MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

# Tagungsbericht 7/1988

## FUNKTIONENTHEORIE

14.2. bis 20.2.1988

Die Tagung fand unter der Leitung der Herren F. Gehring (Ann Arbor), E. Mues (Hannover) und K. Strebel (Zürich) statt. Von den 39 Teilnehmern hielten 25 Teilnehmer Vorträge. Wie aus der Teilnehmerliste ersichtlich ist, hatte die Tagung einen ausgesprochen internationalen Charakter.

Im Vordergrund der diesjährigen Tagung über Funktionentheorie standen die beiden Schwerpunkte 'Riemannsche Flächen, Teichmüller-Räume, diskontinuierliche Gruppen' einerseits und 'Quasikonforme Abbildungen' andererseits. Aber auch andere Problemkreise wurden in einigen Vorträgen behandelt. Spontan eingerichtete Seminare und rege Diskussionen rundeten den Verlauf dieser interessanten Tagung ab.

Teilnehmer

L.V. Ahlfors, Cambridge/USA
K. Astala, Helsinki/SF
A. Beardon, Cambridge/GB
J. Becker, Berlin
T. Betker, Berlin
C. Earle, Ithaca/USA
H. Epheser, Hannover
R. Fehlmann, Helsinki/SF
J.L. Fernândez, Madrid/E
G. Frank, Dortmund
D. Gaier, Gießen
J. Garnett, Los Angeles/USA

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F. Gehring, Ann Arbor/USA
K. Hag, Trondheim/N
W.K. Hayman, Heslington, York/GB
A. Huber, Zürich/CH
J. Jenkins, St. Louis/USA
P.W. Jones, New Haven/USA
H. Köditz, Hannover
I. Kra, Stony Brook/USA
R. Kühnau, Halle/DDR
A. Marden, Minneapolis/USA
H.H. Martens, Trondheim/N
O. Martio, Jyväskylä/SF
B. Maskit, Stony Brook/USA
R. Mortini, Karlsruhe
E. Mues, Hannover
A. Pfluger, Zürich/CH
E. Reich, Minneapolis/USA
H.M. Reimann, Bern/CH
M. von Renteln, Karlsruhe
S. Rickmann, Helsinki/SF
G. Schmieder, Würzburg
N. Steinmetz, Karlsruhe
K. Strebel, Zürich/CH
H. Tietz, Hannover
J. Väisälä, Helsinki/SF
J. Winkler, Berlin
M. Zinsmeister, Paris/F

## Vortragsauszüge

### K. ASTALA: $H^p$-theory for quasiconformal mappings

If f is a quasiconformal mapping in the unit ball $B^n$, $n \geq 2$, the radial boundary values $f(w) = \lim_{r\to 1} f(rw)$ exist for almost all $w \in S^{n-1}$. Hence it is natural to ask when is the boundary function f(w) contained in some of the basic function spaces like $L^p(S^{n-1})$, $BMO(S^{n-1})$, etc., and how does the behaviour of f inside $B^n$ affect the properties of f(w).

For holomorphic functions in the unit disk these questions are part of the well known $H^p$-theory. It turns out that many of the holomorphic $H^p$-results hold as such also for the n-dimensional quasiconformal mappings. However, the proofs must now be very different; we replace the Poisson integral and subharmonicity by more geometric arguments such as Möbius invariance, Modulus estimates and Calderon-Zygmund theory. As an example we describe a proof for the quasiconformal counterpart of the theorem of Burkholder, Gundy and Silverstein.

A.F. BEARDON: Iteration of self-maps of the unit disk

In 1926, J. Wolff proved that a holomorphic map f of the unit disk $\Delta$ into itself is either a hyperbolic rotation, or has the property that the n-th iterate $f^n$ converges to a point $\zeta$ in the closed disk $\bar{\Delta}$. If f is a Möbius map (a hyperbolic isometry of $\Delta$), the iterates of f can be computed and we ignore this case. In all other cases f is a contraction (distance decreasing in $\Delta$). We prove that a contraction f of a visibility manifold has the property that the iterates converge to a point in the natural closure of the manifold: this generalises Wolff's theorem to higher dimensions and variable curvature.

J. BECKER: Hölder continuity of conformal maps with quasiconformal extension (Joint work with C. Pommerenke)

Let f be a bounded univalent function in the unit disk $\mathbb{D}$ which has a K-quasiconformal extension to $\mathbb{C}$, then it is known that f satisfies the Hölder condition

$$|f(z_1)-f(z_2)| \le M|z_1-z_2|^{1-\kappa} \qquad (z_1,z_2 \in \overline{\mathbb{D}}),$$

where $\kappa = (K-1)/(K+1)$. It is easy to show that the Hölder exponent $1-\kappa$ is best possible.

Under suitable normalizations estimates of the Hölder constant $M = M(\kappa)$ are given implying that $M(\kappa) \to 1$ as $\kappa \to 0$ $(\kappa \to 1)$.

T. BETKER: Criteria for univalence and quasiconformal extensibility

Let $G \subset \mathbb{C}$ be a bounded quasidisk, $\lambda$ a $\frac{1+q}{1-q}$-quasiconformal reflection in $\partial G$ and f analytic (or meromorphic) in G. We consider some expression $\phi(z,\lambda) = f + (\lambda-z)[\dots]$ in terms of z, $\lambda$, f and the derivatives of f. Defining F by $F(\lambda) = \Phi(z,\lambda)$, we ask whether the condition $|F(\lambda)_{\bar{z}}| \le k|F(\lambda)_z|$ a.e. in G implies that f is quasiconformally extensible to $\hat{\mathbb{C}}$ by

F, and whether f is univalent, if $k = 1$.

Let in addition $\lambda$ be generated by a Löwner chain $h(z,t)$, i.e. $\lambda(h(e^{-t\zeta})) = h(\zeta,t)$ $(|\zeta| = 1,\ t \geq 0)$; an example in the case $G = \mathbb{D}$ is $\lambda(z) = \frac{1}{1-c}\frac{1}{z} - \frac{c}{1-c}z$, $|c| < 1$. Then the question can be answered in the affirmative for $\Phi(z,\lambda) = f + (\lambda-z)\frac{f'}{g'}$ and $\Phi(z,\lambda) = f + (\lambda-z)f'/[1 - \frac{1}{2}(\lambda-z)(\frac{f''}{f'} - \frac{g''}{g'})]$, where g is analytic in G, $g'(z) \neq 0$. The corresponding conditions

$$|(\lambda-z)[\frac{f''}{f'} - \frac{g''}{g'}] + (g'-1) + \lambda_z| \leq k|\lambda_{\bar{z}}| \quad \text{a.e. in } G\,,$$

$$|\frac{1}{2}(\lambda-z)^2[S_f - S_g] + (\lambda-z)\frac{g''}{g'} + \lambda_z| \leq k|\lambda_{\bar{z}}| \quad \text{a.e. in } G$$

generalize several known criteria due to Ahlfors, Becker, Epstein, Krzyz and Nehari.

The proof uses Löwner theory and a property of so-called q-chains: For a.e. $t \geq 0$, $\frac{1}{h'(z,t)} \in H^{\varepsilon}$ (and $h'(z,t) \in H^{2+\varepsilon}$) for $0 < \varepsilon < \varepsilon_0(q)$.

C. EARLE: Holomorphic motions and Teichmüller spaces

Let E be a closed subset of $\mathbb{C}' = \mathbb{C} \setminus \{0,1\}$. Two of the basic results of Bers and Royden about holomorphic motions of E over the open unit disk $\Delta$ are:

1) If E is a finite set, the holomorphic motions of E over $\Delta$ are in natural bijective correspondence with the holomorphic maps $f: \Delta \to \text{Teich}(\mathbb{C}' \setminus E)$ with $f(0) = 0$.

2) If $E = \mathbb{C}'$, the holomorphic motions of E over $\Delta$ are in natural bijective correspondence with the holomorphic maps f from $\Delta$ to the open unit ball $M(E) \subset L^{\infty}(E,\mathbb{C})$ s.t. $f(0) = 0$.

A Cornell graduate student, G. Lieb, has found an extension of these results to the general case by considering holomorphic maps into $\text{Teich}(\mathbb{C}' \setminus E) \times M(E)$ for arbitrary closed E. This requires a new definition if $\mathbb{C}' \setminus E$ is not connected.

R. FEHLMANN: Minimal slit sets in the metric of a quadratic differential

The classical notion of minimal slit sets which arises in conformal mappings of domains of arbitrary connectivity carries over to slit sets on trajectories of quadratic differentials. Another characterization in terms of the Main Inequality of Reich and Strebel has been given in [R. Fehlmann, Extremal quasiconformal mappings with free boundary components in domains of arbitrary connectivity. Math. Z. 184 (1983), 109-126]. While it is an open question if these slit sets are the same in general, we sketch a proof that this is indeed true in case of quadratic differentials which correspond to Teichmüller n-gon mappings.

J.L. FERNANDEZ: Level sets of covering maps

A few years ago Hayman-Wu showed that the level sets of conformal mappings from the unit disk $\Delta$ onto simply connected domains can not be arbitrarily long. We study for which multiply connected domains does this result hold.

Say, (HW) holds if

$$\text{length } (f^{-1}(\Omega \cap L)) \leq C(\Omega)$$

for all f universal coverings of $\Omega$ and lines L.

Let $\Gamma$ be the universal covering group of $\Omega$ and define

$$U_t(z) = \sum_{T \in \Gamma} \left(1 - \left|\frac{r-Tz}{1-\bar{z}Tz}\right|^2\right)^t .$$

The infimum of t's for which $U_t$ is finite is the exponent of convergence $\delta$. We have

A) $U_{1/2} \in L^\infty \implies$ (HW) holds $\implies U_1 \in L^\infty$.

Also

B) $U_1 \in L^\infty \implies \delta < 1$, while there are $\Omega$'s for which $\delta$ is as small as desired while $U_1 \notin L^\infty$.

D. GAIER: On the convergence of the Bieberbach polynomials

The Bieberbach polynomials $\pi_n$ were introduced by Bieberbach 1914 to approximate the conformal mapping $f_o$ of a Jordan region G onto $\{w: |w| < r\}$, $f_o(0) = 0$, $f_o'(0) = 1$. They minimize $\iint_G |f'(z)|^2 d\sigma$ in the class of all polynomials P of degree $\leq n$, $P(0) = 0$, $P'(0) = 1$. Estimates are given for $E_n = \max_{\bar{G}} |f_o(z) - \pi_n(z)|$ for regions with piecewise smooth boundary.

Theorem 1. If $\partial G$ is piecewise smooth, and $\lambda\pi$ is the smallest exterior angle, $0 < \lambda \leq 1$, then $E_n = O(n^{-\gamma})$ for every $\gamma < \min\left(\frac{\lambda}{2-\lambda}, \frac{1}{2}\right)$.

A result of Mergelyan, for smooth $\partial G$, is thus extended.

Theorem 2. If $\partial G$ is quasiconformal, $f_o \in \mathrm{Lip}\,\alpha$, and the exterior map $\psi$ is in Lip $\beta$, then $E_n = O(n^{-\gamma})$ for every $\gamma < \frac{\alpha\beta}{2}$.

We have also an inverse theorem which shows that the bound in Theorem 1 is sharp whenever $0 < \lambda \leq \frac{2}{3}$. In particular, if G is the L-shaped domain, $E_n = O(n^{-\gamma})$ for all $\gamma < \frac{1}{3}$ but for no $\gamma > \frac{1}{3}$. The proofs rely on previous work of Andrievskii (1983). Numerical experiments indicate that some improvements might be true in case that $\partial G$ is piecewise analytic.

J. GARNETT: Almost isometric maps
(Joint work with M. Papadimitrakis)

Let the unit disk $\Delta$ be endowed with the hyperbolic metric and let $f: \Delta \to \Delta$ be analytic. Following C. McMillan say $f \in M(d)$, d constant, if

$$\operatorname{diam} f(B(z,R)) \geq 2R-d, \quad \forall\, z \in \Delta, \quad \forall\, R > 0 .$$

Theorem: $\exists\, A(d)$, $A'(D)$ such that $f \in M(d) \iff$ for all I on $\partial\Delta$ and vertex $z_I$

$$\inf_I |f'(\theta)| \leq \frac{A}{\operatorname{meas}(I)}(1 - |f(z_I)|^2)$$

where $f'(\theta)$ = angular derivative at $e^{i\theta}$.

Other characterizations are also given.

W.K. HAYMAN: Functions of locally bounded characteristic
(Joint work with C. Pommerenke)

In 1976 B. Korenblum and the author considered functions $f(z)$ regular in $\{D: |z| < 1\}$ and satisfying there a growth condition

$$\log |f(z)| \leq k(r), \quad |z| \leq r, \quad 0 \leq r < 1.$$

We proved that $f(z)$ has locally bounded characteristics in D if

(1) $$\int_0^1 \sqrt{\left(\frac{k(r)}{1-r}\right)} dr < \infty .$$

It turns out that the same conclusion obtains under the more general hypothesis that f is meromorphic in D and that there exists a positive decreasing function $\delta(r)$ and a positive increasing function $k(r)$ for $0 \leq r < 1$, such that for $|z_0| < r$, the image by $f(z)$ of $\left|\frac{z-z_0}{1-\bar{z}_0 z}\right| < \delta(r)$ onto the Riemann sphere leaves out a spherical cap of chordal radius $\exp(-k(r))$, where

(2) $$\int_0^1 \sqrt{\left(\frac{k(r)}{1-r}\right)} \frac{dr}{\delta(r)} < \infty .$$

Taking $\delta(r) = 1/2$, we see that (1) implies (2). A significant generalisation of the class of normal functions also satisfies (2).

A. HUBER: Curves generated by conformal welding

Let $\Phi: e^{i\Theta} \to e^{i\phi(\Theta)}$ $(\Theta, \phi \in \mathbb{R})$ be an analytic, sense-preserving homeomorphism of $C = \{z \mid |z| = 1\}$ onto itself, $\Phi' \neq 0$ on C. Then there exist an analytic Jordan curve $\Gamma$ and conformal mappings F (of the interior of C onto the interior of $\Gamma$) and G (of the exterior of C onto the exterior of $\Gamma$) such that $F(e^{i\Theta}) = G(\Phi(e^{i\Theta}))$ for all $\Theta \in \mathbb{R}$ (conformal welding). We normalize $G(\infty) = \infty$.

THEOREM 1. For all $\Theta \in \mathbb{R}$

$$\frac{d}{d\Theta} \arg F'(e^{i\Theta}) = \psi(\Theta),$$

where

$$\psi(\Theta) = \frac{[\phi'(\Theta)]^2-\phi'(\Theta)+(\partial U/\partial n)(e^{i\Theta})+K}{\phi'(\Theta)+1}.$$

Here U is the solution of the Dirichlet problem for the interior of C with boundary values log $\phi'(\Theta)$ on C, $\partial/\partial n$ denotes the derivation in the direction of the outer normal to C and K the (by $\Phi$ uniquely determined) number, for which

$$\int_0^{2\pi} \psi(\Theta)d\Theta = 0.$$

THEOREM 2. For all z, $|z| < 1$,

$$F(z) = K_1 \int_0^z \exp\{\frac{i}{2\pi}\int_0^{2\pi}[\frac{e^{i\Theta}+w}{e^{i\Theta}-w}\int_0^{\Theta}\psi(t)dt]d\Theta\}dw + K_2\ ,$$

where $\psi$ is defined in Theorem 1, and $K_1, K_2 \in C$, $K_1 \neq 0$.

J.A. JENKINS: Some estimates for harmonic measures II

In the first paper with this title the author characterized possible extremal configurations arising in the following problem. Find a continuum in the closed unit disk which meets every radius and for which the harmonic measure at the origin is minimal. This is a particular case of a problem raised by W.H.J. Fuchs. Up to normalization these configurations consist of an arc on the unit circumference from 1 described in the clockwise sense to $e^{i\phi}$ with $0 < \phi \leq \pi$ together with an arc of a trajectory of a quadratic differential from the latter point to a point $\mathfrak{z}^*$ with $0 < \mathfrak{z}^* < 1$. Each continuum is the unique minimizing continuum in its homotopy class for arcs joining 1 and $\mathfrak{z}^*$ in $|\mathfrak{z}| \leq 1$. The unique minimizing configuration for all $\mathfrak{z}^*$ was recently shown by Marshall and Sundberg to consist of the arc on $|\mathfrak{z}| = 1$ from 1 to -1 in the above sense plus the image under $\mathfrak{z} = \frac{1-z}{1+z}$ of the half-branch of the hyperbola $x^2 - \frac{x^2}{3} = 4$ (z = x + iy) in the lower right half-plane. This is now proved by a very simple argument using threefold symmetric symmetrization.

P. JONES: <u>Function theory on Riemann surfaces</u>

Let $\Omega$ be a Riemann surface with universal covering surface the disk, U, an let $\Omega \cong U/\Gamma$. Also let $\Gamma^*$ denote the character group of $\Gamma$. Consider the following conditions on $\Omega$:

(A) There is a fundamental domain F in U such that arc-length on $\bigcup_{\gamma \in \Gamma} \gamma(\partial F)$ is a Carleson measure.

(B) There is $P(z) \in H^\infty(U)$ such that $\sum_\gamma P(\gamma(z)) \equiv 1$ and $\sum_\gamma |P(\gamma(z))| \leq c \quad \forall \; z \in U$ <u>and</u> $\exists \; c < \infty$ such that $\forall \; \zeta_o \in \Omega$, $\{\zeta : G(\zeta, \zeta_o) > c\}$ is simply connected (G = Green's function).

(C) For $\lambda \in \Gamma^*$ let $H^\infty_\lambda = \{f \in H^\infty(U) : f(\gamma(z)) = \lambda(\gamma) f(z)\}$. There is $\varepsilon > 0$ such that $\forall \; \lambda \in \Gamma^* \;\; \exists \; f \in H^\infty_\lambda$, $\varepsilon \leq |f(z)| \leq 1$ $\forall \; z \in U$.

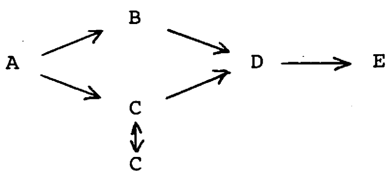
(C') $\forall \lambda \in \Gamma^* \quad \exists \; f \in H^\infty_\lambda$, $0 < \varepsilon_\lambda \leq |f(z)| \leq 1 \quad \forall \; z \in U$.

(D) $\exists \; c < \infty$ such that $\forall \; \zeta_o \in \Omega$,

$$\int_0^\infty 1^{st} \text{ Betti } \# \{\zeta : G(\zeta, \zeta_o) > t\} dt \leq c .$$

(E) $\forall \; \zeta_o \in \Omega \;\; \exists \; \Omega_{\zeta_o} \subset \Omega$, $\Omega_{\zeta_o}$ simply connected such that $\omega(\zeta_o, \partial\Omega, \Omega_{\zeta_o})$ = harmonic measure at $\zeta_o$ of $\partial\Omega$, in $\Omega_{\zeta_o} \geq \varepsilon > 0$.

Theorem:




It is easy to see that if $\Omega \subset \mathbb{C}$ is a Denjoy domain, $\partial\Omega \subset \mathbb{R}$, then E $\Longrightarrow$ A. It is tempting to conjecture that E $\Longrightarrow$ A for all surfaces $\Omega$. Unfortunately there is a counter-example, modeled on Brian Cole's example of a surface for which the corona theorem fails. It appears, however, that the conjecture may be true in the genus 0 (planar) case.

I. KRA: <u>Coordinates for Teichmüller and Riemann spaces</u>

The Teichmüller space T(p,n), of n-punctured surfaces of genus p, can be represented as the deformation space T(G) of a finitely generated Kleinian group G; for example, G can be chosen to be a terminal regular b-group of type (p,n). From T(G), one can read off intrinsic coordinates for T(p,n). It is of interest to choose coordinates in which certain paths in Teichmüller space can be explicitly described; for example, an infinitesimal Dehn twist or the shrinking of a closed curve. Traces of elements of groups and cross ratios of fixed points of loxodromic or parabolic elements of these groups provide appropriate coordinates for such studies. They also serve as good local coordinates for the Riemann space R(p,n). It is also possible to describe some degenerations of Riemann surfaces in terms of these coordinates. Interesting Kleinian groups appear on the boundary of T(G).

R. KÜHNAU: <u>Zur möglichst konformen Spiegelung an Jordankurven</u>

Zu einem gegebenen Quasikreis werden die möglichst konformen Spiegelungen studiert, das sind diejenigen quasikonformen Spiegelungen mit kleinstmöglicher Dilatationsschranke ≥ 1. Diese kleinstmögliche Dilatationsschranke heiße "Spiegelungskoeffizient" $Q_{\check{\mathcal{L}}}$. Es werden hierzu äquivalente Fragen formuliert.

Sodann wird bei n gegebenen Punkten $z_1,\dots,z_n$ nach Quasikreisen hierdurch (bei gegebener Homotopieklasse) gefragt, deren Spiegelungskoeffizient kleinstmöglich ist. Dies läßt sich auf eine alte Aufgabe von O. Teichmüller zurückführen. Es zeigt sich, daß die Lösungskurven $\check{\mathcal{L}}$ i.a. Knicke besitzen.

Schließlich werden noch (sinngemäß analog zu definierende) möglichst konforme Spiegelungen an abgeschlossenen Jordanbögen studiert. Es gibt bisher fast keine Beispiele, für die der Spiegelungskoeffizient explizit bekannt ist. Aber es lassen sich asymptotische Formeln für "kleine" Bögen aufstellen.

H.H. MARTENS: Haupt's characterization of period vectors

Considering the difficulty of characterizing period matrices of a closed surface (Schottky's problem) it is interesting to observe that Otto Haupt in 1920 gave a complete characterization of period vectors. In the case of vectors of the form $(1,0,\dots,0,z_1,\dots,z_g)$ this reduces to the condition that $\mathrm{Im}\,(z_1) > 0$ and that the vector not be symplectically equivalent to one with only two non-vanishing entries. The proof involves a construction of independent interest and some intriguing open problems.

O. MARTIO: Generalized harmonic measure

An elliptic partial differential equation $\nabla \cdot A(x,\nabla u(x)) = 0$ in a bounded domain G of $\mathbb{R}^n$ with $A(x,h) \cdot h \approx |h|^p$, $1 < p \le n$, induces an A-harmonic measure $\omega = \omega(E,G;A): G \to \mathbb{R}$ on subsets E of $\partial G$. If $A(x,h) = h$, then $\omega$ is the ordinary (outer) harmonic measure. It is shown that there is a class of sets E in $\partial G$ such that $\omega \doteq 0$ for all A and p; the condition of E is formulated in terms of quasihyperbolic distance. For $G = B^n$ this class includes sets E whose Hausdorff-dimension can be arbitrarily close to n-1.

B. MASKIT: Some special Kleinian groups

Gehring and Martin asked the following question (oral communication). Let G be the subgroup of $PSL(2;\mathbb{C})$ generated by the elliptic element a of order 6, and the parabolic element b, where b maps one fixed point of a to the other; is G discrete? This question is answered affirmatively, and some related questions are discussed.

R. MORTINI: F-ideals in Banach algebras of analytic functions

In 1971 V.P. Khavin introduced the notion of the "F-property" for subspaces of the Hardy space $H^1$. Here we shall generalize this concept to ideals in subalgebras of $H^\infty$. This enables us to give a complete characterization of those closed ideals in $H^\infty$ which can be lifted to closed ideals in $L^\infty$. As a corollary we obtain the result that every closed F-ideal in $H^\infty$ has the form

$$I = I(E,H^\infty) := \{f \in H^\infty : f|_E = 0\},$$

where E is a closed subset of the Shilov boundary of $H^\infty$. The proof is based on a Beurling-Rudin type theorem for $H^\infty$, which tells us that every closed ideal in $H^\infty$ with inner factor 1 is the trace of a unique closed ideal in $H^\infty+C$. Similar questions - including prime and radical ideals - are also investigated for the spaces $QA_B$, where B is a Douglas algebra.

Finally we remark that the author has obtained these results in cooperation with Pamela Gorkin (and Håkan Hedenmalm for the $H^\infty+C$ case).

A. PFLUGER: On a method of Georg Faber

Proving Koebe's 1/4-theorem in the class S is the same as to show that a function of class $\Sigma$ omits a continuum of diameter not larger than 4, or, that for a function $f(z) = az + a_o + a_1/z$ univalent in $\{|z| > 1\}$ and omitting the points 1 and -1 we have $|a| \geq 1/2$. G. Faber proved this in 1916 by considering the functions

$$g(z) = f(z) + f^2(z) - 1 = 2az + a_o' + \frac{a_1'}{z} + \dots .$$

They have the property that $g(z) \cdot g(z') \neq 1$ for any two points of $\{|z| > 1\}$, hence the omitted set of g has area measure $\geq \pi$ and the aerea theorem yields $2|a| \geq 1$.

This method of Faber applies if $\Omega$ is an arbitrary multiply connected domain containing $\infty$ and allows to find the range of functionals $\log \frac{f(a)-f(b)}{a-b}$ over $\Sigma(\Omega)$.

M. von RENTELN: Some remarkable properties of the Wiener-algebra $W^+$ (Joint work with R. Mortini)

Let $W^+$ denote the Banach algebra of all absolutely convergent Taylor series in the open unit disk, i.e.

$$W^+ := \{f(z) = \sum_{n=0}^{\infty} a_n z^n : \|f\| = \sum_{n=0}^{\infty} |a_n| < \infty\} .$$

An open problem is a characterization of the closed ideals in $W^+$. This seems difficult because $W^+$ has unusual properties, e.g.

1) $W^+$ has not the F-property (factorization property) in the sense of Havin (Gurarii, 1972).

2) There is no characterization known of the zero sets in $W^+$.

3) $W^+$ has not the bounded inverse property (H.S. Shapiro, 1979).

On the positive side the following result is proved:

Theorem: The closed finitely generated nontrivial ideals in $W^+$ are just the principal ideals generated by the finite Blaschke products.

The method in the proof can be used to show that there exist finitely generated ideals I, J such that $I \cap J$ is not finitely generated. This answers a question of Rubel and McVoy.

S. RICKMAN: Growth of quasiregular mappings

Let $f: R^n \to M$ be a quasiregular mapping into a Riemannian n-manifold M. If $\omega$ is the volume form of M, we measure the growth of f by the integral of the pullback $f^*\omega$ over balls in $R^n$. Simple growth estimates give the result that if the isoperimetric dimension of M is greater than n, then f is constant.

With more delicate methods growth can be studied also in more general cases. For example, the growth in the case $M = S^n \setminus \{a_1, \ldots, a_q\}$ can be shown to depend on q.

H. TIETZ: Anwendungen der Laurent-Trennung

Die Zerlegung einer in einem Ringgebiet holomorphen Funktion als Summe von Funktionen mit größeren Definitionsgebieten läßt sich systematisieren.

Anwendungen sind einerseits die klassischen Faber-Entwicklungen und andererseits eine Charakterisierung der meromorphen Funktionen, die von ihrer Partialbruchreihe dargestellt werden.

J. VÄISÄLÄ: John disks

Let E be an arc in the extended plane $\dot{R}^2$. For an interior point x of E let $\rho(x)$ be the diameter of the smaller component of $E \setminus \{x\}$. For $c \geq 1$ the union of all disks $B(x, \frac{\rho(x)}{c})$ is written as $\text{cig}_d\ (E,c)$. A conformal disk $D \subset \dot{R}^2$ is a c-John disk if each pair of points of D can be joined by an arc E with $\text{cig}_d\ (E,c)$.

Every quasidisk and the domain  are John disks. Results:

1. A Jordan curve $\gamma \subset \dot{R}^2$ is a quasidisk $\Longleftrightarrow$ both components of $\complement\gamma$ are John disks. Thus a John disk can be regarded as a one-sided quasidisk.

2. A domain $D \subset \dot{R}^2$ is a quasidisk $\Longleftrightarrow$ $\exists$ c such that $\phi D$ is a c-John disk for every Möbius map $\phi$.

There are numerous characterizations for John disks. For example: A conformal disk $D \subset \dot{R}^2$ is c-John $\Longleftrightarrow$ each pair of points $a,b \in \complement D$ can be joined by an arc $E \subset \complement D$ with $d(E) \leq c_1 |a-b|$. Here c and $c_1$ depend only on each other.

The results and characterizations are partly folklore, partly contained in papers of Gehring, Pommerenke, Näkki, Palka. A survey article by Näkki and me is in preparation.

M. ZINSMEISTER: Fejér-Riesz property

Fejêr and Riesz proved that $\int_{-1}^{1} |f(x)|dx \le 2\|f\|_{H^1(D)}$ for every $f \in H^1(D)$, the usual Hardy space. Later, Carleson characterized the positive measures $\mu$ on the unit disk D such that there exists $C > 0$ with $\iint_D |f|d\mu \le C\|f\|_{H^1(D)}$, they are the measures for which $\mu(D(z_0,r)) \le Cr \ \forall z_0 \in D, \ \forall r > 0$.
In particular ds/E is such a measure if E is regular in the sense of Ahlfors, that is if $\exists C > 0, \Lambda(E \cap D(z,r)) \le Cr$ $\forall z \in \mathbb{C}, r > 0$, where $\Lambda$ stands for one-dimensional Hausdorff measure. The purpose of the lecture is to investigate the following problem: Given a domain $\Omega$ on which one can define a "reasonable" notion of Hardy space $H^1$, decide whether for every regular set in the plane $\exists C > 0; \int_E |f||dz| \le C\|f\|_{H^1(\Omega)}$ for $f \in H^1(\Omega)$. Call "good" a domain having this property.
1) Simply connected case. Let $\Omega$ be a Jordan domain with rectifiable boundary and $\phi: D \to \Omega$ the Riemann map. Define $H^1(\Omega) = \{f;\ f \circ \phi\phi' \in H^1(D)\}$. G. David has shown that $\Omega$ is good if $\partial\Omega$ is regular. The first result is the construction of a rectifiable quasicircle which is not a good domain.
2) Denjoy domains $\equiv \Omega = \mathbb{C} \setminus K$, K compact $\subset \mathbb{R}$. Define $H^1(K) = \{f \in L^1(K), \int f = 0, Hf \in L^1(K)\}$ (H = Hilbert transform), $H^1(\Omega) = \{$Cauchy integrals of functions in $H^1(K)\}$. Result: $\Omega$ is good iff K is Carleson-homogeneous.

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