MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 2/1988

Modelltheorie

10.1. bis 16.1.1988

Die Tagung fand unter der Leitung von W.Hodges (London), A.Prestel (Konstanz) und M.Ziegler (Bonn) statt. Wie frühere Modelltheorietagungen hatte sie den Charakter einer Arbeitstagung. Der Schwerpunkt lag auf der Modelltheorie der Moduln und der Entscheidbarkeit der Theorie von Moduln über endlich dimensionalen Algebren. Zu diesen Themen wurden in den Vormittagssitzungen ausführliche Vorträge* von A.Pillay, M.Prest, C.M.Ringel und Ph.Rothmaler gehalten. In den Nachmittagssitzungen, die thematisch nicht fixiert waren, fanden weitere Vorträge zu verschiedenen Fragestellungen der Modelltheorie statt.

Vortragsauszüge

Andreas Baudisch:

On two hierarchies of dimensions

Let T be a countable complete, ω-stable and non-multidimensional theory. By Lascar in T^{eq} there is in every dimension of T a type with Lascar rank ω^{α} for some α. We give sufficient conditions for coinciding of α with level of that dimension in Pillay's RK-hierarchy of dimensions computed in T^{eq}. In particular this is fulfilled for modules.

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* In den folgenden Vortragsauszügen mit "*" gekennzeichnet.

Steven Buechler:

Vaught's conjecture for weakly minimal theories

After giving some basic definitions I state a conjecture representing Saffe's approach to Vaught's conjecture for weakly minimal theories (the simplest class of superstable, non ω-stable theories). By results of Hrushovski and myself it suffices to prove this conjecture for weakly minimal abelian structures. I then describe what is known about such structures: that they are descending chains of vector spaces over increasingly large finite fields with some additional partially defined homomorphisms. A special case under which Saffe's conjecture can be proved for such structures is discussed.

Gregory Cherlin:

Homogeneous Tournaments

A directed graph is homogeneous if any isomorphism between two of its finite subgroups is induced by an automorphism. The homogeneous tournaments were classified by Lachlan, and using his methods I have classified all homogeneous directed graphs. There are $2^{\aleph_0}$ such directed graphs, of which five are tournaments. Most of them had been constructed previously. I present the essential features of Lachlan's approach in a forthcoming article in Geometriae Dedicata.

Lou van den Dries:

Elimination Theory for the ring of algebraic integers

Let R be the ring of algebraic integers. Making essential use of Rumely's Density Theorem on absolutely irreducible varieties one can derive:

Theorem: Each formula $\phi(A)$, $A = (A_1, \ldots, A_M)$ in the language of rings is effectively equivalent in the ring R to a finite disjunction of formulas of the form $\exists Z (Z^e + p_1(A)Z^{e-1} + \cdots + p_e(A) = 0 \wedge \psi(A, Z))$ where $p_i(A) \in \mathbb{Z}[A]$ and $\psi(A, Z)$ is a quantifier free formula in the language of rings augmented with predicates $rad_{k,l}$ $(k, l \geq 1)$ to be interpreted in R as follows:

$$rad_{k,l}(a_1, \ldots, a_k, b_1, \ldots, b_k, c_1, \ldots, c_l, d_1, \ldots, d_l)$$
$$\iff ((a_1) : (b_1)) \cdot \ldots \cdot ((a_k) : (b_k)) \subset \text{radical}((c_1) : (d_1) + \ldots + (c_l) : (d_l)).$$

Corollary: The ring R is decidable.

This answers a question posed by Julia Robinson in 1965.

Paul C. Eklof:

Incompactness Spectra of Modules

Say that an R-module A is *almost free* if there is a closed unbounded family of free submodules of A each of cardinality $< |A|$. Let $Inc'(R) = \{\kappa \geq \aleph_1$: there is an almost free module of cardinality κ which is not free$\}$. Using the machinery developed by Shelah in his paper "Incompactness in Regular Cardinals", Mekler and I show that if R is not left perfect then $Inc'(\mathbb{Z}) \subseteq Inc'(R)$. On the other hand, if R is left perfect, then $Inc'(R) = \emptyset$.

Yu.L.Ershov:

Σ-definable abelian groups

The notion of Σ-definability was introduced by the author in: *Σ-definability in admissible sets.* Dokl.A.N.USSR **285**(1985),792-795.

Let T be a (fixed) complete decidable $\omega(\omega_1)$-categorical theory,$\mathcal{N}, \mathcal{N}' \in \text{Mod}\,\mathcal{T}$.

Proposition 1: If $\mathcal{M}$ is Σ-definable in HF ($\mathcal{N}$) then $|\mathcal{M}| \leq \omega$ or $|\mathcal{M}| = |\mathcal{N}|$.

Proposition 2: If an uncountable $\mathcal{M}$ is Σ-definable in HF ($\mathcal{N}$) then for every $\mathcal{N}'$ there is $\mathcal{M}' \simeq \mathcal{M}$ such that $\mathcal{M}'$ is Σ-definable in HF ($\mathcal{N}'$) and $|\mathcal{M}'| = |\mathcal{N}'|$.

Fact: A countable $\mathcal{M}$ is Σ-definable in HF ($\mathcal{N}$) iff $\mathcal{M}$ is constructivizable.

Proposition 3: If an abelian group $\mathcal{A}$ (a Boolean algebra $\mathcal{B}$) is constructivizable then the conclusion of proposition 2 is true for $\mathcal{A}$ ($\mathcal{B}$).

Proposition 3 does not hold for some constructive $\mathcal{M}$.

Conjecture: If T_{dl} is the theory of dense linear orders (without ends) then for every model $\mathcal{M}$ with ω_1-categorical decidable theory $\mathcal{M}$ is Σ-definable in HF ($\mathcal{L}$) for some $\mathcal{L} \in \text{Mod}\,T_{dl}$.

David M. Evans:

The Small Index Property for ω-categorical Structures

A (countable, ω-categorical) first-order structure $\mathcal{A}$ on a set Ω having automorphism group Γ has the *small index property* if, whenever H is a subgroup of index less then 2^ω in Γ, there exists a finite subset X of Ω such that H contains the pointwise stabiliser in Γ of X. Examples of countable ω-categorical structures having the small index property include a vector space of dimension $\aleph_0$ (over a finite field); symplectic, orthogonal and unitary spaces (dimension $\aleph_0$ over a finite field);

and countable ω-categorical abelian groups. The proofs of these results follow the same pattern, and we sketch the proof for vector spaces.

Rüdiger Göbel:

Vector spaces with distinguished subspaces

Let F be a free R-module over a commutative ring R. Let $\underline{F} = (F, F_1, \ldots, F_5)$ denote a free R-module F with 5 distinguished subspaces F_i. Let fin denote the finite topology on $\mathrm{End}_R F$ which is generated by all annihilators of finite subsets of F. Clearly, (End F, fin) is a complete Hausdorff R-algebra and End $\underline{F} = \{\sigma \in$ End $F; F_i\sigma \subseteq F_i\}$ is a closed subalgebra. We prove the converse:

Theorem: Let A be a closed subalgebra of (End V, fin) and F, V any free R-modules with rank$(F) \geq$ rank(V). Then we can find 5 subspaces of $V \otimes F$ such that End $(\underline{V \otimes F}) = A \otimes id_F$.

This has many applications in module theory which partly have been discussed.

Frieder Haug:

Cancellation and elementary equivalence of groups

We prove the following Theorem:

Theorem: Let A, B be groups and C a torsion free abelian group of finite rank with $A \oplus C \simeq B \oplus C$. Then $A \equiv B$.

By a Theorem of Goodearl $A \oplus C \simeq B \oplus C$ for a torsion free abelian group of finite rank C implies that there exists a $t \in \mathbb{N}$ with $A^t := \bigoplus_{1 \leq i \leq n} A \simeq B^t$. So the Theorem above is a further step to a conjecture of Sabbagh (s.Manevitz, Isr.J.Math. 1985) which says that $A^t \simeq B^t$ for a $t \in \mathbb{N}$ implies $A \equiv B$. It generalizes the following result of Oger: if $A \oplus \mathbb{Z} \simeq B \oplus \mathbb{Z}$ then $A \equiv B$. We sketch the proof of the Theorem for groups of rank 1.

Franz-Viktor Kuhlmann:

Model theory of valued fields: Ax-Kochen-Ershov-principles in characteristic $p > 0$

Definition: An elementary class $\mathcal{K}$ of (nontrivially) valued fields satisfies the Ax-Kochen-Ershov-principle (AKE) if for every pair $(K, v) \subset (L, v)$ with (K, v), $(L, v) \in \mathcal{K}$ the following holds: $\bar{K} \prec_\exists \bar{L}$ and $v(K) \prec_\exists v(L)$ implies $(K, v) \prec_\exists (L, v)$.

Here $\prec_\exists$ denotes "existentially closed in", $\bar{K}$ denotes the residue field and $v(K)$ the value group.

It is well-known that $\mathcal{K}_1 = \{$henselian fields (K,v) with char $\bar{K} = 0\}$, $\mathcal{K}_2 = \{$henselian finitely ramified fields$\}$ and $\mathcal{K}_3 = \{$algebraically maximal Kaplansky-fields$\}$ satisfy the AKE-principle.

Theorem 1: (F.Pop–K.) $\mathcal{K}_4 = \{$alg. maximal perfect fields (K,v) with char $K > 0\}$ satisfies the AKE-principle.

Question: Does $\mathcal{K}_5 = \{$alg. complete (K,v) with char $K > 0\}$ satisfy the AKE-principle? (If so, $\mathbb{F}_p((t))$ would have decidable theory.)

Theorem 2: It does iff it satisfies the weak AKE-principle: for every immediate extension $(K,v) \subset (L,v)$ with $(K,v),(L,v) \in \mathcal{K}$ holds $(K,v) \prec_\exists (L,v)$.

This Theorem is also true more generally. Using it, we give the idea of the proof of Theorem 1 and summarize the difficulties that occur if one tries to generalize this proof to the class $\mathcal{K}_5$. In particular, we explain the role of structure theorems for henselizations of function fields over algebraically complete fields.

Alistair Lachlan:

Complete $\exists\forall$-theories over relational languages

A first order theory is called $\exists\forall$ if it has a set of axioms each member of which is a $\exists\forall$-sentence. $\aleph_0$-categorical $\exists\forall$-theories have been characterized independently by J.Schmerl and the author. In the general case it can be shown that a complete $\exists\forall$-theory is tree-decidable in the sense of Baldwin/Shelah "Second-order quantifiers and the complexity of theories", Notre Dame J. 1985.

Helmut Lenzing:

Ultraproducts of indecomposable modules

The talk deals with the question whether the ultraproduct $\prod_{\alpha\in\mathcal{F}} M_\alpha/\mathcal{F}$ of a family of indecomposable modules M_α over R_α will be indecomposable again, viewed as a module over $R^* = \prod R_\alpha/\mathcal{F}$, or alternatively over R if $R_\alpha = R$ for any R. Besides some positive results that require additional assumptions on the M_α the talk mainly concentrated on the determination of the structure of $P^* = \prod_{k\in\mathbb{N}} P_k/\mathcal{F}$, where (P_k) is the sequence of preprojective indecomposable Kronecker modules.

E.A.Palyutin:

Spectra of varieties

The spectrum $I(\kappa, K)$ of a class K of algebraic systems is the number of K-systems of cardinality κ (up to isomorphism). The following Theorem describes the spectra $I(\omega_\alpha, K)$, $\alpha \geq 1$, of varieties K.

Theorem 1: A function $I(\omega_\alpha), \alpha \geq 1$, is the spectrum of some variety of countable language if and only if $I(\omega_\alpha, K) = min\{2^{\omega_\alpha}, f(\alpha)\}$, where $f(\alpha)$ comes from the following list:

(1) 2^{ω_α}, (2) n, (3) ω, (4) 2^ω, (5) $2^\omega|\alpha|$, (6) $I_\gamma(|\alpha+\omega|^\mu)$.
Here $\mu \in \{1, \omega, 2^\omega\}$, γ is finite or a countable *non-limit* ordinal, n is natural number, which satisfies some conditions.

In the proof of this Theorem we use

Theorem 2: If K is a variety with non-maximal uncountable spectrum, then every formula ϕ is equivalent in K to a Boolean combination of positive primitive formulas and formulas with one free variable.

Anand Pillay:

***Model Theory of Modules I**

We give an introduction to the model theory of modules. We cover the following material: pp-quantifier-elimination, pure-injective modules (compact modules), pure-injective hulls, indecomposable pure injectives, existence and uniqueness of decomposition of a pure injective module into indecomposable factors and a continuous part, space of indecomposables, Ziegler invariants, decidability question for the theory of all R-modules.

Bruno Poizat:

Groups of finite Morley rank

Theorem (Borovik& Poizat): The maximal 2-subgroups of a group of finite Morley rank are locally finite and conjugated.

Mike Prest:

***Representation type and decidability**

If Γ is a wild quiver without relations then the theory of modules over its path algebra $K\Gamma$ is undecidable. Over any wild algebra there exist nonzero continuous pure-injectives. It is conjectured that over tame quivers the theory of modules is decidable (assuming the base field sufficiently recursive) and that there are no continuous pure-injectives. The decidability conjecture is verified for path algebras of quivers without relations: the proof depends on obtaining an explicit description of the space of indecomposable pure-injectives. For tame, nondomestic algebras the situation is considerably more complicated: examples considered have m-dimension= ∞ – so the space of indecomposables does not have CB-rank. Nevertheless the decidability/classification problem (for pure-injectives) over these algebras remains open.

Claus Michael Ringel:

***Representations of tame quivers**

As an introduction to the talks of M.Prest on the space of pure injectives for the path algebra of a tame quiver, we first gave a report on the basic notions of representation theory of finite dimensional algebras over a field. The topics covered included the existence of Auslander-Reiten sequences, their relationship to irreducible maps, the Auslander-Reiten quiver and its components, in particular preprojective components, preinjective components and stable tubes. Also, the relationship between quivers and their representations on the one hand, and algebras and modules on the other, was discussed. With these preperations, we presented the structure theory for the category of tame quivers (one of type $\tilde{A}_n, \tilde{D}_n, \tilde{E}_6, \tilde{E}_7, \tilde{E}_8$). The results presented are due to several mathematicians: the first case considered was that of a quiver of type $\tilde{A}_1$, its representations are called matrix pencils or Kronecker modules, and the full classification was published by Kronecker in 1890.

Philipp Rothmaler:

***Model Theory of Modules II**

In the first part we study the relation between indecomposable types in the context of Ziegler's space of indecomposable pure injectives.

The second part deals with Prest's general lattice-theoretic setting of elementary Krull dimension and a variant of Ziegler's width.

In the third part we present Prest's elegant treatment, using part two, of Ziegler's analysis of the interaction of lattice-theoretic and topological features of the space of indecomposables. Ziegler's refined decomposition theorem for pure injectives with elementary Krull dimension is presented.

Karsten Schmidt-Göttsch:

Polynomial bounds in polynomial rings

By a modification of van den Dries's nonstandard approach to bounds in the theory of polynomial rings over fields it is shown that some of these bounds can be taken polynomial in d, the degree of the polynomials constituting the data of the problem, the exponent depending only on the number n of variables. The cases treated are:

(1) solutions of systems of linear equations (this was already proved in a paper of G.Hermann of 1926)

(2) testing primality of $I = (f_1, \ldots, f_m)K[X_1, \ldots, X_n]$, $\deg f_i \leq d$: it suffices to test products of polynomials of degree $\leq d^{\alpha(n)}$

(3) for I as in (2), the number of associated prime ideals is $\leq d^{\beta(n)}$ and each of them is generated by polynomials of degree $\leq d^{\gamma(n)}$.

Gabriel Srour:

Forking relative to a set of equations

Let $\mathbb{M}$ be a very large saturated model of a complete first-order theory T. A formula $\phi(\bar{x}; \bar{y})$ is said to be an "equation" in T if for every family $\langle \phi(\bar{x}; \bar{a}_i) : i \in I \rangle$, there is $J \subset I$, J finite such that

$$\bigwedge_{i \in I} \varphi(\bar{x}; \bar{a}_i) \equiv \bigwedge_{j \in J} \varphi(\bar{x}; \bar{a}_j).$$

Fix a set of equations E. If p is a type and $A \subset \mathbb{M}$, let

$$p_A^E = \{\varphi(\bar{x}; \bar{a}) \in E(A) : p \vdash \varphi(\bar{x}; \bar{a})\}.$$

Given $p \in S(A), q \in S(B), A \subset B \subset \mathbb{M}$, we say that q is an E-minimal extension of p to B if $q \supset p$ and there is no $r \in S(B), r \supset p$, such that $r_{\mathbb{M}}^E$ is a proper subset of $q^E_{\mathbb{M}}$. If A, B, C are three subsets of $\mathbb{M}$ with $A \subseteq B \cap C$, we write $B \perp^E_A C$ (and read "B is independent from C over A relative to E") if $tp(B;C)$ is an E-minimal extension of $tp(B;A)$.

Then, under general conditions on E, $\perp^E$ satisfies natural properties similar to those satisfied by the independence relation $\perp$ in stability theory.

Simon Thomas:

Shelah's proof of the van der Waerden theorem

Shelah's proof that the Hales-Jewett numbers $HJ(n,c)$, and also the van der Waerden numbers $W(n,c)$, are primitive recursive was presented.

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