Theorie der vielfachen Kontinuität, § 29 and § 31) and by H.S.M. Coxeter with use of differential formulas.

G. FEJES TÓTH:

Packing and covering r-convex domains with unit circles

Let $h(x)$ be the area of the intersection of a circle of unit area and a regular hexagon of area x concentric with the circle. For positive values of d we give a function $r(d)$, $4<r(d)<17$, so that the following holds:

If R is the complement of the union of a set of circles of radius $r(d)$ and S is a system of unit circles such that the density of S with respect to R equals d and each component of R is met by at least two elements of S, then the density σ of the part of R which is covered by the circles of S with respect to R is at most $df(1/d)$.

L. FEJES TÓTH:

Densest packing of translates of a domain

It is known that the density of a packing of translates of a convex domain cannot exceed the density of the densest lattice-packing. An interesting field of research presents itself by trying to find non-convex domains which share this property with convex domains. A few initial results in this direction have been discussed.

A. FLORIAN:

On a metric for the class of convex bodies

Let $\mathcal{K}_n$ be the class of all non-empty compact convex subsets of E^n. Define a distance function on $\mathcal{K}_2$ by setting

$$\rho(K_1,K_2)=2p(\mathrm{conv}(K_1 \cup K_2))-p(K_1)-p(K_2),$$

where $p(M)$ denotes the perameter of M. This definition is extended to $\mathcal{K}_n$ by

$$\rho^W(K_1,K_2)=2W(\mathrm{conv}(K_1 \cup K_2))-W(K_1)-W(K_2), \qquad (1)$$

where

$$W(K) = \frac{1}{|S^{n-1}|} \int_{S^{n-1}} w(K,u)\,d\sigma$$

is the average width of K. It is shown that (1) is infact a metric on $\mathcal{K}_n$. Some properties of this metric are developed. e.g., it is proved that $(\mathcal{K}_n,\rho^W)$ is a complete and locally compact space, and a metricsegment space as defined by K. Menger.

H. HARBORTH:

Packing with regular n-gons

Packings of K regular n-gons in the plane are discussed, where every n-gon has an edge in common with one of the other n-gons, and the packing is connected by edges.

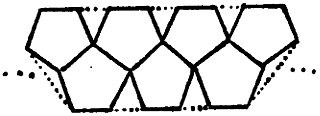
(1) The numbers of different shapes (polynominoes) are hopeless to determine.

(2) The strict Newtonian number $\bar{N}_n$ (maximum number of n-gons, which have an adge in common with a fixed n-gon): $\bar{N}_3=\bar{N}_7=3$, $\bar{N}_4=\bar{N}_8=\bar{N}_9=\bar{N}_{13}=\bar{N}_{14}=\bar{N}_{19}=4$, $\bar{N}_n=6$ for $n\equiv 0 \pmod 6$, $\bar{N}_n=5$ otherwise.

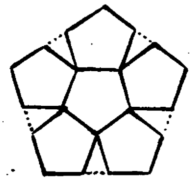
(3) Maximum number $B_n(K)$ of common edges: $B_3(K)=2K-\{\frac{1}{2}(K+\sqrt{6K})\}$, $B_4(K)=2K-\{2\sqrt{K}\}$, $B_6(K)=3K-\{\sqrt{12K-3}\}=B_n(K)$ for $n\equiv 0 \pmod 6$.

Conjecture: $B_n(K)=B_4(K)$ for $n\equiv 4,8 \pmod{12}$, $B_n(K)=B_3(K)$ otherwise.

(4) Minimum area of the convex hull: For triangles only sausages (in linear sequence) are possible only for $K=3$ and $K=5$ ($\Delta\Delta$,$\Delta\Delta\Delta$). For squares only sausages are possible only for K a prime number. For $K=3$ sausages exist only for $n\equiv 1 \pmod 2$ if $K=7,13,19$, for $n\equiv 4 \pmod 6$ if $K=4,10,16$, for $n\equiv 2 \pmod 6$ if $K=8,14,20,26,32,38$. For 5-gons the minimum area is conjectured to occur as follows:



with the only exception:



A. HEPPES:

Packing with rounded figures

One of the well known results of discrete geometry is the upper bound for the packing density of equal circles on the sphere (see e.g. L.Fejes Tóth, Lagerungen in der Ebene, auf der Kugel und im Raum, 1953). It can be reformulated in the following manner:

If $k>1$ circles of radius r are packed on the sphere, then the surface area of the uncovered part is at least $2.(k-2).t$, where t denotes the surface area of the domain surrounded by three mutually touching circles of radius r.

It will be shown that the validity of this theorem can be extended to cases when the domains we pack are not circles and not necessarily congruent. The domains in question have a simply connected interior and a boundary that consists of a finite sequence of (convex or concave) circular arcs of radius r that join under non-convex (inner) angles.

This estimate is sharp in innumerable cases, and proves to be useful in deriving density estimates in even more general cases.

E. HERTEL:

A problem of discrete geometry (Vertex-invariant embedding of regular simplexes in cubes and the geometry of Hamming space)

Let (R,G) be a general Kleinian space, that means, $R \neq \emptyset$ is a set with a group G of transformations of R onto R. In case that R is a metric space any collection $\underline{M}$ of subsets of R is said to be discrete if $\underline{M}$ is locally finite, that is, such that every bounded set meets only a finite number of $\underline{M}$-elements. The group G is said to be discrete if each orbit Gx $(x \in R)$ consists of isolated points. Discrete geometry is the theory of discrete (or finite) systems $\underline{M}$ of geometric objects the theory of discrete (or finite) groups G of transformations the theory of discrete (or finite) spaces R respectively.

The talk discusses a nice example for discrete geometry in this sense namely the connection between the vertex-invariant embedding of a regular d-simplex in the d-cube and the geometry of the finite space $\{0,1\}^d$ with the Hamming metric.

A. IVIC WEISS:

Some infinite families of finite incidence-polytopes

A type of partially ordered structures called incidence-polytopes generalize the notion of polytopes in a combinatorial sense. We discuss the possibility of constructing n-dimensional incidence-polytopes $\{P_1, P_2\}$ with preassigned facets P_1 and vertex-figures P_2. In particular when the facets are taken to be isomorphic to the maps $\{2q,4\}_4$ and $\{2q,3\}_6$ (on surfaces of genus $\binom{q-1}{2}$ and $(q-1)^2$ respectively) and their duals we obtain several infinite families of finite incidence-polytopes.

G.KERTÉSZ:

Packing with translates of a special domain

Let S be a connected union of two translates of a convex domain. Then the density of the densest packing of translates of S equals the density of the densest lattice-packing of translates of S.

W. KUPERBERG:

Packings and coverings of R^2

For every convex plane body K (not necessarily centrally symmetric), let $d(K)$ and $D(K)$ denote the packing and the covering density of K, respectively.

Theorem 1: For every K, $d(K) \geq \sqrt{3}/2=0.866025...$

Theorem 2: For every K, $D(K) \leq 8(2\sqrt{3}-3)/3=1.237604...$

The proofs of these theorems are constructive, and the packings and coverings that this construction produces are of a so called binary-lattice type. Some packings and covering with odd-sided regular polygons are presented to illustrate the method.

E. MAKAI JR:

A lower bound on the number of sharp shadow boundaries of convex polytopes (joint contribution with H. Martini)

The results of our joint paper (same title, submitted to Period. Math. Hungar.) will be presented. Let P be a convex polytope in R^d $(d\geq 2)$, with n facets. We consider light sources x outside P, but on no facet hyperplane of P. By illumination from x some subset $I(x)$ of $Bd\ P$ will be illuminated; its boundary (relative to $Bd\ P$) will be called the shadow boundary of P w.r.t. x. This is a $(d-2)$-complex $C(x)$ (a subcomplex of

P). Denote $\sigma(P)$ the number of all such subcomplexes, as x varies arbitrarily (under the above restriction). We prove

$\sigma(P) \geq 2^{d-2} \sum_{i=0}^{2} \binom{n-d+1}{i} - 1$, with equality if and only if P is a $(d-2)$-fold pyramid over a planar convex $(n-d+2)$-gon. If Q is an unbounded convex polytopal set with $n(\geq 3)$ facets, we have for $\sigma(Q)$ defined in the same way $\sigma(Q) \geq \sum_{i=0}^{2} \binom{n-1}{i} - 1$,

while if the intersection of Q with the infinite hyperplane has dimension $t(\leq d-3)$ $\sigma(Q) \geq 2^{d-t-3} \sum_{i=0}^{2} \binom{n-d+t+2}{i} - 1$. The cases of equality are characterized.

P. McMULLEN:

Realizations of regular polytopes

In a talk in Salzburg a couple of years ago, it was shown that the equivalence classes under congruence of realizations in euclidean spaces of a regular polytope form a closed convex cone. But the theory presented then was incomplete, and consequently to some extent incorrect. The earlier account can now be supplemented, and a finer description of the structure of the realization cone given. In particular, there are new numerical relationships between the symmetry group and realization cone of the polytope.

B. MONSON:

Flexible uniform polytopes

The regular icosahedron $\{3,5\}$ is quite rigid in $\mathbb{R}^3$; but embedded in $\mathbb{R}^4$ it looses its rigidity and can easily be folded into a (starry) $\{3,5/2\}$. In $\mathbb{R}^6$ it is even possible this folding so that at each stage the (skew) icosahedron is regular. Likewise consider a $\{3,3,5\}$ and $\{3,3,5/2\}$ with concentric circumspheres of radii τ and 1. When embedded in $\mathbb{R}^8$ these polytopes can be simultaneously folded and unfolded into a concentric {3,3,5/2} and {3,3,5}. At each stage the (skew) polytopes are regular; "half-way" they are inscribed in the E_8 polytope 4_{21}. These and many other examples follow at once from 'A Family of Uniform Polytopes with Symmetric Shadows', Geometriae Dedicata, 1987(?). A simple manipulation of Coxeter diagrams provides the orthogonal projections necessary to start the folding.

J. PACH:

Regions enclosed by convex plates

Let $f_1,\ldots,f_m$ be (partially defined) piecewise linear functions of d variables whose graphs consist of n d-simplices altogether. We show that the maximal number of d-faces comprising the upper envelope (i.e. the pointwise maximum) of these functions is $O(n^d\alpha(n))$, where $\alpha(n)$ denotes the inverse of the Ackermann function, and that this bound is tight in the worst case. If, instead of the upper envelope, we consider any single connected component C enclosed by n d-simplices in $\mathbb{R}^{d+1}$, then we show that the overall combinatorial complexity of the boundary of C is at most $O(n^{d+1-\varepsilon(d+1)})$ for some fixed constant $\varepsilon(d+1)>0$.

R. POLLACK:

Hadwiger's transversal theorem in higher dimensions

Hadwiger's transversal theorem states that if n disjoint convex compact sets $B_1,\ldots,B_n$ in the plane have the property that every 3 can be met by a directed line consistent with the order $1,\ldots,n$, then there is a line which meets all of them. We (J.E.Goodman and myself) prove that if n "separated" convex compact sets $B_1,\ldots,B_n\subset R^d$ and a labelled configuration of points $C=\{P_1,\ldots,P_n\}\subset R^{d-1}$ have the property that any d+1 of the sets can be met by an oriented hyperplane consistent with the "order type of C" then there is a hyperplane which meets all of them. "Separated" means that no d of the sets are met by a $d-2$ flat. Two configurations $\{P_1,\dots,P_n\}$ and $\{Q_1,\dots,Q_n\}$ have the same "order type" in R^{d-1} if any corresponding d of them $(P_{i_1},\dots,P_{i_d})$ and $Q_{i_1},\dots,Q_{i_d})$ have the same orientation. These are the natural generalizations of disjointness in the plane and order on the line. The key element in the proof is an exchange lemma for minimal Radon partitions.

G. PURDY:

Inequalities involving points lines and planes

Given n points in R^3 forming ℓ lines and p points, we discuss the evidence for our conjecture that $n-\ell+p\geq 0$ if the points do not all lie on two skew lines or one plane. We also can prove this in C^3 if no three points are collinear and n is sufficiently large, using an inequality due to F. Hirzebruch.

Given n points in R^3 and t arbitrary planes, we can show that the number of incidences I satisfies $I\leq c(\log n)^{1/4}n^{3/4}t^{3/4}$ if no three points are collinear, but we think that $I\leq c\, n^{3/4}t^{3/4}$.

P. SCHMITT:

Polymorphic prototiles

A prototile is called k-morphic if it admits precisely k distinct tilings of the plane (see GRÜNBAUM-SHEPHARD, Tillings and patterns. 1986). Since neither the types of the tilings admitted nor the combined incidence symbol provide a satisfactory classification the following definition is proposed:

Two prototiles T and T' are topologically equivalent iff:

(a) The tilings $\tau_i=\{T_{i1}, T_{i2},...\}$ admitted by T and the tilings $\tau_i=\{T_{i1}, T_{i2},...\}$ admitted by T' are topologically equivalent $(i=1,...,k)$: $h_i:\tau_i \to \tau_i'$

(b) there exists a homemorphism $h:T\to T'$ such that for each homemorphism $h_i:\tau_i \to \tau_i'$ $T_{ik} \to T_{ik}'$ and each k there are homeomorphisms φ and ψ such that the following diagram is commutative:

$$\begin{array}{ccc} T_{ik} & \xrightarrow{h_i} & T_{ik}' \\ \varphi \downarrow & & \downarrow \psi \\ T & \xrightarrow[h]{} & T' \end{array}$$

Using some examples of dimorphic prototiles obtained by cutting a centrally symmetric Z-formed prototile into two congruent halves it is shown how their types can be distinguished.

J.J.SEIDEL:

Nice sets in Enclidean $\mathbb{R}^d$

Measures of strength t in $V=\mathbb{R}^d$ generalize spherieal t-designs and cubature formulae for the unit sphere S. In the case of finite support Y on p concentric spheres

$M:=\bigcup_{i=1}^{p} r_i S$, the definition amounts to the isometry of the spaces $Pol_e(Y)$ and $Pol_e(M)$, for $t=2e$. This implies a lower bound for $|Y|$, as a consequence of

$$Pol_e(M) = \sum_{k=0}^{2p-1} Hom_{e-k}(V).$$

Joint work with A. Neumaier and with P. Delsarte.

G.C. SHEPHARD:

Rigid plate frameworks

This lecture gave a partial answer to a problem posed by A. Ehrenfeucht and J. Mycielski in 1981 (American Math. Monthly unsolved problem 6367).

A plate framework in the plane is a collection of plates pivolted together that satisfies the following conditions:

(i) Each plate is a regular n-gon, and all plates are mutually congruent.
(ii) The number of plates is finite.
(iii) No two plates coincide.
(iv) Every vertex of a plate is a pivot.
(v) Every pivot is a vertex of exactly two plates.
(vi) No two pivots coincide.

The problem is to find plate frameworks which are rigid. (In answer to the original problem, a construction was given for a plate framework with 2n n-gons that was not rigid.) The following theorem was proved:

Theorem: There exist infinitely many rigid plate prameworks using triangles, and also infinitely many such frameworks using squares.

It is not known if there exist any rigid plate frameworks using n-gons for $n \geqq 5$.
Generalisations to three dimensional plate frameworks were also briefly mentioned.

J.B. WILKER:

Tiling $\mathbb{R}^3$ with circles and disks

A collection of circles or of disks gives a tiling of $\mathbb{R}^3$ if each point of $\mathbb{R}^3$ belongs to one and only one of the sets in question. We review a number of constructions and results selected to give some idea of the constraints that these tilings can satisfy. For example, while it is not possible to tile the plane with homeomorphs of the closed unit disk, it is possible to tile 3-space with hexagonal polyhedral tiles of this sort.

J.M. WILLS:

<u>On finite packing and covering</u>

The main problens of finite packing and covering in Euclidean d-space E^d are: Determine for a given $k \in \mathbb{N}$ A) the minimal volume of all convex bodies, into which k unit-balls can be packed, B) the maximal volume of all convex bodies, which can be covered by k unit-balls.
It turns out that the expected answer for $d \geq 5$ is completely different from that for $d \leq 4$. For $d \geq 5$ it is conjectured (L. Fejes Tóth, J.M. Wills) that linear arrangements of balls (sausages) are best possible. For $d \leq 4$ there is no such elegant statement. But the socalled sausage-catastrophes (1983) and the space-conjecture (1987) describe the expected behaviour in a satisfactory way.
In the talk we give some new results, which support the above conjectures.

T. ZAMFIRESCU:

<u>Hamiltonian lattice graphs</u>

This joint work with Cristina Zamfirescu desribes sufficient conditions for a grid graph to be hamiltonian. Here a grid graph is a subgraph of the usual planar infinite lattice graph which (i) is connected, (ii) has connected complement, (iii) has connected intersection with any vertical or horizontal infinite path. For one of the three main types of grid graphs our conditions are close to a characterization.

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