MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 1/1988

Mathematische Optimierung

3.1. bis 9.1.1988

Leitung: Bernhard Korte (Bonn)

Klaus Ritter (München)

Auch diese Tagung über mathematische Optimierung hat wieder eine große Anzahl an Forschern aus allen Gebieten der mathematischen Optimierung nach Oberwolfach gebracht. Insgesamt nahmen 66 Teilnehmer an dieser Tagung teil. Etwa die Hälfte der Teilnehmer kamen aus Übersee, ohne finanzielle Unterstützung hiesigerseits, was wiederum den hohen Stellenwert der Oberwolfach-Tagung auch in Nordamerika demonstriert.

In 51 Vorträgen wurde das gesamte Spektrum der mathematischen Optimierung berücksichtigt. Ein Schwerpunkt lag bei Entwicklungen, die sich durch Karmarkars Algorithmus ergeben haben. Entwicklungen, die für die diskrete und für die kontinuierliche Optimierung gleichermaßen von Bedeutung sind.

Der quadratischen Optimierung, dem trust-region-Ansatz und der stochastischen Optimierung waren weitere Beiträge gewidmet. Schließlich wurden auch Optimierungsprobleme, die bei der Lösung von Differentialgleichungen auftreten, und Probleme der automatischen Differentiation, die bei vielen Optimierungsproblemen auftreten, behandelt.

Mehrere Einzelvorträge beschäftigten sich mit Netzwerkflußproblemen, einem Gebiet der diskreten Optimierung, das gerade in den letzten Jahren durch das Zusammenspiel von Problemen der Anwendung und theoretischer Forschung bedeutende Ergebnisse hervorgebracht hat. Darüber hinaus wurden Arbeiten aus dem Gebiet der polyedrischen Kombinatorik vorgestellt und ihre Bedeutung für die Graphentheorie (Spannende Bäume, Travelling-Salesman-Problem) und Kodierungstheorie aufgezeigt. Weitere Vorträge beschäftigten sich mit Matroid- und Ramseytheorie und deren Beziehung zur Optimierung.

Veranstalter und Teilnehmer danken dem Direktor des Mathematischen For-

schungsinstituts Herrn Professor Dr. M. Barner und seinen Mitarbeitern für die gastliche Aufnahme und die ausgezeichnete Betreuung.

Vortragsauszüge

M.J.D. Powell: *An SQP method for linearity constrained optimization*

It is usual to preserve feasibility in algorithms for linearity constrained optimization. Therefore constraint boundaries can cause short steps to be taken by many algorithms that use line searches, making some active set methods inefficient on certain problems. The alternative of taking account of *all* constraints when generating search directions is unattractive, partly because of the extra work. Therefore an algorithm is proposed whose "active set" contains all constraints whose "relative residuals" are less than a tolerance that is large initially and that is reduced automatically. Inequality constraints in this set are treated as inequalities with zero right hand sides. A numerical example illustrates the main purpose of the technique. A Fortran implementation will be available in the IMSL library.

J.E. Dennis: *Trust region subproblems for constrained optimization*

This talk will concentrate on the ideas at issue in extending to constrained optimization the successful trust region methods for unconstrained optimization. The emphasis will be on the relationships between joint work of the speaker with Celis and Tapia and work of Byrd, Schnabel, and Shultz and of Powell and Yuan.

A.R. Conn: *A projection method for the uncapacitated facility location problem*

Several algorithms already exist for solving the uncapacitated facility location problem. The most efficient are based upon the solution of the strong linear programming relaxation. The dual of this relaxation has a condensed form which consists of minimizing a certain piecewise linear convex function. This talk presents a new method for solving the uncapacitated facility location problem based upon the exact solution of the condensed dual via orthogonal projection. The amount of work per iteration is of the same order as that of a simplex iteration for a linear program in m variables and constraints, where m is the number of clients. Moreover, one is able to exploit the inherent problem structure whilst carrying out the numerical linear algebra.

The method is flexible as it can handle side constraints. In particular, when there is a duality gap, the linear programming formulation can be strengthened by

adding cuts. Numerical results for some classical test problems are included as well as some extensions to other problems.
(This is joint work with Gerard Cornuèjols.)

Grötschel: *Stable sets and bounds for block codes*

One of the central problems in coding theory is the following. Given an alphabet with q letters, a word length n, and a minimum distance d, find a block code C (a set of words of length n) such that each two words in C have Hamming distance at least d and such that $|C|$ is as large as possible. This maximum value is denoted by $A(n,d,q)$. The problem of determinig $A(n,d,q)$ can be viewed as a stable set problem for a certain graph $G(n,d,q)$. We show how polyhedral results about the stable set polytope of $G(n,d,q)$ can be used to obtain good upper bounds for $A(n,d,q)$.
(This work is joint with E. Zehendner (Augsburg).)

L.E. Trotter: *Discrete Weyl–Minkowski duality*

An abstract duality model is presented which has as special cases several duality settings of interest in combinatorial optimization: subspace orthogonality, cone polarity, geometric lattice duality, "Hilbert cone" polarity, blocking polyhedra and antiblocking polyhedra. Of particular interest in the algebraic duality of specifying a set using either constraints or generators. Properties of Weyl, Farkas, Minkowski, Lehman and Fulkerson are defined for the general model in analogy with classical results of cone polarity, blocking polyhedra and antiblocking polyhedra. All logical implications among these properties are derived and examples of well–studied duality settings validating these logical relations are given.

The example of polarity of "Hilbert cones" is examined in some detail and it is indicated that the class of finitely generated integral monoids for which linear Weyl–Minkowski duality holds is characterized by homogeneous totally dual integral linear systems. It is also indicated that Weyl–Minkowski duality holds generally for finitely generated integral monoids and finite systems of "Chvátal restrictions"; this latter type of duality is shown to lead naturally to a generalization of the notion of total dual integrality.

The results presented represent joint work with P. Carvalho and J. Ryan.

J. Stoer: *Generalized quadratic programs and ellipsoidal approximations*

Consider generalized quadratic programs

$$\lambda^* = min\ \frac{1}{2}x^T B_o x + b_o^T x \equiv q_o(x)$$
$$x : q_i(x) \equiv \frac{1}{2}x^T B_i x + b_i^T x - \beta_i \leq 0, i = 1, \dots, m,$$

where the B_i are positive semidefinite and the feasible set $P_i := \cap_{i=1}^m K_i$, $K_i := \{x|q_i(x) \leq 0\}$, is compact and has a nonempty interior. Then the *analytic center* $\overline{x} := argmax_{x\in P}\Psi(x) := [\Pi_{i=1}^m(-q_i(x))]^{1/m}$ and the associated ellipsoid $\overline{E} := \{x| -\frac{1}{2}x^T\Psi''(\overline{x})\ x \leq \Psi(\overline{x})\}$ provides an easily computable twosided ellipsoidal approximation for P, which depends analytically of the data of the problem

$$\overline{x} + r_i\overline{E} \subset P \subset \overline{x} + r_a\overline{E}, \quad \text{where} \quad r_i = \frac{1}{2\sqrt{m}}, r_a := m\sqrt{2}, \frac{r_a}{r_i} = \sqrt{8m^3}.$$

This generalizes a result for polyhedra (where one can achieve $\frac{r_a}{r_i} = m - 1$), which was used by Karmarkar to prove the polynomiality of his method to solve linear programs. In a different way the above result can be used to prove the polynomiality of a method for solving generalized quadratic programs, which consists in following the path $x(\lambda), \lambda \downarrow \lambda^*$, of analytic centers of the set $P_\lambda := P \cap \{x|q_o(x) \leq \lambda\}$ belonging to such problems.
(This is joint work with G. Sonnevend)

D.F. Shanno: *A primal–dual interior point method for linear programming*

Karmarkar's projective method for linear programming can be shown to be a special case of the logarithmic barrier method applied to problems in standard form and with a positive barrier parameter. Megiddo has suggested applying the barrier method to the primal and dual problems simultaneously. This generates a primal–dual path following algorithm. The talk discusses numerous issues in implementing this algorithm, including choice of the barrier parameter, step length, artificial variable cost coefficients, incorporation of bounds, and numerical linear algebra considerations. Computational experience will be given and compared with a standard simplex code.

C.L. Monma: *An implementation of a primal–dual interior point method for linear programming*

In this talk we present an implementation and computational results for a primal–dual barrier method for linear programming. This method is compared to our implementation of the dual affine method, and to the simplex method (MINOS 5.0). (This is joint work with Kevin McShane and David Shanno.)

H. Fischer: *Automatic differentiation*

Gradient and Hessian matrix of a real function of several variables play an important role in many numerical methods, especially in nonlinear optimization. But little effort has been devoted to the computation of these entities so far. "The Hessian is not available", this statement used to be an axiom in the optimization folklore for decades. It led to the construction of well–known algorithms for nonlinear optimization problems where the Hessian matrix respectively its inverse is approximated. Nevertheless, we will show how to obtain gradient and Hessian matrix "automatically" in an easy and straightforward manner. No quotients of differences are used. And no manipulation of symbols is involved. Complexity considerations show that "automatic differentiation" is competitive and efficient.

Ch. Kredler: *PADMOS an optimization system using automatic differentiation*

A TURBO–PASCAL program for unconstraint optimization running on PC's under MS–DOS is presented. The derivatives are computed automatically. The function to be minimized has to be typed into a text file. The syntax used is quite similar to the notation of higher programming languages. The infix notation given by a user is transformed into computer adjusted prefix notation which implies a sequence of automatic differentation procedure calls. Gradient and Hessian can be computed accurately up to machine precision. Hence the program provides modified Newton besides BFGS. If the Hessian at some iterate is not positive semidefinite we compute the smallest eigenvalue and move along an according direction of negative curvature.

W.R. Pulleyblank: *Minimum Steiner trees on rectangular grids*

We are given an $m \times n$ rectangular grid G with t terminal nodes designated, all belonging to the boundary of the outer face. A *Steiner tree* is a tree whose node set includes all terminals. A *Steiner cut* is a set S of edges such that each component of $G - S$ contains at least one terminal node. The *rank* of S is one less than the number of these components. We show that the minimum number of edges in a Steiner tree equals the maximum sum of the ranks of a packing of pairwise disjoint

Steiner cuts. In order to prove this result we obtain a structural characterization of minimum Steiner trees which enables us to find such a tree in time $O(t\ log(m \times n))$.

A. Frank: *Disjoint homotopic paths*

N. Robertson and P. Seymour proved the following theorem. Let G be a planar graph with a specified inner face I. Let us be given k pairs of distinct nodes $(s_1, t_1), \ldots, (b_k, t_k)$ so that each s_i is on the outer face O and each t_i is on I. Moreover the order of s_i's are the same as the order of the t_i's. There are k disjoint paths of given homotopy connecting the corresponding terminal pairs iff any node–dual path P contains at least as many nodes as the number of terminal pairs (counted with multiplicity) separated by P.

Here we provide two generalizations of the theorem of Robertson and Seymour.
Thm. 1: Under the conditions above there are $l (l \leq k)$ disjoint homotopic paths iff any system of disjoint node–dual paths, that separate each terminal pair d times, contains at least $l \cdot d$ nodes.
Thm. 2: Given a graph on a torus, there are k disjoint circuits of given homotopy iff the node–dual path condition holds.

A. Schrijver: *Finding disjoint paths in a graph*

We discuss the following theorem, conjectured by Lovász and Seymour: Let $G = (V, E)$ be a planar graph embedded in the plane $\mathbb{R}^2$, let $I_1, \ldots, I_p$ be some of its faces (including the unbounded face), and let $P_1, \ldots, P_k$ be paths in G with end points on the boundary of $I_1 \cup \ldots \cup I_p$. Then there exist pairwise vertex – disjoint simple paths $\tilde{P}_1, \ldots, \tilde{P}_k$ where $\tilde{P}_i$ is homotopic to P_i in the space $\mathbb{R}^2 \backslash (I_1 \cup \ldots \cup I_p)$ if and only if each curve in $\mathbb{R}^2 \backslash (I_1 \cup \ldots \cup I_p)$ intersects "enough" points of G.

Moreover, there exists an $O(|V|^3)$ algorithm finding these paths (if existing), and the result can be extended to trees (instead of paths).

A.H.G. Rinnooy Kan: *Duality and decomposition*

In the framework of the general duality theory developed by Tind and Wolsey, we characterize two canonical decomposition procedures, show that they subsume known special cases such as Benders decomposition and Dantzig–Wolfe decomposition, establish noncycling properties and show that the procedures are mutually dual as well.

J.J. Moré: *Convergence of trust region methods for linearly constrained problems*

The typical convergence result for a trust region method for linearly constrained problems assumes that the subproblems are solved exactly and that the constraints are linearly independent. In this talk we show how to avoid these assumptions and yet obtain strong convergence results. Our approach is geometric. The main tools are the notion of the projected gradient and (dual) nondegeneracy.

E.W. Sachs: *Sequential quadratic programming methods in optimal control*

Optimal control problems can be interpreted as optimization problems in infinite dimensional spaces with nonlinear equality constraints. We extend one version of the sequential quadratic programming methods to cover this class of problems. A convergence analysis is given and the relation to some other approaches is discussed.

K. Truemper: *A matroid technique for analysis of certain minimal violation matrices*

We propose a new matroid technique for the analysis of certain minimal violation matrices. Application of this method to a case of particular interest yields surprisingly simple constructions for several matrix classes, for each of which no construction has been known todate, save complete enumeration. Specifically, constructions are given for the minimal nonregular matrices, the minimal non–totally unimodular matrices, the complement totally unimodular matrices, and the ternary matrices that are almost binary. In addition, we construct a large subclass of balanced matrices.

U. Faigle: *On the clique polytope of a graph*

Consider the collection $\mathcal{K}$ of all subcliques on at least two nodes of a complete graph with edge set E. Each member of $\mathcal{K}$ is represented by a $(0,1)$ – incidence vector of edges in $\mathbf{R}^E$. The polytope $\mathbb{P}$ is defined as the convex hull of those vectors. Then $\mathbb{P}$ has diameter one. This may be proved by embedding $\mathbb{P}$ into a suitable polytope $\mathbb{P}^*$ for which a convenient description by linear inequalities exists.

R.H. Möhring: *A duality theorem for the bump number of a partial order*

The bump number $b(P)$ of a partial order P is the minimum number $b(L,P)$ of comparable adjacent pairs in some linear extension L of P. Its determination is equivalent to *maximizing* the number of jumps in some linear extension of P, for which the corresponding *minimization* problem (the jump number problem) is NP–hard. We derive a polynomial algorithm for determining $b(P)$. It is based on a min–max theorem involving special series–parallel partial orders contained in P as dual objects. The algorithm constructs both a linear extension L and a dual object Q such that

$b(L, P) = b(Q)$, thus proving optimality. As a corollary, we obtain a characterization of all minimal partial orders (with respect to the deletion of comparable pairs) with fixed bump number.

P.L. Hammer: *Completely separable graphs*

We define a property of Boolian functions, called separability, and specialize it for a class of functions naturally associated to graphs. Completely separable graphs are defined and characterized by the existence of two crossing chords in any cycle of length at least five, implying the perfectness of these graphs. Polynomial algorithms are provided for the recognition of completely separable graphs, and for the detection of maximum weighted stable sets, maximum weigthed cliques, and for the chromatic number of such graphs.

R.E. Burkard: *On shortest polygonal lines*

We consider the following problem: Let m points $P_1, P_2, \ldots, P_m$ be given in $\mathbb{R}^n$ with $P_i \neq P_{i+1}, P_{i+2}$. Find points $Q_i \in [P_i, P_{i+1}]$ such that $\sum \|Q_{i+1} - Q_i\| \rightarrow min$. We consider two versions of this problem: the open problem and the closed problem where we have the additional requirement $P_{m+1} = P_1, Q_{m+1} = Q_1$. As special case this problem contains the task to find an inpolygon of a convex polygon with shortest circumference. This special case has been solved by J. Focke (optimization, 1986) using Schwarz' principle of reflective evolution. We show here that using this principle also the general (open or closed) problem can be solved by a low–order algorithm and that even side constraints like fixed points can be taken into account.
(This is joint work with Yao En–yu.)

B. Simeone: *Most uniform path partitioning*

Let P be a node–weigthed path with n nodes. Given $b > a > 0$, can one find a partition of P into subpaths, such that the total weigth of every subpath lies between a and b? The number of subpaths may be prescribed in advance or not. We present linear–time algorithms for these problems. Our approach combines a *pre–processing* procedure, which detects "obstructions", if any, via a sequence of node compressions; and a *greedy* procedure, which actually finds the desired partition. We also describe an $O(n^3)$ algorithm, relying on the above procedures, for finding a partition that minimizes the difference between the largest and the smallest weight of a subpath.
(Joint work with M. Lucertini and Y. Perl)

S.M. Robinson: *Generalized equations: implicit functions and continuation methods*

We give an overview of several areas of analysis of generalized equations (inclusions of the form

$$0 \in f(x) + N_C(x)$$

where f is C^1, C is polyhedral in $\mathbf{R}^n$, and N_C is the normal cone) currently being investigated by the lecturer and his students. These include some numerical methods developed from the analytical foundation already in place.

R. Fletcher: *Low storage methods for unconstrained optimization*

These methods aim to improve on c.g. methods in circumstances where there are more than $4n$ storage locations available, but less than the $\frac{1}{2}n^2 + O(n)$ required to run the BFGS method. Two new approaches are derived. One is a non–cyclic method which uses the most recent m difference pairs. A particular advantage of this approach is that it provides a mechanism for limiting the number of difference pairs if the quality of the information is poor. The second approach is interesting in that it can be equivalent to the BFGS method if less than $\frac{1}{2}n^2$ locations are available but some symmetry is present. It is based on updating partial factors of a reduced inverse Hessian matrix, and requires half the storage of the first approach. Evaluation of the methods is currently in progress but there are indications that the second method should be used in conjunction with periodic restarts.

M. Fukushima: *A successive quadratic programming method for a class of constrained nonsmooth optimization problems*

In this paper we present an algorithm for solving nonlinear programming problems where the objective function contains a possibly nonsmooth convex term. The algorithm successively solves direction finding subproblems which are quadratic programming problems constructed by exploiting the special feature of the objective function. An exact penalty function is used to determine a step–size, once a search direction thus obtained is judged to yield a sufficient reduction in the penalty function value. The penalty parameter is adjusted to a suitable value automatically. Under approximate optimal solution to the problem with any desirable accuracy in a finite number of iterations.

M.J. Todd: *A centered projective algorithm for linear programming*

We describe a projective algorithm for linear programming that shares features with Karmarkar's projective algorithm and its variants and with the path – following methods of Gonzage, Kojina – Mizuro – Yoshise, Monteiro – Adler, Renegar, Vaidya and Ye. It operates in a primal–dual setting, stays close to the central trajectories and converges in $O(\sqrt{n}L)$ iterations like the latter methods. (Here n is the number of variables and L the input size of the problem.) However, it is motivated by seeking reduction in a suitable potential function as in projective algorithms, and the approximate centering is an automatic byproduct of our choice of potential function. (This is joint work with Yingu Ye.)

R.E. Bixby: *A compact 'C' implementation of the simplex method*

A *'C'* implementation of the simplex method, developed by the speaker, was described. Limited test results run on a SUN 3/50 workstation showed the code using about 1/2 as many iterations and less than 1/2 the time in comparison to Roy Marston's XMP. Topics discussed were: (1) Factorization and factorization update, (2) choice of leaving, variable (which uses a piecewise linear objective in phase 10), (3) crash procedure, (4) pricing, and (5) scaling.

L. Collatz: *Optimierungsprobleme bei Differentialgleichungen mit Anwendungsbeispielen*

Die Lösung von Randwertaufgaben bei linearen und nichtlinearen gewöhnlichen und partiellen Differentialgleichungen, bei Integralgleichungen und anderen Funktionalgleichungen kann oft auf semiinfinite Optimierungsaufgaben zurückgeführt werden. Bei nicht zu komplizierten Problemen, bei denen Monotoniesätze gelten, sind bei Verwendung der *punktweisen* Monotonie und der Tschebyscheff – Approximation die Optimierungsmethoden die einzigen, welche mit erträglichem Rechenaufwand auf Computern *garantierbare untere und obere Schranken für die Lösung* liefern. Das wird an vielen in neuerer Zeit behandelten Problemen vorgeführt. Numerische Testbeispiele bei elliptischen, hyperbolischen (Blow up – Probleme), parabolischen Gleichungen, bei Eigenwertaufgaben, freien Randwertproblemen, Rand – Singularitäten u.a. illustrieren die Methode.

J.B. Rosen: *Minimum norm solution to the linear complementarity problem*

The general Linear Complementarity Problem (LCP) is considered, with no special properties assumed for the $n \times n$ matrix M. An equivalent $0-1$ mixed integer program (MIP) is formulated which always has an optimal solution. This optimal solution either shows that no solution to LCP exists, or gives a minimum norm solution to the LCP. Computational results for 280 randomly generated problems are presented which demonstrate the practical implementation of this equivalent MIP, and illustrate some interesting properties of these problems as a function of n.

D. Pallaschke: *Higher order derivatives for quasi–differentiable functions*

Let $U \subseteq \mathbb{R}^n$ be an open set, $x_o \in U$ and $f : U \longrightarrow \mathbf{R}$ a continuous function. Then f is said "quasi– differentiable in x_o" if the following two conditions are satisfied: i) for every $y \in \mathbf{R}^n$ the directional derivative $\frac{d\,f}{d\,y}|_{x_o}$ exists and ii) there exist two compact convex sets $\underline{\partial} f|_{x_o}, \overline{\partial} f|_{x_o} \subseteq \mathbb{R}^n$, such that

$$\frac{d\,f}{d\,y}|_{x_o} = sup_{v \in \underline{\partial} f|_{x_o}} < v, y > + inf_{w \in \overline{\partial} f|_{x_o}} < w, y > .$$

The theory of quasi–differentiable functions is developed by V. Demyanov and A. Rubinov. One of the most essential points in this theory is the existence of a full differential calculus.

If we set $\mathcal{D}(\mathbb{R}^n) := \{\varphi = p - y \mid p, y \underset{sublinear}{:\mathbf{R}^n \to \mathbb{R}_+}\}$ which is a lattice, then obviously this definition is equivalent to the fact, that $y \longmapsto \frac{df}{dy}|_{x_o} \in \mathcal{D}(\mathbb{R}^n)$. The following result holds:

Theorem: *Let $U \subseteq \mathbb{R}^n$ be an open set, $x_o \in U, f : U \to \mathbb{R}$ locally lipschitz.*

The following assertions are equivalent

(i) f is quasi–differentiable in x_o

(ii) there exists a $\varphi \in \mathcal{D}(\mathbb{R}^n)$ with the following property: for every $\epsilon > o$ there exists a $\delta > o$, such that for all $h \in \mathbf{R}^n$ with $\| h \| \leq \delta$ and $x_o + h \in U$:

$$|f(x_o + h) - f(x_o) - \varphi(h)| \subseteq \epsilon \, \| h \| \, .$$

This leads to the following notation: "$df|_{x_o} := \varphi$." For $n = 1, \mathcal{D}(\mathbb{R}) = \{\varphi(x) = ax + b\,|x| \mid a, b \in \mathbb{R}\}$. If $f : [a, b] \to \mathbf{R}$ is quasi–differentiable, then we write

$$df \Big|_{x_o}(x) = \Big(\frac{\delta f}{\delta x}\Big|_{x_o}\Big)x + \Big(\frac{\delta f}{\delta\,|x|}\Big|_{x_o}\Big)\Big| \;\; x\Big| .$$

In this notation, the mean value theorem is: there exists a $c \in (a, b)$ such that

$$\left|\frac{f(b) - f(a)}{b - a} - \frac{\delta f}{\delta x}\Big|_c\right| \;\; \leq \left|\; \frac{\delta f}{\delta\,|x|}\Big|_c \;\;\right| .$$

For the second order directional derivative, we propose the following definition:

Let $f : U \to \mathbb{R}$ quasi–differentiable, then

$$df : U \to \mathcal{D}(\mathbb{R}^n)$$

and for $y \in \mathbb{R}^n$, we put:

$$\frac{d}{dy}(df)\mid_{x_o} := lim_{\alpha\to 0_{\alpha>0}} \frac{1}{\alpha}\left(df\Big|_{x_o+\alpha y} - lim_{\beta\to 0_{\beta>0}} df\Big|_{x_o+\beta y}\right)$$

If $\mathcal{D}_2(\mathbb{R}^n) := \{\Phi = P - Q \mid P, Q : \underset{sublinear}{\mathbb{R}^n \to \mathcal{D}(\mathbb{R}^n)}\}$ then we require $y \longmapsto \frac{d}{dy} df\mid_{x_o} \in \mathcal{D}_2(\mathbb{R}^n)$.

Theorem: *For every $\varphi \in \mathcal{D}(\mathbb{R}^n)$ we have*

$$d^2\varphi = o.$$

T.M. Liebling: *Radio channel assignment in Latin America*

This talk is based on joint work with S. Buden, H. Telley and M. Giroux (VIT). We consider the problem of allotting c adjacent frequency channels to zones of a number of countries. The allotment should be "equitable", while minimizing inter – country interferences. The combinatorial optimization problems arising, are akin to node coloring problems on fairly large sized graphs. The supplementary conditions give heuristics with greater efficiency than originally expected: Allotments for Latin America were determined s.t. no "strong" inference occured. This amounts to coloring the nodes of graphs with over 1000 nodes and 10000 edges with 10 colors. Getting the combinatorial optimization problem involves using state of the art methods from computational geometry.

P. Bruckner: *K–optimal solution sets for some polynomially solvable scheduling problems*

A k–optimal solution set $\Omega = \{S_1, \ldots, S_K\}$ of a given scheduling problem is a set of schedules such that no schedule S which is not contained in Ω is better than any $S_i, i = 1, \ldots, k$. K–optimal sets are useful in offering alternatives to a single optimal schedule. Algorithms are presented which compute k–optimal sets for various scheduling problems for which the optimal schedule can be computed in polynomial time. These algorithms are based on general approaches due to Murty (1968) and Gabow (1977).

H.J. Prömel: *Sparse Ramsey theorems and a conjecture of Spencer*

The by now classical theorem of Hales and Jewett asserts that for every finite alphabet A and for every color–number r there exists n such that for every coloring $\Delta : A^n \longrightarrow r$ there exists a line $L \subseteq A^n$ which is monochromatic. We prove a sparse version of this theorem showing that for every k, r there exists already n and $\mathcal{N} \subseteq A^n$ satisfying girth $(A\binom{\mathcal{N}}{1}) > k$, hence being sparse, such that still the conclusion of Hales – Jewett's theorem is valid. Thereby $A\binom{\mathcal{N}}{1}$ denotes the set of all lines completely by contained in $\mathcal{N}$. As an application of this result we confirm a conjecture of J. Spencer. (This work was done jointly with B. Voigt (Bielefeld).

R.A. Tapia: *The Karmarkar algorithm for linear programming problems*

In this work we demonstrate that the now famous Karmarkar algorithm for linear programs results from the classical approach of first transforming nonnegativity constraints into equality constraints by adding squared – slack variables and then applying the method of successive linear programming.

D. Goldfarb: *An $O(n^3L)$ primal interior point algorithm for convex quadratic programming*

We present a primal interior point method for convex quadratic programming which is based upon a logarithmic barrier function approach. This approach generates a sequence of problems which are approximately solved by taking one projected Newton step in each. It is shown that the method requires $O(\sqrt{n} \cdot L)$ iterations. By using modified projected Newton steps the amount of arithmetic operations required by the algorithm can be reduced to $O(n^3L)$.

V. Kovacevic–Vujicic: *The rate of convergence of Karmarkar's algorithm*

The asymptotic behaviour of Karmarkar's method is studied. It is shown that the direction vector $c_p(x^K) \to O, K \to \infty$, and

$$\frac{c_p(x^K)}{c^T x^K} \to (\underbrace{-\frac{1}{n}, \ldots, -\frac{1}{n}}_{r}, \underbrace{\frac{1}{n-r} - \frac{1}{n}, \ldots, \frac{1}{n-r} - \frac{1}{n}}_{n-r}), K \to \infty,$$

where $\bar{x} = (\overline{x_1}, \ldots, \overline{x_r}, 0, \ldots, 0), \overline{x_1} > 0, \ldots, \overline{x_r} > 0$ is the unique nondegenerate solution of the problem. Using this result we derive the rate of the objective function value decrease:

$$\frac{c^T x^{K+1}}{c^T x^K} \to \frac{\sqrt{(n-1)(n-r)r} - \alpha r}{\sqrt{(n-1)(n-r)r} + \alpha(n-r)}, K \to \infty,$$

where $\alpha < \frac{1}{3}$ is the step size of Karmarkar's method. Some results on the ill – conditioness arising in the case of degeneracy are also presented.
(jointly work with M.D. Asic and Mirjana Radosavljeviċ–Nikoliċ)

R.J.B. Wets: *Quantitative stability theory for optimization problems*

Much of the research on stability has been concerned with topological properties: continuity of the infimum value, semicontinuity of the argmin set–function. In this presentation, I am interested in results that can be used to estimating rates of convergence, error bounds for approximating problems, etc. This requires a notion of "distance" between optimization problems. Various candidates are examined, and then used to obtain lipschitzian and hölderian properties for the solution set.

P. Kall: *Upper bounds for the expected recourse function in stochastic programming, which are needed to design solution procedures*

Because the well–known difficulties to evalute the expected recourse function, solution methods have been proposed using – in the convex case – Jensen's inequality for lower bounds and Edmundson–Madansky's and other integral inequalities for upper bounds, which amounts to an outer and inner linearization, respectively, of the expected recourse. Here the upper bounds correspond to the solution of various (generalized) moment problems. We shall review therefore results on moment problems relevant for our problem with the aim to develop upper bounding techniques which are efficiently manageable and still tight enough for computational purposes.

A. Prekopa: *Linear programming solution of moment problems and applications to stochastic programming*

The moment problem is formulated as an optimization problem where the probability content or some other characteristic of the distribution is minimized (maximized) subject to constraints prescribing a finite number of moments. The vertices of the dual problem are found and an elegant dual algorithm is proposed to solve the problem. As byproduct, we get lower and upper bounds for the expectation of functions of random variables. The results can efficiently be used to solve stochastic programming problems where the right hand side vector in the underlying deterministic LP Problem is random. The problem, where a probabilistic constraint is also prescribed can be solved too by the use of the same methodology.

T. Magnati: *Polyhedral structure of the capacitated minimal spanning tree problem*

The uncapacitated minimal spanning tree problem, which is one of the few core

models in combinatorial optimization, has been extensively studied and is well solved. In practical settings – for example, computer network design or vehicle routing – this model arises with the added wrinkle: the number of nodes (more generally the total "node weight") in any subtree off the root is limited by a specified capacity K. This more general capacitated version of the problem is computationally difficult (NP–hard). We consider the polyhedral structure of this capacitated problem, introducing several facets for it and giving a complete polyhedral description of the problem for the special case $K = 2$ (which can be solved efficiently as a nonbipartite matching problem).
(This talk reports on joint work with Leslie Hall).

J. Fonlupt: *Disconnecting stable sets in perfect graphs and parity graphs*

Let $G = (V, E)$ be a graph with a minimal disconnecting stable set S. Let us assume that $G(V|S)$ has two connected components $G(V_1)$ and $G(V_2)$. $G_1 = G(V_1 \cup S)$ and $G_2 = G(V_2 \cup S)$. We note $G = G_1 \Phi G_2$. A. Tucker proved that if G_1 and G_2 are perfect graphs G is perfect provided that G has no odd chordless cycle. We prove that if a certain condition holds on the parity of the minimal chains linking any pair (s, t) of vertices of S, there exist parity graphs $H_1 = (W_1 \cup S, F_1)$ and $H_2 = (W_2 \cup S, F_2)$ such that $G = G_1 \Phi G_2$ is perfect if and only if $H_1 \Phi G_2$ and $H_2 \Phi G_1$ and $H_1 \Phi H_2$ are perfect.
(jointly work with D. Corneil, University of Toronto.)

L.A. Wolsey: *Economic lot–sizing with start–up costs*

We consider the uncapacitated lot–sizing problem with the addition of start–up costs. A family $\mathcal{F}$ of strong valid inequalities is presented, as well as two reformulations of the facilities location and shortest path type that are at least as strong as that obtained from $\mathcal{F}$. Computational experiments with the linear programming relaxations of these models always lead to solutions of the original problem, and it is conjectured that the family $\mathcal{F}$ gives the convex hull of solutions. Secondly we examine multi–item problems with changeover costs between items where exactly one item can be produced per period. A formulation including start–up variables and the family $\mathcal{F}$ turns out to be computionally effective.

U. Zimmermann: *Search directions for the projective method of DeGhellinck and Vial*

The projective method of DeGhellinck and Vial in equivalent to Karmarkar's algorithm when starting with a feasible point, but it allows to treat feasibility and optimality simultaneously. We discuss the quality of search directions differing from the usual Newton direction in order to enable large step sizes. It turns out that close to the Newton direction complexity remains unchanged. The more surprising result seems to be that any direction when projected to some affine subspace yields a polynomial algorithm (complexity better than $O(h^3 l)$ iterations). Thus, in a practical implementation one may choose from a large set of "polynomial" directions. Using the best among several directions will improve bounds as well as step size.

E. Tardos: *Generalized network flows*

We consider the generalization of the usual network flow problem to the case where there is a loss or gain factor associated with each edge. We present two combinatorial approaches to solve the maximum flow problem in generalized networks in polynomial time. Previously the only polynomial time algorithm known used general linear programming techniques. The running time of the combinatorial algorithms is comparable to the LP algorithms. The talk is based on joint work with Andrew Goldberg and Serge Plotkin.

A. Recski: *Inequality systems and bipartite matching*

Consider a 1–story building consisting of $k \times l$ square rooms. If the vertical rods are fixed to the earth via joints and all the four external vertical walls contain a diagonal rod then $k+l-2$ diagonal rods or $k+l-1$ diagonal cables in the roof are known to be enough to make the whole building globally rigid (Bolker and Crapo, 1977 for rods; Chakravarty, Holman, McGuinness and Recski; 1986 for cables). In the former case (where rods imply equalities among certain deformations) the minimum systems were identified with certain 2–component forests of the complete bipartite graph $K_{k,l}$. In the latter case (where cables imply inequalities only) the characterization of the minimum systems seems to be unsolved. Two partial results are presented: when the corresponding subgraph is not circuit–free, and when all the cables are parallel. In this latter case a Hall–type condition is deduced and the problem reduces to the existence of a perfect matching in an auxiliary bipartite graph.

C. Lemaréchal: *A combination of bundle and trust region approaches*

To minimize a function f, the trust region approach consists in moving from the current iterate x_k to the solution $x_{k+1} = x(\delta)$ of

(1) $$min \{M_k(x) : 1/2 |x - x_k|^2 \leq \delta\}$$

where M_k is a *model* function approximating f near x_k. A suitable value of δ cannot be guessed and thus must be computed "on line" for each k, using observations of

f–values. When f is not C^1, the model is a piecewise linear function:

(2) $$M_k(x) := max\ \{f(x_i) + (g_i, x - x_i) : i = 1 \ldots k\}.$$

An equivalent formulation of (1) is then (u is the multiplier)

(3) $$min\ M_k(x) + 1/2\ u\,|x - x_k|^2$$

which can be interpreted as minimizing a refined model ((2) is a poor approximation of f). In (3), u must again be chosen at each iteration. Numerical experiments indicate a definite improvement over classical line–searches. The implementation is delicate, however, because $f[x(\delta)]$ is highly oscillating.

G. Di Pillo: *Exact penalty functions and exact augmented Lagrangian functions in NLP*

We introduce formal definitions of exactness for penalty functions and we state sufficient conditions for a penalty function to be exact according to these definitions. In this framework we analyse the best know classes of exact penalty functions and we establish new results concerning the correspondence between the solutions of the constrained problem and the unconstrained minimizes of the penalty functions. Finally we show that exact augmented Lagrangian functions can also be analysed in the same framework.

M.D. Grigoriadis: *ONETHIRD: a new class of matching heuristics*

We consider complete graphs of n vertices, n even, and edge weights that satisfy the triangle inequality. For each nonnegative integer $k \leq log_3 n$ we state an $O(max\ \{n^2, t_A (3^{-k}n)\})$–time heuristic matching algorithm with an error bound of $(\frac{1}{3})^k(1+f_A(3^{-k}n))-1$. The time $t_A(n)$ and the error bound $f_A(n)$ refer to those for any perfect matching algorithm applied to a problem of n vertices. We also consider the special case of matching n points in $\mathbb{R}^p$ in the L_q–norm. The class of heuristic algorithms we present in this paper have better running times and/or error bounds than existing matching heuristics.
(Joint work with B. Kalantari).

T. Ibaraki: *Resource allocation problems*

The resource allocation problem addressed here is a simple nonlinear (integer) programming problem:

$$\text{minimize}\quad f(x_1, \ldots, x_n)$$
$$\text{subject to}\quad \sum x_j = N,\ x_j \geq 0, j = 1, \ldots, n.$$

Variables can be continuous or integer. Due to its simple structure, there are various applications, e.g., load distribution, production planning, computer scheduling, portofolio selection and apportionment. Efficient algorithms are known, depending upon the form of objective function, e.g., separable, convex, minimax, maximin, fair or fractional, and the form of constraint, continuous or discrete. This talk surveys algorithms for these problems, putting emphasis on discrete cases with minimax, maximin and fair objective functions.

F. Barahona: *Compact systems for some combinatorial optimization problems*

We prove that the following problems can be formulated as a linear program with a polynomial number of variables and a polynomial number of inequalities:

- Max Cut in graphs not contractible to K_5.
- Chinese Postman Problem in planar graphs.
- Perfect Matching in planar graphs.

W. Cook: *On the travelling salesman polytope*

Gomory's cutting–plane technique can be viewed as a recursive procedure for proving the validity of inequalities over the set of integer vectors in a given polyhedron. The number of rounds of cutting planes needed to obtain all valid inequalities is known as the rank of a polyhedron. We show that the rank of the subtour relaxation of the travelling salesman problem grows at least linearly with the number of cities, settling conjectures of Chvátal and Grötschel and Pulleyblank.
This talk is based on joint work with V. Chvátal and M. Haremann.

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