MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 4/1988

ONE PARAMETER SEMIGROUPS
AND
DIFFERENTIAL OPERATORS

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This conference, which was organized by Rainer Nagel (Tubingen), Jerome A. Goldstein (New Orleans) and E. Brian Davis (London), was the first one on this subject to be held at Oberwolfach. The fifty-one participants represented thirteen countries. The thirty-nine scheduled talks treated many topics ranging from the theory of analytic, integrated, sub-Markovian and C-semigroups to the application of semigroups to nonlinear evolution equations, reaction-diffusion systems, semilinear parabolic and elliptic systems, population dynamics, pertubation theory, scattering theory and Markov processes. Mid-afternoons and evenings were devoted to private discussions, and the time thus spent was extremely productive. For the younger mathematicians this conference was a unique opportunity to consult with some of the world's foremost authorities in the field. The large number of participants and the diversity of the applications of the theory demonstrate both the interest in and the importance of this area of mathematics.

Vortragsauszuge

W. ARENDT:

KATO'S INEQUALITY FOR SUBMARKOVIAN SEMIGROUPS

This is a joint work with Ph. Benilan (Besancon). Let Ω be locally compact, $J = \{ j: \mathbb{R} \longrightarrow \mathbb{R}$ convex, $j(0) = 0 \}$, $SL: = \{j \in J : j$ semilinear$\}$. Note that $j \circ u \in C_o(\Omega)$. Whenever $u \in C_o(\Omega)$.

(A) Let $T : C_o(\Omega) \to C_o(\Omega)$ be linear. Then $T \geq 0$ iff $j \circ Tu \leq T(j \circ u)$ $(u \in C_o(\Omega))$ for all $j \in SL$.

Theorem 1 TFAE:

(i) T is submarkovian (i.e. $T \geq 0$, $\|T\| \leq 1$)

(ii) $j(Tu) \leq T(j \circ u)$ $(u \in C_o(\Omega))$ for all $j \in J$

(iii) There exists $j \in J/SL$ s.t. $j(Tu) \leq T(j \circ u)$ $(u \in C_o(\Omega))$.

(B) Let $\tau = (T(t))_{t \geq 0}$ be a semigroup on $C_o(\Omega)$ with generator A.

Theorem 2 If τ is submarkovian and $j \in J \cap C^1$, then for all $u \in D(A)$, $0 \leq \mu \in D(A')$

$$(K_j) \qquad \int (j' \circ u)\, Au \, d\mu \leq \int j \circ u \, dA\mu .$$

An example shows that (K_j) for all $j \in J \cap C^1$ does not imply that τ is positive or contractive. However, converse versions of Theorem 2 can be obtained under additional assumptions.

H. AMANN:

ELLIPTIC SYSTEMS AND ANALYTIC SEMIGROUPS

We discuss the problem of the generation of analytic semigroups by general elliptic systems under appropriate general boundary conditions. We showed the main step in the derivation of the resolvnt estimates, namely the estimation of H_p^{s+2n}-norms for a half-space problem. This is done by using the theory of analytic semigroups and a characterization of Besov spaces by means of analytic semigroups.

C.J.K. BATTY:

TAUBERIAN THEOREMS AND STABILITY OF SEMIGROUPS

Theorem 1: Let $(T(t))_{t>0}$ be a bounded C_0-semigroup on a Banach space X with generator A, and suppose that $R\sigma(A^*) \cap i\mathbb{R} = \phi$ and $\sigma(A) \cap i\mathbb{R}$ is countable. Then $\|T(t)x\| \to 0$ as $t \to \infty$, for all x in X.

The proof of this theorem depends on estimating a contour integral, and applying a transfinite induction. The estimate may also be used to prove the following Tauberian theorem:

<u>Theorem 2:</u> Let $f: [0,\infty] \longrightarrow X$ be bounded and strongly measurable with Laplace transform g. Suppose that the singular set E of g is (Lebesque) null and that

$$\sup_{i\eta\in E} \sup_{t>0} \| \int_0^t e^{i\eta s} f(s)\, ds \| < \infty.$$

then

$$\| \int_0^t e^{-i\xi s} f(s)\, ds - g(i\xi) \| \longrightarrow 0 \text{ as } t \longrightarrow \infty$$

for all regular points $i\xi$.

Theorem 2 is an analogue of a Tauberian theorem for power series due to Allan, O'Farrell, and Rousford. It is possible to lift Theorem 2 to obtain a Tauberian theorem for Laplace-Stieltjes transforms, which includes Theorem 2, the power series version, and versions to Dirchlet series.

PH. BENILAN, M.G. CRANDALL AND A. PAZY

APPLICATION OF THE NONLINEAR THEORY TO LINEAR EVOLUTION EQUATIONS

In the nonlinear theory we consider the equation in a Banach space X

$$(*) \qquad f \in \frac{du}{dt} + Au \quad \text{on} \quad [0;T[$$

when $A: X \longrightarrow P(X)$, $f \in L^1_{loc}([0,T[;X)$ and define a mild solution of (*) as a continuous $u: [0,T[\longrightarrow X$ which can be approximated by solution of the discretisation of (*) by an implicit scheme, namely for all $\varepsilon > 0$, $0 < \bar{T} < T$, there exist

$$t_o = 0 < t_1 < \dots \leq t_{n-1} < \bar{T} < t_n < T$$

$$x_o, \dots, x_n, y_1, \dots, y_n \in X \text{ s.t.}$$

$$y_i \in \frac{x_i - x_{i-1}}{t_i - t_{i-1}} + Ax_i \text{ for } i = 1, \dots, n$$

$$t_i - t_{i-1} < \varepsilon, \| u(t) - x_i \| < \varepsilon \text{ for } t \in [t_{i-1}, t_i], i = 1, \dots, n$$

and

$$\Sigma \int_{t_{i-1}}^{t_i} \| f(t) - y_i \| dt < \varepsilon.$$

In the case when A is linear, that is the graph of A (= $\{(x,y) \in X \times X;\ y \in Ax\}$) is a linear subspace of $X \times X$, we have the following characterizations

<u>Theorem</u> Let $A : X \rightarrow P(x)$ be linear,

$f \in L^1_{loc}([0, T[; X)$ and $u \in C([0,T[; X)$.

Then the following properties are equivalent:

(i) u is a mild solution of (*).

(ii) u is an "integerated solution" of (*) in the sense $U(t) = \int_0^t u(s)ds$ is a classical solution of $F(t) \in U'(t) + \bar{A}U(t)$ with $F(t) = u(0) + \int_0^t f(s)(ds)$ where $\bar{A}$ is the operator (linear) whose graph is the closure of the graph of A.

(iii) u is a "weak solution" of (*) in the sense

$$\frac{d}{dt} \langle v, u(t) \rangle - \langle w, u(t) \rangle = \langle v, f(t) \rangle \text{ in } D'(]0,T[)$$

for any $w \in A^* v$, that is $w, v \in X^*$ s.t.

$$\langle w, x \rangle = \langle v, y \rangle \text{ for all } y \in Ax.$$

Some applications of these characterizations are given.

K.N. BOYADZHIEV:

STRONG STABILITY OF CONTRACTION C_0 SEMI-GROUPS ON HILBERT SPACE

Let $e^{TA}, t \geq 0$, be a completely non-unitary contraction on the Hilbert space H. Consider the condition:

$$\sqrt{\alpha}\,(A - i\beta - \alpha)^{-1} x \to 0 \text{ when } \alpha > 0,\ \alpha \to 0$$

for all $x \in D(A) \cup D(A^*)$ and a.e. $\beta \in R$.

This condition implies:

$$e^{tA} x \to 0 \ (t \to \infty) \text{ for all } x \in H,$$

and in a certain sense, the converse is also true. The proof is based on the Nagy-Foias functional model for completely non-unitary Hilbert space contractions.

O. BRATTELI:

HEAT SEMIGROUPS AND INTEGRATION OF LIE ALGEBRAS

Let B be a Banach space, $\underline{G}$ of a finite dimensional Lie algebra with universal covering Lie group G, and let V be a representation of $\underline{G}$ as (unbounded) closed operators on B such that

$$B_\infty = \bigcap_{n\geq 1} \bigcap_{x_1..x_n \in \underline{G}} D(V(x_1)\, V(x_2) \ldots V(x_n))$$

is dense in B. Let $x_1, \ldots, x_d$ be a basis for the vector space of $\underline{G}$ and define the corresponding Laplacian by

$$\Delta = - \sum_{i=1}^{d} V(x_i)^2.$$

We say that V integrates if there exists a strongly continuous representation U of G as bounded operators on B such that $V(x) = \frac{d}{dt} U\,(\exp(tx))\big|_{t=o}$ for each $x \in \underline{G}$.

<u>Theorem</u> (Bratteli-Goodman-Jorgensen-Robinson 87) The following two conditions are equivalent

1. V integrates

2. (A) The $V(x)$, $x \in \underline{G}$, are weakly conservative, i.e. for each $x \in \underline{G}$ or there exists a $M > 1$, $w > 0$ such that $\|(1 + \alpha\, V(x))^n a\| > M^{-1}(1-|\alpha|w)^n \|a\|$ for all $a \in B_\infty$, $n = 0, 1, 2, \ldots, \alpha \in \mathbb{R}$.

(B) Δ is closable and $\overline{\Delta}$ generates a continuous semigroup $S_t = e^{-t\overline{\Delta}}$.

(C) $S_t\, B \subseteq B_\infty$ for $t > 0$.

(D) For each $x \in \underline{G}$ there is a $C > 0$ such that

$$\|V(x)\, S_t\| \leq C\, t^{-1/2}$$

for $0 < t < 1$.

Condition (C) can be replaced by $S_t\, B \subseteq B_4$ (Robinson 87). If $\underline{G}$ is a C*-algebra and each $V(x)$ is a derivation, then if $-\Delta$ is dissipative $V(x)$ is automatically conservative on $D(\Delta)$, i.e. $\|(1 + \alpha\, V(x))\, a\,\| \geq \|a\|$ for all $\alpha \in \mathbb{R}$, $a \in D(\Delta)$, and hence condition (A) may be replaced by: $e^{-t\bar{\Delta}}$ is a contraction semigroup and $D(\Delta)$ is a joint cone for all $V(x)$, $x \in \underline{G}$. (Bratteli-Jorgensen 87). Note that a derivation δ on a C*-algebra $\underline{G}$ is not automatically conservative, even when the resolvent $(1 - \alpha\,\delta)^{-1}$ exists as a bounded operator for all $\alpha \in \mathbb{R}$ (Batty-Bratteli-Robinson 87).

COULHON:

DIMENSIONS OF A SEMIGROUP AND EMBEDDINGS BETWEEN LIPSCHITZ AND SOBOLEV SPACES

This talk is about a joint work with Laurent Saloff-Coste. Let $(T_t)_{t>0}$ be a submarkovian symmetric semigroup on $L^2(X,\xi)$. If there exists $C > 0$ such that $\|T_t f\|_\infty \leq C\, t^{-\frac{n}{2}} \|f\|_1$, for all $f \in L^1(X,\xi)$. , we say, following Varopoulos, that $(T_t)_{t > 0}$ is of dimension n. We consider some similar notions of dimension for

t small and t going to infinity, which we call local dimension, d, and dimension at infinity, D, of the semigroup. We consider the classical Besov and Sobolev norms associated to a semigroup, e.g.

$$\Lambda_\alpha^{p,q}(f) = \left[\int_0^{+\infty} \left(t^{1-\frac{\alpha}{2}} \| A T_t f \|_p\right)^q \frac{dt}{t} \right]^{1/q}$$

for $0 < \alpha < 2$, and $L_\alpha^p(f) = \| A^{\frac{\alpha}{2}} f \|_p$, where $-A$ is the generator of $(T_t)_{t>0}$. Using the Littlewood-Paley-Stein function g, we show that the relations between the scales of Besov and Sobolev spaces in $\mathbb{R}^n$, established by Taibleson, remain true in our setting. Supposing that $(T_t)_{t>0}$ is of dimension n, we study embeddings of Sobolev spaces L_α^p into Lipschitz and L^p spaces, according to the position of αp with respect to n. We treat the case when the local dimension and the dimension at infinity differ, by introducing appropriate spaces. We finally show that $L_\alpha^p \cap L^q \subset L^\infty$. These results put together draw a fairly complete picture of the different Sobolev embedding theorems for spaces associated with a semigroup which has two dimensions, according as $d < D < +\infty$, $D < d < +\infty$, or $d < D = +\infty$. This applies to spaces of functions on a unimodular Lie group, since Varopoulos showed that the heat semigroup generated by a sublaplacian on such a group indeed has two dimensions.

E.B. DAVIES:

HEAT KERNEL BONDS AND LOG SOBOLEV INEQUALITIES

Log sobolev inequalities are closely related to $L^p \to L^q$ boundedness of the heat kernel. This may be used to prove a uniform bound on the heat kernels of many second order elliptic operators on manifolds. By using weighted L^p spaces one may get a gaussian upper bound on the heat kernels, which are close to optimal in many cases. Applications to Laplace-Beltrami operators are of particular interest.

G. DI BLASIO

REGULARITY RESULTS FOR SOME ANALYTIC SEMIGROUPS

Let $A: \quad D(A) \subseteq X \to X$ be the infinitesimal generator of an analytic semigroup $S(t)$ on a Banach space X. We denote by $D_{\theta,p}$ ($\theta \in]0,1[$, $1 < p < \infty$) the interpolation space $(X,D(A))_{\theta,p}$ between $D(A)$ and X. Then if we set $u_o(t) = S(t)x$ and $u_1(t) = \int_o^t S(t-s)f(s)\,ds$, for $x \in X$ and $f \in L^1(0,T;X)$, we can prove the following regularity results

(i) $\quad u_o \in C(0,T;X) \cap W^{\theta,1}(0,T;X) \cap L^1(0,T;D_{\theta,1}) \quad 0 < \theta < 1.$

(ii) if $x \in D_{\alpha,1}$, then $u_0 \in C(0,T;D_{\alpha,1})$ and u_0', $Au_0 \in W^{\alpha,1}(0,T;X) \cap L^1(0,T;D_{\alpha,1})$.

(iii) $u_1 \in C(0,T;X) \cap W^{\theta,1}(0,T;X) \cap L^1(0,T;D_{\theta,1})$, $0 < \theta < 1$.

(iv) if $f \in L^1(0,T;D_{\alpha,1})$, then $u_1 \in C(0,T;D_{\alpha,1})$, u_1', $Au_1 \in L^1(0,T;D_{\alpha,1})$ and $Au_1 \in W^{\alpha,1}(0,T;X)$.

(v) if $f \in W^{\alpha,1}(0,T;X)$ then $u_1 \in C(0,T;D_{\alpha,1})$, u_1', $Au_1 \in W^{\alpha,1}(0,T;X)$ and $u_1' \in L^1(0,T;D_{\alpha,1})$.

Since the function $u = u_0 + u_1$ is the solution of the Cauchy problem (C.P) $u'(t) = Au(t) + f(t)$, $u(0) = x$, properties (i)-(v) give regularity results for solutions of (C.P)

Now let A be the L^1 realization of an elliptic operator in R^n. Then the following characterization can be proved

$$D_{\theta,1} = \begin{cases} W^{2\theta,1}(R^n) & \text{if } \theta < 1/2 \\ B^{1,1}(R^n) & \text{if } \theta = 1/2 \\ u \in W^{1,1}(R^n) \ : \ D_i u \in W^{2\theta-1,1}(R^n) & \text{if } \theta > 1/2 \end{cases}$$

Therefore using (i)-(v) we obtain regularity results for solutions of parabolic equations in $L^1(R^n)$.

O. DIEKMANN:

PERTURBATION THEORY FOR DUAL SEMIGROUPS (WITH APPLICATIONS TO STRUCTURED POPULATION MODELS)

This is a story about suns and stars. Let T_0 be a strongly continuous semigroup with generator A_0 on a non-reflexive Banach space X. Define $X^{\odot} = \overline{D(A_0^*)}$ and let $C : X^{\odot} \to X^*$ be linear and bounded. The variation-of-constants equation

$$T^{\odot}(t)x^{\odot} = T_0^{\odot}(t)x^{\odot} + \int_0^t T_0^*(t-\tau)C\, T^{\odot}(\tau)x^{\odot}d\tau$$

(where the integral is a weak * Riemann integral) yields a "perturbed" strongly continuous semigroup $T^{\odot}$ on $X^{\odot}$ which can be extended to the whole space X^* by the intertwining formula

$$T^{\times}(t) = (\lambda I-A^{\times})\, T^{\odot}(t)(\lambda I-A^{\times})^{-1}$$

where $D(A^{\times}) := D(A_0^*)$ and $A^* := A_0^* + C$. In general $T^{\times}$ is not a dual semigroup. However, introducing $X^{\odot *}$ and $X^{\odot\odot}$ as well as $T^{\odot *}$ and $T^{\odot\odot}$ we have

$$[T^{\odot\odot}(t)x^{\odot\odot}, x^*] = [x^{\odot\odot}, T^{\times}(t)x^*] \qquad \text{etc.}$$

where the pairing $[\ ,\ ]$ between $X^{\odot\odot}$ and X^* is defined by

$$[x^{\odot\odot}, x^*] = \lim_{t\downarrow 0} \frac{1}{t} < x^{\odot\odot}, \int_0^t T_0^*(\tau)x^*d\tau >$$

The variation-of-constants equation can be used to prove linearized stability for semilinear problems. A motivating example from physiologically structured population theory is discussed in some detail. The lecture is based on joint work with

Ph. Clement, M. Gyllenberg, H.J.A.M. Heymans, J.A.J. Metz, and H.R. Thieme.

G. GREINER

SEMILINEAR BOUNDARY CONDITIONS

We consider initial-boundary value problems in Banach spaces of the following form

$$\dot{u} = Au + F(t,u) \;;\; Lu = \Phi(t,u),\; u(0)=u_0$$

where $A : D(A) \to X$ is closed linear, $F:\mathbb{R} \times X \to X$ is continuous $L : D(A) \to \partial X$ is a linear A-bounded surjection, $\Phi : \mathbb{R}_+ \times X \to X$ is continuous. Moreover, it is assumed that the homogenous linear problem is well-posed, i.e., the restriction $A_0 := A|_{\ker L}$ generates a C_0-semigroup $(T_0(t))$.

Assuming that Φ is Frechet differentiable and $D\Phi(t,x) \circ A$ is bounded one can prove all the results concerning existence, uniqueness, blow up, regularization, dependence on initial data, and linearized stability respectively which are well-known for initial value problems (i.e. in case L=0, Φ=0).

If A_0 generates an analytic semigroup one can prove existence and uniqueness if the following conditions are satisfied. There exist $0 < \alpha < \beta < 1$ such that $D((-A_0)^{\beta}) \supseteq D(A)$; $F : D((-A_0)^{\alpha}) \to X$ and $\Phi : D((-A_0)^{\alpha}) \to \partial X$ are both locally Lipschlitz; $u_0 \in D((-A_0)^{\alpha})$.

J. HEJTMANEK:

SEMIGROUPS AND NONSTANDARD ANALYSIS

Methods from Nonstandard Analysis, especially Loeb integration theory, has been applied by Leif Arkeryd from Goteborg to construct a solution for the nonlinear Boltzmann equation. The linear Boltzmann equation, especially the neutron transport equation, is a paradigma for a strongly continuous semigroup of positive operators. The following problem is posed: If we know the spectrum of the neutron transport operator A being $\sigma(A) = \{\lambda_0 = 0\} \cup \{\lambda \in \mathbb{C} : \mathrm{Re}\,\lambda < \lambda_1\}$ for some $\lambda_1 < 0$, is it possible to describe the asymptotic behavior of the semigroup $[U(t) : t > 0]$, as $t \to \infty$? If $\lambda_o = 0$ is an eigenvalue of multiplicity one and eigenprojection P, then the semigroup splits into an asymptotic part $e^{\lambda_o t} P$ and a transient part (I-P) U(t). What is the asymptotic behavior, as $t \to \infty$, of this asymptotic part? A. Huber [1987] proposed to replace the "continuous" semigroup $[U(t) : t > 0]$ by the "discrete" nonstandard semigroup $[(I-hA)^{-j} : j \in {}^*N]$, where $0 < h \approx 0$. Then, in the nonstandard world we have $(I-hA)^{-j} \approx P$, for all $j > j_o$, for some $j_o \in {}^*\mathbb{N}\setminus\mathbb{N}$.

I. HERBST

SOBOLEV SPACES, KAC-REGULARITY, AND THE FEYNMAN-KAC FORMULA

Given a Borel set $M \subset \mathbb{R}^d$ let $\tilde{H}^1_o(M) = L^2(M) \cap H^1(\mathbb{R}^d) = \{f \in L^2(\mathbb{R}^d): \nabla f \in L^2(\mathbb{R}^d)\}$, $L^2(M) = \{f \in L^2(\mathbb{R}^d): f = 0$ a.e. on $M^c\}$. The space $\tilde{H}^1_o(M)$ arises naturally. For example for $t > 0$ and $n \to \infty$

$$e^{t(\Delta - n\chi_{M^c})} \xrightarrow{s} e^{tG_M} P_M$$

where G_M is the Laplacian with form domain $\tilde{H}^1_o(M)$, and $P_M f = \chi_{M^*} f$ with $M^* = \{x \in \mathbb{R}^d: P^x(\gamma_M > 0) = 1\}$. Here γ_M is the "penetration time" of Brownian motion into M^c (Stroock's definition): $\gamma_M = \inf\{t > 0 : \int_0^t \chi_{M^c}(x_s)\, ds > 0\}$. If $M = D$ is open

$$H^1_o(D) \subset \tilde{H}^1_o(D) \subset \tilde{H}^1_o(\bar{D})$$

where $H^1_o(D)$ is the "usual" Sobolev space. If τ_M is the hitting time for M^c then

$H^1_o(D) = \tilde{H}^1_o(D) \iff \tau_D = \gamma_D, P^x\text{-a.s.} \iff P^x(\tau_D < \infty,\ x(\tau_D) \in D^*) = 0$

for all $x \in D$. Also

$$\tilde{H}^1_o(D) = \tilde{H}^1_o(\overline{D}) \iff m(\{ x \in \partial D: P^x(\tau_{\overline{D}} > 0) = 1\}) = 0$$

with m = Lebesgue measure. In general

$$e^{tG_M} P_M f(x) = E^x(f(x_t); \gamma_M > t).$$

P. HESS:

ON DISCRETE STRONGLY ORDER-PRESERVING SEMIGROUPS

In the order-bounded closed subset D of a suitable Banach lattice X the discrete-time strongly order-preserving semigroup $(S^n)_{n \in \mathbb{N}}$ is considered. In particular the question of stabilization is studied: when is it true that $S^n x \longrightarrow q$ $(n \longrightarrow \infty)$, with $Sq = q$? A sufficient condition is given and the result is applied to periodic-parabolic initial-boundary value problems. A passage to the limit in the period yields results also for continuous-time strongly order-preserving semigroups and extends well-known work of M. Hirsch. Finally perturbation results (asymptotically autonomous discrete-time dynamical processes) are presented.

R.M. KAUFFMAN

A STRONG CONTRACTIVITY PROPERTY FOR SEMIGROUPS GENERATED BY DIFFERENTIAL OPERATORS

We study the question of semigroups in $L_{2,p}(R^k)$ generated by the restriction A of L to $C_0^\infty(\mathbb{R}^k)$ where L is a formally positive partial differential expresion of arbitrary order, and ρ is C weight function which may be very small at infinity, and which is computed in terms of the coefficient of L. It is shown that often in the discrete spectrum case −A generates an analytic semigroup u_t in $L_{2,p}$, such that $u_t f \in L_{2,1/p}$ for all $t > 0$. For the heat equation in $\mathbb{R}^2$, with appropriate coefficients, this means that temperatures which are initially very large at $|x|=\infty$ become immediately very small at infinity.

F. KAPPEL

A UNIFORMELY DIFFERENTIABLE APPROXIMATION SCHEME FOR DELAY EQUATIONS

Using spline functions an approximation scheme for delay equations is developed which is uniformly differentiable in the sense that

there exists a fixed exponential sector in the complex plane which contains the spectra of all approximating infinitesimal generators. Using the fact that a differentiable semigroup can be obtained by integration of the resolvent along the boundary of an exponential sector, we can prove optimal convergence rates for our scheme. The results presented in the talk are joint work with K. Ito(Brown University).

J. KISYNSKI

MARKOV SEMIGROUPS GENERATED BY DIFFEROINTEGRAL BOUNDARY SYSTEMS OF VENTCELI-WALDENFELS

Let $(N_t)_{t>0}$ be a Feller semigroup on a compact C^∞ manifold M with boundary, generated by an elliptic differointegral boundary system (W,Γ,δ) of Ventcel'-Waldenfels. Let $\mathbb{D} = \mathbb{D}(0,\infty;M_\Delta)$ be the Skorochod space, and $(P^x)_{x\in M_\Delta}$ the Markov system of probability measures on $\mathbb{D}$, corresponding to the transition semigroup $(N_t)_{t>0}$. Under assumption of ellipticity of all the considered systems (W,Γ,δ), for $\alpha\in(0,1)$, we prove continuity of the map

$$(W,\Gamma,\delta,x)\in L(C^{2+\alpha}(M),C^\alpha(M))\times L(C^{2+\alpha}(M),C^\alpha(\partial M))\times C^\alpha(\partial M)\times M_\Delta \to P^x\in P(\mathbb{D})$$

where $P(\mathbb{D})$ = {Borel probability measures on $\mathbb{D}$}, the spaces of operators being equipped with their norm topologies, and $P(\mathbb{D})$ with topology of weak convergence of measures. As a consequence of this result, we can prove (in rigorous measure theoretical formulation) that before the time of first exit from $M \setminus \partial M$ the behaviour of the Markov process generated by (W,Γ,δ) is independent of Γ and δ. Importance of the former result can be confirmed by some calculations concerning Brownian motion on $\mathbb{R}^+$ submitted to Feller's boundary conditions at 0.

M.A. KON

SEMIGROUPS GENERATED BY ELLIPTIC OPERATORS

(i) We consider semigroups generated by elliptic operators

$$A = \sum_{|\alpha| \leq m} b_\alpha(x) D^\alpha,$$

where α is a multiindex and $D^\alpha = \left(\frac{1}{i}\right)^{|\alpha|} \frac{\partial^\alpha}{\partial x^\alpha}$.

We assume that the leading term of A is positive and with smooth coefficients, and that for $|\alpha| < m$,

$$b_\alpha \in L^{r_\alpha},$$

where $\frac{n}{r_\alpha} + |\alpha| < m$ (these are analoges of appropriate L^p conditions for the potential of a Schrödinger operator to yield an operator with most standard regularity properties).

<u>Theorem:</u> A generates a semigroup e^{-tA} which is analytic in the right half t-plane. This semigroup maps $L^p \to L^q$ for all $q > p$, including $q = \infty$.

(ii) Let L^p_s denote the space of functions with s derivatives in L^p (s may be any real number). Schrödinger semigroup mappings between these spaces have recently been studied by Simon and others.

<u>Theorem:</u> Let $V(x) \in L^{\frac{n}{2}+\varepsilon}_{loc}$, $1 < p < \infty$, $s > 0$, and $A = -\Delta + V$ on $\mathbb{R}^n$. Then $e^{-tA}: L^p \in L^p_{s+2,loc}$ is bounded iff $V \in L^p_{s,loc}$.

This is proved using a Leibnitz rule for fractional derivatives, and some lemmas on the composition properties of functions in Sobolev spaces. This can be partially extended to higher order operators.

(iii) Here we consider the case $V(x) \in L^p$, where $p < \frac{n}{2}$. In this case (where the potential is an unbounded relative to the Laplacian), essentially any kind of smoothing can occur. That is, there exist such highly singular potentials whose semigroups are infinitely smoothing in the scale of Sobolev spaces, and others in this class whose semigroups add only two derivatives to functions.

B. KUMMERER

DILATIONS OF SEMIGROUPS AND NON-COMMUTATIVE MARKOV PROCESSES

In classical probability theory to a Markov process there corresponds a semigroup of transition operators and conversely, to any such semigroup there corresponds uniquely a Markov process. If the process is stationary with state space Ω, inducing there is an invariant probability measure μ, then the transition operators may be realized as doubly stochastic operators on $L^\infty(\Omega, \Sigma, \mu)$.

In developing a theory of stationary Markov processes which is suitable also for quantum mechanical applications we generalize $L^\infty(\Omega,\Sigma,\mu)$ to a pair (M,ϕ) consisting of a W^*-algebra equipped with a faithful normal state ϕ. The notion of a Markov process is reformulated in terms of a dilation for the semigroup of transition operators. A doubly stochastic semigroup T_t of transition operators on (M, ϕ) is also called a dynamical system (M, ϕ, T_t).

<u>Theorem</u> (Kummerer-Maassen). For a dynamical system (M_n,tr,T_t) (M_n the nxn-matrices, tr the normalized trace on M_n) the following conditions are equivalent

(a) (M_n, tr, T_t) admits a Markov process generating an algebra of the form $M_n \otimes L^\infty(\Omega, \Sigma, \mu)$

(b) For all $t>0$: $T_t \in co \cdot Aut(M_n)$, the convex hull of automorphisms of M_n.

(c) There exists a weak*-continuous convolution semigroup S_t on Aut (M_n) such that $T_t = \int_{Aut(M_n)} \alpha \, dS_t(\alpha)$

(d) $T_t = e^{Lt}$ where $L(x) = i(hx - xh) + \sum_j a_j x a_j - \frac{1}{2}(a_j^2 x + a_j x) + \sum_k \lambda_k (u_i \, x \, u_i - x)$ where h, a_j are self-adjoint, u_k unitary, $\lambda_k \in \mathbb{R}^+$ for all j,k.

(e) $T_t = e^{Lt}$ where L can be approximated by generator of the form $\Sigma_{k>0} \lambda_k (u_k^* \, x \, u_k - x)$, u_k unitary, $\lambda_k > 0$ for all k.

The implication (e)=>(a) can be generalized to the following:

<u>Theorem:</u> For a dynamical system (M,ϕ,T_t) the following conditions are equivalent:

(a) (M,ϕ,T_t) has a dilation

(b) for all t_0, (M,ϕ,T_{t_0}) has a dilation in discrete time

(c) $T_t = \text{weak}^*\text{-}\lim_j e^{\alpha_j (T_j - jd)t}$ for all $t > 0$ where for all j the discrete dynamical system (M,ϕ,T_j) has a dilation and $(\alpha_j)_j \subseteq \mathbb{R}_+$.

R. LEIS:

SCATTERING PROBLEMS IN THERMOELASTICITY

The linear systems of elasticity and thermoelasticity are formulated first. Initial-boundary value problems in $\mathbb{R}^3$ are solved using semigroup approach. The solutions obtained have finite total energy. Afterwards the asymptotic behavior of the solutions for $t \to \infty$ is discussed with emphasis on exterior boundary value problems. The free space case with homogeneous isotropic medium can be calculated explicitly, and one notices that the equations split into a vibrating component and into a component with damping. Special boundary value problems can also be treated. Finally more results on the corresponding nonlinear equations are indicated concerning the existence of global solutions for small and smooth data. An exterior boundary value problem in $\mathbb{R}^1$ has been studied by Jiang and the free space problem in $\mathbb{R}^3$ has been treated by Racke.

G. LUMER:

SINGULAR MULTIPLICATIVE PERTURBATION, OPERATORS OF FINITE TYPE AND "WEIGHTED" NORMS

We study a very general class of ("homotopy-like") perturbations. We give general stability results concerning the density of the image of the perturbed operators (stability of "$\beta(\cdot) = 0$", $\beta(\cdot)$ an appropriate deficiency index) for singular and nonsingular perturbations. We give applications to singular multiplicative perturbations of generators of contraction semigroups, which extend considerably results known so far (example: Theorem. Let A be the generator of a contraction semigroup on X (Banach space), $B \in B(X)$ be accretive with $I(B^*)$ dense. Then BA pregenerates a contraction semigroup on X iff it is dissipative). We extend some of the latter results to generators of "finite type" (operators which "less some constants" are dissipative). One way in which operators of finite type arise naturally in applications is by using weights (weighted norms) on usual function spaces, and studying operators which in the original spaces (without the weights) are dissipative.

N. MANDOUVALOS

HEAT KERNEL BOUNDS AND DISCRETE GROUPS

This is work done jointly with E.B. Davis. We have obtained sharp upper and lower bounds for the heat kernel on hyperbolic space $\mathbb{H}^{n+1}$ for all $n \geq 1$ and for real and complex time. We have used these bounds to obtain upper bounds for the heat kernel on hyperbolic manifolds of the form $\Gamma\setminus\mathbb{H}^{n+1}$, where Γ is a Kleinian group on $\mathbb{H}^{n+1}$. The most important of our results are described in the following two theorems. (We use the notation $f \sim g$ for functions f and g when there is a constant $c>0$ such that $c^{-1}f(s) \leq g(s) \leq cf(s)$ for all s).

Theorem 1. Let $L_{n+1} \leq 0$ be the hyperbolic Laplacian on $\mathbb{H}^{n+1}$ and $K_{n+1}(t,\rho)$ be the kernel of the operator $e^{L_{n+1}t}$, $t>0$. If $n \geq 1$ is any integer then

$$K_{n+1}(t,\rho) \sim t^{-(n+1)/2}\, e^{-(n^2t/4)-(\rho^2/4t)-(n\rho/2)} (1+\rho+t)^{n/2-1}(1+\rho), \text{ uniformly for}$$

$0 \leq \rho < \infty$ and $0 < t < \infty$, where ρ is the hyperbolic distance in $\mathbb{H}^{n+1}$.

Let now $w = (x,y) \in \mathbb{H}^{n+1}$, $x \in \mathbb{R}^n$, $y > 0$,

$\sigma(w,w') = \dfrac{|x-x'|^2 + (y+y')^2}{4yy'}$ be the standard point-pair invariant in $\mathbb{H}^{n+1}$ and let $Con(n+1)$ be the group of isometrics of $\mathbb{H}^{n+1}$. Let Γ be a discrete subgroup of $Con(n+1)$ and $\delta(\Gamma)$ be the invariant $\delta(\Gamma) = \inf\{s>0 : \sum_{\gamma\in\Gamma} e^{-s\rho(\gamma x,y)} < +\infty\}$. Let also

$\mu_\alpha(x)^2 = \sum_{\gamma\in\Gamma} e^{-\alpha\rho(x,\gamma x)}$, $\alpha>\delta(\Gamma)$, and $\tilde\rho(x,y) = \min_{\gamma\in\Gamma}\rho(\gamma x,y)$ be the distance function in $\Gamma\setminus \mathbb{H}^{n+1}$. Then the following theorem holds.

Theorem 2. Let Γ be a geometrically finite Kleinian group on $\mathbb{H}^{n+1}$. Then the following hold for the heat kernel $\tilde K(t,x,y)$ of $\Gamma \quad \mathbb{H}^{n+1}$:

(i) If $0 \leq \delta(\Gamma) \leq \frac{n}{2}$, $0 < t < \infty$ and $0 < \varepsilon < \infty$, then

$$0<\tilde K(t,x,y)<C_\varepsilon t^{-(n+1)/2} e^{-(\frac{n^2}{4} - 2\varepsilon)t} e^{-\tilde\rho(x,y)^2/4(1+\varepsilon)t} \mu_\alpha(x)\mu_\alpha(y).$$

(ii) If $\frac{n}{q} \leq \delta(\Gamma) \leq n$, $0 < t < \infty$, $\alpha > \delta(\Gamma)$ and $0 < \varepsilon < \infty$, then

$$0<\tilde K(t,x,y)<C_\varepsilon t^{-(n+1)/2}\mu_\alpha(x)\mu_\alpha(y)e^{-[\delta(\Gamma)(n-\delta(\Gamma))-2\varepsilon]t}\cdot e^{-\tilde\rho(x,y)^2/4(1+\varepsilon)t}.$$

Moreover if $\alpha > n$ then $\mu_\alpha(x) \to 1$ as x approaches an end and $\mu_\alpha(x)\sim e^{p(x,a)r/2}$ as x approaches a cusp of rank r, where a is any point in $\mathbb{H}^{n+2}$. In particular $\mu_\alpha(x)$ is bounded if $\alpha > n$ and the manifold has no cusps.

I. MIYADERA:

SEMIGROUPS AND EXPONENTIALLY BOUNDED C-SEMIGROUPS

We first discuss the relationship between semigroups and exponentially bounded C-semigroups. It is shown that if $\{T(t);\ t > 0\}$ is a semigroup of classes $(C_{(k)})$ of growth order

α , then $\{CT(t);\ t > 0\}$ becomes an exponentially bounded C-semigroup with a suitable C and its C-complete infinistesimal generator coincides with the complete infinitesimal generator of $\{T(t);\ t > 0\}$.

Next, a characterization for the C-complete infinitesimal generator of an exponentially bounded C-semigroup is given, and then the generation of semigroups is discussed. By using our generation theorem, it is shown that the known generation theorems for semigroups of the above-mentioned classes are obtained in a unified way.

R. NAGEL:

HOW TO INVERT A 2X2-MATRIX

Systems of linear evolution equations often lead to operator matrices $\underline{A} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ or a product Banach space E x F. If D is invertible one obtains the inverse of A as

$$\underline{A}^{-1} = \begin{pmatrix} \Delta^{-1} & -\Delta^{-1}BD^{-1} \\ -D^{-1}C\Delta^{-1} & D^{-1}(F + C\Delta^{-1}BD^{-1}) \end{pmatrix}$$

for $\Delta := A - BD^{-1}C$. It is shown how the above formula can be used in order to compute the spectrum $\sigma(\underline{A})$ for $\underline{A}$ with unbounded entries.

B. NAJMAN:

RESOLVENT ESTIMATES FOR SINGULARITY PERTURBED ELLIPTIC OPERATORS

Let Ω be a smooth bounded domain, A an elliptic operator of order $2m$, B an elliptic operator of order $2m'$, $m > m'$, and let $A_\varepsilon = \varepsilon A + B$. Let $1 < p < \infty$ and let $\| \ \|_{s,p}$ be the norm of the Lebesque space $H^{s,p}(\Omega)$. The estimate

$$\|u\|_{s,p} < C(\varepsilon, s, t, \lambda) (\|(A_\varepsilon + \lambda) u\|_{t,p} + \|u\|_{t,p})$$

is proved for λ from a sector of the complex plane, $C(\varepsilon, s, t, \lambda)$ explicitly given.

F. NEUBRANDER:

C-SEMIGROUPS AND THEIR APPLICATION TO THE COMPLETE SECOND ORDER PROBLEM

A natural way to study well posedness of the complete second order equation

$$u''(t) - Bu'(t) - Au(t) = 0 \ ; \ u(0) = x, \ u'(0) = y$$

is to reduce it to a first order system $w'(t) = Mw(t)$; $w(0) = (x,y)$, $M = \begin{pmatrix} 0 & I \\ A & B \end{pmatrix}$ on $E \times E$ where one has to construct a phase-space $F_0 \subset E \times E$ on which $M|_{F_0}$ generates a C_o-semigroup on F_0. For unbounded, closed operators A,B on an arbitrary Banach space E the construction of these phase-spaces is often difficult if not impossible. We show how the theory of "integrated semigroups" (i.e. C-semigroups with $C = R(\lambda_0,A)^n$) can be used to bypass these difficulties.

S. OHARU

NONLINEAR PERTURBATIONS OF ANALYTIC SEMIGROUPS

Relatively continuous perturbations of analytic semigroups in Banach spaces are discussed from the point of view of the nonlinear semigroup theory. Necessary and sufficient conditions are given for a semilinear operator A + B to be the full infinitesimal generator of a nonlinear semigroup which provides mild solutions of the semilinear evolution equation u' = Au + Bu, where the linear operator A generates an analytic semigroup in a Banach space X and B is continuous with respect to the graph norm of a fractional power of -A. First, nonlinear quasidissipative perturbations of analytic contraction semigroups are treated and a Hille-Yosida type theorem for nonlinearly perturbed quasicontractive analytic semigroups is presented. Next, locally Lipschitz continuous perturbations of bounded analytic semigroups are investigated. The local Lipschitz continuity is stated in terms of a lower semicontinuous functional on X. Finally, applications to semilinear partial differential equations are examined.

N. OKAZAWA:

HOLOMORPHIC FAMILIES OF M-ACCRETIVE OPERATORS IN A REFLEXIVE BANACH SPACE

First we present L^p generalizations of some fundamental estimates in $L^2(\mathbb{R}^m)$ such as the Hardy and Rellich inequalities (we assume $2 \leq p < \infty$): Setting $A = -\Delta$, $B = |x|^{-2}$, we have

$$|\operatorname{Im}(Au, |u|^{p-2}u)| \leq \frac{p-2}{2} \operatorname{Re}(Au, |u|^{p-2}u),$$

$$\frac{p-1}{p^2}(m-2)^2 (Bu, |u|^{p-2}u) \leq \operatorname{Re}(Au, |u|^{p-2}u), \quad m > 2,$$

$$\frac{(p-1)m(m-2p)}{p^2} \|Bu\| \leq \|Au\|, \quad m > 2p.$$

Then we consider properties of the family of operators $A + \kappa B$ with complex κ in a reflexive Banach space X. Two abstract theorems and two examples will be given.

Example 1. $X = L^p(0,\infty)$, $A = d/dx$, $B = |x|^{-1}$.

Example 2. $X = L^p(\mathbb{R}^m)$, $A = -\Delta$, $B = |x|^{-2}$.

D. PFEIFER:

THE PROBABILISTIC REPRESENTATION THEORY FOR ONE-PARAMETER SEMIGROUPS

It is shown that most of the known product representation formulas for (C_o)-semigroups $\{T(t);\ t \geq 0\}$ on a Banach space X are of the form

(1) $$\{\psi_N(E[T(\tfrac{Y}{n})J)\}^n\ f \longrightarrow T(t)f\ \ (f \in X)$$

when $N \geq 0$ is an integer valued random variable with expectation $E(N) = \xi$, $Y \geq 0$ a real random variable with $E(Y) = \gamma$, and $t = \xi_\gamma$ (N,Y fulfilling some regularity conditions). There ψ_N denotes the probability generating function (p.g.f.) of N, and E(.) means quasi-weak Pettis integration (of the quasiweak random operator $T(\frac{Y}{n})$). It is also shown that if for a general representation function ψ it is analytic in some internal $[o,\delta]$, $\delta > 1$, with non-negative coefficients, then ψ is already a p.g.f. of some random variable N. Investigations concerning the role of convergence in (1) are also made, either specifying smoothness conditions on f or by means of the modified modulus of continuity.

M. PIERRE:

ABOUT GLOBAL EXISTENCE FOR REACTION-DIFFUSION SYSTEMS

We are interested in studying global existence of solutions for reaction-diffusion systems which present two main properties:

(1) The nonnegativity of the initial data is preserved.

(2) The total mass of the components is nonincreasing in time.

Properties (1) and (2) lead to an a priori estimate of the L^1-norms of the solutions uniformly in time. It is well-known that existence of an a priori uniform L^∞-bound is a sufficient condition for global existence of solutions. Our goal is to try to understand how a similar bound in L^1 can be exploited. This is motivated by applications to many situations of interest where no L^∞-bounds are expected: let us mention for instance the case when the initial data and the source-terms are L^1-functions.

We describe on a 2x2 system the techniques that can be used: Under an extra structural assumption, the reactive terms can be bounded in L^1. This yields compactness of approximate solutions. The convergence then follows from the uniform integrability of the nonlinear terms.

Y. PINCHOVER:

ON POSITIVE SOLUTIONS OF PARABOLIC EQUATIONS WITH PERIODIC COEFFICIENTS IN SOME UNBOUNDED DOMAINS

We determine all the minimal positive solutions of the parabolic equation Lu = 0 in $\mathbb{R}^n \times \mathbb{R}_-$ (or $\mathbb{R}^n \times \mathbb{R}$), where L has time independent coefficients or L has coefficients which are periodic in the space and the time variables.
Assuming further that L is in divergence form we obtain an integral representation theorem for solutions of the following problem

$$Lu = 0,\ u > 0 \text{ in } F \times \mathbb{R}_-,\ u = 0 \text{ on } \partial (F \times \mathbb{R}_-)$$

where F is a convex cone or a cylinrical domain in R^n.

J. PRUSS:

ANALYTIC RESOLVENTS FOR LINEAR VOLTERRA EQUATIONS OF SCALAR TYPE

Consider equations of the form

(I) $$u(t) = g(t) + \int_0^t a(t-s)Au(s)ds \quad , t>0$$

in a Banach space X, where A is a closed linear densely defined operator in X, $a \in L^1_{loc}(\mathbb{R}_+)$ is a scalar kernel and

$g \in C(\mathbb{R}_+,X)$. By means of complex analysis methods, we characterize those A and a(t) for which (I) admits an analytic resolvent S(t) and obtain the complete analogue of Hille's theorem on generation of analytic semigroups. As an application of this result we study integrability properties of the resolvent for (I) which are important for the solvability behavior of

$$(I') \qquad u(t) = g(t) + \int_{-\infty}^{t} a(t-s)Au(s)ds \quad , \quad t \in \mathbb{R},$$

the limiting equation associated with (I).

R. RACKE:

GLOBAL SOLUTIONS TO SEMILINEAR PARABOLIC SYSTEMS FOR SMALL DATA

We consider semilinear parabolic systems $u_t + Au + f(u) = g$, $u(0) = u_0$, $-A$ being the generator of an analytic semigroup with spectrum $\sigma(A)$ in the right half plane. For appropriate small data (g,u_0) global solutions are obtained if 0 belongs to the resolvent set and local existence is proved if $0 \in \sigma(A)$. The inverse-function theorem is used in a class of functions which are Hölder continuous with respect to time t. The asymptotic behaviour of the solution is given too.

As first application we prove the existence of global (respectively local) solutions to the Navier-Stokes equations in

L_p-spaces in bounded (respectively unbounded) domains. Secondly we consider the case where A is an elliptic operator of order 2m and f has bounded growth. Thirdly a quasilinear example where the nonlinearity involves derivatives of the same order as A is studied. Then an example of a nonautonomous nonlinearity is given and finally the convergence of solutions of the time-dependent system to solutions of the stationary (semilinear elliptic) one is derived for $0 \in \rho(A)$.

D.W. ROBINSON:

INTEGRATION OF LIE ALGEBRAS, THE HEAT SEMIGROUP, LIPSCHITZ SPACES, ETC.

Let (B,G,U) denote a continuous isometric representation of the Lie group G by linear operators $U(g)$, $g \in G$, on the Banach space B. Further let B_n denote the C_n-elements of the representation and $\|\cdot\|_n$ the corresponding C_n-norm. An operator K is defined to be a Lipschitz operator if

$B_\infty \subseteq D(K)$ and

$$\|(U(g)KU(g)^{-1} - K)a\| \leq c|g| \, \|a\|_1 \quad , \quad a \in B_\infty, \; |g| \leq 1,$$

for some $c > 0$. It follows that the closure of a dissipative Lipschitz generator generates a contraction semigroup. This result incorporates and extends many well-known and seemingly disparate results in PDE's, harmonic analysis and mathematical physics. We will discuss the proof of the theorem together with some its applications.

M. RÖCKNER:

DIRICHLET FORMS AND SEMIGROUPS ON TOPOLOGICAL VECTOR SPACES

Let E be a locally convex Hausdorff topological vector space which is Souslin; μ a probability measure on its Borel sets. We study forms of the type

(1) $$\underline{E}_k(u,v) = \int_E \frac{\partial u}{\partial k} \frac{\partial v}{\partial k}\, d\mu \quad \text{on} \quad L^2(E;\mu)$$

where $K \in E\setminus\{0\}$, $\frac{du}{dk}$ means Gateaux derivative of u in the direction k and the domain $D(\underline{E}_k)$ of $\underline{E}_k$ is taken to be the μ-classes in $L^2(E;\mu)$ which are induced by

$\underline{F}\, C_b^\infty : = \{u : E \longrightarrow \mathbb{R}$: there exist $L_1, .., L_m \in E'$ and $f \in C_b^\infty(\mathbb{R}^n)$ such that $u(z) = f(L_1(z), \ldots, L_m(z))$, $z \in E\}$.

If we suppose the $(\underline{E}_k, D(\underline{E}_k))$ is well-defined (i.e. "respects μ-classes") the first crucial question is whether it is closable. We have proved the following necessary and sufficient condition (on μ):

<u>Theorem:</u> Let E_0 be a closed linear subspace of E such that $E = E_0 + \mathbb{R}\ k$ and $\pi : E \longrightarrow E_0$ the natural "projection". Let $\rho: E_0 \times B(\mathbb{R}) \longrightarrow [0,1]$ be a kernel such that for all $u: E \longrightarrow R$, measurable and bounded,

$$\int_E u(z)\ \mu(dz) = \iint_{E_0 R} u(x + sk)\ \ \rho(x,ds)\ \nu(dx),$$

where ν is the image measure of μ under π. Then $(\underline{E}_k, D(\underline{E}_k))$ is well-defined and closable if and only if for γ - a.e., $x \in E_0$, $\rho(x,ds) = \rho(x,s)\ ds$ for some measurable function $\rho(x,.)$: $\mathbb{R} \longrightarrow \mathbb{R}^+$ which satisfies Hamza's condition:

(H) $\qquad \rho(x,.) = 0$ ds-a.e. on $\mathbb{R} \setminus R\ (\rho(x,s))$

where

$$R(\rho(x,\cdot)) := \{t \in \mathbb{R} : \int_{t-\varepsilon}^{t+\varepsilon} \rho(x,s)^{-1}\ ds < +\infty \text{ for some } \varepsilon > 0\}.$$

This theorem provides many examples of sub-Markovian semigroups on $L^2(E;\mu)$ for very general μ. It is namely easy to see that a countable sum $(\underline{E}, D(\underline{E}))$ of closable forms of type (1) is closable. Its closure $(\overline{\underline{E}}, D(\overline{\underline{E}}))$ is then a Dirichlet form, i.e. $u \in D(\overline{\underline{E}}) \Rightarrow |u| \in D(\overline{\underline{E}})$ and $\overline{\underline{E}}(|u|, |u|) \leq \overline{\underline{E}}(u,u)$. The semigroup $T_t = e^{tA}$, $t > 0$, generated by the negative definite self-adjoint operator A associated with $\overline{\underline{E}}$ (note that $D(\underline{E})$ is dense in $L^2(E;\mu)$) is therefore sub-Markovian, i.e. $0 \leq u \leq 1 \Rightarrow 0 \leq T_t u \leq 1$ μ-a.e., $t > 0$. This in turn implies that (under some additional assumption on E) there exists an associated Markov process with state space E. Furthermore, the theorem has applications both to finite dimensional situations like e.g.

operators in divergence form on an open set in $\mathbb{R}^n$ and to infinite dimensional settings, in particular to quasi-invariant measures like Gaussian measures or measures in 2-dimensional Euclidean quantum field theory. In these cases the theorem also implies a Cameron-Martin-Girsanev-Maryana type formula.

W. SCHEPPACHER

HYPERBOLIC EQUATIONS WITH DELAY IN THE BOUNDARY CONDITIONS

We consider the hyperbolic system

$$w_t = \Lambda w_x + Uw + Vw$$

where $\Lambda = \mathrm{diag}\,(\Lambda_- , \Lambda_+ , \Lambda_0)$, U and V are appropriate matrices. These equations are models for flexible structures (i.e. strings, beams, antennas ...).We assume that we observe the structure at the boundary and feed this information back into a control strategy, where we suppose that there is some delay. It is shown that this problem gives rise to a C_0-semigroup on an appropriately chosen state space.

WOLF VON WAHL

APPLICATIONS OF THE THEORY OF SEMIGROUPS WITH NON-DENSELY DEFINED GENERATOR

This is a report on joint work with E. Sinestrari. Let us consider a parabolic equation

$$u' + A(t)u = f,\ u(0)=\psi$$

over a cylindrical domain $[0,T] \times \bar{\Omega} \subset \mathbb{R}^{n+1}$.

$A(t) = \sum_{|\tilde{\alpha}| \leq 2m} a_{\tilde{\alpha}}(t,x) D^{\tilde{\alpha}}$, $a_{\tilde{\alpha}} \in C^{\alpha/2m,\alpha}([0,T] \times \bar{\Omega})$, is uniformly strongly elliptic, f is in $C^{\alpha/2m,\alpha}([0,T] \times \bar{\Omega})$. We impose Dirichlet-0-conditions on $\partial\Omega$ and assume that the compatibility conditions of order 0 and 1 hold in $t=0$. We then show that the so called Schauder estimates (due to Solonnikov) can be proved easily by application of abstract semigroup theory (with non-densely defined generator) and Schauder-theory for elliptic equations. Finally semilinear equations $u' + A(t)u + M(u) = f$ are treated where M has quadratic growth with respect to $|D^m u|$.

GLENN WEBB

INVESTIGATION OF A NONLINEAR INTEGRODIFFERENCE EQUATION

A discrete nonlinear semigroup is used to investigate a nonlinear integrodifference equation. The equation has the form $N_{t+1}(x) = \int_\Omega k(x,y)f(N_t(y))dy$, $x \in \Omega \subset \mathbb{R}^n$, $t = 0,1,2,\ldots$. The model applies to populations that grow and disperse in separate phases. The growth phase is a nonlinear process that allows for the effects of local crowding. The dispersion phase is a linear process that distributes the population throughout its habitat. The issues of survival and extinction are studied by analyzing the existence and stability of nontrivial equilibria. A comparison of various dispersion strategies is made. The analysis used recent results from the theory of positive operators in Banach lattices.

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