MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 3/1988

Wahrscheinlichkeitsmaße auf Gruppen

17.1. bis 23.1.1988

Die Tagung wurde von Herrn H. Heyer (Tübingen) und Herrn L. Schmetterer (Wien) geleitet.

Von den 52 Teilnehmern aus zehn europäischen Ländern, Indien, Japan und den USA wurden 34 Vorträge gehalten. Als Schwerpunkte der dabei behandelten Themen sind zu nennen: Harmonische Analyse und Wahrscheinlichkeitstheorie auf Hypergruppen; stochastische Prozesse und stochastische Integrale auf Gruppen; Grenzmaße von Faltungsprodukten von Wahrscheinlichkeitsmaßen; strukturelle Charakterisierung von speziellen Familien von Verteilungen.

Am Vormittag wurde jeweils ein einstündiger Hauptvortrag über ein dem zentralen Thema der Tagung benachbartes Gebiet abgehalten: Statistische Behandlung von sphärischen Daten; zentrale Grenzwertsätze auf symmetrischen Räumen; Verwendung von Taubersätzen in der Wahrscheinlichkeitstheorie; unendlichdimensionale Drehungsgruppen und deren Anwendungen in der Wahrscheinlichkeitstheorie; Bimaße und harmonisierbare Prozesse. Anschließend wurde in halbstündigen Pausen Gelegenheit zu ausführlicher Diskussion geboten. Die in den Hauptvorträgen dargelegten Anregungen aus mathematischer Statistik und harmonischer Analyse wurden mit besonderem Interesse aufgenommen.

Vortragsauszüge

M.S. BINGHAM

An approximate martingale functional central limit theorem on locally compact abelian groups

Let G be a locally compact second countable abelian group. All random variables mentioned will be assumed to be defined on the same probability space $(\Omega,\mathcal{F},P)$. Let $\{S_{nj},\mathcal{F}_{nj}\ ;\ 0\leqq j\leqq k_n\ ,\ n\geq 1\}$ be an adapted triangular array of G-valued random variables and set $S_n(t) = S_{n,[k_nt]}$ for $0\leqq t\leqq 1$; let Φ_t be a random continuous nonnegative quadratic form on the dual group of G. Then, under certain restrictions (involving an approximate martingale condition with respect to a local inner product on $G\times\hat{G}$), for every F with $P(F)>0$ the conditional distribution of $(S_n(t_1),\dots,S_n(t_k))$ given F converges weakly to the distribution on G^k with characteristic function $(y_1,\dots,y_k)\mapsto E(\exp(-\frac{1}{2}\sum_{j=1}^{k}(\Phi_{t_j}-\Phi_{t_{j-1}})\sum_{i=j}^{k}y_i)|F)$ if $0\leqq t_1<\dots<t_k\leqq 1$.
Corollary: Given a collection of continuous nonnegative quadratic forms ϕ_t on $\hat{G}$ with $\phi_0\equiv 0$, $\phi_{t_1}\leqq\phi_{t_2}$ for $t_1\leqq t_2$ and $t\mapsto\phi_t(y)$ continuous for each $y\in\hat{G}$, there is a continuous G-valued Gaussian process X with independent increments such that $X(0)=e$ and $X(t)$ has characteristic function $y\mapsto\exp(-\frac{1}{2}\phi_t(y))$. (Cf. J.Theoretical Prob.1(1988))

N.H. BINGHAM

Tauberian theorems in probability theory

The aim of the talk is to show how Wiener Tauberian theory may be fruitfully applied in probability theory. We discuss Wiener's First and Second Tauberian theorems and Beurling's Tauberian theorem. Applications include: (i) a five-line proof of Blackwell's Renewal Theorem from Wiener's Second Theorem; (ii) limit theorems for occupation times for random walks, linking $\lim P(S_n\in A)$ with density properties of A, for positive drift (Beurling) and zero drift (Wiener).

T. BYCZKOWSKI

Functional methods in the theory of stochastic processes with values in locally compact groups

We discuss an application of functional methods developed in the context of probability theory on vector spaces for some problems in the theory of stochastic processes. (1) Invariance Principle: Suppose that $\{\xi_j^{(n)};\ j=1,\dots,n,\ n\geq 1\}$ is a triangular array of i.i.d. random variables with values in a locally compact separable group G. Assume that $S_n^{(n)}=\xi_1^{(n)}\dots\xi_n^{(n)}$ converges in distribution to a Gaussian measure μ. Then the random walk $S_{[nt]}^{(n)}$ converges in distribution to the (left) Brownian Motion on G generated by μ. (2) Zero-One Law: Let (μ_t) be a symmetric convolution semigroup on a complete separable metric group G and let H be a Borel subgroup of G such that $\mu_t(H)>0$ for all $t>0$. Then the generating functional A is of the form $A=A^H+c(\nu-\delta_e)$, where A^H is a generating functional of a convolution semigroup concentrated on H, $c\geq 0$ and $c\nu(H)=0$. (3): Let E be a complete separable metric vector space and μ be a Gaussian measure (in the sense of Bernstein) on E. Then for every measurable additive functional ϕ there exists $x\in E$ such that $d(\mu * x)/d\mu = \exp(\phi-\frac{1}{2}\|\phi\|^2)$ and the mapping $\phi\mapsto x$ is one-to-one, continuous and linear.

A. DERIGHETTI

Some remarks concerning the induction of convolution operators

As a generalization of a theorem of de Leeuw concerning the p-multipliers ($p>1$) of T, we defined in 1979 an isometric map i of $CV_p(H)$ into $CV_p(G)$ where H is a closed subgroup of a locally compact group G. In this talk we essentially prove the following results: (1) i is an algebra homomorphism; (2) i depends only on the choice of the Haar measures of H and G; (3) i commutes with the action of the Herz - Figà-Talamanca algebra $A_p(G)$ onto $CV_p(G)$.

E. DETTWEILER

Representation of Banach space valued martingales as stochastic integrals

Let E be a real separable Banach space and suppose there are given two functions $g:E\to E$ and $\rho:E\to G(E)$ (where G(E) denotes the space of Gaussian measures) which are assumed to be continuous; ρ is also assumed to have an additional smoothness property. An E-valued continuous adapted process X_t is called a diffusion if the process Y defined by $Y(t) = X(t)-x_0-\int_0^t g(X(s))ds$ is a continuous local square integrable martingale with (tensor) quadratic variation $[Y](t) = \int_0^t Q(X(s))\,ds$, where $Q:E\to L(E',E)$ denotes the covariance function associated with ρ. Theorem: On an extended probability space there exists an infinite-dimensional Brownian motion (β_k) and there exists a sequence of continuous functions $T_k:E\to E$ such that a.s.

$$X(t) = x_0 + \int_0^t g(X(s))ds + \sum_k \int_0^t T_k(X(s))d\beta_k(s) .$$

The method of proof is quite different from the classical argument for $E=R^d$ and is based on a joint paper with G. Little in Stud. Math. 1987.

Ph. FEINSILVER (R. SCHOTT)

Operators, stochastic processes, and Lie groups

Starting with a process with stationary independent increments on R^d, we construct a process on a Lie group via the exponential map. The expectation values for the group-valued process may be expressed in terms of operator-valued processes (deterministic). We present the construction and various examples. The main technical ingredients are the use of limit theorems on Lie groups (probabilistic side) and the Trotter product formula (on the operator side).

O. GEBUHRER

L^1 ergodic theorems on ${}^K A(G)^K$ algebras

The main properties of the Fourier Eymard algebra A(G) of a locally compact group are the following: (i) A(G) is a commutative regular Banach algebra. (ii) A(G) has the Wiener property: the Gelfand spectrum can be identified with G and, if I is a closed ideal of A(G) such that for each $x \in G$ there exists $\mu \in I$ with $\mu(x)=0$, then I= A(G). (iii) All points in G are of spectral synthesis.

Here analogues are studied in the case of a Gelfand pair (G,K). This is interesting in three directions: The proofs of these properties are not modifications of the proofs of (i),(ii),(iii) and they shed light on the structure of G in terms of amenability. A completely new proof of spectral synthesis of the trivial character is given. It is reasonable to study random walks on the hermitian part of the Gelfand spectrum of $L^1(K\backslash G/K)$; in this way one can obtain a strong ergodic theorem.

W. HAZOD

The compactness lemma for simply connected nilpotent Lie groups

The "convergence-of-types-theorem" or "compactness lemma" was first proved via vector valued random variables by Billingsley 1966 and M. Sharpe 1969: Let μ_n, μ, ν be probability measures on R^d, let τ_n be a sequence of automorphisms of R^d and $a_n \in R^d$. Suppose $\mu_n \to \mu$ and $\tau_n \mu_n * \varepsilon_{a_n} \to \nu$. If μ and ν are full, i.e. not concentrated on a hyperplane, then $\{\tau_n\}$ and $\{a_n\}$ are relatively compact. This lemma is the key for investigating the domain of attraction of stable, semistable, and selfdecomposable measures. If R^d is replaced by a group two examples are known: One by P. Baldi for the motion group and one by Drisch and Gallardo for the Heisenberg group. It is shown here that the compactness lemma holds for any simply connected nilpotent Lie group. A measure is called (S-)full in this case, if the support is not contained in (a coset of) a proper connected subgroup.

T. HIDA

Infinite dimensional rotation group and unitary group

We start with a white noise $\dot{B}(t)$, which is the time-derivative of a Brownian motion $B(t)$. The probability distribution of $\{\dot{B}(t)\}$ is defined on the dual space of a nuclear space E; it is called white noise measure and denoted by μ. The Hilbert space $L^2(E^*,\mu)$ is a realization of functionals of $\dot{B}(t)$'s with finite variance. The rotation group $O(E)$ is a group of linear homeomorphisms g of E such that $\|g\xi\| = \|\xi\|$ for every ξ. Let g^* be the adjoint of g; then $g^*\mu=\mu$ holds. This property indicates the intimate connection between white noise and the rotation group. In fact, the rotation group is a powerful tool in the analysis of white noise. There are three interesting types of subgroups of $O(E)$: Finite dimensional rotation groups; the Lévy group which is defined by permutations of a given complete orthonormal system in $L^2(R)$; and "whiskers" which stem from a one-parameter group of diffeomorphisms of the time parameter space. These subgroups have probabilistic interpretations in the study of white noise functionals. Complex white noise and the unitary group can be treated similarly.

J. HILGERT

Lie semigroups and their applications to Gaussian semigroups

Let (μ_t) be a Gaussian semigroup on a connected Lie group G with infinitesimal generator $N = x_0+x_1^2+\ldots+x_r^2$, $x_i \in L(G)$. Then the closure S_μ of the union of the supports of the μ_t is a subsemigroup of G. It is the smallest closed subsemigroup of G containing $\exp M$ and $\exp \mathbb{R}^+ x_0$ where M is the Lie algebra generated by $x_1,\ldots,x_r$. Let G_μ be the analytic subgroup of G generated by $\exp M$ and $\exp \mathbb{R}^+ x_0$; then $\mu_t(G\backslash G_\mu)=0$ for all t and so we may assume $G_\mu=G$. Then S_μ is contained in a maximal semigroup S_{max} unless S_μ is all of G. It is known that μ_t is absolutely continuous with respect to Haar measure for some t iff it is for all t iff there exists no subalgebra of $L(G)$ invariant under ad x_0 and containing M except $L(G)$ itself. This implies the following propositions: If μ is absolutely continuous and $S_\mu \neq G$, then $S_{max} \cap S_{max}^{-1}$ cannot be a normal subgroup of G; if μ is not absolutely continuous and G is simply connected, then $S_{max} \cap S_{max}^{-1}$ is a normal subgroup of codimension 1. The knowledge of maximal subsemigroups of Lie groups now yields $S_\mu=G$ for any absolutely continuous Gaussian semigroup, provided $G=KN$ with $K<G$ compact

and $N \triangleleft G$ either nilpotent or complex. On the other hand under the above hypotheses it follows immediately that any Gaussian semigroup on a simply connected semisimple Lie group is absolutely continuous.

F. HIRSCH

On the flow of a stochastic differential equation

The following problems are considered and some answers (obtained in joint work with N. Bonban) are given:

(1) What can be said about the differentiablity of the flow of a stochastic differential equation with respect to the initial condition x in $\mathbf{R}^n$, if the coefficients are only Lipschitz continuous ?

(2) Is it possible to define some "precise" versions of the flow such that the usual properties of the flow hold quasi everywhere instead of almost everywhere (quasi everywhere referring to the Ornstein - Uhlenbeck capacity on the Wiener space) ?

J. KISYŃSKI

A local unique determination property for some classes of Feller infinitesimal generators

Let M be a compact C^∞ manifold with boundary ∂M. A Hölderian elliptic Ventcel boundary system on M is a triple (W,Γ,δ), where $W: C^{2+\alpha}(M) \to C^\alpha(M)$, $\Gamma: C^{2+\alpha}(M) \to C^{2-r+\alpha}(\partial M)$ are differointegral operators and $\delta \in C^{2-r+\alpha}(\partial M)$ is a non-negative function. Here $0 \leqq \alpha \leqq 1$, the highest order homogeneous differential part of W has order 2 and is elliptic and the highest order homogeneous differential part of Γ has order r=0,1 or 2. According to Bony, Courrège and Priouret (Ann.Inst.Fourier 18(1968)), the closure G of the restriction of W to the set of C^2-functions u with $\Gamma u=\delta Wu$ on ∂M is the infinitesimal generator of a Feller semigroup on M. To every such semigroup there corresponds a function space type (cadlag) Markov process with state space $M_\Delta = M \cup \{\Delta\}$. We prove that if $U \subset M \setminus \partial M$ is open, G_i (i=1,2) are Feller generators arising from systems (W_i,Γ_i,δ_i) of the described class, and $1_U W_1 = 1_U W_2$ when restricted to $C_c^2(U)$, then $\overline{\mathcal{N}}_{\tau-}^{G_1} = \overline{\mathcal{N}}_{\tau-}^{G_2}$ and the restrictions of $P_x^{G_i}$ (i=1,2) to this σ-algebra coincide for all $x \in M_\Delta$. Here τ is the first exit time from U and $\overline{\mathcal{N}}^G_{\tau-} = \sigma\{X_0^{-1}(B), A\cap\{t<\tau\} : B\in\mathcal{B}(M_\Delta),$ $t\geq 0$, $A\in\overline{\mathcal{N}}^G_{t+0}\}$, where the filtrations $(\overline{\mathcal{N}}^G_{t+0})_{t\geq 0}$ (depending on the generators G) are that of Dynkin, Markov Processes I, p.108 .

A. KUMAR

A dichotomy theorem for random walks on hypergroups

We define possible and recurrent elements for random walks on commutative hypergroups and show that for a large class of random walks either no element is recurrent or all possible elements are recurrent and they form a closed subhypergroup.

R. LASSER

Weakly stationary stochastic processes indexed by hypergroups

An example from statistics is given that motivates the study of weakly stationary processes indexed by hypergroups. Appropriate analogues of moving average processes and autoregression processes are introduced and investigated. Moreover the family of corresponding shift operators is studied.

G. LETAC

A characterization of the Wishart exponential families by an invariance property

A natural exponential family on a vector space E is defined by $\{L_\mu(\theta)^{-1} e^{\langle\theta,x\rangle}\mu(dx) ; \theta\in\Theta(\mu)\}$ where μ is a positive Radon measure on E, L_μ is its Laplace transform and $\Theta(\mu)$ is the interior of $\{\theta ; L_\mu(\theta)<\infty\}$. For $p=\frac{1}{2},\ldots,\frac{d-1}{2}$ or $\frac{d-1}{2}<p<\infty$ the Wishart family F_p is defined on the space E of d×d symmetric matrices as generated by μ_p which is defined by $\int_E \exp(-\mathrm{tr}(\Delta x))\mu_p(dx) = (\det\Delta)^{-p}$ for every positive definite matrix Δ. The family F_p is invariant under the action of GL(d) on E defined by $g_a(x) = {}^t a\, x\, a$. We prove that this fact is characteristic of Wishart families: Let F be an exponential family on E that is invariant under all g_a. Then there exists p such that either $F=F_p$ or F is the image of F_p by $x\mapsto -x$.

M. McCRUDDEN

Recent progress on the embedding problem

Let G be a locally compact group and let P(G) be the topological semigroup of probability measures on G; in the following all measures $\lambda,\mu,\ldots$ are elements of P(G). The embedding problem asks for a characterization of those groups G which have the property that every infinitely divisible μ is continuously embedded (i.e. there is a continuous homomorphism $t\mapsto\mu_t$ of $\mathbf{R}_+$ into P(G) such that $\mu=\mu_1$). We write $G(\mu)$ for the closed subgroup generated by the support of μ and $Z(\mu)$ for the centralizer of $G(\mu)$. A connected Lie group G is called weakly algebraic iff (i) Ad G is an almost algebraic subgroup of the general linear group of the Lie algebra of G and (ii) any compact subgroup of the centre of G contains only finitely many commutators. Our main theorem (joint work with S.G. Dani) is as follows: Let G be a weakly algebraic group with compact centre. Then $\{\lambda:$ there exists ν such that $\lambda\nu=\nu\lambda=\mu\}/Z(\mu)$ is relatively compact for every μ. Corollary: Any homomorphism $t\mapsto\mu_t$ of a real directed semigroup S into P(G), where G is a connected Lie group, is locally tight (i.e. $\{\mu_t : t\leqq t_0\}$ is relatively compact for all t_0).

A. MUKHERJEA

Some observations on the weak convergence of convolution products of non-identical distributions on a set of d×d stochastic matrices

We consider composition convergent sequences (μ_n) of probability measures on a compact semigroup G and, in particular, on semigroups of finite dimensional stochastic matrices. Let G be a compact abelian second countable topological semigroup and let μ_n $(n\geqq 1)$ be probability measures on G such that, for all $k\geq 1$, $\mu_{k+1}\cdots\mu_n$ converges weakly to a probability measure ν_k. Then (ν_k) converges weakly to the normed Haar measure of a compact subsemigroup H, $\sum(1-\mu_n(UU^{-1}))$ is finite for every open set U containing H, and the set of all x such that $\sum\mu_n(N(x))=\infty$ for all open neighbourhooeds N(x) of x is contained in HH^{-1}.

N. OBATA

Harmonic functions on Hilbert space and the Lévy Laplacian

Let H be a real Hilbert space with a complete orthonormal system $(e_n)_{n\geq 1}$. For a C^2-function F defined on an open subset of H put $\Delta F(\xi) = \lim \frac{1}{N}\sum_{n=1}^{N}(F''(\xi)e_n, e_n)$ if the limit exists; Δ is called the Lévy Laplacian. The mean of F on the sphere at center ξ with radius ρ is defined by $MF(\xi,\rho) = \lim \int_{S^{n-1}} F(\xi+\rho h)dS_{n-1}(h)$ if the limit exists; dS_{n-1} denotes the uniform probability measure on S^{n-1} which is regarded as a subset of H. Then under suitable conditions we have $\Delta F(\xi)/2 = \lim_{\rho\to 0} (MF(\xi,\rho)-f(\xi))/\rho^2$. A function F is called regularly analytic at ξ_0 if there exist $a_n \in S^nH$ (n-th symmetric power of H) such that the power series $\sum \|a_n\| t^n$ has positive radius of convergence and $F(\xi) = \sum (a_n, (\xi-\xi_0)^{\otimes n})_{S^nH}$ in a neighbourhood of ξ_0. Theorem: If F is regularly analytic at ξ_0, then there exists $R>0$ such that $MF(\xi_0,\rho)=F(\xi_0)$ whenever $|\rho|<R$. Corollary: If F is regularly analytic on an open subset U of H then F is harmonic, i.e. $\Delta F\equiv 0$ on U. Application: Let $E\subset H\subset E^*$ be a Gelfand triple and μ be the standard Gaussian measure on E^*. For each $f\in L^2(E^*,\mu)$ we put $Sf(\xi) = \int f(\xi+x)d\mu(x)$. Then from the above result we obtain $\Delta Sf\equiv 0$ ("harmonicity of ordinary Brownian functionals").

M.M. RAO

Bimeasures and harmonizable processes

Let G be a locally compact abelian group and $L_0^2(P)$ be the Hilbert space of square integrable scalar functions on a probability space with zero means. A mapping $X:G\to L_0^2(P)$ is a harmonizable random field if it is the Fourier transform of a vector measure $Z:\mathcal{B}(\hat{G})\to L_0^2(P)$ $\hat{G}$ being the dual of G. If $\beta(A,B) = E(Z(A)\overline{Z(B)})$ then $\beta:\mathcal{B}(\hat{G})\times\mathcal{B}(\hat{G})\longrightarrow \mathbb{C}$ is a bimeasure. If β has finite Vitali variation then X is called strongly harmonizable and otherwise it is called weakly harmonizable. If $X:G\to L_0^2(P;H)$, H a Hilbert space, new problems arise. Also the case where G is not necessarily abelian but separable and unimodular is treated. Operator-valued harmonizable fields on locally compact abelian groups are briefly discussed. Finally, if the field X has α-moments for $1\leq\alpha\leq 2$ then in analogy to strictly stationary fields "strictly harmonizable" fields can be introduced.

P. RESSEL

A conjecture concerning mixtures of characters from a given closed subsemigroup in the dual

Let S be an abelian semigroup with involution * and let K be a closed *-subsemigroup of the set $\hat{S}$ of bounded characters of S; K is assumed to contain the neutral element. A function $\phi:S\to\mathbb{C}$ is called K-positive definite iff $(\phi(s_{jk}))$ is positive semidefinite for each matrix (s_{jk}) such that $(\rho(s_{jk}))$ is positive semidefinite for all $\rho\in K$. It is easy to see that each such function is bounded and positive definite, and the set P(S,K) of all K-positive definite functions is a closed multiplicative convex cone, containing the closed multiplicative convex cone $P^b(S|K)$ of all positive definite bounded functions whose representing measure on $\hat{S}$ is supported by K. Our conjecture is that (P) $P^b(S|K)=P(S,K)$. Theorem: $\{\phi\in P(S,K):\phi(0)=1\}$ is a Bauer simplex whose extreme boundary is the smallest $\tilde{K}$ containing K such that (P) holds. The conjecture holds for many examples, including all abelian groups, the closed unit interval, and certain K in $[-1,1]\cong\hat{\mathbb{N}}_0$. If the conjecture holds for all finite powers of $\mathbb{N}_0\times\mathbb{N}_0$ with usual addition and $(m,n)^*=(n,m)$ then it holds in general by a Hahn-Banach argument.

H. RINDLER

Groups of measure preserving transformations

Let G be a locally compact group acting as a group of measure preserving transformations on a standard probability measure space $(X,\mathcal{B},\mu)$; K always means a compact subset of G. We say that property (ε,δ) holds iff for every K there is $A\in\mathcal{B}$ such that $0<\mu(A)<\delta$ and $\mu(gA\,\Delta\,A)<\varepsilon\mu(A)$ for all $g\in K$. Let V_δ be the infimum of all $\delta>0$ such that (ε,δ) holds for all $\varepsilon>0$; let V_ε be the infimum of all $\varepsilon>0$ such that (ε,δ) holds for all $\delta>0$; and set $V=\sup_K \inf_A \sup_{g\in K} \frac{\mu(gA\,\Delta\,A)}{\mu(A)(1-\mu(A))}$. The main result states that either (i) $V_\delta=V_\varepsilon=V=0$; (ii) $V_\varepsilon=V=2$, $V_\delta=1$; or (iii) $0<V_\delta<1$, $V_\varepsilon=2$, $V=0$. (ii) implies that μ is the only topological G-invariant mean on $L^\infty(X,\mu)$; for σ-compact groups the converse also holds. $V_\varepsilon=2$ is equivalent to a certain almost disjointness property; $V_\delta=1$ holds iff there are no nontrivial almost invariant subsets; $\underline{V}=2$ is in many cases (but not always) equivalent to a certain almost independence property. Some examples are presented which might be of interest.

I.Z. RUZSA (G.J. SZÉKELY)

Decomposition of probability measures on groups

Inspired by the Delphic theory of Kendall and Davidson, we develop a theory of decomposition on certain semigroups. A commutative topological semigroup S is said to be Hun, if a~b implies a=b and the set T_s of divisors of s is compact for all s. We call S Hungarian if a~b implies a=bu for some unit u and S/~ is Hun. In every Hun semigroup and in every M_1 Hungarian semigroup we obtain a decomposition of elements into irreducible factors and an extra factor that has no irreducible factors. If S is also normable (i.e. for every s with $s^2 \neq s$ there is a homomorphism $\Delta_s : T_s \to [0,\infty)$ that is continuous at e and $\Delta_s(s) \neq 0$) then this extra factor is infinitely divisible. We also prove that (under some technical conditions) the limit of an infinite triangular array is infinitely divisible. The applicability of the theory follows from the following theorem: If G is any commutative topological group then the semigroup D(G) of tight probability measures on G (with the operation of convolution and the weak topology) is Hungarian; if G is separated by its continuous characters then it is also normable. Several further examples can be given.

M. SCHÜRMANN

Representations given by infinitely divisible linear functionals on -bialgebras

A $*$-bialgebra is a $*$-algebra B which is also a coalgebra (cf. the book of Sweedler on Hopf algebras) such that the comultiplication $\Delta : B \to B \otimes B$ and the counit $\delta : B \to \mathbb{C}$ are $*$-algebra homomorphisms. The convolution product of linear functionals on B is defined by $\phi_1 * \phi_2 =$

$(\phi_1 \otimes \phi_2) \circ \Delta$. If ψ is a conditionally positive hermitian linear functional on B with $\psi(1)=0$ then the convolution exponential $\phi=\exp_* \psi$ is an infinitely divisible state (i.e. for every n there exists $\phi_n \geqq 0$ such that $\phi_n(1)=1$ and $\phi=\phi_n^{*n}$). One can associate $*$-representations of B to both of ψ and ϕ. The question arises how these $*$-representations are related to each other. If B is the free $*$-algebra generated by hermitian indeterminates $x_1,\ldots,x_d$ then it is well known that every infinitely divisible state ϕ on B is of the form $\phi = \exp_* \psi$ with ψ conditionally positive hermitian and $\psi(1)=0$. Using the $*$-representation associated to ψ it can be shown that the GNS-representation of an infinitely divisible state maps the generator x_k to an operator of the form $\Gamma(A_k)+\alpha_k \mathrm{id}+(a+a^+)(f_k)$ where A_k is a self-adjoint operator on a Hilbert space H, $\alpha_k \in \mathbb{R}$, $f_k \in H$, $\Gamma(A_k)$ denotes the self-adjoint operator on the Bose-Fock space $\Gamma(H)$ which is the differential second quantisation of A_k, and a and a^+ denote the annihilation and creation operators on $\Gamma(H)$ respectively. A similar result holds for the much more general case of a cocommutative graded $*$-bialgebra.

H. SÉNATEUR (J.L. DUNAU)

Characterization of the type of some generalizations of the Cauchy distribution

Let G be a closed connected subgroup of $GL(n,\mathbb{R})$, $\mathfrak{g}$ its Lie algebra and suppose G is semi-simple. Consider a Cartan decomposition $\mathfrak{g} = \mathcal{K} \oplus \mathcal{P}$, a maximal abelian subalgebra $\mathcal{A}$ of $\mathcal{P}$, and let Δ be the set of roots; then $\mathfrak{g}=\oplus(\mathfrak{g}_\alpha;\ \alpha\in\Delta)$, where $\mathfrak{g}_\alpha=\{x\in\mathfrak{g};\ Hx-xH=\alpha(H)x \text{ for all } H\in\mathcal{A}\}$. Choosing a Weyl chamber w_0 we define $\Delta_+=\{\alpha\in\Delta\setminus\{0\};\ \alpha(H)>0 \text{ for } H\in w_0\}$, Δ_- similarly, $\mathcal{N}=\oplus(\mathfrak{g}_\alpha;\ \alpha\in\Delta_-)$, and $\tilde{\mathcal{N}}=\oplus(\mathfrak{g}_\alpha;\ \alpha\in\Delta_+)$. Let $N,\tilde{N},A,K$ be the connected Lie subgroups of G corresponding to the Lie algebras $\mathcal{N},\tilde{\mathcal{N}}$, $\mathcal{A},\mathcal{K}$. Consider the centralizer M of A in K and the normalizer M' of A in K; then M'/M is the Weyl group. Let $B = G/\tilde{N}AM$. For any $g\in G$, let $\bar{g}$ denote the corresponding element of B. In each coset of B there is at most one element of N; define B_N to be the set of elements of B corresponding to an element of N. By the type of a probability measure μ of B we mean the set of images of μ by elements of AN. Since $B\cong K/M$ is compact, the unique probability on B invariant by K will be called Cauchy distribution on B; it is denoted by γ. Then the type of γ is invariant by G. Conversely: If the type of a probability measure μ on B is invariant by G and $\mu(g^{-1}(B\backslash B_N))=0$ for every $g\in G$, then μ is of the type of γ. As an illustration, note that for $G=SL(n,\mathbf{R})$ B is the set of flags on $\mathbf{R}^n$, and so we obtain a characterization of the type of the Cauchy distribution on the flags.

G.J. SZÉKELY

Generalized generating functions

$g(t)=\sum p_k h_k(t)$ with $p_k\geq 0$ for all $k\geq 0$ and $\sum p_k \leq 1$ is called generalized generating function if (i) all h_k are real valued continuous functions on an interval $[a,b]$, (ii) $h_0(t)\equiv 1$, (iii) $h_k(c)=1$ for all k and some fixed c, $a\leq c\leq b$, (iv) $h_k h_m = \sum c_n(k,m)h_n$ where $c_n(k,m)\geq 0$, and (v) $\int |h_k(t)|\,dt \to 0$ as $k\to\infty$. These conditions imply that $|h_k(t)|\leq 1$ for all k and t; thus g(t) is defined by a uniformly convergent series. Theorem 1: A generalized generating function is infinitely divisible iff it is of the form $\exp(q(f-1))$ where $q\geq 0$ and f is a generalized generating function. Theorem 2: The set of generalized generating functions with the uniform topology is a stable normable Hun semigroup.

A. TERRAS

Harmonic analysis on symmetric spaces and central limit theorems

A naive harmonic analyst's approach to central limit theorems for rotation-invariant independent identically distributed random variables X_n on the symmetric spaces of $G=SL(2,\mathbf{R})$ and $GL(3,\mathbf{R})$ is given. We use properties of the Fourier transform on G/K, K=SO(2) and O(3), respectively. One also needs the first few terms of Taylor's expansion for spherical functions on G/K. These are similar to expansions obtained by James for matrix argument ${}_0F_0$. One surprise is that the Fourier transform of the limiting density for the normalized product of the X_n is somewhat different from the Fourier transform of the fundamental solution of the heat equation on $GL(3,\mathbf{R})/O(3)$.

G. TURNWALD

Roots of Haar measure on a compact group

Let G be a compact group with Haar measure U. Then the following assertions are equivalent: (a) There is a probability measure $P \neq U$ such that $P*P=U$; (b) G is neither abelian nor isomorphic to $Q \times E$ where Q denotes the quaternion-group and E is a product of 2-element groups. It is shown that (a) is equivalent to (c): For some irreducible unitary representation ρ of G, the real algebra R_ρ generated by all matrices $\rho(x)$ has nilpotent elements. If (c) (and (a)) fails, then G must be Hamiltonian (i.e. every closed subgroup of G is normal). Then Strunkov's classification of (locally compact) Hamiltonian groups can be used to prove that (b) implies (a). In the case of separable G, the above results are due to Diaconis and Shahshahani (Proc.AMS 98(1986); their proof contains several gaps which are closed here).

K. URBANIK

Limit sets of probability measures

The talk is devoted to the study of limit sets consisting of cluster points of normalized powers under a generalized convolution of probability measures. A relationship between probabilistic and topological properties of these sets is established. In particular it is shown that an irreducible limit set is generated by a quasi-stable measure iff it is either a one-point set or homeomorphic to a circle.

M. VOIT

Negative definite functions on commutative hypergroups

For any convolution semigroup on a strong commutative hypergroup K there exists an associated negative definite function f on the dual hypergroup $\hat{K}$ which satisfies $\mathrm{Re}\, f \geq f(e) \geq 0$. Conversely, for any negative definite function f on $\hat{K}$ with $\mathrm{Re}\, f \geq f(e) \geq 0$ such that for all $t>0$ $\exp(-tf)$ is positive definite, we obtain an associated convolution semigroup on K. We give some conditions on negative definite functions f with $\mathrm{Re}\, f \geq f(e) \geq 0$ which imply that $\exp(-tf)$ is positive definite for all $t>0$. The conclusion holds if f is bounded or if $\hat{K}$ is a polynomial hypergroup which satisfies some restrictions; examples are given by the double coset hypergroups $SO(n)//SO(n-1)$ for $n \geq 3$.

R. VREM

L^p-improving measures on hypergroups

Let K be a compact abelian hypergroup with dual $\hat{K}$ and Haar measure m. We call a measure μ on K L^p-improving $(1<p<\infty)$ if there exists $\varepsilon>0$ such that $\mu * L^p \subset L^{p+\varepsilon}$. Characterizations of such measures are given as well as some contrasts with the group case. In particular, it is shown that point masses can be L^p-improving. Connections with Λ_p sets are discussed and applications to compact nonabelian groups are studied.

G.S. WATSON

The statistics of rotations in three dimensions

Orthogonal transformations play a large role in statistics because of the prevalence of the Gaussian distribution. But in many fields of application (geology , geophysics) one is required to estimate rotations in three dimensions. In one class of problems one has pairs of points (x_i, y_i) on S^2 where x_i is u_i measured with error, and y_i is v_i measured with error, $v_i = Au_i$ and all u_i, v_i have norm 1. From such data one is required to estimate A ($\det A = \pm 1$, $A'A = I_3$). We discuss the case where $x_i = u_i$ and the y_i are independent and have Fisher density on S^2. As shown by Mackenzie, if $M_n = (u_1 y_1' + \ldots + u_n y_n')/ /n = L\Lambda R'$, then the maximum likelihood estimator of A is $\hat{A} = RL'$. By writing $A = \hat{A} \exp S$, $S' = -S$ will have small elements (a 3-vector h), if the parameters are large; h is known to be asymptotically Gaussian. Our proofs are much simpler than the previous ones and allow many further results (e.g. significance tests) to be obtained easily.

W. WOESS

A converse to the mean value property on homogeneous trees

Motivation from classical potential theory: At each point x of the open unit disk D fix a radius $r(x)>0$ such that the ball $B(x,r(x))$ is contained in D. The mean value property (MVP) says that if h is a harmonic function on D then, for all x, h(x) is equal to the mean of h, taken over $B(x,r(x))$ with respect to Lebesgue measure. A converse to the MVP is then a statement which gives conditions on the radius function $r:D\to\mathbb{R}^+$, such that a positive (or bounded) function h on D which has the MVP with respect to r must be harmonic. The homogeneous tree $T=T_q$ of degree q+1 ($q\geq 2$) may be considered as a discrete analogue of D. The discrete Laplacian on T is given by $\Delta f(x) = f(x) - \frac{1}{q+1}\sum f(y)$ where the sum is extended over all neighbours of x; h is called harmonic if $\Delta h\equiv 0$. We now consider a radius function r which assigns a positive integer to each vertex. In this setting, the MVP holds for harmonic functions on the tree. Theorem: If $|r(x)-r(y)| \leq \Phi(d(x,y))$ and $\limsup \Phi(n)/\log_q n < 2/3$ then every positive function on T which has the MVP with respect to r is harmonic. We also have a radius function r on T which does not satisfy a Lipschitz-type condition as above but grows linearly, such that the converse to the MVP does not hold. (Joint work with M.A. Picardello)

K. YLINEN

Completely bounded and related random fields on locally compact groups

Let G be a locally compact group and H be a Hilbert space. Various classes of random fields $\phi:G\to H$ are defined in terms of their covariance functions $(s,t)\mapsto R(s,t)=(\phi(s)|\phi(t))$. If $(s,t)\mapsto R(s,t^{-1})$ is the Fourier transform of a so-called completely bounded bilinear form on $C^*(G)\times C^*(G)$, ϕ is said to be completely bounded. If $(s,t)\mapsto R(s,t^{-1})$ belongs to the Fourier-Stieltjes algebra $B(G\times G)$, ϕ is said to be strongly harmonizable. Strong harmonizability implies complete boundedness, which in turn implies V-boundedness (i.e., weak harmonizability). Complete boundedness turns out to have the same type of dilation relation to right homogeneity as V-boundedness is known to have to hemihomogeneity.

Hm. ZEUNER

Limit theorems for Chébli-Trimèche hypergroups

For every second countable hypergroup K the notion of concretization is introduced which allows the definition of the random sum $X \overset{\wedge}{+} Y$ of two independent K-valued random variables, which is distributed as $P_X * P_Y$. Specializing to the case of Chébli-Trimèche hypergroups the analogue of Kolmogorov's three series theorem (which in this example is a two series theorem) is stated, as well as strong laws of large numbers for random walks. In order to do this it is necessary to introduce the $*$-expectation and $*$-variance which satisfy $E_*(X \overset{\wedge}{+} Y) = E_*(X) + E_*(Y)$ and $V_*(X \overset{\wedge}{+} Y) = V_*(X) + V_*(Y)$ respectively.

Berichterstatter: G. Turnwald

Tagungsteilnehmer

Prof.Dr. C. Berg
Matematisk Institut
Kobenhavns Universitet
Universitetsparken 5

DK-2100 Kobenhavn

Prof.Dr. H. Carnal
Mathematische Statistik
und Versicherungslehre
Universität Bern
Sidlerstraße 5

CH-3012 Bern

Prof.Dr. M. S. Bingham
Dept. of Mathematical Statistics
University of Hull
Cottingham Road

GB- Hull , HU6 7RX

Prof.Dr. A. Derighetti
Institut de Mathematiques
Universite de Lausanne

CH-1015 Lausanne -Dorigny

Prof.Dr. N. H. Bingham
Dept. of Mathematics
Royal Holloway and Bedford New
College
Egham Hill

GB- Egham, Surrey TW20 0EX

Prof.Dr. Y. Derriennic
Dept. de Mathematiques
Universite de Bretagne Occidentale
6, Avenue Victor Le Gorgeu

F-29283 Brest Cedex

Prof.Dr. Ph. Bougerol
UER Sciences Mathematiques
Universite de Nancy I
Boite Postale 239

F-54506 Vandoeuvre les Nancy Cedex

Dr. E. Dettweiler
Unterer Haldenweg 6

7410 Reutlingen 26

Prof.Dr. T. Byczkowski
Instytut Matematyki
Politechniki Wroclawskiej
Wyb. Wyspianskiego 27

50-370 Wroclaw
POLAND

Prof.Dr. J. L. Dunau
Laboratoire de Statistique et
Probabilites
Universite Paul Sabatier
118, route de Narbonne

F-31062 Toulouse Cedex

Prof.Dr. L. Elie
U. E. R. de Mathematiques et
Informatiques, T. 45-55, 5etage
Universite de Paris VII
2, Place Jussieu

F-75251 Paris Cedex 05

Prof.Dr. P. Gerl
Institut für Mathematik
Universität Salzburg
Hellbrunnerstr. 34

A-5020 Salzburg

Prof.Dr. P. J. Feinsilver
Dept. of Mathematics
Southern Illinois University

Carbondale , IL 62901
USA

Prof.Dr. W. Hazod
Fachbereich Mathematik
der Universität Dortmund
Postfach 50 05 00

4600 Dortmund 50

Prof.Dr. A. Figa-Talamanca
Dipartimento di Matematica
Universita degli Studi di Roma I
"La Sapienza"
Piazzale Aldo Moro, 2

I-00185 Roma

Prof.Dr. H. Heyer
Mathematisches Institut
der Universität Tübingen
Auf der Morgenstelle 10

7400 Tübingen 1

Prof.Dr. L. Gallardo
Dept. de Mathematiques
Universite de Bretagne Occidentale
6, Avenue Victor Le Gorgeu

F-29283 Brest Cedex

Prof.Dr. T. Hida
Dept. of Mathematics
Faculty of Sciences
Nagoya University
Chikusa-Ku

Nagoya 464
JAPAN

Prof.Dr. O. Gebuhrer
Institut de Mathematiques
Universite Louis Pasteur
7, rue Rene Descartes

F-67084 Strasbourg Cedex

Dr. J. Hilgert
Fachbereich Mathematik
der TH Darmstadt
Schloßgartenstr. 7

6100 Darmstadt

Prof.Dr. F. Hirsch
Ecole Normale Superieure de Cachan
61, Av. du President Wilson

F-94230 Cachan

Prof. Dr. A. Kumar
Institut de Recherche
Mathematique Avancee
7, rue Rene Descartes

F-67084 Strasbourg

Prof. Dr. G. Högnäs
österlanggatan 20B 43

SF-20520 Abo

Dr. R. Lasser
Medis - Institut
Gesellschaft für Strahlen- und
Umweltforschung mbH
Ingolstädter Landstraße 1

8042 Oberschleißheim

Prof.Dr. E. Kaniuth
Fachbereich Mathematik
der Universität Paderborn
Warburger Str. 100

4790 Paderborn

Prof.Dr. E. Le Page
Laboratoire de Probabilite
Universite de Rennes 1
B. P. 25A
Avenue du General Leclerc

F-35031 Rennes Cedex

Prof.Dr. F. Kinzl
Institut für Mathematik
Universität Salzburg
Hellbrunnerstr. 34

A-5020 Salzburg

Prof.Dr. G. Letac
Mathematiques
Universite Paul Sabatier
118, route de Narbonne

F-31062 Toulouse Cedex

Prof.Dr. J. Kisynski
Department of Mathematics
Technical Univiersity of Lublin
ul. J. Dabrowskiego 13

20109 Lublin
POLAND

Prof.Dr. M. McCrudden
Dept. of Mathematics
University of Manchester
Institute of Science and Technology

GB- Manchester M60 1QD

Prof.Dr. A. Mukherjea
Dept. of Mathematics
University of South Florida

Tampa , FL 33620
USA

Prof.Dr. I. Z. Ruzsa
Mathematical Institute of the
Hungarian Academy of Sciences
Realtanoda u. 13 - 15, Pf. 127

H-1364 Budapest

Prof. Dr. N. Obata
Dept. of Mathematics
Faculty of Sciences
Nagoya University
Chikusa-Ku

Nagoya 464
JAPAN

Prof.Dr. L.K. Schmetterer
Institut für Mathematik
Universität Wien
Strudlhofgasse 4

A-1090 Wien

Prof.Dr. M. M. Rao
Dept. of Mathematics
University of California

Riverside , CA 92521
USA

Prof.Dr. R. Schott
UER Sciences Mathematiques
Universite de Nancy I
Boite Postale 239

F-54506 Vandoeuvre les Nancy Cedex

Prof.Dr. P. Ressel
Mathematisch-Geographische
Fakultät
der Universität Eichstätt
Ostenstr. 26 - 28

8078 Eichstätt

Dr. M. Schürmann
Institut für Angewandte Mathematik
der Universität Heidelberg
Im Neuenheimer Feld 294

6900 Heidelberg

Prof.Dr. H. Rindler
Institut für Mathematik
Universität Wien
Strudlhofgasse 4

A-1090 Wien

Prof. Dr. H. Senateur
Laboratoire de Statistique et
Probabilites
Universite Paul Sabatier
118, route de Narbonne

F-31062 Toulouse Cedex

Prof.Dr. E. Siebert
Mathematisches Institut
der Universität Tübingen
Auf der Morgenstelle 10

7400 Tübingen 1

M. Voit
Mathematisches Institut
der TU München
PF 20 24 20, Arcisstr. 21

8000 München 2

Prof.Dr. G. Szekely
Department of Probability and
Statistics
Eötvös University
Muzeumkörut 6 - 8

H-1088 Budapest

Prof.Dr. R. C. Vrem
Department of Mathematics
Humboldt State University

Arcata , CA 95521
USA

Prof.Dr. A. Terras
Dept. of Mathematics
University of California, San Diego

La Jolla , CA 92093
USA

Prof.Dr. W. von Waldenfels
Institut für Angewandte Mathematik
der Universität Heidelberg
Im Neuenheimer Feld 294

6900 Heidelberg

Dr. G. Turnwald
Mathematisches Institut
der Universität Tübingen
Auf der Morgenstelle 10

7400 Tübingen 1

Prof.Dr. G. S. Watson
Department of Statistics
Princeton University
Fine Hall, P. O. Box 708

Princeton , NJ 08544
USA

Prof.Dr. K. Urbanik
Instytut Matematyczny
Uniwersytet Wroclawski
pl. Grunwaldzki 2/4

50-384 Wroclaw
POLAND

Prof.Dr. W. Woess
Institut für Mathematik
und Angewandte Geometrie
Montanuniversität Leoben
Franz-Josef-Str. 18

A-8700 Leoben

Prof.Dr. K. Ylinen
Institute of Mathematical Sciences
University of Turku

SF-20500 Turku

Dr. H.M. Zeuner
Mathematisches Institut
der Universität Tübingen
Auf der Morgenstelle 10

7400 Tübingen 1