MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Numerical Linear Algebra and Parallel Computation

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The conference directed by W. Niethammer (Karlsruhe), G. Rodrigue (Livermore, California) and R.S. Varga (Kent, Ohio) was attended by 41 individuals from 11 countries. The 33 contributions to this meeting offered a broad view of large scientific computing problems in numerical linear algebra. They primarily dealt with different aspects of either the solution of large linear algebraic systems or the algebraic eigenproblem on sequential and parallel machines. These problems were mainly discussed in the context of the numerical solution of partial differential equations based on finite difference approximations or on finite element methods, but other areas of applications, e.g., large least squares problems arising in signal processing and image reconstruction, were also considered.

In connection with the solution of linear algebraic systems, one major topic was the investigation of preconditioners for the conjugate gradient method (e.g., domain decomposition, hierarchical basis functions, incomplete Cholesky factorization and polynomial preconditioners). Theoretical results for these

algorithms as well as examples for their actual performance on different parallel architectures were presented. Other subjects treated at the meeting were multisplittings, the solution of interval equations, SSOR, the Schwarz alternating principle, cyclic reduction, Gaussian elimination and LU factorization. For direct solvers, special emphasis was again put on those issues relevant to the efficient implementation on parallel computers. As far as the algebraic eigenproblem is concerned, the topics ranged from modifications of the QR and LR algorithms, Raleigh-quotient minimization problems, the usage of block reflectors to 'divide-and-conquer' techniques for the computation of the singular value decomposition.

This conference brought together theoreticians in numerical linear algebra as well as experts in solving large scientific computing problems on parallel computers. The intensive exchange of ideas and informations between these groups and the interplay between theory and computation emphasized in several contributions will certainly stimulate further research in the area.

Finally, we wish to express our gratitude to the staff of the Oberwolfach Mathematical Research Institute for providing a pleasant and cordial atmosphere which was very conducive to scientific communication.

VORTRAGSAUSZÜGE

L.M. ADAMS:

Implementation of the Hierarchical Basis Preconditioner on a Shared Memory Machine

The time required to solve a system of linear equations by the preconditioned conjugate gradient method on a parallel machine depends on the choice of preconditioner. In this paper, we give a parallel implementation of the hierarchical basis preconditioner on the flexible - 32 shared memory machine. Our results show that this preconditioner can be efficiently implemented on this machine. We also show that this preconditioner is much more efficient than the incomplete Cholesky preconditioner on this machine. (Joint work with E.J. Ong)

O. AXELSSON:

A Smoothing-Correction V-Cycle Method Based on Domain Decomposition

Given an elliptic difference equation a new approximate factorization preconditioner is computed on a coarse mesh and used as a correction operator in a two-level V-cycle method. The matrix is reordered to a block tridiagonal form based on a decomposition of the mesh into strips. Using an indirect approximation of the Schur complements arising during the approximate factorization based on consistently chosen vectors, a favourable eigenvalue distributation is achieved. The preconditioner which is of optimal (lowest) order of computational complexity, has therefore the same error reduction properties of Fourier components as the elliptic operators on the coarse mesh. Numerical tests show the robustness of the method. (Joint work with B. Polman)

O. AXELSSON:

A W-Cycle Multilevel-Preconditioning Method

Given a sequence of finite element meshes for a polygonal domain, defined by refinements and using standard nodal basis functions, we recursively define a preconditioner for the stiffness matrices on each mesh, using the preconditioner on the previous mesh as a correction operator in a ν-fold V-cycle method. For $\nu=2$, the preconditioner has a relative condition number $O(1)$, $h \to 0$ if $\gamma^2<\frac{1}{2}$ and for $\nu=3$, if $\gamma^2<\frac{2}{3}$. Here γ is the constant in the strengthened C.B.S. inequality. No elliptic regularity assumption is required. (Joint work with P. Vassilevski)

A. BUNSE-GERSTNER:

A Quaternion QR Algorithm

This paper extends the Francis QR algorithm to quaternion and antiquaternion matrices. It calculates a quaternion version of the Schur decomposition using quaternion unitary similarity transformations. Following a finite step reduction to a Hessenberg-like condensed form, a sequence of implicit QR steps reduces the matrix to triangular form. Eigenvalues may be read off the diagonal. Eigenvectors may be obtained from simple back substitution. For serial computation, the algorithm uses only half the work and storage of the unstructered Francis QR iteration. By preserving quaternion structure, the algorithm calculates the eigenvalues of a nearby quaternion matrix despite roundings errors. (Joint work with R. Byers and V. Mehrmann)

D.-Y. CAI:

A New Algorithm for Eigenvalue Problems of a Matrix

Q.I.F. (Quadrant Interlocking Factorization) is a factorization technique for a matrix. It was presented by Evans in the 70's. This factorization can be used to solve a system of linear equations parallely. Essentially speaking, it is a block LR factorization of the permuted matrix PAP^T, where P is a permutation matrix.

In this paper, a new algorithm for the eigenvalue problem of a matrix by means of Q.I.F. is given. Several numerical examples show that it may be an effective method. The convergence of our algorithm is proved for some special cases.

Finally, open questions are cited. (Joint work with H. Jing)

T.F. CHAN:

Fourier Analysis of Iterative Methods for Elliptic Problems

We present a Fourier method for analyzing stationary iterative methods and preconditioners for discretized elliptic boundary value problems. Similar to the von Neumann stability analysis of hyperbolic and parabolic problems, the approach is easier to apply and reveals more details about convergence properties than standard techniques, and can be applied in a systematic way to a wide class of numerical methods. Although the analysis is applicable only to periodic problems, the results essentially reproduce those of classical convergence and condition number analysis for problems with other boundary conditions, such as the Dirichlet problem. In addition, they give suggestive new evidence of the strengths and weaknesses of methods such as incomplete factorization preconditioners in the Dirichlet case. (Joint work with H.C. Elman)

I.S. DUFF:

Multitasking a Sparse Matrix Code on the Alliant FX/8

We have recently multitasked a code for the direct solution of sparse linear equations on the Alliant FX/8. We discuss several issues which are involved, all of which are of relevance to any shared memory multiprocessor. Among these issues are the dynamic allocation of data, the management of task queues, task spawning, and the effect of controlling the granularity. We also show runs of our code under the Schedule package from Argonne which presents a portable interface to users of parallel machines, allows the user to define his computational graph, and has very useful graphic output to a SUN workstation. Our tailored code attains a speed–up of about 6 on the eight processors of the Alliant. We suggest ways of improving it further. We also discuss our experience with the same code on the 4 processor CRAY 2.

M. EIERMANN:

"Chebyshev-like" Iteration Methods for Nonsymmetric Systems of Linear Equations

Consider a nonsingular system of linear algebraic equations $\mathbf{Ax} = \mathbf{b}$, or in fixed point form $\mathbf{x} = \mathbf{Tx} + \mathbf{c}$, where the eigenvalues of T are contained in some compact subset Ω of the complex plane. Using Faber polynomials, iteration methods of the form

$$\mathbf{y}_m := \mu_{m,0}(\mathbf{c} + \mathbf{T}\mathbf{y}_{m-1}) + \mu_{m,1}\mathbf{y}_{m-1} + \cdots + \mu_{m,k}\mathbf{y}_{m-k},$$

$\mu_{m,0} \neq 0$, $\Sigma \mu_{m,j} = 1$ (for all $m \geq 1$), are constructed which include the Chebyshev iterative method as well as the stationary k-step methods. These schemes have the following properties

- they allow a recursive and efficient computation of the approximants,

- they are "near–best" in the sense that the asymptotic rate of convergence is best possible and moreover, error estimations can be derived which show that these methods are nearly optimal in every iteration step,
- as in the Chebyshev case, the "extreme" eigenvalues of T can be estimated during the iteration. This information can be used to adapt the iteration parameters $\mu_{m,j}$.

L. ELSNER:

Comparisons of Convergence Rates for Multisplitting Methods

For a linear system $Ax = b$, k splittings $A = M_\ell - N_\ell$ and k matrices $0 \le E_\ell \sum_{\ell=1}^{k} E_\ell = I$, a multisplitting iterative method is given by $x_{i+1} = \left[\sum_\ell E_\ell M_\ell^{-1} N_\ell\right] x_i + \sum_\ell E_\ell M_\ell^{-1} b$. For certain cases involving regular splittings related to the standard decomposition $A = D-L-U$ of an M-matrix, Neumann-Plemmons have given upper bounds for the convergence rates. Here we give a new proof and find also lower bounds, as conjectured in the above mentioned paper. For a "chaotic" version of the multisplitting method

$$x_{j+r_j} = (I-E_{i_j})x_{j+r_j-1} + E_{i_j} M_{i_j}^{-1}(N_{i_j} x_j + b) \qquad j = 0,1,\ldots,$$

where $\{i_j\}$ is a regulated sequence, $1 \le i_j \le k$, Bru-Elsner-Neumann have proven convergence in the situation considered above. In a special case we can in addition show that by increasing r_j, i.e. using more processors, the convergence rate is increased. (Joint work with M. Neumann)

J. GARLOFF:

Block Methods for the Solution of Linear Interval Equations

We generalize feasibility results for interval arithmetical versions of total step, single and symmetric single step methods to block methods. We give convergence results and compare the quality of the enclosure and speed of convergence with respect to the fineness of the partition into blocks of the given matrix.

J.A. GEORGE:

QR Factorization of a Dense Matrix on a Hypercube Multiprocessor

An algorithm for computing a QR factorization of a rectangular matrix on a hypercube multiprocessor is described. The scheme involves the embedding of a two-dimensional grid in the hypercube network. A communication scheme which uses redundant computation in order to maintain data proximity is employed, and the data mapping strategy is such that for a fixed number of processors the processor idle time is either constant or grows linearly with the dimension of the matrix. A complexity analysis indicates what the aspect ratio of the imbedded grid should be in terms of the shape of the matrix and the relative speeds of communication and computation. Numerical experiments performed on an Intel hypercube multiprocessor support the theoretical results. (Joint work with E. Chu)

A. GREENBAUM:

Comparison of Linear System Solvers Applied to Diffusion-Type Finite Element Equations

Various iterative methods for solving the linear systems associated with finite element approximations to self-adjoint elliptic differential operators are compared based on their performance on serial and parallel machines. The

methods studied are all preconditioned conjugate gradient methods, differing only in the choice of preconditioner. The preconditioners considered arise from diagonal scaling, incomplete Cholesky decomposition, hierarchical basis functions, and a Neumann-Dirichlet domain decomposition technique. The hierarchical basis function idea is shown to be especially effective on both serial and parallel architectures. (Joint work with C. Li, H.Z. Chao)

M.H. GUTKNECHT:

Iterative (k,ℓ)-Step Methods for Linear and Nonlinear Systems of Equations

Given a fixed point equation $x = \phi x$ of a Fréchet-differentiable mapping ϕ, we discuss its solution by iterative methods of the form

$$x_m := \frac{1}{\beta_{mm}} \left\{ \phi \left[\sum_{i=1}^{m} \gamma_{m-1,i}\, x_i \right] - \sum_{j=1}^{m} \beta_{m,j}\, x_j \right\}$$

defined by two infinite regular lower triangular matrices $B = (\beta_{mj})$ and $C = (\gamma_{mi})$ with row sums 1. We show that two such methods, given by B,C and $\tilde{B},\tilde{C}$, respectively, are equivalent if and only if $BC^{-1} = \tilde{B}\tilde{C}^{-1}$. Hence, $\hat{B} := BC^{-1}$ and I define an equivalent semi-iterative method. We introduce (k,ℓ)-step methods as those, where B and C have lower bandwidth k and ℓ, respectively. Stationary methods are defined by B and C having Toeplitz structure except in their first column. They are equivalent to Euler methods and are (k,ℓ)-step methods if and only if the underlying exterior conformal map is a rational function of the form $z(\mu_0 + \ldots + \mu_k z^{-k})/(\nu_0 + \ldots + \nu_{\ell-1} z^{-(\ell-1)})$. An application of the non-stationary case may consist of the economization of procedures such as ICCG and ORTHORES for linear systems with nonsymmetric matrices.

Y.A. KUZNETSOV:

Algebraic Multigrid Domain Decomposition Methods

To solve the finite element grid systems a multilevel domain decomposition method is considered. Each step of this method consists of a set of imbedded Chebyshev iterative procedures with special preconditioners. For the two- and three-dimensional diffusion equations in rectangular domains with mixed boundary conditions and piece-wise constant coefficients, a multilevel domain decomposition method with partitioning into small subdomains is built which is a variant of the algebraic multigrid method. The convergence of this method is proved to be independent of the grid step and the coefficient jump. Numerical experiment results are given.

T.A. MANTEUFFEL:

The Role of Boundary Conditions in Preconditioning with Equivalent Operators

We say that the linear operators A and B are equivalent in norm on the set D if $\|Ax\|/\|Bx\|$ is bounded above and below for x included in D. If the set D is "large enough" then the operator B^{-1} can be used to postcondition the linear system

$$Ax = b$$

to yield the new system

$$(AB^{-1})y = b, \quad Bx = y$$

where the condition in the given norm, $\mathrm{Cond}(AB^{-1})$, is bounded. Likewise, if A^{-1} is equivalent to B^{-1} in norm on a sufficiently large set, then B^{-1} can be used as a preconditioning to the same system to yield

$$(B^{-1}A)x = (B^{-1})b,$$

where the condition in the given norm, $\mathrm{Cond}(B^{-1}A)$, is bounded. Discrete

approximants, A_h and B_h, cannot yield pre- or postconditionings bounded independent of h unless their limits are equivalent.

This talk will focus on second-order uniformly elliptic partial differential operators. We will present necessary and sufficient conditions for operators and their inverses to be equivalent in various norms. Boundary conditions will play an important and surprising role. The implications to the iterative solution of discrete approximations will be discussed. Both analytical and numerical examples will be presented.

G. MAYER:

Parallel Interval Multisplittings

We present an iterative process to enclose the solutions of systems of linear equations by an interval vector. The coefficients of these equations are allowed to vary within given intervals. The method is based on interval multisplittings appropriate to be used on a parallel computer. We investigate the feasibility, the global convergence, the quality of enclosure and the speed of convergence of this method. (Joint work with A. Frommer)

E. MAYR:

The Parallel Complexity of Gaussian Elimination

We show that Gaussian elimination with (partial or full) pivoting is complete for P (= polynomial time) under log space reduction. It is therefore considered very unlikely that Gaussian elimination can be performed by an NC-algorithm NC-al gorithms are parallel algorithms running in polylogarithmic time on apolynomial number of processors. NC is thought to characterize the class of problems efficiently solvable in parallel. Gaussian elimination is an example for a P-complete algorithm since there are other algorithms for solving linear systems of equations

which are in NC.

G. MEURANT:

Domain Decomposition Preconditioners

We present a new class of preconditioners to be used with the conjugate gradient method for solving sparse linear systems arising from the discretization of elliptic partial differential equations. The construction of these preconditioners is based on domain decomposition ideas and block preconditioning techniques. The rectangular domain is divided into strips, within each subdomain we choose a block preconditioner called INV and then we generate an approximation for the Schur complement that is the interface matrix. The main tool used for these approximations is how to construct a sparse approximate inverse for a tridiagonal matrix.

Numerical experiments show that these methods are really efficient. Moreover, for a fixed mesh size, the increase of the number of iterations as a function of the number of subdomains is very small, making these methods really efficient for parallel computers.

Finally, these preconditioners can be extended straightforwardly to nonsymmetric problems.

A. NEUMAIER:

Scaling in Gauss Elimination

An I-matrix is a matrix A such that all $|A_{ik}| \leq 1$ and all $A_{ii} = 1$.

Theorem. For every structurally nonsingular $n \times n$ matrix A there are a permutation matrix P and two diagonal matrices D_1, D_2 such that PD_1AD_2 is an I matrix.

P, D_1,D_2 can be found in $O(n^3)$ operations (and usually even with $O(n^{2.25})$) by solving a related weighted matching problem.

Numerical examples were discussed comparing Gauss elimination with partial pivoting on a row equilibrated DA and on the above PD_1AD_2. (Joint work with M. Olschowka)

M. NEUMANN:

Domains of Convergence of the Block SSOR Method for Block p–Cyclic Matrices

Suppose that the block p–cyclic Jacobi matrix J^A has spectral radius $\mu = \rho(J^A) \in (0,1)$. We apply Rouché's theorem to determine the range of ω's in the interval (0,2) for which the block SSOR method converges. (The domain thus obtained differes from the domain of convergence of the block SSOR method subject to the constraint that $\nu = \rho(|J^A|) \in (0,1)$ which has been investigated by Varga, Cai, Hadjidimos and Saridakis.) (Joint work with A. Hadjidimos)

D.D. OLESKY:

Inheritance of Matrix Entries in LU Factorizations

For an $n \times n$ matrix $A \equiv (a_{ij})$, which has a unique unit LU factorization $A = LU$ with $U \equiv (u_{ij})$, we determine combinatorial circumstances under which $u_{ij} = a_{ij}$ for a given pair $i \leq j$ or for all $i < j$. Analogous results are stated for the LU factorization of a principal submatrix of A. The relationship of our results to the well-known "fill-in" results of sparse matrix analysis are discussed. (Joint work with C.R. Johnson and P. van den Driessche)

R.J. PLEMMONS:

Parallel Algorithms in Signal Processing

We are concerned with the investigation of parallel algorithms for least squares modifications in signal processing. The first part of the talk involves a comparison of three methods for least squares downdating on the hypercube: (1) an orthogonal scheme used in LINPACK, (2) a modified hyperbolic scheme, and (3) a hybrid scheme combining the first two. Although the computational complexity of (1)-(3) decreases, the communication complexity for the parallel implementations surprisingly increases. Computational tests on an iPSC 64 hypercube confirm our analysis and indicate a slight preference for (1) on the hypercube. (Joint work with C.E. Henkel and M.T. Heath)

The second part of the talk involves parallel least squares modifications using inverse factorizations. The process of modifying least squares computations by updating the covariance matrix has been used in control and signal processing for some time in the context of linear sequential filtering. Here we give an alternative derivation of the process and provide extensions to downdating. Our purpose is to develop algorithms that are amenable to implementation on modern multiprocessor architectures. We have attempted to provide some new insights into least squares modification processes and to suggest parallel algorithms for implementing Kalman type sequential filters in the analysis and solution of estimation problems in control and signal processing. (Joint work with C.-T. Pan)

L. REICHEL:

The Ordering of Tridiagonal Matrices in the Cyclic Reduction Method for Poisson's Equation

Discretization of Poisson's equation on a rectangle by finite differences using the standard five-point stencil yields a linear system of algebraic equations, which can be solved rapidly by the cyclic reduction method. In this method a sequence of tridiagonal linear systems of equations is solved. The matrices of these systems commute, and we investigate numerical aspects of their ordering. In particular, we present new ordering schemes that avoid overflow and loss of accuracy due to underflow. These ordering schemes are easy to implement and improve the numerical performance of the subroutine HWSCRT of FISHPAK. Our orderings are also applicable to the solution of the Helmholtz equation by cyclic reduction, and to related numerical schemes, such as FACR methods.

G. RODRIGUE:

The Generalized Schwarz Alternating Principle

The classical Schwarz Alternating Procedure (SAP) for solving elliptic partial differential equations maps very nicely into a multiprocessing computer where the solution of the equation on each subdomain is assumed by an individual processor. In this talk, we extend the SAP to multi-domains and show how the numerical analog of the SAP is a block-Gauss-Seidel method on a positive definite non-symmetric problem. Inner-outer iterations are defined and results are presented on the effect of the convergence of the outer iteration by varying the number of inner iterations. Also, results are presented comparing the convergence rates of different SAP's when different interval pseudo-boundary conditions are used.

A. RUTTAN:

Parallel LU Factorizations

We describe an algorithm for producing the LU factorization of an m bounded, n×n upper Hessenberg matrix. Using P processors such a matrix can be factorized in $O\left[m^2 \frac{n}{P}\right]$ operations. In the tridiagonal code, the algorithm requires $19 \frac{n}{P} + O(1)$ operations as $n \to \infty$.

Y. SAAD:

Polynomial Preconditioners and Their Performances on Parallel and Vector Computers

This talk will present the practical aspects of polynomial preconditioning techniques for solving large sparse linear systems of equations. We will consider both the symmetric case and the more difficult nonsymmetric case. Moreover, the emphasis will be on realistic general unstructered sparse matrices such as those issued from finite element models. We will see why polynomial preconditioners can be attractive on parallel and vector machines and also explain some of their weaknesses. Numerical experiments for the Alliant FX/8 will be reported.

P. SAYLOR:

A Taxonomy for Conjugate Gradient Methods

From the Faber-Manteuffel theory, a conjugate gradient method for the solution of the pre-conditioned system $CAx = Cb$ is characterized by matrices C, A and B, where B is an Hermitian positive definite matrix defining an inner product and norm with respect to which the error is minimized. These matrices must be such that CA is B-normal (1) in order that the CG method be generated by a three term recursion. The ways in which these methods combine with certain choices of B, C, and A yield patterns of conjugate gradient methods. One such

pattern is the pattern describing the preconditioned conjugate residual method. For this particular pattern, the most general domain of applicability is described by an algebraic relation connecting the preconditioning matrix, C, and the real and imaginary parts of a complex symmetric matrix. This relation shows the necessary and sufficient conditions for the CG solution of such a system. (Joint work with T.A. Manteuffel and S.F. Ashby)

R. SCHREIBER:

Block Reflectors and How to Compute Them

A block reflector is a symmetric, orthogonal matrix that reverses a subspace of dimension greater than one. We show how, given a matrix $E \in \mathbb{R}^{m \times n}$, $m>n$, to construct a reflector $H = I - GG^T$, $G^TG = 2\,I_n$, such that

(*) $$HE = (F^T,0)^T$$

where $F \in \mathbb{R}^{n \times n}$. First we show that the conditions

(i) $F^TF = E^TE$,

(ii) F^TE_1 = symmetric (where $E = (E_1^T,E_2^T)^T$, $E_1 \in \mathbb{R}^{n \times n}$)

are necessary and sufficient for the existence of such H. Second, we show that there are, in general, 2^n choices of F satisfying (i) and (ii). Given such F, H may not be unique: we give necessary and sufficient conditions for H as well . Finally, we give stable algorithms for the construction of G such that H(G) satisfies (*), for any F satisfying (i) and (ii).

H.R. SCHWARZ:

Raleigh Quotient Minimization with Preconditioning

The smallest eigenvalues and corresponding eigenvectors of $Ax = \lambda Bx$, where A and B are symmetric, positive definite and sparse matrices of high order, can

be computed by minimizing the Rayleigh quotient by means of the method of conjugate gradients. The convergence can essentially be improved by an appropriate preconditioning of the eigenvalue problem. If the preconditioning is performed in an implicit way on the base of a partial Cholesky decomposition, the computational effort per iteration step is only slightly increased. The resulting algorithm is efficient with respect to computational effort and to storage requirements because the sparsity of the matrices is fully exploited. Moreover, the algorithm is vectorizable to a great extent.

A substantial improvement of the convergence is attained if a complete Cholesky decomposition of A is used and the technique of rank one modifications is applied for the computation of the higher eigenpairs.

A. VAN DER SLUIS:

SIRT and CG Methods for Sparse Least Squares Problems

SIRT methods (simultaneous iterative reconstruction techniques) are always convergent iterative methods for solving least squares problems. Their speed of convergence will be discussed as well as the way the iterands respond to perturbations of the right hand side and it will turn out that in the early stages there will be quite a marked regularizing effect. The same will be done for CG methods (conjugate gradients). It will appear that for large classes of problems virtually the same approximate solutions as produced by SIRT can be obtained by CG if properly applied, however with an amount of work proportional to the square root of that of SIRT.

D.C. SORENSEN:

Parallel Algorithms for Eigenvalue Problems

This paper discusses divide and conquer algorithms for the symmetric eigenvalue problem and for computing the singular value decomposition. The numerical techniques for the symmetric eigenvalue problem are reviewed and extended to the singular value decomposition. There it is shown how to divide and conquer the bidiagonal matrix by deleting a column. This induces a rank one change to the product of the bidiagonal with its transpose which can be used without explicit formation. Formulae are given for direct simultaneous computation of the left and right singular vectors and deflation rules are given which operate on the original data rather than squared quantities. It is shown how to pipeline the computation of the singular values and vectors with the initial reduction to bidiagonal form. Finally, a theorem concerning the orthogonality of computed singular vectors is given which assures orthogonal singular vectors whenever the solution procedure for finding the roots of the scalar equation of the updating problem are sufficiently accurate in the calculated differences between old and new roots.

R.S. VARGA:

On Hybrid Semiiterative Methods

In this paper, we investigate *hybrid semiiterative methods*, which consist of two independent steps:

i) transform $x = Tx + c$ (where $1 \notin \sigma(T)$) into $x = \hat{T}x + \tilde{c}$ where $\hat{T} := t_n(T)$, where $t_n(z)$ is a polynomial of degree n in z;

ii) we consider *asymptotically optimal* SIM's (semiiterative methods) with respect to $x = \hat{T}x + \tilde{c}$.

We geometrically characterize those $t_n(z)$ for which the asymptotically optimal hybrid SIM for $\hat{\Omega} := t_n(\Omega)$, gives the *same* asymptotic convergence rate as do asymptotically optimal SIM applied to the original problem $x = Tx + c$ and Ω.

We also give three examples (one arising from neutron transport theory) to show how the above theory and numerical implementation, apply. (Joint work with M. Eiermann and X. Li)

D.M. YOUNG:

Vector and Parallel Methods for the Numerical Solution of Partial Differential Equations

The paper is concerned with the numerical solution of partial differential equations based on finite difference methods or on finite element methods. Iterative methods are used to solve the large, sparse linear systems of equations which arise. The emphasis is on the use of algorithms which are suitable for use on vector and parallel processors. The focus is on methods which are based, not upon the decomposition of the physical domain, but rather on the decomposition of the eigenvalue spectrum of the coefficient matrix A or of a preconditioned matrix associated with some basic iterative method such as the Jacobi method or the symmetric SOR method. These spectral decomposition methods are related to multigrid methods as well as to additive correction methods used by Adams [1985] and to multisplitting methods of O'Leary and White [1985]. They involve first splitting the initial residual corresponding to an initial approximate solution into several components each of which is primarily associated with a part of the eigenvalue spectrum. Each such component can then be treated independently in parallel and the results combined to minimize the norm of the error.

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