MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 11/1988

Diophantische Approximationen

13.3. bis 19.3.1988

Die Tagung, die unter der Leitung von P. Bundschuh (Köln) und R. Tijdeman (Leiden) stand, führte 43 auf dem Gebiet der Diophantischen Approximationen arbeitende Wissenschaftler zusammen. Bei ihren Vorträgen standen arithmetische Fragestellungen (Irrationalität, Transzendenz, algebraische Unabhängigkeit) sowie diophantische Gleichungen im Vordergrund des Interesses. Einen weiteren Schwerpunkt bildeten Probleme aus der Gleichverteilungstheorie.

Auf die Durchführung der in der Vergangenheit üblichen "Problem Session" wurde diesmal verzichtet; stattdessen enthält der vorliegende Bericht im Anschluß an die Vortragsauszüge eine Sammlung offener Probleme, die den Tagungsleitern von verschiedenen Teilnehmern eingereicht wurden. Darunter befindet sich auch ein Problem von K. Mahler, das dieser kürzlich - wenige Tage vor seinem Tod - noch P. Erdös mitgeteilt hatte.

Vortragsauszüge

J. BECK:

Irregularities of distribution (An improvement of Roth's theorem)

In 1954 K.F. Roth proved the following result: Let $P_1, P_2, \dots, P_N$ be N points in $[0,1]^k$. Then there is a box $B(\underline{x}) = \prod_{i=1}^{k} [0, x_i]$ such that

$$|D(\underline{x})| = \Big| \sum_{P_i \in B(\underline{x})} 1 - N x_1 x_2 \cdots x_k \Big| \gg (\log N)^{\frac{k-1}{2}} .$$

The case k = 2 was improved by W.M. Schmidt. In 1972 he proved the best possible result log N.

For $k \geq 3$ we can write $(\log N)^{\frac{k-1}{2}} \cdot (\log\log N)^{c_k}$ $(c_k > 0)$ instead of $(\log N)^{\frac{k-1}{2}}$. This improvement enables us to prove e.g. the 2-dimensional analogue of van Aardenne-Ehrenfest's theorem.

P.-G. BECKER:

P-adic continued fractions

In the year 1968 Schneider introduced a special type of p-adic continued fractions with properties similar to those known in the real case. Although, as recently found by de Weger, the analogon to the theorem of Lagrange is not true for p-adic continued fractions, one can prove that for periodic p-adic continued fractions an analogon to the theorem of Legendre holds.

Theorem: Let p be an odd prime and let c be a rational number with $|c|_p = 1$ and $\sqrt{c} \in \mathbb{Q}_p \setminus \mathbb{Q}$.

i) If the p-adic continued fraction of $\sqrt{c}$ is periodic, then it is nearly purely periodic (i.e. there are

$$a_i \in \mathbb{N},\ b_j \in \{1,\dots,p-1\} \text{ with } \sqrt{c} = b_0 + \overline{\frac{p^{a_0}|}{|b_1} + \dots + \frac{p^{a_{h-1}}|}{|b_h}}).$$

ii) Furthermore, if $b_h = 2b_0$, then the p-adic continued fraction is symmetric (i.e. $b_v = b_{h-v}$ for $v=1,\dots,h-1$ and $a_v = a_{h-v-1}$ for $v=0,\dots,h-1$).

V.I. BERNIK:

Die inhomogenen metrischen Sätze über Approximationen auf Mannigfaltigkeiten

Es sei $P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$, $a_i \in \mathbb{Z}$, $H = \max_{0 \le i \le n} |a_i|$ und $L_n(w,c)$ die Menge der $x \in \mathbb{R}$, für welche die Ungleichung $|P(x)+c| < H^{-w}$ unendlich viele Lösungen hat. Es ist bekannt, daß für $c \in \mathbb{Q}$ und $w > n$ gilt: $\mu L_n(w,c) = 0$. Hier bezeichnet μA das Lebesquesche Maß von A.

Satz: Für $w > n$ und beliebige $c \in \mathbb{R}$ gilt

$$\mu L_n(w,c) = 0.$$

D. BERTRAND:

Heights and volumes in Baker's theory

Let G be a commutative algebraic group defined over a number field K, and embedded in some projective space. We shall study the behaviour of the associated height function:

a) on algebraic subgroups of G and b) on quotients of G, in suitable projective embeddings.

a) yields estimates for the linear dependence relations linking n-tuple $(P_1,\dots,P_n)$ of points in G(K), extending the results of Loxton, van der Poorten and Masser in the multiplicative or elliptic case.

b) provides a new proof of Baker type inequalities for linear forms in logarithms or abelian integrals in the rational case.

F. BEUKERS:

A p-adic proof of the Lindemann-Weierstrass theorem following Bézivin and Robba

The statement (LW) of the Lindemann-Weierstrass theorem is well-known. Consider now the following statement:

(BR) Let $L \in \mathbb{Q}[x, \frac{d}{dx}]$ be the differential operator defined by $Ly := x^2y'(x)+(x-1)y(x)$; and let $u(x) \in \mathbb{Q}[[x]]$ be a locally convergent power series with rational coefficients. If Lu is an element of $\mathbb{Q}(x)$, then so is u.

Bézivin and Robba prove that (LW) and (BR) are equivalent assertions. Moreover, they prove (BR) either directly or by applying a general result of theirs on differential operators with many large p-adic radii of convergence. The proof ultimately relies on F. Bertrandias' sharpening of the Borel-Polya-Dwork criterion

for rationality of power series.
In addition, during the conference it turned out that an even simpler proof of (LW) exists. It is based on the ideas of the above approach, but requires no p-adic analysis.

D.W. BOYD:

Multiplication modulo one by Salem numbers

We consider the mapping of the interval defined by $Tx = \beta x \pmod 1$, where $\beta > 1$. The question of interest is whether 1 is a periodic point of T. This is known to be the case if β is a Pisot number but it is unknown whether it must be true if β is a Salem number. If β is a Salem number of degree 4, then the result is true and the periods depend in a rather regular way on the coefficients of the minimal polynomial of β. If β has degree 6, we describe some computations which show that this is not the case. There are some very small β which have very large periods and pre-periods. We give a heuristic explanation of the differences betwenn $d = 4$ and $d > 4$.

B. BRINDZA:

On Thue's equation

Let $F(X,Y) = \prod_{i=1}^{n} (X-\alpha_i Y)$ be an irreducible binary form with rational integer coefficients $(n > 2)$ and m be a non-zero rational integer. Further, let H, D and d denote the height, discriminant and the degree of the splitting field of F, respectively.

Theorem: All the rational integer solutions x and y of the equation

$$F(x,y) = m$$

satisfy

$$|y| < \max\{2n\,H^{1+\frac{3}{n-2}}\,|m|^{\frac{1}{n}},\ \exp(C(\log|2D|)^3(\log|2m|)^3)\}$$

where C is an effectively computable constant depending only on n and d.

W.D. BROWNAWELL:

Criteria for measures of algebraic independence

P. Philippon and the speaker have given criteria furnishing measures of algebraic independence in dimension d. Philippon's criterion has recently been rendered effective by E.M. Jabbouri. We show that these criteria are the two extreme cases of a family of criteria in which increasing radii of zero-free regions (of polynomial ideals in $\mathbb{Z}[x_1,\ldots,x_n]$) correspond to ever shorter sequences of ideals, until finally the sequence degenerates to a single ideal.

P. BUNDSCHUH:

Linear independence of certain algebraic numbers

Let k > 0 and q > 2 be integers, and let T denote a set of $\frac{1}{2}\varphi(q)$

representatives mod q such that $T \cup (-T)$ is a complete set of residues mod q coprime with q. Then we sketch a new proof of

Theorem 1: The numbers $(\cot \pi z)^{(k-1)}|_{z=a/q}$ $(a \in T)$ are linearly independent over $\mathbb{Q}$.

The same method can be applied to show

Theorem 2: If $k \geq 2$, then the numbers $(\frac{1}{\sin \pi z})^{(k-1)}|_{z=a/q}$ $(a \in T)$ are linearly independent over $\mathbb{Q}$. In the case $k = 1$ this is true iff $\chi(2) \neq 1$ for every Dirichlet-character χ with $\chi(-1) = -1$.

Theorem 1 is due to T. Okada and K. Wang and contains earlier results of S. Chowla, H. Hasse, and H. Jager-H.W. Lenstra. We give also (best possible) linear independence measures for the numbers involved in the above theorems. Furthermore we give a Baker-Birch-Wirsing-style result concerning non-vanishing of the values of certain Dirichlet-series at positive integers.

W.W.L. CHEN:

Irregularities of distribution (Joint work with J. Beck)

Let P be a distribution of N points in the unit torus $U^{K+L} = [0,1]^{K+L}$, where $K \geq 2$ and $L \geq 1$. For every compact and convex body A in U^K, any number $\lambda \in (0,1]$, any proper orthogonal transformation τ in $\mathbb{R}^K$ and any vector $\underline{u} \in U^K$, let $A(\lambda,\tau,\underline{u}) = \{\tau(\lambda\underline{x})+\underline{u}: \underline{x} \in A\}$. For every $\underline{y} = (y_1,\ldots,y_L) \in U^L$, let $B(y) = [0,y_1)\times\ldots\times[0,y_L)$. Write

$$D[P;A(\lambda,\tau,\underline{u})\times B(\underline{y})] = \sum_{\underline{p}\in P\cap(A(\lambda,\tau,\underline{u})\times B(\underline{y}))} 1 - N\cdot\mu_K(A(\lambda,\tau,\underline{u}))\mu_L(B(\underline{y})),$$

where μ_K and μ_L denote respectively K- and L-dimensional volume. Further, let $\mathcal{T}$ denote the group of all proper orthogonal transformations τ in $\mathbb{R}^K$, and $d\tau$ the volume element of the invariant measure on $\mathcal{T}$, normalized such that $\int_{\mathcal{T}} d\tau = 1$.

<u>Theorem</u>: Let A be a given compact and convex body in U^K. For every natural number N, there exists a distribution P of N points in U^{K+L} such that

$$\int_0^1 \int_{\mathcal{T}} \int_{U^K} \int_{U^L} |D[P;A(\lambda,\tau,\underline{u})\times B(\underline{y})]|^2 d\underline{y}\, d\underline{u}\, d\tau\, d\lambda \ll_{A,L} N^{1-1/K}$$

P. COHEN:

<u>Some transcendence problems for automorphic functions</u>

One knows that if A is a projective group (or abelian) variety defined over the field $\bar{\mathbb{Q}}$ and one imposes on the exponential map $\exp: \mathbb{C}^n \to A(\mathbb{C})$, $n = \dim_{\mathbb{C}} A$, the normalisation condition that $\exp(\underline{0})$ and $d\exp(\underline{0})$ be algebraic, that $\exp(P)$ is transcendental for P algebraic, $P \neq \underline{0}$.

We propose the following conjecture whose solution would give an analogous result for projective varieties which are not necessarily group varieties:

<u>Conjecture</u>: Let X be a compact connected complex manifold carrying a (Hodge) metric $\| \ \|_H^2$, endowing it with the structure of a projective variety definable over the field of algebraic numbers.

Suppose that the universal covering $\tilde{X}$ of X may be realized as a domain in complex Euclidean space of dimension $\dim_{\mathbb{C}} X$, with covering group freely acting bi-holomorphic automorphisms of $\tilde{X}$, and corresponding holomorphic covering map $\varphi: \tilde{X} \to X$.

Then not all the following are algebraic:

(I) for any $u,v \in \tilde{X}$, $u \neq v$:

$u, \varphi(u), d\varphi(u), v, \varphi(v)$

(II) for any $u \in \tilde{X}$, $\underline{e} \in T(\tilde{X})$:

$u, \varphi(u), d\varphi(u), c_H \cdot (\varphi^* \| \underline{e} \|_H^2)(u)$

where $\| \underline{e} \|_E^2(u) = 1$ with $\| \ \|_E^2$ the natural Euclidean metric of the ambient space of $\tilde{X}$ and

$$c_H = 1 \quad \text{if} \quad \text{curvature } (\| \ \|_H^2) \equiv 0$$
$$= \pi \quad \text{if} \quad \text{curvature } (\| \ \|_H^2) \not\equiv 0.$$

P. DEBES:

<u>Arithmetic variation of fibers in families of curves</u>

(Joint work with M. Fried, Univ. of Florida)

Simple data given entirely by Group Theory, called a Nielsen class, determines families of covers of the affine line that are complete with respect to a simple moduli problem. Hurwitz monodromy action determines much about the geometry of the family: field of définition K, the possibility that the parameter space for the family is a rational variety. We will explain an arithmetical use of the monodromy action: a criteria for the existence of rational points on each K-fiber of the family. We will illustrate the theory with an example which originates in an exceptional case of the Hilbert-Siegel problem. Exploration of application of this kind of results to the elliptic curve rank problem will be continued in later work.

J.M. EVERTSE:

Weighted unit equations (Joint work with K. Győry)

Let $\mathbb{A}$ be the field of algebraic numbers, let K be an algebraic number field of degree d, and let S be a finite set of places on K containing all infinite places. let U_S be the group of S-units, $\bar{U}_S := \{\xi \in \mathbb{A}^* | \exists n \in \mathbb{Z} \text{ with } \xi^n \in U_S\}$ ($\bar{U}_S$ is called the group of division points of U_S).

We consider the equation:

(WU_n): $\alpha_1 x_1 + \dots + \alpha_n x_n = 1$ in $x_1, \dots, x_n \in \Gamma$, where $\Gamma = U_S$ or $\Gamma = \bar{U}_S$, and $\alpha_1, \dots, \alpha_n \in (\mathbb{A}^*)^n$.

(WU_2) has only finitely many solutions. If $\Gamma = U_S$ then (WU_2) has at most $3 \cdot 7^{d+2|S|}$ solutions. This can be generalized as follows:

Theorem 1: If $\Gamma = \bar{U}_S$ then (WU_2) has at most $3 \cdot 7^{d+2|S|} + (12\, d \log\log 3\, d)^4$ solutions.

If $n \geq 3$ then (WU_n) has infinitely many solutions in general, but the minimal number of (n-1)-dimensional subspaces of $\mathbb{A}^n$ containing all solutions of (WU_n) is finite. Denote this number by $B(n, \underline{\alpha}, \Gamma)$. Call two tuples $\underline{\alpha}, \underline{\beta} \in (\mathbb{A}^*)^n$ Γ-equivalent if $\alpha_i / \beta_i \in \Gamma$ for $i = 1, \dots, n$ (notation $\underline{\alpha} \overset{\Gamma}{\sim} \underline{\beta}$). If $\underline{\alpha} \overset{\Gamma}{\sim} \underline{\beta}$ then $B(n, \underline{\alpha}, \Gamma) = B(n, \underline{\beta}, \Gamma)$, hence we may write $B(n,\mathfrak{a},\Gamma)$ for any Γ-equivalence class $\mathfrak{a}$.

Then we have

<u>Theorem 2</u>: For all but finitely many Γ-equivalence classes $\mathfrak{a}$

$$B(n,\mathfrak{a},\Gamma) \leq (n!)^{2n+2}.$$

K.GYŐRY:

<u>Unit equations with rational coefficients</u> (Joint work with B. Brindza)

Let K be a normal extension of $\mathbb{Q}$ of finite degree n, U_K the unit group of K, and $a,b,c \in \mathbb{Z} \setminus \{0\}$. We deal with the unit equation

$$(1) \qquad ax + by = c \quad \text{in} \quad x,y \in U_K.$$

One may assume without loss of generality that a, b, c are coprime positive integers with $c \geq \max(a,b)$. We do not distinguish between conjugate solutions of (1).

<u>Theorem 1</u>: There exists an effectively computable number $C_1(K)$ such that if $c > C_1(K)$ then with at most one exception, each solution of (1) satisfies

$$(2) \qquad \min\{\overline{|x|},\overline{|y|}\} < c^{-1}\exp(((\log 2a)(\log 2b)(\log c))^{2/3}).$$

Further, if $ab > 1$ or n is odd then each solution of (1) satisfies (2).

If $ab = 1$ and n is even then (1) can have a solution for which (2) does not hold. Theorem 1 implies that if $c > C_2(K,a,b)$ then (1) has at most one solution, and if $ab > 1$ or n is odd then (1) is not solvable.

Theorem 2: Apart from finitely many triples $(a,b,c) \in \mathbb{N}^3$ with coprime a, b, c, (1) is solvable if and only if either

a) $c = a+b$ when $x = y = 1$ is the only solution of (1) or

b) $c = \epsilon + \epsilon'$ for some real quadratic units ϵ, ϵ' which are conjugates to each other. In this case $x = \epsilon$, $y = \epsilon'$ is the only solution of (1).

M. LAURENT:

Some new results on the non vanishing of p-adic regulators

Using tools from transcendental number theory we give lower bounds for the p-adic rank of certain matrices whose entries are logarithms of algebraic numbers, in terms of the Galois action on these numbers.

J. LOXTON:

Zeros of Newman polynomials

A Newman polynomial is a polynomial

$$P_n(z) = \sum_{m=0}^{n} c_{nm} z^m \text{ with } c_{nm} = 0 \text{ or } 1.$$

These polynomials are named after some conjectures of D.J. Newman on the order of magnitude of

$$\min_{P_n} \max_{|z|=1} |P_n(z)| \quad \text{and} \quad \max_{P_n} \min_{|z|=1} |P_n(z)|.$$

Clearly the zeros of such $P_n(z)$ lie in the annulus $\frac{1}{2} < |z| < 2$. Computer experiments suggest that these zeros fill out a rather interesting shape S. The extreme zeros include the omnipresent number $\frac{1}{2}(\sqrt{5}+1)$.
In fact $\overline{S \cap \mathbb{R}} = [-\frac{1}{2}(\sqrt{5}+1), -\frac{1}{2}(\sqrt{5}-1)]$ and the convex hull of S is tangent to the positive real axis at 1.

H. LUCKHARDT:

Herbrand-Analysen zweier Beweise des Satzes von Roth: polynomiale Anzahlschranken

A previously unexplored method, combining logical and mathematical elements, is shown to yield substantial numerical improvements in the area of diophantine approximations. Kreisel illustrated the method abstractly by noting that effective bounds on the number of elements are ensured if Herbrand terms from ineffective proofs of Σ_2-finiteness theorems satisfy certain simple growth conditions. Here several efficient growth conditions for the same purpose are presented that are actually satisfied in practice, in particular, by the proofs of Roth's theorem due to Roth himself and to Esnault/ Viehweg. The analysis of the former yields an exponential bound of order $\exp(70\varepsilon^{-2}d^2)$ in place of $\exp(285\varepsilon^{-2}d^2)$ given by Davenport/ Roth in 1955, where α is (real) algebraic of degree $d \geq 2$ and $|\alpha - pq^{-1}| < q^{-2-\varepsilon}$. (Thus the new bound is less than the fourth root of the old one.) - The new bounds extracted from the other proof are polynomial of low degree (in $\varepsilon^{-1}, \log d$).

Corollaries: Apart from a new bound for the number of solutions of the corresponding diophantine equations and inequalities (among them Thue's inequality), $\log\log q_\nu < C_{\alpha,\varepsilon}\nu^{5/6+\varepsilon}$, where q_ν are the denominators of the convergents to the continued fraction of α.

M. MENDES FRANCE:

On Euler like products (Joint work with A. van der Poorten)

Consider the dynamical system $\mathbb{R}^2 \to \mathbb{R}^2$, $(x_0, y_0) \neq (0,0)$,

$$x_{n+1} := \varphi(x_n, y_n)$$
$$y_{n+1} := \Psi(x_n, y_n)$$

where we assume

(i) $y_{n+1}/x_{n+1} = F(y_n/x_n)$,

(ii) $y_{n+1}/y_n = G(y_n/x_n)$,

(iii) $x_n \to x_\infty$,

(iv) there exists a continuous V with $V(\varphi(x,y), \Psi(x,y)) = V(x,y)$.

Then y_n converges to y_∞

$$y_\infty := y_0 \prod_{n=0}^{\infty} G(F^n(y_0/x_0))$$

where F^n is the n^{th} iterate of F. The value of y_∞ is determined by $V(x_\infty, y_\infty) = V(x_0, y_0)$.

Several examples are discussed:

1. $\varphi(x,y) = \frac{x^2}{x+y}$, $\Psi(x,y) = \frac{y^2}{x+y}$, $V(x,y) = y-x$.

Then one obtains the well known Euler product

$$\prod_{n=0}^{\infty} (1+\xi^{2^n}) = (1-\xi)^{-1} \qquad |\xi| < 1$$

2. $\varphi(x,y) = \frac{(x+y)(2y^2-x^2)}{xy}$, $\Psi(x,y) = y \cdot \frac{x+y}{x}$, $V(x,y) = y^2 \frac{x^2-4y^2}{(x-y)^2}$.

Then

$$\prod_{n=0}^{\infty} \left(1+\frac{1}{F^n(\xi)}\right) = \frac{\sqrt{\xi^2-4}}{|\xi-1|}, \quad |\xi| \geq 2$$

where $F(\xi) = 2-\xi^2$.

The value of the last product was discovered by R. Ostrowski.

M. MIGNOTTE:

Arithmetical applications of a theorem of Erdös-Turan

A theorem of Erdös-Turan says that the arguments of the roots of a complex polynomial are well-distributed when the coefficients of the polynomial are "not to big".

We apply this result to integer polynomials, but after a preparation via some Siegel's lemma. Our result contains a theorem of Michel Langevin.

J. MUELLER:

On the number of good rational approximations to algebraic numbers

Recently, Bombieri and van der Poorten obtained a result (H. Luckhart obtained a similar result in 1984 but unpublished) on the number of exceptional approximations in Roth's theorem: Let $0 < \delta < \delta_0$ where δ_0 is a fixed number less than 1. Let α be an algebraic number of degree n and of absolute height $H(\alpha)$ which is defined as the largest coefficient in absolute value of the defining polynomial of α over $\mathbb{Q}$. Denote by $N(\frac{x}{y})$ the number of rational approximations to α with

$$|\alpha - \frac{x}{y}| < \frac{1}{64 \max(|x|,|y|)^{2+\delta}},$$

then

$$N(\frac{x}{y}) < \frac{\log\log 4H(\alpha)}{\log(1+\delta)} + 3000\,\frac{(\log n)^2}{\delta^5}\,\log(\frac{50\log n}{\delta^2}).$$

Schmidt and I observed that the first term in the above inequality is best possible.

Theorem: (Mueller & Schmidt) There are infinitely many algebraic numbers α of degree n such that

$$N(\frac{x}{y}) > \frac{\log\log H}{\log(1+\delta)} + \frac{\log(n\delta(4+2\delta)^{-1})}{\log(1+\delta)}.$$

H. NIEDERREITER:

New low-discrepancy sequences

We generalize and improve earlier constructions of low-discrepancy sequences given by Sobol', Faure, and the speaker. For any $s \geq 2$ this yields sequences in the s-dimensional unit cube with the smallest discrepancy that is currently known. The construction is based on the theory of nets and (t,s)-sequences developed by the speaker and on algebraic techniques using formal Laurent series.

K. NISHIOKA:

Algebraic independence of gap series

We shall discuss the necessary and sufficient conditions of algebraic independence of certain power series using Evertse theorem.

Example 1. Let $f(z) = \sum_{k=1}^{\infty} z^{k!}$ and $\alpha_1,\ldots,\alpha_n$ be algebraic numbers with $0 < |\alpha_i| < 1$. If no α_i/α_j $(i \neq j)$ is a root of unity, then $f(\alpha_1),\ldots,f(\alpha_n)$ are algebraically independent.

Example 2. Let $f(z) = \sum_{k=1}^{\infty} z^{k!+k}$ and $\alpha_1,\ldots,\alpha_n$ be algebraic numbers with $0 < |\alpha_i| < 1$. If $\alpha_1,\ldots,\alpha_n$ are all distinct, then $f(\alpha_1),\ldots,f(\alpha_n)$ are algebraically independent.

Example 3. Let $f(z) = \sum_{k=1}^{\infty} [ke]z^k$ and $\alpha_1,\ldots,\alpha_n$ be algebraic numbers with $0 < |\alpha_i| < 1$. If $\alpha_1,\ldots,\alpha_n$ are all distinct, then $f(\alpha_1),\ldots,f(\alpha_n)$ are algebraically independent.

P. PHILIPPON:

Zero estimates in finite characteristic

We show that the classical zero estimates on commutative algebraic groups extend when the base field is not required to be of characteristic zero provided that the notion of order of vanishing is appropriately modified.

G. RHIN:

Transfinite diameter and irrationality measures

We use some results on the "integral transfinite diameter" of a real interval to get a new method that provides effective irrationality measures for logarithms of rational numbers. We prove that if a, b are positive integers such that $(\sqrt{a}-\sqrt{b})^2 \leq 0,6$ then it is possible to give an effective irrationality measure of $\log(\frac{a}{b})$. The previous bound was $\frac{1}{e} = 0,367\ldots$. We give some information on the limit of the irrationality measure given by this method for log 2.

A. SCHINZEL:

A decomposition of integer vectors

A proof is outlined of the following theorem.

For all positive integers $k > 1$ and $\ell \leq k$ and for every integer vector $\bar{n} = [n_1,\ldots,n_k] \neq 0$ with the height $h(\bar{n}) = \max_{1\leq i\leq k} |n_i|$ there exists a decomposition

$$\bar{n} = \sum_{i=1}^{\ell} u_i \bar{p}_i$$

where $u_i \in \mathbb{Q}$, $\bar{p}_i \in \mathbb{Z}^k$ and linearly independent vectors $\bar{p}_i$ satisfy the inequality

$$\prod_{i=1}^{\ell} h(\bar{p}_i) \leq h(\bar{n})^{\frac{k-\ell}{k-1}} \ell!.$$

H.P. SCHLICKEWEI:

The subspace theorem

Generalizing W. Schmidt's recent quantitative version of the subspace theorem we prove.

Theorem 1 : Let K be a number field of degree d. Let $s = \{0, p_1, \ldots, p_s\}$, where the p_i are rational primes. Write $|\;|_j$ for the p_j-adic valuation ($|\;|_0$ denoting the standard absolute value). Suppose that for each j ($0 \leq j \leq s$) we are given linearly independent linear forms $L_1^{(j)}, \ldots, L_n^{(j)}$ in n variables with coefficients in K. Consider the inequality

$$(1) \qquad \prod_{j=0}^{s} |L_1^{(j)}(\underline{x}) \ldots L_n^{(j)}(\underline{x})|_j \leq \left(\prod_{j=0}^{s} |\det(L_1^{(j)}, \ldots, L_n^{(j)})|_j \right) |\underline{x}|^{-\delta}$$

where $0 < \delta < 1$ is given, where $\det(L_1^{(j)}, \ldots, L_n^{(j)})$ denotes the determinant of the coefficient matrix of the $L_i^{(j)}$ and where $|\underline{x}| = (x_1^2 + \ldots + x_n^2)^{1/2}$.

There exist proper subspaces $T_1, \ldots, T_t$ of $\mathbb{Q}^n$ with

$$t \leq (8(s+1)d!)^{2^{26n}(s+1)^6 \delta^{-2}}$$

such that each rational integral solution $\underline{x}$ of (1) either lies in one of $T_1,\dots,T_t$ or has norm

$$|\underline{x}| < \max\{(n!)^{8/\delta}, H(L_1^{(0)}),\dots,H(L_n^{(s)})\}$$

where $H(L_i^{(j)})$ is a suitable defined height.
As an application we obtain an explicit upper bound for the number of solutions of S-unit equations.

We can give a version of the above result where the variables x_i lie in K instead of $\mathbb{Q}$.

J. SCHOISSENGEIER:

On the number of lattice-points in a right-angled triangle

Die Anzahl der Gitterpunkte in einem rechtwinkligen Dreieck der Form $\{(x,y) \mid 0 \le x \le N, 0 \le y \le \alpha x\}$ hängt wesentlich von der Summe $\sum_{n=1}^{N} B_1(n\alpha)$ ab, wobei $B_1(x) = \{x\} - \frac{1}{2}$. Es genügt, den Fall $\alpha \in (0,1) \setminus \mathbb{Q}$ zu betrachten. Ist $\alpha = [0;a_1,a_2,\dots]$ die Kettenbruchentwicklung von α mit den Näherungsbrüchen $\frac{p_n}{q_n}$,

$$s_{ij} = q_{\min(i,j)}(q_{\max(i,j)} - p_{\max(i,j)}) \text{ für } i,j \ge 0 \quad \text{und}$$

$$\kappa_i = \frac{1}{2}(1-(-1)^{a_{i+1}}) \prod_{\substack{j=0 \\ j\equiv i(2)}}^{i-1} (-1)^{a_{j+1}} \text{ für } i \ge 0,$$

so gelten die Abschätzungen

$$\min_{1\le N<q_{m+1}} \sum_{n=1}^{N} B_1(n\alpha) = -\frac{1}{8}\Big(\sum_{2|i\le m} a_{i+1} - \sum_{\substack{2|i\le m \\ 2|j\le m}} \kappa_i\kappa_j s_{ij}\Big) + O(1),$$

$$\max_{1\leq N<q_{m+1}} \sum_{n=1}^{N} B_1(n\alpha) = \frac{1}{8}\Big(\sum_{2\nmid i\leq m} a_{i+1} + \sum_{\substack{2\nmid i\leq m\\ 2\nmid j\leq m}} \kappa_i\kappa_j s_{ij}\Big)+0(1).$$

Daraus ergeben sich eine Reihe von Folgerungen. Z.B. ist für $\alpha = \frac{1}{2}(\sqrt{t^2+4}-t)$ (t eine positive ganze Zahl) $\overline{\lim}_{N\to\infty} \frac{1}{\log N} \sum_{n=1}^{N} B_1(n\alpha)$ $= \frac{t}{16\log(t+\alpha)}$ für gerades t, und für ungerades t ergibt sich $\frac{t}{16\log(t+\alpha)} \frac{t^2+2}{t^2+3}$.

J. SILVERMAN:

Heights and integral points

Lang has conjectured:

I $\quad \hat{h}(P) \gg_K h(E)$ for non-torsion points $P \in E(K)$.

II $\quad |E(R)| \leq c_K^{1+\mathrm{rank}E(K)}$ for integral points on a minimal equation for E.

Earlier I proved that I $\Rightarrow$ II. Recently, Marc Hindry and I proved a version of I in which the $\gg$ constant depends also on the Szpiro ratio $\sigma_E = h(\Delta_E)/h(N_E)$. Since it is known that $\sigma_E \ll_K 1$ for function fields, we obtain an unconditional proof of I and II in this case.

C.L. STEWART:

On the number of solutions of the Thue equation

Let $F(x,y) = x^n+a_{n-1}x^{n-1}y+\ldots+a_0y^n \in \mathbb{Z}[x,y]$ with non-zero discriminant and with $n \geq 3$. Let $\varepsilon > 0$, let h be a positive integer and let h_1 be any divisior of h with $h_1 > h^{2/n+\varepsilon}$. We shall show that the number of solutions of the equation

$$F(x,y) = h,$$

in coprime integers x and y is at most $C_1 n^{\omega(h_1)+1}$ for $h > C_2$, where C_1 is a number which is effectively computable in terms of ε and C_2 is a number which is computable in terms of n, ε and $H := \max_i |a_i|$. Here $\omega(h_1)$ denotes the number of distinct prime factors of h_1.

R.F. TICHY:

<u>The discrepancy of digit-depending sequences</u>

For a wide class of number-theoretical functions $t(n)$ depending on the digit representation of n to a certain base q, distribution properties of the sequences $(t(n)\cdot x)_{n=0}^{\infty}$ are studied and best possible discrepancy-estimates are given. The estimates depend on the approximation type of x. First results are due to M. Mendes France, J. Coquet et al. Several refinements and generalizations are obtained.

<u>Theorem</u>: Let x be of finite approximation type η. Then the discrepancy D_N of $(t(n)x)$ satisfies the estimate

$$D_N \ll (\log N)^{-\frac{1}{2\eta}+\varepsilon} \qquad (\varepsilon > 0),$$

and this estimate is best possible if x is not of type $\eta' < \eta$.

R. TIJDEMAN:

Representations as sums of elements of finitely generated groups

Report on the following results proved jointly with Lianxiang Wang.

1. (To appear in Pacific J.Math.)

   Every sufficiently large integer has at most four representations of the form

$$N = 2^a 3^b + 2^c + 3^d \quad \text{with} \quad a,b,c,d \in \mathbb{Z}_{\geq 0}.$$

   If N is large and admits four representations, then it is of the form $2^\alpha + 3^\beta$ with $\alpha, \beta \in \mathbb{Z}$.

2. (To appear in Indag.Math.)

   Let G and H be finitely generated subgroups of $\mathbb{C} \setminus \{0\}$ and $m,n \in \mathbb{Z}^+$. Let $a_1,\dots,a_m$ and $b_1,\dots,b_n$ in $\mathbb{C}$. Put $F = G \cap H$. If α admits non-degenerate representations

$$\alpha = a_1 g_1 + \dots + a_m g_m = b_1 h_1 + \dots + b_n h_n \quad \text{with}$$

$$g_1,\dots,g_m \in G, \; h_1,\dots,h_n \in H,$$

   then there are finite subsets V of G and W of H, the cardinalities of which depend only on G, H, m and n, such that $g_i = v_i f_i$, $h_j = w_j f'_j$ with $v_i \in V$, $w_j \in W$, $f_i, f'_j \in G \cap H$ for all i and j. Moreover, the representations have a common contraction over $G \cap H$.

K. VÄÄNÄNEN:

On arithmetic properties of G-functions

Let K be an algebraic number field of finite degree over $\mathbb{Q}$. For every place v of K we denote by $| \ |_v$ the corresponding normalized absolute value. Let $g_1(z),\dots,g_n(z)$ be algebraically independent (over K(z)) KG-functions satisfying $g_i'(z) = \sum_{j=1}^{n} A_{ij}(z)g_j(z)$ $(i = 1,\dots,n)$, where $A_{ij}(z) \in K(z)$. Then we have the following

Theorem: Let $P \in K[x_1,\dots,x_n]$, $P \not\equiv 0$, be a polynomial of degree $\leq \lambda$ and height h(P). Then there exist positive constants c, Λ, depending only on the functions $g_i(z)$ and n such that for any $\Theta \in K$ of height $h(\Theta) \leq h$ $(\geq e^e)$ satisfying

$$\log h \geq (1+\max(3,\lambda))^{4n} \log\log h,$$
$$|\Theta|_v < e^{-c\lambda(\log h)^{(4n-1)/4n}(\log\log h)^{1/4n}},$$

we have $|P(g_1(\Theta),\dots,g_n(\Theta))|_v > h(P)^{-\Lambda(\log h)^{1/4}(\log\log h)^{-1/4}}$

for all h(P) large enough.

P. VOJTA:

A refinement of Schmidt's subspace theorem

Schmidt's subspace theorem asserts that a certain diophantine inequality pertaining to hyperplanes in projective space holds for all points outside a finite set of linear subspaces. As originally proved, this set depends on the number field, the set of excluded primes, and on $\varepsilon > 0$, as well as the set of hyperplanes. We show that Schmidt's inequality holds for all points outside $Z_1 \cup Z_2$, where Z_1 is a finite set of points (depending on all data, as before) and Z_2 is a finite set of linear subspaces depending on the set of hyperplanes but independent of the number field, the set of excluded primes and $\varepsilon > 0$.

G. WAGNER:

On means of distances on the surfache of a sphere

On the surface S of the unit sphere in 3-dimensional euclidean space we study two extremal problems. Let $x_1,\dots,x_N$ be a fixed set of points on S and let $x \in S$ be a variable point. $|\dots|$ denotes the euclidean distance.

1. We obtain sharp lower (upper) bounds for the maximum (minimum) on S of the α-Riesz potentials $\sum_{j=1}^{N} |x-x_j|^\alpha$ $(-2 < \alpha < \infty)$ and the logarithmic potential $\sum_{j=1}^{N} \log|x-x_j|$.
2. We obtain sharp lower (upper) bounds for the α-energies $\sum_{j<k} |x_j-x_k|^\alpha$ $(-2 < \alpha < 2)$ and $\sum_{j<k} \log|x_j-x_k|$.

To give an example, we prove the inequality

$$\prod_{j<k} |x_j-x_k| \leq \left(\frac{2}{\sqrt{e}}\right)^{\frac{N(N-1)}{2}} N^{\frac{N}{4}(1+o(1))} .$$

I. WAKABAYASHI:

Algebraic values of functions on Riemann surfaces and the unit disk

First we generalize the Schneider-Lang theorem to functions on punctured Riemann surfaces by using Jensen's fomula for Riemann surfaces. Next we improve the Schneider-Lang theorem which was generalized to functions on the unit disk by Gramain-Mignotte-Waldschmidt. This improvement is achieved by estimating more precisely the Blaschke terms in Jensen's formula.

M. WALDSCHMIDT:

Some topics in transcendental number theory

We first give irrationality results for theta functions by Gel'fond-Schneider's method (joint work with P. Bundschuh). Next we consider the algebraic independence of Liouville numbers (joint wirk with Zhu Yao Chen). We mention recent results on lower bounds for linear forms in logarithms (work of Blass, Glass, Manski, Meronk and Steiner on Baker's method, and joint work with M. Mignotte on Schneider's method for linear forms in two logarithms). Finally we state some results due to D. Ray on finitely generated dense subgroups of $\mathbb{R}^n$.

B.M.M. DE WEGER:

The sum of two S-units being a square

Let $p_1,\ldots,p_s$ be fixed primes. The set of S-units is

$$\zeta = \{\pm p_1^{x_1} \cdots p_s^{x_s} \mid x_i \in \mathbb{Z}\}.$$

We study the diophantine equation

(1) $$x + y = z^2$$

in $x,y \in \zeta$, $z \in \mathbb{Q}$. Without loss of generality we may study (1) with conditions

(2) $$\begin{cases} x \in \zeta \cap \mathbb{N}, \quad y \in \zeta \cap \mathbb{Z}, \quad z \in \mathbb{Z}, \\ x \geq y, \quad z > 0, \\ \gcd(x,y) \text{ is squarefree.} \end{cases}$$

The generalized Ramanujan-Nagell equation $x+k = z^2$ (k fixed) is a special case of (1).

Theorem A: The solutions of equation (1) with conditions (2) satisfy

$$\max(x,|y|,z) < C,$$

where C depends on $p_1,\ldots,p_s$ only, and is effectively computable. The proof is based on (Yu's) bounds for p-adic linear forms in logarithms.

Theorem B: For $s = 4$, $p_1,\ldots,p_4 = 2,\ldots,7$, equation (1) with conditions (2) has exactly 388 solutions. The one with largest $\max(x,|y|,z)$ is $x = 3^{13}\cdot 5^3$, $y = -2\cdot 7^3$, $z = 14117$.

The method of proof works in priciple for any $p_1,\ldots,p_s$. It uses computational p-adic diophantine approximation techniques, based on the L^3-algorithm for lattice case reduction.

(Ref.: B.M.M. de Weger, Algorithms for diophantine equations, PhD thesis, Univ. of Leiden, 1987, Chapter 7)

J. YU:

Transcendence theory in char p. Special zeta values

We propose to prove the transcendence of Carlitz zeta values at all positive integers. These are the values in $\mathbb{F}_q((T))$ given by

$$\zeta(m) = \sum_{\text{monic } N \in \mathbb{F}_q[T]} N^{-m} \qquad m \geq 1 \text{ integer.}$$

Let E_c be the Carlitz module, with period $\tilde{\pi}$. If m is "even", i.e., $m \equiv 0 \bmod q-1$, Carlitz showed that $\zeta(m)/\tilde{\pi}^m$ is essentially the m-th Bernoulli-Carlitz numbers in $\mathbb{F}_q(T)$. If $m \not\equiv 0 \bmod q-1$, we manage also to prove that $\zeta(m)/\tilde{\pi}^m$ is transcendental. Our starting point is the tensor products of Carlitz modules introduced by G. Anderson and a formula of G. Anderson and D. Thakur relating the zeta values with the exponential maps parametrizing Anderson's tensor products. We shall prove versions of Hermite-Lindemann and qualitative Baker's theorems for the algebraic groups in question, which imply the transcendence of $\zeta(m)$, and $\zeta(m)/\tilde{\pi}^m$ (when m "odd") respectively.

K. YU:

Linear forms in p-adic logarithms

Let $\alpha_1,\dots,\alpha_n$ $(n \geq 2)$ be non-zero algebraic numbers $K_0 := \mathbb{Q}(\alpha_1,\dots,\alpha_n)$,

p an odd prime number and set $K := K_0(\zeta_4)$. ($\zeta_h := e^{2\pi i/h}$). Let $D := [K:\mathbb{Q}]$, $m := \max\{t \in \mathbb{Z},\ t \geq 0 \mid \zeta_{2^t} \in K\}$. Let $\mathfrak{p}$ be a prime ideal of the ring of integers in K, lying above p; denote by $e_{\mathfrak{p}}$ and $f_{\mathfrak{p}}$ the ramification index and residue class degree, respectively. For $\alpha \in K^*$ denote by $\mathrm{ord}_{\mathfrak{p}}\alpha$ the order to which $\mathfrak{p}$ divides the fractional ideal (α) generated by α. Let $V_1 \leq V_2 \leq \dots \leq V_n$ be real numbers such that for $1 \leq j \leq n$

$$V_j \geq \max(h(\alpha_j), f_{\mathfrak{p}}|\log \alpha_j|(2\pi D)^{-1}, f_{\mathfrak{p}} D^{-1} \log p),$$

where $h(\alpha)$ denotes the logarithmic absolute height of α. If $\log\alpha_n$ is linearly dependent on $\pi i, \log\alpha_1, \dots, \log\alpha_{n-1}$ over $\mathbb{Q}$, we assume, in addition, that $V_n \geq (f_{\mathfrak{p}} D^{-1} \log p) \cdot \max(1, \log(f_{\mathfrak{p}} \log p))$. Let $b_1, \dots, b_n \in \mathbb{Z}$, not all zero, and $B \geq \max_j |b_j|$. Set

$$V := \begin{cases} V_{n-1}, & \text{if } \mathrm{ord}_p b_n = \min_j(\mathrm{ord}_p b_j) \\ V_n, & \text{if } \mathrm{ord}_p b_n > \min_j(\mathrm{ord}_p b_j). \end{cases}$$

<u>Theorem</u>: Suppose that $\mathrm{ord}_{\mathfrak{p}}\alpha_j = 0$ $(1 \leq j \leq n)$ and $\alpha_1^{b_1} \cdots \alpha_n^{b_n} \neq 1$. Then we have

$$\mathrm{ord}_{\mathfrak{p}}(\alpha_1^{b_1} \cdots \alpha_n^{b_n} - 1) < \frac{2844000}{2^m} 25^n \cdot (n+1)^{2n+2.25} \frac{p^{f_{\mathfrak{p}}}}{(f_{\mathfrak{p}} \log p)^n} \cdot D^{n+2} \cdot V_1 \cdots V_n \cdot$$

$$\max(\log(D^2 B), \log(2^{12} n D)) \log(2^{11} n^2 D V).$$

Remark: A similar result holds for the case $p = 2$.

Probleme

V. BERNIK:

Man zeige:

1. Die Ungleichung $|P(X)| < \Psi(H)\cdot H^{-n+1}$ hat für fast alle X unendlich viele Lösungen, wenn $P(X) \in \mathbb{Z}[X]$, $\partial P = n$, $H = H(P)$ und $\Psi(H)$ eine monoton fallende Funktion mit $\Psi(H) \to 0$ für $H \to \infty$ und $\sum_{H=1}^{\infty} \Psi(H) = \infty$ ist.
2. Die schwache Vermutung: In 1. setze man $n = 2$ und $\Psi(H) = H^{-1}(\log H)^{-\gamma}$ mit $0 < \gamma < 1$.

P. ERDÖS:

1. (Mahler's last problem)

   A few days before his death Mahler told me that he conjectures that for every $k > 4$

$$\sum_{i=0}^{\ell} \varepsilon_i k^i = x^2, \quad \varepsilon_i \in \{0,1\}, \quad \sum_{i=0}^{\ell} \varepsilon_i > 1$$

   has only a finite number of solutions. He showed that for $k \leq 4$ the number of solutions is infinite. Mahler found only one non-trivial solution for $k > 4$: $1+7+7^2+7^3 = 400$. (Remark: to avoid a "trivial" solution of this problem one has to assume $(k,x) = 1$.)
2. Is it true that

$$x^k = \sum_{i=0}^{\infty} \varepsilon_i i!, \quad \varepsilon_i \in \{0,1\}, \quad \sum_{i=0}^{\infty} \varepsilon_i < \infty$$

has only a finite number of solutions? In fact it probably can be powerful (i.e. of the form x^2y^3) only finitely often. A weaker result will be in a forthcoming paper of Brindza and myself.

A. SCHINZEL:

1. Does there exist an absolute constant c with one of the following properties?

For all integers k and ℓ, $k \geq \ell > 1$ and every vector $\bar{n} \in \mathbb{Z}^k \setminus \{0\}$ there exist linearly independent vectors $\bar{p}_1, \ldots, \bar{p}_\ell$ such that

$$\prod_{i=1}^{\ell} h(\bar{p}_i) \leq c\, h(\bar{n})^{\frac{k-\ell}{k-1}}$$

and

(a) $$\bar{n} = \sum_{i=1}^{\ell} u_i \bar{p}_i, \quad u_i \in \mathbb{Q}$$

(b) $$\bar{n} = \sum_{i=1}^{\ell} u_i \bar{p}_i, \quad u_i \in \mathbb{Z}$$

2. Estimate (or evaluate) the function $f(\ell)$ defined on positive integers by the formula

$$f(\ell) = \sup_A \inf_U \prod_{i=1}^{\ell} \left(\sum_{j=1}^{\ell} d_{ij} \right),$$

where $[d_{ij}] = U A^{-1}$, A and U run through all lower triangular non-singular integral matrices and all lower triangular unimodular integral matrices of order k, respectively.

Remark: It is known that

$$f(\ell) \geq \left(\frac{5}{2}\right)^{\left[\frac{\ell}{5}\right]}$$

and

$$f(\ell) \leq \frac{(\ell+m+1)!}{4^{\ell-m}(2m+1)!},$$

where

$$m = \left[\frac{1+\sqrt{16\ell+17}}{4}\right]$$

C.L. STEWART:

We conjecture that there exists an absolute constant c such that for any binary form F(x,y) with integer coefficients non-zero discriminant and degree at least three, there exists a number C, which depends on F, such that if h is an integer larger than C then the equation F(x,y) = h has at most c solutions in coprime integers x and y.

R. TIJDEMAN:

Is it true that if $f \in \mathbb{Z}[X,Y]$ is of degree d, then the number of (coprime) solutions $(x,y) \in \mathbb{Z}^2$ of the equation $f(x,y) = 0$ is either infinite or bounded by a number depending only on d?

B.M.M. DE WEGER:

Let $p_1,\dots,p_s$ be fixed prime numbers. For $n \in \mathbb{N}$, let $a_1,\dots,a_s \in \mathbb{N}_0$ be such that $|n!-p_1^{a_1}\cdot\ldots\cdot p_s^{a_s}|$ is minimal. Using p-adic linear forms in logarithms I could prove that

$$|n! - p_1^{a_1}\cdot\ldots\cdot p_s^{a_s}| > \exp\left(C\cdot\frac{n}{\log n}\right)$$

for some constant C. Numerical experiments suggest that

$$|n! - p_1^{a_1}\cdot\ldots\cdot p_s^{a_s}| > \exp(C'\cdot n\cdot\log n)$$

might be true for some constant C'. Is this true?

Berichterstatter: P.-G. Becker

Tagungsteilnehmer

Prof.Dr. J. Beck
Mathematical Institute of the
Hungarian Academy of Sciences
Realtanoda u. 13 - 15, Pf. 127

H-1053 Budapest

Prof.Dr. D. W. Boyd
Dept. of Mathematics
University of British Columbia
2075, Wesbrook Mall

Vancouver , B. C. V6T 1Y4
CANADA

Dr. P. G. Becker
Mathematisches Institut
der Universität Köln
Weyertal 86-90

5000 Köln 41

Prof.Dr. B. Brindza
Institute of Mathematics
Lajos Kossuth University
Pf. 12

H-4010 Debrecen

Prof.Dr. V. I. Bernik
Institute of Mathematics
Academy of Sciences BSSR
ul. Surganova 11

Minsk 220604
USSR

Prof.Dr. W.D. Brownawell
Department of Mathematics
Pennsylvania State University
215 McAllister Building

University Park , PA 16802
USA

Prof.Dr. D. Bertrand
Institut de Mathematiques Pures et
Appliquees, UER 47
Universite de Paris VI
4, Place Jussieu

F-75252 Paris Cedex 05

Prof.Dr. P. Bundschuh
Mathematisches Institut
der Universität Köln
Weyertal 86-90

5000 Köln 41

Prof.Dr. F. Beukers
Mathematisch Instituut
Rijksuniversiteit te Utrecht
P. O. Box 80.010

NL-3508 TA Utrecht

Prof.Dr. W. W. L. Chen
Dept. of Mathematics
Imperial College of Science
and Technology
Queen's Gate

GB- London , SW7 2AZ

Prof.Dr. P. S. Cohen
IHES
Institut des Hautes Etudes
Scientifiques
35, Route de Chartres

F-91440 Bures-sur-Yvette

Prof.Dr. M. Laurent
CNRS
Institut Henri Poincare
11, rue Pierre et Marie Curie

F-75231 Paris Cedex

Prof.Dr. P. Debes
Problemes Diophantiens
Institut Henri Poincare
11, rue Pierre et Marie Curie

F-75231 Paris Cedex 05

Prof.Dr. J. H. Loxton
School of Mathematics ,Physics,
Computing and Electronics
Macquarie University

North Ryde , NSW 2113
AUSTRALIA

Prof.Dr. P. Erdös
Mathematical Institute of the
Hungarian Academy of Sciences
Realtanoda u. 13 - 15, Pf. 127

H-1053 Budapest

Prof.Dr. H. Luckhardt
Fachbereich Mathematik
Mathematisches Seminar
der Universität Frankfurt
Robert-Mayer-Str. 6-10

6000 Frankfurt 1

Dr. J. H. Evertse
Filips van Bourgonoiestr. 41A
3117 SC Schiedam

NL-3117 SC Schiedam

Prof.Dr. M. Mendes France
Mathematiques et Informatiques
Universite de Bordeaux I
351, cours de la Liberation

F-33405 Talence Cedex

Prof.Dr. K. Györy
Institute of Mathematics
Lajos Kossuth University
Pf. 12

H-4010 Debrecen

Prof.Dr. M. Mignotte
Institut de Mathematiques
Universite Louis Pasteur
7, rue Rene Descartes

F-67084 Strasbourg Cedex

Prof.Dr. J. Mueller
464 Riverside Drive, Apt. 121

New York , NY 10027
USA

Prof.Dr. A. Schinzel
Institute of Mathematics of the
Polish Academy of Sciences
ul. Sniadeckich 8

00-950 Warszawa
POLAND

Prof.Dr. H. Niederreiter
Kommission für Mathematik der
österreichischen Akademie der
Wissenschaften
Dr. Ignaz-Seipel-Platz 2

A-1010 Wien

Prof.Dr. H.P. Schlickewei
Abteilungen für Mathematik
der Universität Ulm
Oberer Eselsberg

7900 Ulm

Prof.Dr. K. Nishioka
c/o Prof. Dr. P. Bundschuh
Mathematisches Institut der
Universität
Weyertal 86 - 90

5000 Köln 41

Prof.Dr. A. L. Schmidt
Matematisk Institut
Kobenhavns Universitet
Universitetsparken 5

DK-2100 Kobenhavn

Prof.Dr. P. Philippon
Les Anges

F-27170 Le Plessis Ste Opportune

Dr. J. Schoissengeier
Institut für Mathematik
Universität Wien
Strudlhofgasse 4

A-1090 Wien

Prof.Dr. G. Rhin
Mathematiques
Universite de Metz
Faculte des Sciences
Ile du Saulcy

F-57045 Metz Cedex

Prof.Dr. J. H. Silverman
Dept. of Mathematics
Boston University
111 Cummington Street

Boston , MA 02215
USA

Prof.Dr. C. L. Stewart
Dept. of Mathematics
Unviversity of Waterloo

Waterloo, Ontario N2L 3G1
CANADA

Prof.Dr. G. Wagner
Mathematisches Institut A
der Universität Stuttgart
Pfaffenwaldring 57

7000 Stuttgart 80

Dr. R. F. Tichy
Inst. f. Analysis, Technische
Mathematik u. Versicherungsmathem.
Technische Universität Wien
Wiedner Hauptstr. 8 - 10/114

A-1040 Wien

Prof.Dr. I. Wakabayashi
Department of Mathematics
Tokyo University of
Agriculture and Technology
Fuchu

Tokyo 183
JAPAN

Prof.Dr. R. Tijdeman
Department of Mathematics and
Computer Science
Rijksuniversiteit Leiden
Postbus 9512

NL-2300 RA Leiden

Prof.Dr. M. Waldschmidt
CNRS
Institut Henri Poincare
11, rue Pierre et Marie Curie

F-75231 Paris Cedex

Prof.Dr. K. Väänänen
Dept. of Mathematics
University of Oulu
Linnanmaa

SF-90570 Oulu

Prof.Dr. R. Wallisser
Mathematisches Institut
der Universität Freiburg
Albertstr. 23b

7800 Freiburg

Prof.Dr. P. Vojta
Dept. of Mathematics
University of California

Berkeley , CA 94720
USA

B. M. M. de Weger
Faculteit der Toegepaste Wiskunde
Universiteit Twente
Postbus 217

NL-7500 AE Enschede -Drienerlo

Prof.Dr. J. Wolfart
Fachbereich Mathematik
Mathematisches Seminar
der Universität Frankfurt
Robert-Mayer-Str. 6-10

6000 Frankfurt 1

Prof.Dr. J. Yu
School of Mathematics
Institute for Advanced Study

Princeton , NJ 08540
USA

Prof.Dr. Yu Kunrui
Institute of Mathematics
Academia Sinica

Beijing 100080
CHINA