MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 10/1988

Mathematische Stochastik

6.3. bis 12.3.1988

Die diesjährige Tagung über mathematische Stochastik fand unter Leitung von Professor Herbert Heyer (Tübingen) statt. Sie wurde von insgesamt 56 Teilnehmern aus der Bundesrepublik Deutschland, dem europäischen Ausland und den USA besucht. Die Thematik war bewußt breit angelegt, wobei versucht worden war, vor allem die Gebiete zu betonen, die nicht in regelmäßig stattfindenden Spezialtagungen innerhalb des Institutsprogramms Berücksichtigung finden.

Die inhaltlichen Schwerpunkte der Tagung wurden durch fünf je einstündige Übersichtsvorträge über aktuelle Forschungsprobleme gesetzt. Dabei wurden die folgenden Schwerpunkte behandelt:

- Stochastische Flüsse mit Anwendungen,
- Asymptotik semiparametrischer Modelle,
- Erzeugung von Pseudo-Zufallszahlen,
- Gauß'sche Prozesse,
- Nicht-parametrische Bayes'sche Statistik.

Im weiteren Tagesverlauf folgten dann jeweils halbstündige Einzelvorträge über neue Forschungsergebnisse. Diese umfaßten unter anderem die folgenden Themen: sequentielle statistische Verfahren, Vielteilchensysteme, Verfahren der statistischen Qualitätskontrolle, stochastische Expertensysteme und nichtparametrische Regression. Durch großzügig bemessene Pausen zwischen den Vorträgen war Gelegenheit zu vielfältigen mathematischen Diskussionen gegeben, die von den Teilnehmern ausgiebig genutzt wurde.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Vortragsauszüge:

M. AKAHIRA

Higher order asymptotics in sequential estimation

Under suitable regularity conditions the Bhattacharya type bound for the asymptotic variance of sequential estimation procedures is obtained up to the second order, and it is shown that the modified sequential maximum likelihood estimation procedure attains the bound if the stopping rule is properly determined. The third order asymptotic bounds for the distributions of regular estimators are given, and it is shown that the maximum likelihood estimation procedure combined with an appropriate stopping rule is uniformly third order asymptotically efficient in the sense that its asymptotic distribution attains the bound uniformly in stopping rules up to third order.

B.F. ARNOLD

Economically designed control procedures for a production process with quality deterioration

A model is presented which involves several serially occurring states characterized by increasing values of the process mean. Besides determining the parameters of the $\bar{x}$–chart, the optimum economic design classifies the states into acceptable and unacceptable ones, where the average long term profit per item produced is used as the objective function. A sample discrimination procedure is proposed. Furthermore, it turns out that the no-sampling alternative is in many cases at least approximately optimal.

L. ARNOLD

Survey talk: Ergodic theory of stochastic flows with applications

Let $X_0,\ldots,X_m$ be smooth vector fields on a smooth Riemannian manifold M, $\Omega :=$ $\{\omega \in C(\mathbf{R},\mathbf{R}^m) : \omega(0)=0\}$, B_1, B_2 independent Brownian motions in $\mathbf{R}^m$, $W(t) := B_1(t)$

if $t \geq 0$ and $W(t)=B_2(-t)$ if $t<0$, and P the Wiener measure on Ω, i.e. the distribution of W. Then, under certain conditions, the stochastic differential equation $dx_t = X_0(x_t)\,dt + \sum_{i\leq m} X_i(x_t) \circ dW_i(t)$ $(t\in \mathbf{R})$ defines a *cocycle* of solutions $x \rightarrow \varphi(t,\omega,x)$, i.e. for some $\Omega_0 \subset \Omega$ with $P(\Omega_0)=1$ we have: for $\omega\in\Omega_0$ $\varphi(t,\omega,\cdot)$ is a diffeomorphism of M for all $t\in \mathbf{R}$ and

$$\varphi(t+s,\omega) = \varphi(t,\vartheta_s\omega) \circ \varphi(s,\omega)$$

where $\vartheta_s\omega(t):=\omega(t+s)-\omega(s)$ is the measure preserving ergodic shift in Ω. The multiplicative ergodic theorem is utilized to discuss invariant manifolds on M, hyperbolicity (chaos), and stochastic bifurcation. Several explicit examples are given and their Lyapunov exponents and corresponding Oseledec spaces are calculated.

D. BIERLEIN

On the extremality of measure extensions

Let $\mathbf{F}$ be the set of all extensions P_1 of a probability measure P defined on a σ-algebra $\mathbf{A}$ to a larger σ-algebra $\mathbf{A}_1$ and ex $\mathbf{F}$ be the collection of extremal elements. In the case of $\mathbf{A}_1 = \mathbf{A} \vee Z$ for a countable partition Z of M, ex $\mathbf{F}$ is presented explicitly. For an arbitrary σ-algebra $\mathbf{A}_1$, the extremality of $P_1 \in \mathbf{F}$ is characterized by the maximality of the ideal of P_1-null sets. If $\mathbf{A}_1$ is generated by a finite partition Z, $\mathbf{F}$ is the σ-convex hull of ex $\mathbf{F}$. It is shown by an example that the analogue of this result for a countably infinite partition does not hold. This research is joint work with W. Stich.

P. BOXLER

A stochastic version of the center manifold theorem

Consider a nonlinear dynamical system (i.e. an ordinary differential equation) perturbed by white or real noise. Its solution generates a cocycle ('flow') of diffeomorphisms. The question of stability may be tackled by means of the stochastic counterpart to the real parts of the eigenvalues, the Lyapunov exponents attached to the linearization of the cocycle. If at least one Lyapunov exponent vanishes then the existence of a stochastic analogue to the deterministic center manifold can be proved. It is invariant and tangent to the Oseledec space corresponding to 0. It is shown that the asymptotic behavior of the stochastic center manifold is crucial for the stability of the entire system. The stochastic center manifold may also be characterized dynamically as the collection of all the points with a certain growth behavior in both directions of time.

E. v. COLLANI

A new generation of sampling plans in statistical quality control

In order to replace obsolete acceptance sampling procedures and sorting schemes by simple, adaptable and efficient 'routine testing procedures', the class of α-optimal sampling schemes is introduced. They are based on a detailed cost model and on the assumption that the probability of receiving an unacceptable lot is known. In addition, a suitable side condition prevents the discrimination of both the consumer and the producer by the test procedure.

D. DACUNHA-CASTELLE

Some new results on maximum entropy methods

Classical ME is defined as follows: Let P an a priori distribution on Ω and suppose we have to choose Q such that $\int \varphi \, dQ = c$ where $\varphi : \Omega \to \mathbf{R}^k$ and $c \in \mathbf{R}^k$. Then we take $Q^* := \arg\min K(Q,P)$ where K is the Kullbach-information. If we want to select a function on $\mathbf{R}^k$ we choose P to be the Lebesgue measure, then Q^* has density $\lambda \to \lambda\varphi(x)/z(\lambda)$ with respect to P and $\exp(\lambda\varphi(x)/z(\lambda))$ is the ME function. But this method does not take in account constraints as $a(x)<f(x)<b(x)$. A new construction is proposed which uses a discretization. The connection to the minimization problem for a concave functional is shown and the values of c for which it exists a solution are studied. This improves the Krein theorem on Markov processes. These methods include least squares and entropy. An application to the summation of Fourier series is given.

P.L. DAVIES

Robust regression

We consider the usual linear regression model $y = x^T\beta + \varepsilon$ and look for robust (in the sense of a high finite sample breakdown point) estimates for β. In particular we consider the s-estimates of Rousseeuw and Yohai and prove consistency, and, in the case of smooth 'β'-functions, asymptotic normality. Rousseeuw's least median of squares estimator is also considered and it is shown that the correct norming sequence is $n^{-1/3}$ and that the estimates converge to some functional of a stationary Gaussian process.

P. DIACONIS

Survey talk: Non-parametric Bayesian statistics

Bayesian methods are being applied to a rich variety of new problems. These are typified by Bayesian numerical analysis. In order to calculate the integral of a function $f \in C[0,1]$, one puts a prior on C[0,1], and, after observing the values of f at points x_i, one computes the posterior distribution, and the associated Bayes rule. Taking the distribution of the Brownian motion as the prior leads to the trapezoid rule, while integrated Brownian motion leads to splines. For some problems, the posterior mean characterizes the prior: Thus if π is a prior on C[0,1] that predicts like Brownian motion, π is a scale mixture of Brownian motions. Similar progress has been made in image reconstruction, geodesy, and non-parametric estimates of densities and distribution functions. There is a real need for rigorous results here: our infinite dimensional intuition is not good. A revue of these results can be found in the proceedings of the 3rd Purdue symposium on decision theory.

U. DIETER

Survey talk: Problems in the generation of pseudo-random numbers

In order to test computer generated pseudo-random numbers u_i some properties are compared with the corresponding properties of i.i.d. uniformly distributed random elements of [0,1[: they have to be equidistributed, pairs (u_i,u_{i+1}) should be uniformly distributed on $[0,1[^2$, and the same should be true in higher dimensions. This singles out some of the earlier proposals. For the linear congruential generator $z_{i+1} := a \cdot z_i + r \pmod m$, $u_i := z_i/m$, this leads to a simple condition for the factor a: the continued fraction for a/m has to have small quotients. In higher dimensions results of the theory of number theoretical lattices have to be used. For the discrepancy only bounds are known.

For generating random numbers from given distributions some general principles are known: Direct inversion (for exponential, Cauchy-, and normal variables) and comparison methods (for exponential and normal variables). The 'acceptance-rejection method' can be applied for any distribution. Its validity depends on the choice of the 'hat'-function and of the quality of the 'squeeze'-function. Only for few distributions in higher dimensions effective methods are known. The simulation of a stochastic process is still an open question.

H. DRYGAS

MINQE-theory and the estimation of the residual variance

We consider the linear model $E\, y = X\beta$, $\mathrm{Cov}\, y \in \mathbf{V} = \{ V = \sum_{i \le m} \theta_i V_i \ge 0 \}$ and want to estimate $\sum_{i \le m} f_i \theta_i = f'\theta$ by $y'Ay$ with A symmetric. Due to the lack of uniformly minimum variance unbiased estimators, C.R. Rao proposed the MINQE-principle. It is shown that it is equivalent to the determination of locally best unbiased quadratic estimators. An Aitken-type estimation formula for estimating estimable linear functions in a Gauß-Markov model is derived. Appropriate linear models in some linear functions of yy' are constructed. The application of the results to the case $n=1$ gives new interesting formulae for the estimation of the residual variance in regression analysis.

X.M. FERNIQUE

Survey talk: Gaussian processes

Two aspects of Gaussian processes were presented in this survey talk: In the first part, majorizations and minorizations of the law of $\sup_{t\in T} X_t$ were considered, where X is a Gaussian random function on a finite set T. In the second part the reproducing Hilbert space was introduced and some of its properties and applications.

N. GAFFKE

Iterative cyclic projections and duality

A well-known result of J. von Neumann (1950) is the following: Given r closed linear subspaces of a Hilbert space H and a point $x_0 \in H$, then the projection of x_0 onto the intersection of the subspaces can be obtained as the limit of the iterative cyclic projections onto the individual subspaces. Motivated by restricted least squares problems, Dykstra (1983) gave a modified procedure for $H=R^n$ and closed convex cones instead of subspaces, and Dykstra & Boyle (1987) extended this to the case of translated cones. In papers of Boyle & Dykstra (1986), Han (1988), and Gaffke & Mathar (1988) it has been shown that the result holds for any Hilbert space and any closed convex subsets. The procedure has a natural explanation via Fenchel duality. Some applications (restricted least squares, convex quadratic programming, and MINQE) are also given.

W. GAWRONSKI

On the mode and intersection properties for stable laws with index $\alpha \neq 0$

For stable distribution functions $F(x;\alpha,\gamma)$ and its density $p(x:\alpha,\gamma)$ with index $\alpha \in]0,1[\cup]1,2[$ and asymmetry parameter γ ($|\gamma| \leq \min(\alpha,2-\alpha)$) some analytic properties are proved. In particular the dependance of various quantities on γ is investigated. If $m_\alpha(\gamma)$ denotes the unique mode, then monotonicity properties for $m_\alpha(\gamma)$ and $p(m_\alpha(\gamma);\alpha,\gamma)$ with respect to γ are established and a complete asymptotic expansion for m_α is derived as $\gamma \to 0$. Furthermore the number and the location of the solutions of the equations $p(x;\alpha,\gamma_0)=p(x;\alpha,\gamma_1)$ and $F(x;\alpha,\gamma_0)=F(x;\alpha,\gamma_1)$ $(\gamma_0 \neq \gamma_1)$ are determined.

A. GREVEN

Large systems with locally interacting components

A recipe, how to relate finite and infinite systems with locally interacting components is developed in the case of nonergodic infinite systems. The voter model η_t on Z^d and η_t^N on the torus of side N in Z^d is considered. For $d \geq 3$ it is proved for a large class of initial measures with density that if $T(N) \to \infty$ and $N^{-d}T(N) \to s$ as $N \to \infty$, then $\eta^N_{T(N)}$ converges to $\int_{[0,1]} Q_s(\theta,d\theta')\, \nu_{\theta'}$ in distribution. Here the ν_θ are the extremal invariant measures of the infinite system and $Q_s(\cdot,\cdot)$ is the transition kernel of the process with generator $\frac{2}{G}(x)(1-x)(\frac{\partial}{\partial x})^2$ and absorption in 0 and 1. Some extensions and related results are proved for the branching random walk (critical) and the contact process on **Z**. An outlook on open problems, especially for Isnig models below the critical temperature are also given.

K. HELMES

Drifting Brownian motion with quadratic killing and its applications

Let $X_t = x+\delta(t)+CW_t$ $(t \geq 0)$ where $(W_t : t \geq 0)$ denotes an n-dimensional Brownian motion, $\delta: \mathbf{R}_+ \to \mathbf{R}^n$ a locally square integrable deterministic function with $\delta(0)=0$, C an nxn matrix, and $x \in \mathbf{R}^n$. We consider the process (X_t) killed at a time-varying quadratic rate. A closed form expression (in terms of the given data) of the survival probability of (X_t) is given, i.e. a formula for $E[\exp\{-z \cdot {}_0\int^t |Q(s)\, X_s|^2\, ds\}]$, where Q is a deterministic matrix valued function. The formula obtained is exploited in the case of special drift and weight functions δ and Q, and implications of these results in failure models are discussed.

A. JANSSEN

Locally asymptotically normal and mixed normal families

From the work of LeCam it is known that local asymptotic normality is a very useful tool in asymptotic statistics. Necessary and sufficient conditions for the validity of LAN for independent observations are given, which are used to prove the efficiency of certain rank tests. For dependent observations (for example certain branching processes, autoregressive processes) the LAN condition is often violated. However, the underlying experiments are often weakly convergent to linear or mixed normal limit experiments. For these classes a convergence theorem, an approximation theorem by curved exponential families, and a convolution theorem for estimators in a generalized sense of Hájek are proved.

G. KERSTING

Asymptotic properties of multidimensional diffusion

We consider solutions X_t in $\mathbf{R}^d$ of the stochastic equation

$$dX_t = b(X_t)\,dt + \sigma(X_t)\,dW_t.$$

The long term behavior of X_t is compared with that of $dx_t = b(x_t)dt$ and $dZ_t = \sigma(Z_t)\,dW_t$. Let $\lambda_{max}(x)$ and $\lambda_{min}(x)$ be the largest and smallest eigenvalue of the diffusion matrix $\sigma(x)\sigma(x)^T$. It is claimed that X_t and x_t show similar behavior under the low noise condition $\lambda_{max}(x) = o(|x|\cdot|b(x)|)$ $(|x|\to\infty)$, whereas X_t and Z_t are close under the high noise condition $|x|\cdot|b(x)| = o(\lambda_{min}(x))$. Some theorems are presented to support this claim.

T. KUSAMA

On invariant majorized experiments

Let $(X,\mathbf{A})$ be a measurable space, G a group of bimeasurable transformations, P a probability measure, P_g the image of P under g, and suppose that $\mathbf{P}:=\{P_g : g\in G\}$ is majorized by an invariant measure μ. $\mathbf{B}\subset\mathbf{A}$ is said to be *invariant* if for all $g\in G$, $B\in\mathbf{B}$ there exists $B'\in\mathbf{B}$ such that $\mu(gB \,\Delta\, B')=0$, and $(\mu,G,\mathbf{A})$ is said to be *invasive* if for all $A,B\in\mathbf{A}_o(\mu) := \{A\in\mathbf{A} : 0 < \mu(A) < \infty\}$ there exists $g\in G$ with $\mu(gA \cap B) > 0$. It is shown that if $(X,\mathbf{A},\mathbf{P})$ is weakly dominated then $(\mu,G,\mathbf{A})$ is invasive iff it is ergodic, and the weak completeness of $\mathbf{P}$ implies that $(\mu,G,\mathbf{A})$ is invasive. The smallest pairwise sufficient σ-field containing supports, $\mathbf{B}_0$, is invariant. If $(\mu,G,\mathbf{A})$ is invasive and there exists $B\in\mathbf{B}_o(\mu)$ such that $\frac{dP}{d\mu}$ is strictly positive and bounded above on B, then μ is a pivotal measure. Furthermore, if $(\mu,G,\mathbf{A})$ is invasive, $\mathbf{B}_0$ is the smallest invariant pairwise sufficient σ-field among all regular invariant pairwise sufficient σ-fields.

S. LAURITZEN

Markov random fields and expert systems

The lecture is based on joint work with D. Spiegelhalter and will appear in the Journal of the Royal Statistical Society B, **50**, 1988. Motivated by an application in electromyography methods are developed for handling uncertainty in expert systems of probabilistic nature. Local representations related to Markov structures are used to perform efficient calculations. The scheme was illustrated on a small fictitious example concerning lung diseases, and the application to electromyography was also briefly discussed.

E. MAMMEN

Nonparametric regression under qualitative smoothness conditions

We consider the nonparametric regression model $Y_i = \mu(X_i) + \varepsilon_i$ $(i = 1,\dots,n)$ where Y_i are the observations, the X_i's are the design points, the ε_i are independent with expectation 0, and μ is the unknown regression function. It is well known that μ can be estimated with order $O(n^{-m/2m+1})$ if $\int \mu^{(m)}(x)^2\,dx$ is finite. Generalizing a proposal of Holm & Frisen (1985), it is proposed to assume that the minimal number $T_m(\mu)$ of intervals covering the set, containing the design points, and such that $\mu^{(m-1)}$ is monotone on each of these intervals, is bounded, and to use the least squares estimator. In this talk it is shown that μ can be estimated with the same rate under this assumption as under the assumption above. These estimators are interesting in situations where the shape of μ is more important than the accurate pointwise estimation.

P. MANDL

Applications of functional central limit theorems in stochastic control

To establish the consistency of least squares estimates in discrete time ARMAX models usually conditions are imposed implying the so called persistent excitation. In this lecture continuous time linear systems were considered and an analogue of this property was presented together with sufficient conditions for its validity proved by control theory methods. In the self-tuning continuous time control models not only the consistency of the least squares identification, but also the validity of limit theorems for quadratic cost functionals can be demonstrated under persistent excitation.

H. MILBRODT

Sampling with varying probabilities from superpopulation models with tangent vectors

Continuing the investigations of Milbrodt (1985,1987) concerning the local asymptotic normality of experiments obtained by sampling with varying probabilities from finite sets of independent superpopulation models, experiments associated with Poisson sampling, successive sampling, rejective sampling, and its Sampford-Durbin modifications are considered. Under very general assumptions on the superpopulation models, the inclusion probabilities, and the drawing probabilities, the LAN is established. LAN-results for sampling experiments are of general importance when the interest of the statistician focusses on the superpopulation parameter. Conjointly with the Hájek-LeCam minimax theorem they give risk bounds when estimating functionals of the superpopulation parameter.

P. W. MILLAR

Stochastic goodness of fit tests

Let $x_1,\dots,x_n$ be i.i.d. $\mathbf{R}^d$-valued random vectors with common distribution G and $\{P_\theta : \theta \in \Theta\}$ be a possibly nonparametric family of probability measures. A stochastic test is proposed for the null hypothesis H_0: $G \in \{P_\theta : \theta \in \Theta\}$. The test statistic has the form $\min_{i \le j_n} \max_{j \le k_n} n^{1/2} |Q_n(A_j) - P_{\theta_i}(A_j)|$ where $\{A_j : j \le k_n\}$ is a (random) collection of half-spaces in $\mathbf{R}^d$, and Q_n is the empirical measure. Because of the infinite dimensionality of Θ, the θ_i have to be chosen with care; it is proposed to use j_n bootstrap replicas of a preliminary $n^{1/2}$-consistent estimator.

It is shown that, under suitable hypotheses, the stochastic test statistic has the same limit as the ideal, but uncomputable, test statistic $\inf_{\theta \in \Theta} \sup_A n^{1/2} |Q_n(A) - P_\theta(A)|$. It is further shown that critical values for the stochastic test statistic may be obtained by a *conditional* bootstrap method.

U. MÜLLER-FUNK

Conjugate exponential families

The validity of the credibility formula for exponential type sampling distributions and the corresponding conjugate priors is discussed. It is shown that a bias term has to be added in order to render the correct result correct for exponential classes. This result extends earlier findings by Diaconis and Ylvisaker (1979). The talk is based on a joint paper with F. v. Pukelsheim.

M. NAGASAWA

On Schrödinger processes

A Schrödinger process $(X_t, \mathbf{P})$ is a diffusion process prescribed by a transition density $p(s,x;t,y)$ ($\int p(s,x;t,y)\,dy \le 1$ is *not* required) and a pair of functions $(\phi(b,\cdot), \psi(a,\cdot))$ such that the distribution of X_t is given by

$$E[f(X_t)] = \int \psi(t,x)\, \phi(t,x)\, f(x)\, dx,$$

with $\psi(t,x) = \int \psi(a,z)\, p(a,z;t,x)\, dz$ and $\phi(t,x) = \int p(t,x;b,y)\, \phi(b,y)\, dy$.

Schrödinger processes are constructed in terms of a transformation of Markov processes (by means of a multiplicative functional) which is a generalization of Doob's space-time harmonic transformation.

G. NEUHAUS

Some linear and nonlinear rank tests under random censorship

Let $X_{11},\ldots,X_{1m},\ldots,X_{1N}$, $N=n+m$, be independent random variables with $X_{1i}\sim G_1$ for $1\leq i\leq m$ and $X_{1i}\sim G_2$ for $m+1\leq i\leq N$. These random variables are not observable, but only $X_i := X_{1i}\wedge X_{2i}$ and $\Delta_i := 1_{\{X_{1i}\leq X_{2i}\}}$ where $X_{21},\ldots X_{2N}$ are censoring random variables. For some 'generalized shift functions' $D_\rho : \mathbf{R}\rightarrow[0,\infty[$, $\rho=1,\ldots,r$ we assume that $G_1(x) = F(x - \sum_{\rho\leq r} \vartheta_\rho D_\rho(x))$ and $G_2(x) = F(x + \sum_{\rho\leq r} \vartheta_\rho D_\rho(x))$, F given, and consider the testing problem $H_0{:}\vartheta=0$ versus $H_1{:}\vartheta\geq 0$, $\neq 0$. By means of the likelihood principle we derive a class of linear and nonlinear rank tests for the above testing problem. The resulting tests are distribution free under the larger null-hypothesis $H_0^*: G_1=G_2$ and are asymptotically optimal for some fixed F,F_2 if $r=1$.

U. OPPEL

Multiple LIDAR back-scattering

A LIDAR is a LASER RADAR. Its return signal (due to multiple backscattering) generates a non-linear mapping which assigns a return signal measure to the scattering distribution of the scattering particles. In order to identify these particles, the non-Markovian process of multiple scattering is introduced and the non-linear LIDAR double backscattering transform R is derived. Properties concerning continuity and injectivity of R and its inverse are discussed. It turns out that the reconstruction problem is ill-posed. Finally, numerical results of a properly designed inversion technique are shown.

J. PFANZAGL

Survey talk: Recent advances in asymptotic statistics

After introducing some general concepts (differentiability of paths of probability measures and of functionals) the concept of minimal asymptotic variance is introduced. Certain results of Schick and Klaassen are presented which could be used to obtain asymptotically efficient sequences of estimators for semi-parametric models by using the Newton-Raphson improvement procedure. The survey concludes with certain theoretical and numerical results for semi-parametric mixture models.

G. PFLUG

Sampling derivatives of probability measures

We consider the problem of minimizing $G(x) = \int g(y)\, \mu_x(dy)$ where $\{\mu_x : x \in S\}$ is a family of probability measures depending on a real parameter x. It is supposed, that G cannot be calculated and Monte Carlo simulations must be used. It is the aim to find a *stochastic gradient*, i.e. random variables Z_x such that $E(Z_x) = \frac{\partial}{\partial x} \int g(y)\, \mu_x(dy)$. The simplest possibility is to use *process derivatives*, i.e. Z_x is defined as $g'(F_x^{-1}(U)) \cdot \frac{\partial}{\partial x} F_x^{-1}(U)$ where U is uniformly distributed on [0,1] and F_x^{-1} is the inverse distribution function of μ_x. A better way is to use the weak derivative $\mu_x' = c_x(\nu_x - \eta_x)$ where ν_x and η_x are probability measures and $c_x \geq 0$. The case where μ_x is the stationary law of a discrete Markov, semi-Markov or diffusion process is of particular interest. A method for defining stochastic gradients when only the transitions and not the stationary laws are known is presented.

H. ROST

Diffusion-reaction in the régime of moderate interaction

We study the model of n independent Brownian motions in $\mathbf{R}^d$ with binary reaction (i.e. killing of one member of a pair), which is supposed to happen at a rate $r(i,t) = \frac{1}{n}\sum_{j\neq i} q_n(X_j(t)-X_i(t))$. Here q is a given positive function, $q_n(x)=a_n{}^d q(a_n x)$ for some sequence (a_n) tending to infinity, and $c := \int q(x)\,dx$. We ask for conditions on (a_n) which ensure that the limiting behavior of the empirical measure of particles alive at time t is governed by $\frac{\partial}{\partial t}u = \frac{1}{2}\Delta u - cu^2$. If $q\in H^{-1}$, in the case d=1 no condition is needed, and it has to be required that $\log a_n=o(n)$ in the case d=2 and $a_n=o(n^{1/d-2})$ in the case $d \geq 3$. In view of recent results of Sznitman it can be seen that these conditions are sharp.

J. STEFFENS

Excessive measures and the existence of right Markov processes

Let $(U_\alpha : \alpha > 0)$ be a substochastic resolvent with proper potential kernel U on a Lusin space E such that the constant function 1 is excessive and such that the class of excessive functions generates the σ-algebra on E. Then the following conditions on the class Exc of excessive measures are necessary and sufficient for the existence of a right continuous Markov process on E with resolvent (U_α): 'unicity of charges' (i.e. $\mu U = \nu U \in$ Exc implies $\mu=\nu$), and the 'natural solidity of potentials' (i.e. if $\eta\in$ Exc is dominated by a potential μU then there exists ν such that $\eta=\nu U$).

F.W. STEUTEL

On integer-valued fractions

Let X be a nonnegative, integer-valued random variable and $\alpha\in[0,1]$. Then $\alpha\otimes X$ can be defined in distribution as $Z_1+\ldots+Z_X$, where Z_j are i.i.d. Bernoulli random variable with mean α. In terms of the generating function this means that $G_{\alpha\otimes X}(z) = G_X(1-\alpha(1-z))$ for $0 \le z \le 1$. This multiplication can be generalized to $G_{e^{-t}\otimes X}(z) = G_X(F_t(z))$ where $(F_t: t > 0)$ is a semigroup of generating functions such as occur in inherited Markov branching processes; equivalently this means that $e^{-t}\otimes X = Z_1(t)+\ldots+Z_X(t)$, where the Z_j are i.i.d. branching processes. In the generalization to $\mathbf{R}^d$, α is replaced by a substochastic matrix S, and $G_{S\cdot\otimes X}(z) = G_X(e - S(e-z))$ for $z \in [0,1]^d$ (with $e:=(1,\ldots,1)'$). There are applications in branching processes, queuing theory, ARMA-models, and unimodality.

H. STÖRMER

Overlapping random sets

Let $n,r > 1$, $m_1,\ldots,m_r \in \{1,\ldots,n-1\}$, and $X_1,\ldots,X_r$ be independent random subsets of $\{1,\ldots,n\}$ with X_ρ uniformly distributed on the subsets with m_ρ elements for $1 \le \rho \le r$. For any subset K of $\{1,\ldots,r\}$ the intersection set Y_K is defined as

$Y_K := \bigcap_{\rho\in K} X_\rho \cap \bigcap_{\rho\notin K} (\{1,\ldots,n\} \setminus X_\rho)$. The distribution of the size of Y_K is derived, as well as the common distribution of all 2^r sizes $(Y_K : K \subset \{1,\ldots,r\})$. Furthermore, asymptotic results for large $n, m_1,\ldots, m_r$ are given.

H. WALK

On the expected sample size in nonlinear renewal theory

A nonlinear renewal theory has been developed by Lai & Siegmund (1977 / 1979) and Hagwood & Woodroofe (1982), where the deterministic part of the perturbation is of order n^α, $\alpha < 1/2$. The latter result on the expansion for the expected sample size t_a is generalized in so far as in the perturbation term for the renewal process (S_n) in $\mathbf{R}$ an additional deterministic summand g(n) with $g''(x)=O(x^{\tau-2})$, $\tau<1$, $g'(x)\to 0$, and thus $g(x)=O(x^\tau)$ occurs. In the asymptotic expansion then an additional summand $p(a/ES_1)$ appears, where p(s) solves the equation $x=g(x+s)/ES_1$. For the proof a result of Lai-Siegmund type on the asymptotic distribution of the excess over a (with additional perturbation summand g(n)) and two results of Blackwell type are established. An application on tests of power 1 is given.

W. WERTZ

Sequential confidence bounds for probability densities

Let $(X_n : n \geq 1)$ be a sequence of i.i.d. random variables with unknown density f, K a bounded probability density, and $f_n(x) := \frac{1}{n} \sum_{k \leq 1} \frac{1}{b_k} K(x-X_k/b_k)$ a Wolverton-Wagner-Yamato estimator of f. Let f be twice differentiable, $f'' \in L^1$, $b_n = \alpha n^{-\beta}$ ($\alpha > 0$), K symmetric, $t \to t^2 K(t)$ integrable, and $v \to \int K(z)\, K(z+v)\, dz$ decreasing on $[0,\infty[$. Let either $\beta \in]\frac{2}{5},1[$ or $\beta \in]\frac{2}{9},1[$ and $f^{(4)}$ be continuous and bounded. Then for $\gamma \in]0,1[$ stopping times v_ε and v'_ε are given such that

$$1-\gamma \leq \lim_{\varepsilon \to 0} P\{ \int [f_{v_\varepsilon}(x) - f(x)]^2 dx \leq \varepsilon \} \leq \Phi[\sqrt{1+\beta}\, \Phi^{-1}(1-\gamma)]$$

and

$$1-\gamma = \lim_{\varepsilon \to 0} P\{ \int [f_{v_\varepsilon}(x) - f(x)]^2 dx \leq \varepsilon \} .$$

R. WITTMANN

On the law of the iterated logarithm when the variance is unbounded

Let $(X_n : n \geq 1)$ be a sequence of independent identically distributed random variables with $E(X_1^2) = \infty$. For any $0 \leq k \leq n$ let $M_n^{(k)}$ (resp. $S_n^{(k)}$) be the maximum of the $|X_i|$ (resp. the sum of the X_i, $1 \leq i \leq n$) where the k largest terms are omitted. Let further $G(x) := {}_0\int^x 2t\, P\{ |X_1| > t \}\, dt$ and a_n be the solution of the equation $a_n^2 = 2n\, G(a_n) \log\log n$. Finally define
$c_n := n \cdot E(\inf (a_n, \sup(-a_n, X_1)))$. Then it is shown that $\limsup a_n^{-1} M_n^{(k)} = 0$ a.s. if and only if $\lim{}^{\sup}_{\inf}\, a_n^{-1} (S_n^{(k)} - c_n) = \pm 1$ a.s. This generalization of the law of the iterated logarithm is deduced from a general stability criterion involving exponential sums.

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