MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 6/1988

Universelle Algebra

7.2. bis 13.2.1988

Die Tagung fand unter der Leitung von Herrn G. Grätzer (Winnipeg) und Herrn R. Wille (Darmstadt) statt. Von den Teilnehmern kamen 12 aus Deutschland, 24 aus neun anderen europäischen Ländern, 13 aus Nordamerika, und je einer aus Australien und von der Elfenbeinküste. Die 43 Vorträge haben alle wichtige Bereiche der universellen Algebra umfaßt, mit besonderen Schwerpunkten in Verbänden, Varietäten, Clones und algebraischer Logik. Das Programm wurde durch eine sehr erfolgreiche "Problem Session" ergänzt. Die Liste der 42 vorgeschlagenen Probleme ist am Ende dieses Berichtes zu finden. Die Teilnehmer haben die Tagung in ehrendem Gedenken an Evelyn Nelson durchführt, die zu der Tagung eingeladen war und am 1. August 1987 verstorben ist.

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Vortragsauszüge

M.H. ALBERT:

Ehrenfeucht's conjecture and universal algebra

Ehrenfeucht's Conjecture states that if M is a finitely generated free monoid, and L any subset of M, then there is a finite subset T of L such that if two endomorphisms f, g of M agree on T then they agree on L. Ehrenfeucht's conjecture is equivalent to the statement that any system of equations in finitely many variables over a free monoid has a finite equivalent subsystem. We prove that this latter condition is satisfied in a variety iff every finitely generated free algebra has the Ascending Chain Condition on its congruence lattice. As M can be embedded (as a monoid) in a group with this property this proves Ehrenfeucht's conjecture.

M.K. BENNETT:

Bolyai-Lobaschewski geometries

A Bolyai-Lobaschewski space is an incidence space (Hilbert's incidence axioms hold) satisfying: "given $P \notin l$, there are at least two lines through P parallel to l". A model for a B-L space can be obtained as follows: Let D be an ordered division ring and X a bounded open convex subset of D^n. Take as points the vectors in X and as lines the intersections of X with the affine lines in D^n. The flats of such a B-L space form a complete, algebraic, atomistic, geometric, weakly modular lattice in which every coatom has a complementary coatom. Several possible approaches to characterizing the lattices of flats of B-L spaces are discussed - particularly recapturing "betweenness" from collinearity, and embedding such geometric lattices into projective geometries.

J. BERMAN:

Free spectra of finite algebras

Let **A** be a finite algebra with universe A, $|A| = k$. The free spectrum of **A** is the function $s(n)$ = the cardinality of the free algebra on n free generators for the variety generated by **A**. A general project is to investigate how algebraic properties of **A** are related to numerical properties of $s(n)$. The talk provides examples of some results of this nature and will concentrate on gap theorems for free spectra.

W.J. BLOK:

The logic of algebra

For a wide class of deductive systems (protoalgebraic ones) one can show that if they possess a Deduction Detachment Theorem, then their lattice of theories is distributive. On the algebraic side, any variety with Equationally Definable Principal Congruences is congruence distributive. These two assertions amount to the same statement for protoalgebraic deductive systems which are algebraizable (in a precise sense), since these have a DDT if and only if their associated variety has EDPC. We present a more general notion of protoalgebraic deductive system which encompasses the old one, as well as quasi-equational logic. Many results can be transferred to the more general systems. As a by-product we obtain some theorems on quasivarieties, which earlier had been established for varieties only. For example, if the relative principal congruences of a quasivariety are definable by equations, then the lattice of relative congruences of any algebra in the quasivariety is distributive.

B. BOSBACH:

A lemma on representability of lattice-ordered algebras

An algebra A is called lattice ordered if it is of type $(A,\vee,\wedge,f_i)$ and if in addition the operations f_i "behave well" w.r.t. $\leqq$. A term $\tilde{p}$ is called linearly composed if it is a variable x or if it is of the form $f(x_1,\ldots,\tilde{q}(x,y_1,\ldots,y_m),\ldots,x_n)$ where f is a fundamental operation and $\tilde{q}$ is (already) linearly composed. A is called representable if it is a subdirect product of totally ordered factors.

Lemma. A lattice ordered algebra is representable iff it satisfies: $\tilde{p}(a)\wedge\tilde{q}(b) \leqq \tilde{p}(b)\vee\tilde{q}(a)$.

This lemma (and a corresponding one for po-algebras) applies to all lo- (and po-) algebras considered so far - sometimes by changing the type. Moreover it solves problems stated by Fuchs and Evans.

P. BURMEISTER:

An application of formal concept analysis to partial algebras

"Attribute exploration" (AE) is a special method in formal concept analysis based on results of Duquenne & Guigues and Ganter. AE roughly means: Let M be a finite set of attributes for the objects in a class K (like symmetry, etc. for (the class of) all binary relations). Let G' be a finite subset of "starting objects", $I_K := \{(g,m) \in K \times M \mid \text{"g has m"}\}$. In a systematic way one is asked to decide in the n-th step whether every $g \in K$ having all attributes from some $A_n \subseteq M$ also has those from some $B_n \subseteq M$. One has either to accept $A_n \to B_n$ as an "implication" valid in K or to provide a "counterexample" $g_n \in K$ having all attributes in A_n but not all of B_n. When the program stops, $G := G' \cup \{\text{counterexamples } g_n\} \subseteq K$ "separates" M, i.e., $(G, M, I_G := G \times M \cap I_K)$ is a context fully describing the concept lattice of (K, M, I_K). The list of accepted implications is minimal and complete. This method has been applied to investigate properties of homomorphisms between partial algebras.

S.D. COMER:

D-varieties of multi-valued algebras

A function $f: A \to B$ is a D-morphism of multi-valued algebras $\langle A, \Omega \rangle$ into $\langle B, \Omega' \rangle$ if for every $\omega \in \Omega$ of type n and $x_1, \ldots, x_{n+1} \in A$

$$fx_{n+1} \in \omega^B(fx_1, \ldots, fx_n) \iff$$

$$\exists x_1', \ldots, x_n' \in A, \quad fx_i = fx_i' \text{ for all } i \text{ and } x_{n+1} \in \omega^A(x_1', \ldots, x_n').$$

A class of multi-valued algebras is called a D-variety if it is closed under the formation of subalgebras (S), direct products (P), and D-morphism images (Q^2). The classes of multi-valued algebras studied in the literature (e.g., hypergroups, join spaces, and polygroups) are all examples of D-varieties. The notion of a mv-law is described and it is shown that the models of a collection of mv-laws is a D-variety.

S. CRVENKOVIĆ (joint work with R. MADARÁSZ):

<u>Relation algebras and some decidability problems</u>

In general, the unsolvability of the local word problem, for a variety, does not imply undecidability of equational theory. For the variety of relation algebras (RA) we can prove the following. 1. The local word problem for RA is unsolvable. (In the proof we use an embedding (Φ) of a semigroup with unsolvable word problem into the semigroup reduct of a relation algebra.) 2. The class S_Φ of RA, obtained in the proof of 1., is not elementary.
If we use the fact that for every variety the decidability of the theory of quasi-identities is equivalent to the solvability of the global word problem, then 1. and some properties of RA imply a simple proof of the theorem of Tarski that Eq(RA) is undecidable.

B.A. DAVEY (joint work with R. QUACKENBUSH and D. SCHWEIGERT):

<u>Monotone clones and the varieties they determine</u>

I am interested in applying duality theory and tame-congruence theory to the study of varieties generated by finite algebras, $\underline{P}$, whose clone of term functions is a monotone clone. That is there is an order $\leq$ on P such that for all $n \in \mathbb{N}$ the n-ary term functions on $\underline{P}$ are precisely the order-preserving maps from P^n to P; such algebras are said to be order primal.

H. DOBBERTIN:

Congruence lattices of lattices

All known partial solutions of the famous problem "Is every distributive algebraic lattice K representable as the congruence lattice of some lattice?" can be obtained by applying Schmidt's Lemma (1969). The answer is known to be positive if one of the following conditions holds:
(1) K^c, the semilattice of compact elements of K, is locally countable; (2) $|K^c| = \aleph_1$; (3) K^c is a lattice;
(4) K is completely distributive. Surprisingly there is a close connection between the above question and a certain problem arising in the theory of countable Boolean algebras.
I conjecture that the sufficient condition of Schmidt's Lemma for representing a distributive algebraic lattice K as congruence lattice of some lattice is not always satisfied.
I even conjecture that there is a non-representable distributive algebraic lattice K.

H. DRAŠKOVIČOVÁ:

Varieties of modular median algebras

We call a set A with one ternary operation (xyz) modular median algebra (m.m.algebra, and denote it by $(A;(\ ))$) if the following identities are satisfied in A: (1) $(abb) = b$,
(2) $((adc)bc) = (ac(bcd))$.

Denote by U the variety of m.m.algebras given by the identity (U) $((xyz)xt) = (xy(zxt))$.

Denote by W the 4-element m.m.algebra $(\{a,b,c,d\};(\ ))$, where $a=(abc)$, $b=(bca)$, $c=(cab)$ and $d=(adb)=(adc)=(bdc)$.

We prove that the algebra W and the identity (U) form a splitting pair in the lattice of all varieties of m.m.algebras.

J. DUDEK:

The minimal extension of sequences

Let $\underline{A}$ be an algebra. By $p_n(\underline{A})$ we denote the number of essentially n-ary polynomials over $\underline{A}$. First, we deal with the Minimal Extension Property of sequences introduced by G. Grätzer in 1969 and we also give some results on p_n-sequences characterizing the corresponding varieties; e.g.:

Theorem. Let V(.) be the variety of idempotent commutative groupoids. Then we have

(i) $(G,.) \in V(.)$ and $p_2(G,.)=2$ imply that $(G,.)$ contains isomorphically as a subgroupoid the idempotent groupoid $A=(\{1,2,3,4\},.)$ where $xy=1+\max(x,y)$ for $x,y \leqq 3$ $(x \neq y)$ and 4 otherwise.

(ii) The sequence $\langle 0,0,2,\ldots,p_n(A),\ldots\rangle$ is the minimal extension of $\langle 0,0,2\rangle$ in V(.).

(iii) If $(G,.) \in V(.)$, then $(G,.)$ is either a semilattice or an affine space over GF(3) or else $p_n(G,.) \geqq 3^{n-1}$ for all $n \geqq 4$.

G. EIGENTHALER:

Commutative composition semigroups of polynomials

Let $\underline{V}$ be a variety, $A \in \underline{V}$ and X a set, then a $\underline{V}$-polynomial algebra $A(X,\underline{V})$ in X over A is defined to be a coproduct $A \coprod F(X,\underline{V})$, where $F(X,\underline{V})$ denotes the free $\underline{V}$-algebra with free generating set X. Let X be finite, say $X=\{x_1,\ldots,x_k\}$, and $\underline{p}=(p_1,\ldots,p_k)$, $\underline{q}=(q_1,\ldots,q_k) \in A(X,\underline{V})^k$, then the composition $\underline{p}\circ\underline{q}$ is defined in the following way: Take terms $t_i=t_i(a_1,\ldots,a_n,x_1,\ldots,x_k)$, $u_i=u_i(a_1,\ldots,a_n,x_1,\ldots,x_k)$ (where $a_1,\ldots,a_n\in A$) representing p_i, q_i resp., then $\underline{p}\circ\underline{q}=(r_1,\ldots,r_k)$ where $r_i\in A(X,\underline{V})$ is represented by the term $t_i(a_1,\ldots,a_n,u_1,\ldots,u_k)$. Thus we obtain a semigroup $(A(X,\underline{V})^k,\circ)$ with identity $(x_1,\ldots,x_k)$. The problem of finding commutative subsemigroups of $(A(X,\underline{V})^k,\circ)$ has been treated - up to now - only for $\underline{V}$ being the variety of commutative rings with identity. In this case $A(X,\underline{V})$ is the classical polynomial ring $A[x_1,\ldots,x_k]$ in k indeterminates over A.

E. GEDEONOVÁ:

The construction of lattices with special covering graphs

A finite connected graph $G=(V,H)$ is an S-graph if to every vertex $v\in V$ there exists a graded partially ordered set $(V,\leq_v)$ with least element v, such that the covering graph of $(V,\leq_v)$ is isomorphic to G. An S-graph is a CS-graph if $(V,\leq_v)$ is a graded lattice for each $v\in V$. If the covering graph of a lattice is an S-graph then this lattice is called an S-lattice. It is given a construction of S-lattices of arbitrary length from the one-element lattice. It turns out that the covering graphs of these lattices have a special property, we call them ST-graphs. We show that every ST-graph is constructable from the one-element graph. Every CS-graph and every planar S-graph is an ST-graph. But we do not know, if the covering graph of every S-lattice is an ST-graph, and we do not know if every ST-graph is a covering graph of some lattice.

G. GRÄTZER (joint work with E. FRIED):

Pasting infinite lattices

Let A, B, S be lattices, $A \cap B = S$. Then L is a pasting of A and B over S iff $L = A \cup B$ and every amalgamation of A and B over S contains L as a sublattice (in the natural way). Results:

1. $\mathbf{M}$ and $\mathbf{D}$ are closed under pasting (i.e., if A and B are modular (distributive), then so is L).

2. Two structure theorems for pasting, generalizing the results of Slavík, Day, and Ježek for the finite case.

G. HAUSER BORDALO:

Forbidden intervals in classes of lattices

In the present work we enlarge a class of finite distributive lattices that can be characterized by forbidden intervals. We then study the representation of these lattices as subalgebra lattices of finite mono-unary algebras. Since these unary algebras are defined on the poset of join-irreducibles, it is possible, using duality, to define the maps between unary algebras that correspond to the 0-1 lattice homomorphisms. We then give results obtained in the characterization of these maps using the algebraic structure on J(L). We also give some results about the congruence lattice of such algebras and its connections with the subalgebra lattice.

C. HERRMANN:

Decidability of module theories

Burris, McKenzie, and Valeriote reduced the question of decidability of the first order theory of a variety generated by a finite algebraic structure to that of R-Mod (the class of all R-modules) for a finite ring to be constructed from A. Burris observed that Th(R-Mod) is decidable if R is of finite representation type - actually it coincides with Th(R-mod) of all finitely generated R-modules. If R is of wild representation type then, using results of Hutchinson and Slobodskoi, one sees that both Th(R-Mod) and Th(R-mod) are undecidable. For R being the path algebra of a quiver a complete answer has been given by Prest: R is of wild type if and only if Th(R-Mod) is undecidable.The proof is based upon Nazarova's characterization of "wild" by forbidden quivers and the explicit classification of indecomposables in R-mod in the case of finite and "tame" type (due to Ringel and others) which is extended to algebraically compact alias pure injective indecomposables.

J. JEŽEK:

Bounded equational theories

An equational theory T is called bounded if the automorphism group of the term algebra partitions the T-equivalence classes into finitely many orbits. The set of bounded equational theories of a given type is a filter in the lattice of all equational theories. Every bounded equational theory has only finitely many extensions, and is the equational theory of a finite algebra. If the type is finite then every bounded equational theory is finitely based. We introduce special bounded equational theories which we call well-placed; they can be described in a nice way. In case of the type containing a single operation symbol we prove that an equational theory is bounded iff it extends a well-placed equational theory, and that an absorptive equational theory is bounded iff it is a well-placed theory; all the maximal bounded equational theories are described.

H.K. KAISER:

Interpolation and approximation by means of polynomial functions

Let $\langle A,\Omega,\mathcal{T}\rangle$ be a topological universal algebra and $k\in\mathbb{N}$. $F_k(A)$ denotes the set of all k-ary functions over A. On $F_k(A)$ we define the operations $\omega\in\Omega$ pointwise and we endow $F_k(A)$ with the product topology. $P_k(A)$ denotes the subalgebra of k-ary polynomial functions over A. Then $\langle A,\Omega,\mathcal{T}\rangle$ is said to have the approximation property if, for all $k\in\mathbb{N}$, $P_k(A)$ is dense in $F_k(A)$.

Theorem: A topological universal algebra $\langle A,\Omega,\mathcal{T}\rangle$ has the approximation property iff (i) for every nontrivial congruence θ of $\langle A,\Omega\rangle$ we have $\overline{\theta} = A^2$; (ii) there are p, t $:A^2\to A$ having the approximation property such that $p(x,x)=p(y,y)$ and $t(p(x,y),y)=x$ for all $x,y\in A$; (iii) there is an $a\in A$ and a $q:A^2\to A$ having the approximation property such that q is not constant and $q(a,x)=q(x,a)=a$ for all $x\in A$.

T. KATRIŇÁK:

Projective p-algebras

A p-algebra is an algebra of the form $\langle L;\vee,\wedge,{}^*,0,1\rangle$, where $\langle L;\vee,\wedge,0,1\rangle$ is a bounded lattice and * is the pseudocomplement operation on L, that means, $a \wedge b = 0$ if and only if $b \leq a^*$. There are necessary and sufficient conditions given for a p-algebra to be (weak) projective. Essentially simpler conditions are obtained for a finitely generated p-algebra to be projective. One of the equivalent conditions says that a finitely generated p-algebra is projective if and only if it can be embedded into a free p-algebra.

D. KELLY (joint work with R. PADMANABHAN):

Self-dual varieties of lattices

The dual of a lattice polynomial p, denoted by $\tilde{p}$, is obtained from p by replacing join by meet and meet by join simultaneously. $\underset{\sim}{V}$ = finitely based variety of lattices.

I. Bases relative to $\underset{\sim}{L}$ = variety of all lattices.

(A) If $\underset{\sim}{V}$ contains only modular lattices, then $\underset{\sim}{V}$ has a basis relative to $\underset{\sim}{L}$ of the form: $\{p = \tilde{p}\}$.

(B) Many $\underset{\sim}{V}$'s do not have a relative basis of the form $\{p=\tilde{p}\}$.

II. Absolute bases.

(A) $\underset{\sim}{L}$ has an independent basis of the form $\{p=x, \tilde{p}=x\}$.

(B) $\underset{\sim}{V}$ has an independent basis of the form:

$$\{p=x, \tilde{p}=x, q=r, \tilde{q}=r\}.$$

E.W. KISS:

E-minimal algebras of type one

It is a basic problem in universal algebra to decide whether the variety generated by a given finite algebra is residually small. It has been solved by Freese and McKenzie for the congruence modular case. We provide a tool to investigate this question in the general setting. For a subset N of a finite algebra A consider the restriction of every unary polynomial $h(x,c_1,...,c_n)$ of A to N that preserves N and denote by G(N) the group of those such restrictions that are permutations of N. Call two such permutations equivalent if they can be obtained using the same h but (possibly) different constants c_i. If certain natural technical restrictions hold for N, then this equivalence is a congruence relation on G(N), we denote by E(N) the corresponding normal subgroup of G(N). It can be proved, under these restrictions, that if V(A) is residually small, then E(N) must be Abelian.

H.-J. KREOWSKI:

Algebraic specification of partial data types

The concept of data types plays an important part in software development. Equationally specified, total initial algebras are appropriate formalizations of data types in many cases. But what about partial operations that are sometimes very convenient and sometimes unavoidable? In this talk, we introduce and discuss a method for specifying algebras with partial functions while maintaining the simpler framework of total algebras and conventional specifications. For this purpose, an ordinary algebraic specification SPEC is equipped with a subspecification BASE, and each SPEC-algebra is equipped with a BASE-homomorphism distinguishing a BASE-part of the SPEC-algebra. Then one can restrict the SPEC-algebra to its BASE-part yielding a SPEC-algebra with operations that may be partial. As the main result, it turns out that one can get all computable functions in this way.

I.A. MALCEV:

Quasicells of iterative algebras

Let P_A be the set of all operations on a set A, $\mathcal{P}_A$ the iterative Post algebra over A, $B \subseteq A$. An elementary quasicell $\mathcal{K}_B$ is a subalgebra of $\mathcal{P}_A$ satisfying the condition $f \in \mathcal{K}_B \Leftrightarrow \operatorname{Im} f \subseteq B$. A quasicell is a union of elementary quasicells. All quasicells are subalgebras of the Slupecki algebra. In the lecture various properties of the quasicells are considered and in particular it is shown how to solve the completeness problem using identities. E.g., the following theorem is proved: A subalgebra $\mathcal{A} \subseteq \mathcal{P}_{\{0,1,2\}}$ is a quasicell of the form $\mathcal{K}_{\{0,1\}} \cup \mathcal{K}_{\{0,2\}}$ iff (1) $\forall f \in \mathcal{A}\ \operatorname{Im} f \not\supseteq \{1,2\}$; (2) $\mathcal{A} \not\models \gamma * \varphi * \varphi * \gamma(x) = \gamma * \varphi * \gamma(x)$; (3) $\mathcal{A} \not\models \psi(\psi(\psi(\psi(x,x),\psi(x,x),$ $\psi(\psi(x,x),\psi(x,x))),\psi(\psi(\psi(y,y),\psi(y,y)),\psi(\psi(y,y),\psi(y,y)))) =$ $\psi(\psi(\psi(\psi(x,x),\psi(x,x)),\psi(\psi(y,y),\psi(y,y))),\psi(\psi(\psi(x,x),\psi(x,x)),$ $\psi(\psi(y,y),\psi(y,y))))$.

G. McNULTY:

Combinatorial properties of terms with applications in equational logic

This work is still in progress, however the themes of equational logic such as finite axiomatizability, undecidability, and term rewriting systems - as well as various results about the lattice of equational theories, all employ combinatorial properties of terms to good advantage. My presentation would be directed at isolating and developing these combinatorial properties.

P.P. PÁLFY:

Modular subalgebra lattices

We derive some consequences of the modularity of $Sub(A^n)$, the subalgebra lattice of the direct product $A\times\ldots\times A$. If $Sub(A^2)$ is modular, then A is Hamiltonian and has the congruence extension property. If $Sub(A^3)$ is distributive, then A is strongly Abelian (i.e., satisfies the strong term condition). If $Sub(A^4)$ is modular, then A is Abelian (i.e., satisfies the term condition). Though in general the subalgebra lattice of a factor algebra does not belong to the lattice variety generated by the subalgebra lattice of the original algebra, we are able to prove that $Sub(A/\Theta)$ is modular (distributive), whenever $Sub(A)$ is modular (distributive).

H.A. PRIESTLEY:

Natural dualities for varieties of distributive-lattice-ordered algebras and equational bases

In his monograph, Antimorphic Action, W.H.Cornish shows that many varieties of distributive-lattice-ordered algebras (e.g., many subvarieties of Ockham algebras) are of the following form: $\mathbb{ISP}(\underline{P})$, where $\underline{P}$ is a finite algebra whose dual space can be regarded as an ordered monoid $(M=M^+\cup M^-,\cdot,\leqq)$. Elements of M^+ and M^- define via duality operations on $\underline{P}$ which are respectively endomorphisms and dual endomorphisms. One may then in many cases set up a natural duality (in the manner of Davey-Werner, Clark-Krauss) with schizophrenic object a subset of 2^M which is determined by $(M,\leqq)$. This leads to a description of the free algebras in $\mathfrak{a} = \mathbb{I}S\mathbb{P}(\underline{P})$. Further, the identities of $\mathfrak{a}$ are encoded in the schizophrenic object. The duality theory can be extended to handle subvarieties of $\mathfrak{a}$, and provides an algorithm for writing down equational bases for these subvarieties. These ideas are illustrated by discussion of varieties of double MS-algebras.

P. PUDLÁK:

Some results about the length of proofs

I shall discuss some results on Kreisel's conjecture:

$$\exists k\ \forall n\ PA \vdash^{k} \varphi(S^{n}(O)) \Rightarrow PA \vdash \forall x\ \varphi(x),$$

where PA is Peano arithmetic, $\vdash^{k}$ denotes provability by a proof with $\leqq k$ proof lines. I shall consider a unification problem used by M. Baaz to prove Kreisel's conjecture: "Given pairs of terms $(t_1,s_1),\ldots,(t_k,s_k)$, decide whether there exist substitutions $\delta,\sigma_1,\ldots,\sigma_k$ s.t. $t_i\delta\sigma_i \doteq s_i\delta$ for $i=1,\ldots,k$."

I shall show that

(1) the general problem can be reduced to the case $k=2$;

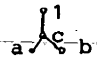
(2) if $k=1$, then the existence of $\delta,\sigma_1,\ldots,\sigma_k$ is decidable.

(In general the decidability is not known. If it were decidable, then one could estimate the length of a proof of $\forall x\varphi(x)$ from k and φ in Kreisel's conjecture.)

R.W. QUACKENBUSH (joint work with J. JEŽEK):

Directoids: Algebraic models for up-directed sets

Let $\langle U;\leqq\rangle$ be an up-directed set (so that $\forall a,b \in U$, $a \leqq c$ and $b \leqq c$ for some $c \in U$); define $\cdot$ on U by $a\cdot b = b\cdot a = b$ if $a \leqq b$ and otherwise $a\cdot b$ is some upper bound of a and b;

$\langle U;\cdot\rangle$ is called a directoid. The class of all directoids forms a variety, $\mathcal{D}$, with basis $x^2=x$, $(xy)y=xy$, $x(xy)=xy$ and $x((xy)z)=(xy)z$. Let $C\mathcal{D}$ be the variety of commutative directoids. If $\langle U;\cdot\rangle$ satisfies $\forall x\ \langle\{u \mid u \geqq x\};\cdot\rangle$ is a semilattice, then $\langle U;\cdot\rangle$ is called a joinoid. The class of all joinoids forms a variety $\mathcal{J}$ defined by $(xy)(xz) \leqq (u(xy))(xz)$ relative to $\mathcal{D}$; $C\mathcal{J}$ is the variety of commutative joinoids. Both $\mathcal{J}$ and $C\mathcal{J}$ are locally finite, but neither is finitely-generated. Let $F=$ ; $\underline{F}$ is the commutative directoid on F such that $ab=1$, and $\underline{F}'$ is the directoid on F such that $ab=1$ and $ba=c$. $V(\underline{F})$ and $V(\underline{F}')$ are finitely based varieties. Every variety of directoids is either trivial, semilattices, or contains $V(\underline{F})$. Every non-commutative variety of directoids contains $V(\underline{F}')$.

A. ROMANOWSKA:

<u>Ordinal products of modals</u>

A mode is an algebra of type $\tau:\Omega\to\mathbb{N}$ which is idempotent and entropic (each singleton is a subalgebra and each operation $\omega: A^{\omega\tau}\to A$ of Ω is a homomorphism). Examples include semilattices, CIM-groupoids, convex sets. A modal is an algebra $(A,+,\Omega)$ with (join) semilattice reduct $(A,+)$ and mode reduct (A,Ω) such that the distributive laws $x_1\ldots(x_j+x_j')\ldots x_{\omega\tau}\omega = x_1\ldots x_j\ldots x_{\omega\tau}\omega+x_1\ldots x_j'\ldots x_{\omega\tau}\omega$ hold for all ω in Ω, $1\leqq j\leqq \omega\tau$. Examples include modals of subalgebras of modes, distributive bisemilattices, real numbers with the operation of max and the mode reduct $(\mathbb{R},(0,1))$. For modals $(D,+,\Omega)$ and $(E,+,\Omega)$, the ordinal product $D\circ E$ is $(D\times E,\Omega,\leqq)$, where $\leqq$ is defined lexicographically. The main theorem gives a sufficient condition for the ordinal product $D \circ E$ to be a modal.

I.G. ROSENBERG:

Primality criterion for finite partial algebras

We give a primality criterion for finite partial algebras on A based on the full list of maximal partial clones on A. One of them is the set of everywhere or nowhere defined operations on A. The others are of the form Pol ϱ for special relations given below, where Pol ϱ consists of all partial operations on A preserving an h-ary ϱ. The relations have a certain reflexivity to which we associate the least such h-ary relation on $\{0,\ldots,h-1\}$ containing $(0,\ldots,h-1)$, called the model of ϱ. The relation is coherent if there is a homomorphism from ϱ onto its model. An h-ary relation ϱ on A determines a maximal partial clone if $\emptyset \neq \varrho \subsetneq A^h$, $0 < h \leq |A|$ and ϱ is either totally symmetric and reflexive or coherent.

J. SCHMID:

Some natural quasivarieties of p-semilattices

We study two series $\{\underline{B}_n\}$ and $\{\underline{C}_n\}$ $(n > 0)$ of quasivarieties of p-semilattices, where $\underline{B}_n = \mathbb{I}SP\{\hat{B}_n\}$ is the semantical and $\underline{C}_n$ the syntactical analogue of the n-th Lee class of distributive p-lattices (i.e., $\underline{C}_n$ is defined by the formal analogue - necessarily a quasi-identity - of the n-th Lee identity). We show that:

(i) $\underline{C}_1 \cap (\underline{B}_n \setminus \underline{B}_{n-1}) \neq \emptyset$ for $n > 0$,

(ii) $\underline{C}_n$ is the largest quasivariety of p-semilattices not containing $\hat{B}_k$ for $k > n$,

(iii) $\underline{B}_n$ may be axiomatized by a single although involved quasiidentity within the class of all p-semilattices.

Here $\hat{B}_k$ stands for the k-atom Boolean algebra augmented by a new greatest element.

D. SCHWEIGERT (joint work with. E. GRACZYŃSKA):

Hypervarieties of a given type

For a given type $\tau = (n_o, n_1, \ldots, n_\gamma, \ldots)$ of positive numbers we associate hypervariables F_γ of arity n_γ and define recursively hyperterms of type τ. By adding an extra substitution to Tarski's rule we define a hyperequational logic and prove a completeness theorem. A variety V of type τ is called solid if every transformation of the identities of V give hyperidentities which hold for V. By this concept we can show that for a set Σ of hyperidentities of type τ which is closed under hyperequational logic there exists a solid variety of type τ which has Σ as hyperidentities. Furthermore, we consider the following operator $\mathbb{D}$ on classes of algebras of type $\tau = (n_1, n_2, \ldots, n_\gamma, \ldots)$. Let $\underset{\sim}{A}$ be an algebra of type τ then $\bar{A} = (A; t_1, t_2, \ldots t_\gamma, \ldots)$ where t_γ is an n_γ-ary term is an algebra contained in $\mathbb{D}(\underset{\sim}{A})$. We prove that V is a solid variety iff $V = \mathbb{HSPD}(V)$.

V. SLAVÍK (joint work with J. JEŽEK):

The free lattices over join-trivial partial lattices

We have proved that the free lattice over a join-trivial partial lattice P is finite iff P is finite and satisfies the following four conditions:

(1) P contains no three-element antichain;

(2) If $a > b$, $c > d$ are two incomparable chains then $ea = b$ for some $e > c$ (or symmetrically);

(3) If a is incomparable with a chain $b < c < d < e$ then $fe \lneq d$ for some $f \gneq a$;

(4) If a is incomparable with a chain $b < c < d < e < f$ then $gf = e$ for some $g > a$.

M.G. STONE:

Ideal homomorphisms of join semilattices

Ideal preserving homomorphisms of join semilattices play a central role in endomorphism representation theory, since each homomorphism of algebras induces an ideal homomorphism of their subalgebra join semilattices, and conversely. Ideal homomorphisms are used here to characterize abstractly those monoids $[f]$ for which f is an endomorphism of an algebra with an arbitrarily given subalgebra lattice L. These are precisely those monoids $[f]$ for which there is some ideal homomorphism of the join semilattice $g: L \rightarrow L$ such that the Cayley diagram for $[g]$ is a "suitable" homomorphic image of that for $[f]$.

Á. SZENDREI:

Strictly simple algebras and minimal varieties

An algebra is called strictly simple if it is finite, simple and has no nontrivial proper subalgebras. One of the main problems concerning strictly simple algebras is which of them generate minimal varieties. Recently it was proved that every idempotent strictly simple algebra generates a minimal variety. This suggests the investigation of strictly simple algebras having "sufficiently many" trivial subalgebras. In the talk several results on Abelian strictly simple algebras with at least two trivial subalgebras are discussed.

G. TARDOS:

Finitely generated pseudosimple algebras

We call an algebra A pseudosimple if any nontrivial homomorphic image of A is isomorphic to A. Henkin, Monk and Tarski proved in their book Cylindric Algebras I that a finitely generated pseudosimple algebra of finite type is simple. We construct non-simple pseudosimple algebras without any proper subalgebra (using of course infinitely many operations). Kollár proved that the congruence lattice of a finitely generated pseudosimple algebra must be a chain of type $\omega^{\alpha}+1$ where α is a limit ordinal. It is possible to get any of these congruence lattices with our construction. The main idea is to construct first some endomorphisms of the algebra and then to select for basic operations those which admit these mappings as endomorphisms.

J. TŮMA:

Group extensions and lattice representations

I have been developing methods for constructing partition and congruence lattice representations, and proved the following theorem: Every algebraic lattice is isomorphic to an interval in the subgroup lattice of an infinite group. The method has also some potential to construct finite lattice representations. Using the method I recently found a new proof of the finite partition lattice representation theorem. The representations constructed in this proof are on much smaller sets than in the original proof. The new proof is based on combinatorial group theory, namely on P. Hall's Basis Theorem and the Kurosh Rewriting Process.

M. VALERIOTE (joint work with R. McKENZIE):

Decidable varieties

We present and discuss the following theorem:
Let V be a locally finite variety of finite type. Then V is decidable (has a recursive first order theory) iff V is equal to $\mathcal{S}\otimes\mathcal{A}\otimes\mathcal{D}$, where $\mathcal{S}$ is a decidable strongly Abelian variety, $\mathcal{A}$ is an affine, decidable variety and $\mathcal{D}$ is a decidable discriminator variety.
Consequences and background of this theorem are discussed.

G.H. WENZEL:

Tolerances on lattices

Toleranzen auf Verbänden sind reflexive, symmetrische, binäre Relationen, die mit den Operationen $\vee$ und $\wedge$ verträglich sind. Sie ergeben sich in natürlicher Weise als Quotienten von Kongruenzen. Ist θ eine solche Toleranz dann ist die Menge L/θ der θ-Blöcke wieder ein Verband (Czédli, 1982). Man kann den Beweis konstruktiv geben und das Auswahlaxiom vermeiden. Der zweite Isomorphiesatz läßt sich auf Toleranzen erweitern, und die dabei auftretenden Ideen haben Relevanz bzgl. eines auf McKenzie zurückgehenden Problems: "Seien V, W Verbandsvarietäten und $\mathbb{H}(V \circ W)$ die vom Varietätenprodukt $V \circ W$ erzeugte Varietät. Ist $L \in \mathbb{H}(V \circ W)$ äquivalent zur Existenz einer Toleranz θ von L, die $L/\theta \subseteq V$ (als Blockmenge) und $L/\theta \in W$ (als Verband) erfüllt?" G. Grätzer und E. Fried zeigten 1987, daß die Antwort auf die gestellte Frage i.a. negativ ist.

R. WILLE:

Tensor products of complete lattices in formal concept analysis

The tensor product of complete lattices L_j $(j \in J)$ is defined to be the concept lattice of the context $(\times_{j \in J} L_j, \times_{j \in J} L_j, \nabla)$ where $\vec{x} \nabla \vec{y}$ iff there is a $k \in J$ with $x_k \leqq y_k$. In formal concept analysis the tensor product occurs as concept lattice of direct product of scales. For measurability and dependency theorems a characterization of tensor products as closure systems is useful. A general characterization is given and specializations to particular scales are shown.

B. WOJDYLO (joint work with P. BURMEISTER):

The meaning of basic category theoretical notions in some categories of partial algebras

For a finitary type τ there are the following restrictions for the existence of categorical constructions in the categories of partial algebras with homomorphisms, closed homomorphisms, quomorphisms, closed quomorphisms and conformisms, respectively:

No.	Notion	$\mathfrak{Hom}(\tau)$	$\mathfrak{C}$-$\mathfrak{Hom}(\tau)$	$\mathfrak{Quom}(\tau)$	$\mathfrak{C}$-$\mathfrak{Quom}(\tau)$	$\mathfrak{Conf}(\tau)$
0	zero object	-	-	+	$\Omega^{(0)}=\emptyset$	$\Omega^{(0)}=\emptyset$
1	terminal object	+	$(1)_{\varphi\in\Omega}$	+	+	+
1d	initial object	+	$\Omega^{(0)}=\emptyset$	+	$\Omega^{(0)}=\emptyset$	$\Omega^{(0)}=\emptyset$
2	product*	+	$(1)_{\varphi\in\Omega}$	$\Omega=\emptyset$	$(0)_{\varphi\in\Omega}$	$(1)_{\varphi\in\Omega}$
2d	coproduct*	+	$(1)_{\varphi\in\Omega}$	$\Omega^{(0)}=\emptyset$	$(1)_{\varphi\in\Omega}$	$(1)_{\varphi\in\Omega}$
3	inverse limit*	+	+	+	+	+
3d	directed colimit*	+	+	+	+	+
4	equalizer*	+	+	+	$(0)_{\varphi\in\Omega}, (1)_{\varphi\in\Omega}$	$(1)_{\varphi\in\Omega}$
4d	coequalizer*	+	$n_\varphi\leq 1$	$(0)_{\varphi\in\Omega}$	$n_\varphi\leq 1$	$n_\varphi\leq 1$
5	(mult.) pullback*	+	+	$\Omega=\emptyset$	$(0)_{\varphi\in\Omega}$	?
5d	(mult.) pushout*	+	$n_\varphi\leq 1$	$\Omega=\emptyset$	$n_\varphi\leq 1$	$n_\varphi\leq 1$
6	limit*	+	$(1)_{\varphi\in\Omega}$	$\Omega=\emptyset$	$(0)_{\varphi\in\Omega}$	$\Omega=\emptyset$
6d	colimit*	+	$(1)_{\varphi\in\Omega}$	$\Omega=\emptyset$	$(1)_{\varphi\in\Omega}$	$\Omega=\emptyset$

Survey on the existence of different category theoretical concepts in some categories of partial algebras ("*" means that the chosen index set should be non-empty)

E. WROŃSKA-GRACZYŃSKA:

Regular identities and hyperidentities

We deal with algebras of a given type, without nullary operations. An identity $p=q$ is called regular if the same variables occur on its both sides. For a variety V, $R(V)$ denotes the variety of the same type as V, which is defined by all the regular identities, satisfied in V.

For a hypervariety V, $R(V)$ denotes the hypervariety, defined by all the regular hyperidentities of V. $L(\tau)$ and $\mathcal{L}(\tau)$ denotes the lattice of all varieties and hyper-varieties of a given type τ, respectively. Theorems:

(1) The variety V covers $R(V)$ in the lattice $L(\tau)$.

(2) The hypervariety V covers $R(V)$ in the lattice $\mathcal{L}(\tau)$.

(3) The word problem for an axiomatic theory V is solvable iff the word problem for $R(V)$ is solvable.

PROBLEMS:

M.H. Albert

1. Find "reasonable" criteria on a variety V which ensure that (*) $Con(F_V(n))$ has Ascending Chain Condition for all finite n.

2. If every system of equations in finitely many variables has a finite equivalent subsystem relative to an algebra M of type τ, is it true, that there is an extension σ of τ, a σ-variety V satisfying (*) and a finitely generated algebra N in V such that $M \in \mathbf{IS}(N|_\tau)$?

3. Is it possible to prove Ehrenfeucht's conjecture without explicit or implicit reference to the Hilbert Basis Theorem?

4. Is it true that the axiom schemes "Every first order definable subset has both a supremum and an infimum" axiomatize the theory of complete lattices?

W.J. Blok

5. Let $\underline{A}_n = \langle\{0,\dots,n-1\},\leftrightarrow\rangle$, $n=1,2,\dots$, $x\leftrightarrow y = \begin{cases} n-1 & \text{if } x=y \\ \min(x,y), & \text{oth.} \end{cases}$

Determine $F_{V(\underline{A}_n)}(k)$. The values are known for $n=3$, k arbitrary, and for $n=4$, $k=1,2$.

S.D. Comer

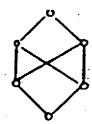
Let G be a group. A conjugacy relation on G is an equivalence relation θ such that (i) $x\theta y \Rightarrow x^{-1}\theta y^{-1}$ and (ii) $z\theta xy \Rightarrow z=x'y'$ for some $x'\theta x$ and $y'\theta y$. A conjugacy relation is special if $x\theta e \Rightarrow x=e$. Let Conj(G) (resp. $\mathrm{Conj}_s(G)$) denote the lattice of all conjugacy relations on G (resp. special conjugacy relations on G).

6. Is every lattice embeddable in Conj(G) for some group? In $\mathrm{Conj}_s(G)$?

7. Does Conj(G) determine G ?

8. Does every maximal element in $\mathrm{Conj}_s(G)$ have at most five blocks?

B.A. Davey

9. The first order sentence $(\forall abcd)\ a,b \leq c,d \Rightarrow (\exists e)\ a,b \leq e \leq c,d$ characterizes the finite members of the order variety generated by $\mathcal{L}$ (i.e., complete lattices). Is this a general phenomenon? In particular, is there a sentence which characterizes the finite members of the order variety generated by  ?

10. Let OPNU denote the class of ordered sets which have an order-preserving near unanimity function. If P, Q are finite members of OPNU which generate the same variety then the varieties Var $\underline{P}$ and Var $\underline{Q}$ generated by the corresponding order-primal algebras are equivalent as categories and conversely. In general, if P and Q generate the same order variety then $\mathbb{I}SP_f(\{\underline{P},\underline{P}/\theta_P\})$ and $\mathbb{I}SP_f(\{\underline{Q},\underline{Q}/\theta_Q\})$ are equivalent as categories (here θ_P is the congruence on $\underline{P}$ whose blocks are the connected components of P). Does it follow that

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$\mathbf{ISP}(\{\underline{P},\underline{P}/\theta_P\})$ is equivalent to $\mathbf{ISP}(\{\underline{Q},\underline{Q}/\theta_Q\})$ or that Var $\underline{P}$ is equivalent to Var $\underline{Q}$?

More generally, for which varieties V and W do we have $V_f \cong_{Cat} W_f$ implies $V \cong_{Cat} W$?

11. Let $\underline{P}$ be a finite non-trivial order-primal algebra. Prove that Var $\underline{P}$ has only a finite number of simple algebras. (I conjecture that in fact there is only one: $\underline{P}$ if $\leq$ is connected and $\underline{P}/\theta_P$ otherwise.)

12. If V is a locally finite affine complete variety then V is congruence distributive. R. McKenzie asks, if we can rub out "locally finite" here.

H. Dobbertin

13. Let L be a distributive lattice. A valuation is a mapping v from L into the non-negative reals which satisfies the identity $v(a\vee b)+v(a\wedge b)=v(a)+v(b)$. Let V(L) denote the set of all valuations on L, and define the compact convex set M(L) = $\{v\in V(L) \mid 0\leq v\leq 1\}$. What are the extreme points of M(L)? (Conjecture: The extreme points of M(L) are precisely the 0-1 valuations.)

G. Grätzer

We say that L is the unique amalgam of the sublattices A, B, with $A\cap B = S$ if for every lattice containing $A\cup B$ as a subposet $K\supseteq L$ holds.

14. Characterize the unique amalgamation property.

15. Is the variety of modular lattices closed for unique amalgams?

C. Herrmann

16. Does Brauer-Thrall II hold for finite modular (non

2-distributive) lattices? (cf. Problem 35.)

17. Let K be a nice quasivariety, e.g., type 1 modular lattices, representable relation algebras. Is there an algorithm deciding $A \in K$ for finite A ?

M. Kamara and K. Keimel

Consider the reals as a lattice ordered ring with identity, i.e., the algebra $A = (\mathbb{R};+,-,0,\cdot,1,\vee,\wedge)$ with the usual ring operations and the lattice operations corresponding to the usual total order. We ask some questions about the variety V generated by A :

18. Give a basis for the equational theory of V.

19. Is there a finite basis?

20. Is the word problem for the free algebras in V decidable?

21. Is there even a normal form for the terms?

T. Katriňák

Let P_n $(1 \leq n < \omega)$ be the subvariety of the variety $P = P_\omega$ of all p-algebras defined by the identity

$$(L_n) \quad (x_1 \wedge \ldots \wedge x_n)^* \vee (x_1^* \wedge \ldots \wedge x_n)^* \vee \ldots \vee (x_1 \wedge \ldots \wedge x_n^*)^* = 1.$$

Let P_0 be the subvariety of P defined by the identity

$$(L_0) \quad (x \wedge y)^* = x^* \vee y^*.$$

It is known that $P_0 \subset P_1 \subset \ldots \subset P_n \subset \ldots \subset P$.

22. Let $L \in P_n$ $(1 \leqq n < \omega)$ be a finitely generated p-algebra, which is a subalgebra of some free p-algebra in P_n. Is L a bounded homomorphic image of a free p-algebra in the class P_n ?

23. Characterize the projective p-algebras in the class P_0.

E.W. Kiss

24. A prime quotient of a finite algebra A is defined to have type zero ("constant type"), if the corresponding minimal algebras have no unary polynomials that are permutations other than the identity map.
Develop a theory of type zero quotients. In particular:
a) give a characterization independent of minimal sets;
b) describe all simple algebras of type zero;
c) describe type zero varieties. When are they residually small?

25. Characterize varieties that admit type 5 only.

W.A. Lampe

26. If L is a lattice of equational theories extending a given theory then it satisfies the following condition:
If $A \subseteq L$, $\bigvee A = 1$ then $\exists a_0, \ldots, a_n \in A$ such that $(\ldots(((a_0 \wedge x) \vee a_1) \wedge x) \ldots \vee a_n) \wedge x = x$ for all $x \in L$.
(M. Erné, G. Tardos). For example, let now $B \subseteq L$, $a_1 \leq \bigvee B$. Then one can prove in the same way that $\exists b_0, \ldots, b_m \in B$ such that $((\ldots(((((\ldots(((a_0 \wedge x) \vee b_0) \wedge x) \ldots) \vee b_m) \wedge ((a_0 \wedge x) \vee a_1) \wedge x) \vee a_2) \wedge x) \ldots) \vee a_n) \wedge x = x$ for all $x \in L$. One can build other terms similarly. Are these conditions for L really stronger than the original one? (Answer: (G. Tardos) Yes.)

G. McNulty

27. For every positive integer n, is there an (n+1)-avoidable word which is not n-avoidable?

28. Describe those sets Σ of equations such that for every finite set Δ of terms

Δ is avoidable with respect to Σ iff every member of Δ is avoidable with respect to Σ.

29. (An old problem) Is it true that if A• is a congruence modular algebra, Con A has finite height and A has a one-element subalgebra then A has the unique factorization property?

P. Pudlák

30. Let a binary operation be given. Consider a finite set of rewriting rules such that every two terms with the same number of occurences of variables (i.e., equivalent in the free commutative semigroup) are equivalent with respect to this system. (For instance, $\alpha\beta \leftrightarrow \beta\alpha$, $\alpha(\beta\gamma) \leftrightarrow (\alpha\beta)\gamma$, $(\alpha\beta)\gamma \leftrightarrow (\beta\alpha)\gamma$.) A standard counting argument shows that it is necessary to use $\Omega(n \cdot \log n)$ rewritings to transform $x_1(x_2(\dots(x_{n-1}x_n)\dots))$ into $x_{\pi 1}(x_{\pi 2}(\dots(x_{\pi(n-1)}x_{\pi n})\dots))$ for some permutation π. Find at least one such permutation.

R.W. Quackenbush

31. What is the smallest first order definable order variety containing $\underset{\circ}{\overset{\circ}{|}}$? Is it the class of bounded lattices? (Answer: (M. Albert) Yes, in fact $= S_\ell P_u RPS_\ell P(\underline{2})$, where S_ℓ = elementary substructure, P_u = ultraproducts. Can we manage without P_u ?)

I.G. Rosenberg

Let A be a finite set, G a permutation group on $\{1,\dots,h\}$ $(h>1)$ and $\rho \subseteq A^h$ such that for all $(a_1,\dots,a_h) \in \rho$ we have (i) $a_i \neq a_j$ for all $1 \leqq i < j \leqq h$ (areflexivity) and (ii) $(a_{\pi 1},\dots,a_{\pi h}) \in \rho \Leftrightarrow \pi \in G$ for all permutations π of $\{1,\dots,h\}$. We say that ρ is strongly colorable if there is a (relational) homomorphism from ρ into $\sigma_G = \{(\pi 1,\dots,\pi h) : \pi \in G\}$.

32. Characterize the strongly colorable relations.

33. Investigate the computational complexity of the problem.

E.T. Schmidt

34. Characterize the rigid quotients in finite modular lattices. (Hint: Let a/b be a prime quotient in a modular lattice M and let $p: a/b \to a/b$ a projectivity. If M is a sublattice of the modular lattice L then p can be taken as a projectivity in L (M-projectivity). a/b is a rigid quotient of M if for every extension L and for every p the restriction of p is the identity mapping.)

35. Let a/b be a prime quotient of a finite modular lattice M and let L be another bounded modular lattice. We put L into the interval $[b,a]$, identifying the zero of L with b and the unit with a. We get a partial lattice P. Give a necessary and sufficient condition for the pair (M,L) that there exists a modular lattice generated by P in which the interval $[b,a]$ is exactly L.

36. Let A, B, S be sublattices of L, $A \cap B = S$, $A \cup B = L$. L pastes A and B together if (i) for all $a \in A$, $b \in B$ $a < b \Rightarrow \exists s: a \leqq s \leqq b$, and dually; and (ii) for $s \in S$ the covers of s in L are either all in A or all in B, and dually. Instead of (ii) we take (ii') for $s \in S$ if $a \succ s$ $(a \in A \setminus B)$, $b \succ s$ $(b \in B \setminus A)$ then $a \vee b \in S$, and dually. What are the properties of this "pasting"?

D. Schweigert

37. Give a completeness criterion for the clone of multi-valued operations.

38. Let us call a clone C binary if C is generated by the set C^2 of operations of arity ≤ 2 in C. Define $C_\alpha \vee C_\beta = \langle C_\alpha^2, C_\beta^2 \rangle$. Describe the lattice of all binary subclones on the set $A=\{0,1,2\}$.

M. Valeriote

39. Does there exist a variety V such that (1) the equational theory of V is recursive, and (2) the equational theory of $\mathbf{HSP}(F_V(\emptyset))$ is not recursive?

H. Werner

40. It would be more reasonable to call a variety V affine complete if all non-simple algebras in V are affine complete. The variety of vector spaces over a field F, the variety of sets, etc. are affine complete in this sense. Investigate the properties of these affine complete varieties.

R. Wille

41. For which direct products of contexts does a common refinement exist?

42. Is every complete lattice isomorphic to the lattice of complete congruence relations of some complete lattice?

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