MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 13/1988

Inverse Spektralprobleme

27.3 bis 2.4.1988

(Marcel Berger gewidmet)

Die Tagung fand unter Leitung von Herrn Brüning (Augsburg), Herrn Guillemin und Herrn Melrose (beide Cambridge, Mass.) statt.

Zu Beginn referierte Herr Guillemin über M. Bergers Arbeit aus dem Jahr 1968 und zeigte, welchen maßgeblichen Einfluß diese auf die weitere Entwicklung der inversen Spektraltheorie in den letzten 20 Jahren gehabt hat. Sein Vorschlag, die Tagung zu Ehren von Herrn Berger abzuhalten, fand einhellige Zustimmung. Die Bandbreite der 19 anschließend gehaltenen Vorträge zeigte den gegenwärtigen Stand der Forschung. Behandelt wurden Beispiele isospektraler, nicht isometrischer Mannigfaltigkeiten, die Topologie der isospektralen Menge für das Sturm-Liouville Rand Eigenwertproblem, Isospektralität für diskrete Schrödingeroperatoren, Streutheorie für Schrödingeroperatoren und Asymptotik von Eigenwertverteilungen, um nur einige Themen zu nennen. Darüberhinaus nutzten fast alle Teilnehmer die Gelegenheit, in intensiven Diskussionen und Einzelgesprächen ihre neuesten Ergebnisse und Projekte zu besprechen.

Bedauert wurde, daß einige der eingeladenen Mathematiker nicht teilnehmen konnten; insbesondere waren keine Mathematiker aus der UdSSR anwesend.

VORTRAGSAUSZÜGE

Y. COLIN DE VERDIÈRE:

Multiplicités des valeurs propres

On s'interesse à la muliplicitè maximale de la première valeur propre d'opérateurs de Schrödinger (avec ou sans champ magnétique) sur une varieté compacte ou de lapaciens discrets sur un graphe fini. On demande que les valeurs propres multiples satisfassent l'hypothèse de stabilité d'Arnold. Après une étude séparée des cas discrets et continus, on étudie le passage discret → continu (théorie de l'effet tunnel) et le passage continu - discret (éléments finis et triangulations). Une application est une critère nécessaire et suffisant de planarité pour un graphe en terme de la multiplicité maximale du λ_1. Ces travaux sont parus ou à paraitre en 86, 87, 88 dans Comm. Math. Helv., Ann. ENS et J. Comb. Theory (B).

T. SUNADA:

Dynamical *L*-functions

In this talk, I present some phenomenon of distribution of closed orbits in hyperbolic dynamical systems. In case of geodesic flows on hyperbolic space forms, a perturbation technique for the first eigenvalues of twisted Laplacians is available in the study. In the general case, I take up a perturbation method for the maximum eigenvalue of twisted Ruelle operators defined on symbolic dynamics associated to the flows. The perturbed eigenvalues are related to the poles of dynamical *L*-functions, so that some classical ideas in number theory can be applied to our problem. The following diagram may illustrate what I wish to discuss:

Dirichlet *L*-functions	$\longleftrightarrow$	density theorem for primes in arithmetic progression
$\updownarrow$		$\updownarrow$
Dynamical *L*-functions	$\longleftrightarrow$	density theorem for closed orbits in homology class

H. KNÖRRER

The geometry of Fermi-curves
(joint work with D. Gieseker and E. Trubowitz)

In the almost free electron approximation the forces acting on an electron in a metal or a crystal are subsumed in a potential $V(x)$ ($x \in \mathbb{R}^n$, $n = 2,3$) which is periodic with respect to a sublattice $\Gamma = \mathbb{Z} \cdot \gamma_1 \oplus \cdots \oplus \mathbb{Z} \cdot \gamma_n$ of $\mathbb{R}^n$. By Bloch's theorem the wave function of such an electron is an eigenfunction $\psi(x)$ of $-\Delta + V$ for which there are complex numbers $\xi_1, \cdots, \xi_n$ of absolute value one such that $\psi(x + \gamma_i) = \xi_i \cdot \psi(x)$. For fixed energy λ the variety

$$F_\lambda := \{(\xi_1, \cdots, \xi_n) \in T^n \mid \exists\, \psi \text{ with } (-\Delta + V)\psi = \lambda\psi,\ \psi(x + \gamma_i) = \xi_i \cdot \psi(x)\}$$

is called the Femi-curve resp.-surface to the energy λ. We define the Bloch manifold $B(V)$ as $\{(\xi, \lambda) \mid \xi \in F_\lambda\}$. It turns out that $B(V)$ is an analytic hypersurface in (ξ, λ)-space. The density of states $\rho_V(\lambda)$ is a function which measures the expectation value for finding an electron with energy λ, it can be computed as

$$\rho_V(\lambda) = \frac{1}{(2\pi i)^{n-1}} \int_{F_\lambda} \frac{\frac{\partial P}{\partial \lambda}}{\xi_1 \frac{\partial P}{\partial \xi_1}} \frac{d\xi_2}{\xi_2} \wedge \cdots \wedge \frac{d\xi_n}{\xi_n}$$

where $P(\xi, \lambda) = 0$ is a (local) equation of $B(V)$.
For a difference approximation of Δ we can show in the case $n = 2$:

Theorem 1: *There is a Zariski-open dense set $\mathcal{O}$ in the space of all potentials such that for $V \in \mathcal{O}$, V' an arbitrary potential the equality $B(V) = B(V')$ implies that V and V' coincide up to the obvious symmetries in the problem.*

Theorem 2: *For $V, V' \in \mathcal{O}$ one has: If the germs of the functions ρ_V and $\rho_{V'}$ coincide at some point λ_0 and are not identically zero there then $B(V) = B(V')$.*

The proofs use methods of complex algebraic geometry. For the first theorem the proof is based on a degree argument. The basic ingredients of the proof of Theorem 2 are an analysis of the monodromy of the family of complexified Fermi-curves over λ-space, Delignes theorem of the fixed part and the Torelli theorem.

A. URIBE

Reduction and the trace formula
(joint work with V. Guillemin)

Let M be a closed Riemannian manifold, and consider the operator $P = \sqrt{\Delta}$. Let A be a first-order, self-adjoint ΨDO commuting with P. Simultaneously diagonalize P and A, and denote the joint eigenvalues by (λ_j, μ_j). Fix $E > 0$, and consider the distribution

$$\mathcal{T}(s) = \sum_i \varphi(\lambda_j - E\mu_j)e^{is\mu_j}$$

where φ is a function on $\mathbb{R}$ with a compactly-supported Fourier transform. This distribution is the F.T. of the weighted counting function

$$N(\mu) = \sum_{\mu_j \leq \mu} \varphi(\lambda_j - E\mu_j)$$

which counts the joint eigenvalues close to the line $\lambda = \mu E$ much more heavily than the rest. We prove a microlocal trace formula for $\mathcal{T}(s)$. In case $e^{2\pi i A} = I$, the bicharacteristic flow of A is 2π-periodic and one can form the Marsden-Weinstein reduction of the bicharacteristic flow of P. $\mathcal{T}$ is then given microlocally by the periodic trajectories of the reduced flow with energy E. Moreover, $\mathcal{T}$ is periodic, and hence we obtain an asymptotic expansion of $\sum_j \varphi(\lambda_{m,j} - mE)$ as $m \nearrow \infty$. As a particular case, take $M = N \times S^1$, $P = \sqrt{\Delta_N - V\frac{\partial^2}{\partial\theta^2}}$ with $V \in C^\infty(N)$ positive, and $A = -i\frac{\partial}{\partial\theta}$. If $\lambda_j^2(\hbar)$ are the eigenvalues of the Schrödinger operator $\hbar\,\Delta + V$ on N, our theorem gives an asymptotic expansion of

$$\sum_j \varphi\left(\frac{\lambda_j(\hbar) - E}{\hbar}\right)$$

as $\hbar$ $(= 1/m)$ goes to zero, in terms of the closed trajectories of the flow of the classical Hamiltonian $H = \sqrt{|\xi|_x^2 + V(x)}$ on the energy surface $\{H = E\}$. This can be generalized to any compact Lie group, and to potentials that include magnetic fields.

G. ESKIN

The inverse backscattering problem for the Schrödinger operator
(joint work with J. Ralston)

For well-behaved potentials q the Schrödinger equation in three dimensions

$$-\Delta u + qu = k^2 u$$

has, for each $\alpha \in S^2$, a unique solution of the form

$$(*) \qquad u(x) = e^{ik\alpha\cdot x} + A(k\beta, k\alpha)\frac{e^{ik|x|}}{|x|} + o\left(\frac{1}{|x|}\right), \quad |x| \to \infty,$$

where $k \geq 0$ and $\beta = x/|x|$. The function $A(k\beta, k\alpha)$ is called the scattering amplitude for q. The scattering amplitude can be defined for a larger class of potentials than those for which $(*)$ holds. We consider the inverse problem of recovering q from the backscattering $A(-k\alpha, k\alpha)$. Let

$$\|f\|_\gamma = \sup_{\xi,\Delta}\left(|f(\xi)| + \frac{|f(\xi+\Delta) - f(\xi))}{|\Delta|^\gamma}\right)$$

be the Hölder norm, $0 < \gamma < 1$, and let $H_{\gamma,N}$ be the closure of $C_0^\infty(\mathbb{R}^3)$ in the norm $\|f\|_{\gamma,N} = \|(1+|\xi|)^N f\|_\gamma$. We show for $N > 1$ and $0 < \gamma < 1$ that for q with Fourier transform $\hat{q}$ in $H_{\gamma,N}$ the mapping

$$\hat{q} \mapsto A(-k\alpha, k\alpha) = A(-\xi, \xi)$$

is an isomorphism on a neighborhood in $H_{\gamma,N}$ of each $\hat{q}$ in an open dense subset in $H_{\gamma,N}$. In particular, for $\hat{q}$ in this open dense set $A(-\xi,\xi) \in H_{\gamma,N}$ and $A(-\xi,\xi)$ is a continuously differentiable function of $\hat{q}$.

V. ENSS

Towards completeness of N-body quantum scattering

We present a strategy to prove asymptotic completeness for N-body Schrödinger operators which simplifies the approach of Sigal and Soffer. We use a time-dependent decomposition of phase-space into cells which are absorbing for all relevant time evolutions. A modification of the standard Cook estimate permits to replace in each of the phase space regions the fully interacting time evolution by a simpler channel evolution.
The procedure works for two- and three-body systems (with long-range forces admitted) and it seems to extend to arbitrary particle number.

S. ZELDITCH

Kuznecov formula on manifolds

The classical Kuznecov formulae on quotients $\Gamma\backslash\mathbb{H}^2$ of the hyperbolic plane are exact formulae of the form

$$\int_{\alpha_1}\int_{\alpha_2} U(t,x,y)dx\,dy = \sum_{[\sigma]\in\Gamma_{\alpha_1}\backslash\Gamma/\Gamma_{\alpha_2}} I_t[\sigma]$$

where U is the wave kernel, α_i are closed geodesics (or horocycles), Γ_{α_i} is the stabilizer in Γ and $I_t[\sigma]$ is a certain double coset invariant. On general manifolds such exact formulae are replaced by singularity expansions at singular times ("sojourn times" = lengths of common orthogonals to α_1 and α_2).
The left side may be actually replaced by more general Lagrangean distributions of the form $\int_{M\times M} U(t,x,y)A(x,y)$ where A is a Lagrangean distribution.
The expansions can be used to give asymptotic formulae for summatory functions $N(T,A) = \sum_{\mu_j\le T}(A,\varphi_j\otimes\bar{\varphi}_j)$ where $\{\mu_j,\varphi_j\}$ is the spectral data of $\sqrt{\Delta}$. When $A=\delta_Y\otimes\delta_Y$ (Y a hypersurface), one derives that $\int_Y\varphi_j \ll 1$, as $j\to\infty$. When A is a FIO kernel associated to a canonical correspondence χ (finitely multiple-valued symplectic map), and χ commutes with the geodesic flow for $\sqrt{\Delta}$, one derives a Szegö formula for the distribution of eigenvalues of A. This applies e.g. to spherical mean operators and has the flavor of the Phillips-Sarnak results on eigenvalues of Hecke operators.
Finally one can modify Sunada's method for constructing isospectral manifolds to construct pairs (M_1,Y_1), (M_2,Y_2) so

$$\int_{Y_1}\int_{Y_1} U_1(t,x,y)dx\,dy = \int_{Y_2}\int_{Y_2} U_2(t,x,y)dx\,dy\,,\ \forall t\,.$$

More precisely, the construction is reduced to a problem of finding a finite group satisfying some double coset conditions (which has not yet been solved). An application would be to construct non-isometric finite area quotients with the same scattering matrix.

R. HEMPEL

Eigenvalues of the Schrödinger operator $H-\lambda W$ in a gap of $\sigma(H)$

In the quantum theory of solids, the understanding of doped semiconductors, lasers and the colour crystals is based on "impurity levels", i.e., energy levels of the Hamiltonian which owe their existence to the presence of impurities. We consider a simple mathematical model, consisting of a (periodic) Schrödinger operator

$H = -\Delta + V$ with a spectral gap (a, b), describing the pure crystal, and a short-range potential W describing the impurity. We then ask the following question (cf. P. Deift, R. Hempel: CMP 103 (1986), 461 - 490):

Given an energy $E \in (a, b)$ does there exist a (real) coupling constant $\lambda = \lambda(E)$ such that $E \in \sigma(H - \lambda W)$?

(So we ask for a coupling constant which produces a certain spectral property). Present work of S. Alama, P. Deift and the author (Habil.-thesis, München 1987) indicates that the answer will be "yes". The proof uses estimates for the asymptotic distribution of eigenvalues μ in eigenvalue problems with weight of the type $(H - E)U = \mu \tilde{W} U$, with $\tilde{W} \geq 0$.

T. KAPPELER

On isospectral potentials on a discrete lattice

Let L be a sublattice of $\mathbb{Z}^d$ of the form $L = p\mathbb{Z} \oplus \cdots \oplus p\mathbb{Z}$ with $p \geq 2$ an integer. Then consider the discrete Schrödinger equation

(1) $(\Delta_{\text{discrete}} + Q)u(x) = \lambda u(x)$ (x in $\mathbb{Z}^d$)
(2) $u(x + \ell) = e^{i\alpha\ell} u(x)$ (x in $\mathbb{Z}^d$, ℓ in L),

where $\Delta_{\text{discrete}} u(x) = \sum_{|y-x|=1} u(y)$ and Q is a L-periodic, possibly complex-valued potential. α is an arbitrary vector in $\mathbb{R}^d$ called the crystal momentum. For a given L-periodic, possibly complex-valued potential Q define $\mathrm{Iso}_\alpha^{\mathbb{C}}(Q)$ to be the set of all L-periodic potentials P such that the spectrum of $\Delta_{\text{discrete}} + Q$ and $\Delta_{\text{discrete}} + P$ with boundary conditions (2) are the same.

Theorem *For $d \geq 2$, $p \geq 2$ and $\alpha \in \mathbb{R}^d$ there exists for generic β in $\mathbb{R}^d$ a Zariski-open dense set U of potentials in $\mathbb{C}^N$ such that*

$$\mathrm{Iso}_\alpha^{\mathbb{C}}(Q) \cap \mathrm{Iso}_\beta^{\mathbb{C}}(Q) = \{Q(\pm x + a) \mid a \in \Gamma\} \quad (Q \text{ in } U)$$

where Γ is a fundamental domain of L.

D. DETURCK

Inaudible geometric invariants
(joint work with C. Gordon, H. Gluck, D. Webb)

On a specific six-dimensional nilmanifold M (compact quotient of a simply connected nilpotent Lie group) with left-invariant metric g_0, there is a deformation g_t

of g_0 which leaves the spectrum of the Laplacian Δ_t of g_t fixed (e.g. an isospectral deformation). This is proved by explicitly constructing the intertwining operator $S_t : L^2(M) \to L^2(M)$ such that $\Delta_t = S_t^{-1}\Delta_0 S_t$. To show that this deformation of metrices is nontrivial (i.e., that g_t is not isometric to g_0), the method of calibrated geometries is used to find closed geodesics and closed area-minimizing surfaces in various homology classes. Even though the length spectrum of (M, g_t) is fixed, the relative positions of the geodesics changes, and the areas of minimizing surfaces changes continuously. This phenomenon also occurs for a wide class of nilmanifolds.

J. RALSTON

Topology of isospectral manifolds
(joint work with E. Trubowitz)

Let spec (q, B) denote the set of λ (with multiplicity) for which the following Sturm-Liouville eigenvalue problem has a nontrivial solution u:

$$-\frac{d^2u}{dx^2} + q(x)u = \lambda u \text{ on } [0,1], \quad B\begin{pmatrix} u(1) \\ u'(1) \end{pmatrix} = \begin{pmatrix} u(0) \\ u'(0) \end{pmatrix}.$$

We are interested in the isospectral sets:

$$M(q_0, B_0) = \{(q, B) \in L^2_{\mathbb{R}}([0,1]) \times SL(2,\mathbb{R}) \setminus \{B_{12} = 0\} \mid \text{spec}(q_0, B_0) = \text{spec}(q, B)\}$$

and

$$M_{B_0}(q_0) = \{q \in L^2_{\mathbb{R}}([0,1]) \mid \text{spec}(q, B_0) = \text{spec}(q_0, B_0)\,.$$

It turns out that for $(q_0, B_0) \in L^2_{\mathbb{R}}([0,1]) \times SL(2,\mathbb{R}) \setminus \{B_{12} = 0\}$ $M(q_0, B_0)$ is always a noncompact analytic manifold with a topological structure independent of (q_0, B_0). To study the subsets $M_{B_0}(q_0)$ of $M(q_0, B_0)$ one needs to study the critical points of tr $B = B_{11} + B_{22}$ as a function on $M(q_0, B_0)$. It turns out that there are always a countable number of these which we index by the finite subsets, I, of the positive integers. Assuming that tr B_0 does not equal tr B at a critical point, the homotopy groups of $M_{B_0}(q_0)$ are determined by $\{I \mid \text{tr } B_0 < \text{tr } B(p_I)$, p_I a critical point$\}$. In addition, if tr $B_0 =$ tr $B(p)$ then $M_{B_0}(q_0) = \{q_0\}$. In other words for the spectra of the problems considered here, given a spectrum there is always a (unique) boundary condition such that the isospectral set reduces to a single point. This joint work with Eugen Trubowitz will appear in Ergodic Theory and Dynamical Systems.

M. ZWORSKI

Counting the poles in potential scattering

We consider scattering in $\mathbb{R}^n$, n odd, by a compactly supported bounded potential

$$\Delta + V\,, \quad V \in L^\infty_c\,.$$

The poles of the scattering matrix can be characterized as $\lambda_j \in \mathbb{C} \setminus \{0\}$ for which there exists an outgoing solution to the reduced wave equation:

$$(\Delta + V - \lambda_j^2)u = 0\,, \quad u(r\theta) = \frac{e^{i\lambda_j r}}{r^{\frac{n-1}{2}}}\left(h(\theta) + O(\frac{1}{r})\right).$$

For the counting function of the poles, $n(r) = \sharp\{\lambda_j \mid |\lambda_j| \leq r\}$, we show $n(r) \leq Cr^n + C'$.

The exponent n is sharp as shown by the example of radial potentials nonvanishing at the boundary of the support, where $n(r) = c_n a^n r^n + (1 + O(1))$, a being the diameter of the support.

T. FRIEDRICH

Riemannian manifolds with killing spinors

Let M^n be a compact Riemannian spin manifold with positive scalar curvature $R > 0$ and denote by D the Dirac operator acting on sections of the spinor bundle. If λ_1 is the first eigenvalue of D we have $\lambda_1^2 \geq \frac{1}{4}\frac{n}{n-1}R_0$, $R_0 = \min R(x)$. Thus, there arises the problem to classify all those Riemannian manifolds where the lower bound $\pm\frac{1}{2}\sqrt{\frac{nR_0}{n-1}}$ is an eigenvalue of D (i.e. this lower bound is attained). M^n must be an Einstein space and the corresponding eigenspinor ψ satisfies the stronger condition $\nabla_X\psi = \pm\frac{1}{2}\sqrt{\frac{R}{n(n-1)}}X \cdot \psi$, $X \in TM$.
In dimension $n = 4$ the only possible manifold is $M^4 = S^4$. In dimension $n = 5$ we first prove that any solution ψ of $D\psi = \pm\frac{1}{4}\sqrt{5R}\psi$ defines an Einstein-Sasaki structure on M^5. Conversely, if M^5 is a simply-connected Einstein-Sasaki space then this equation has a non-trivial solution. Next we classify all regular contact metric structures arising from a non-trivial solution of the equation $D\psi = \frac{1}{4}\sqrt{5R}\psi$. The regularity assumption implies that M^5 is a fibre bundle over an Einstein-Kähler manifold X^4 with positive scalar curvature. Therefore, we know the possible X^4 ($= S^2 \times S^2$, $P^2(\mathbb{C})$ or the del Pezzo surfaces P_k, $3 \leq k \leq 8$) as well as the topological type of the fibration: M^5 is isometric to S^5 or $S^5/\mathbb{Z}_3$, $V_{4,2}$ or $V_{4,2}/\mathbb{Z}_2$ or is the simply connected S^1-bundle with Chern class $c_1^* = c_1(P_k)$ over one of the P_k. In the last case M^5 is diffeomorphic to a connected sum $\Sigma = (S^2 \times S^3)\sharp \cdots \sharp(S^2 \times S^3)$, and there is a one-to-one correspondence between Einstein-Kähler structures on P_k and Einstein metrics with killing spinors on Σ.

H. URAKAWA

Hilbert's 18^{th} problem and eigenvalues of the Laplacian

Hilbert's 18^{th} problem consists of three parts:

(1) The finiteness of isomorphism classes of crystallographic groups (solved by L. Bieberbach, 1912),
(2) The tiling problem (solved by K. Reinhardt for $n = 3$, 1928, and H. Heesch for $n = 2$, 1935, cf. the book of Grünbaum and Shephard),
(3) The (sphere) packing problem (solved by A. Thue for the 2-dimensional disc, 1950, in 3 dimensions still open).

For a given bounded domain $\Omega \subset \mathbb{R}^n$, we call $Z = \{\Omega_i\}_{i\in\mathbb{N}}$ a packing if each Ω_i is congruent to Ω and $\Omega_i \cap \Omega_j = \emptyset$, $i \neq j$. The packing density is defined by $\rho(\Omega) = \sup_Z \rho(Z,\Omega)$, where

$$\rho(Z,\Omega) = \limsup_{S(C)\to\infty} \sum_{\Omega_i\cap C\neq\emptyset} \mathrm{vol}(\Omega_i)\mathrm{vol}(C)\,.$$

with C a n-dimensional "suitcase" with edge-length $S(C)$. It can be shown that $\rho(\Omega) \geq 2(n!)^2/(2n)!$ for every convex domain Ω, and $\rho(\Omega) \geq 3/4$ if $n = 2$.

Theorem 1: *For the Dirichlet eigenvalue problem on Ω, the k-th eigenvalue λ_k can be estimated by (B_n = unit ball)*

$$\lambda_k \geq 4\pi^2\left(k\,\rho(\Omega)\,\mathrm{vol}(\Omega)/\mathrm{vol}(B_n)\right)^{2/n}, \quad k \geq 1\,.$$

The sphere packing density ρ_n is given by $\rho_n = \mathrm{vol}(B_n)(\frac{1}{4})^{n/2}\mu_n^{n/2}$, where $\mu_n = \max_\Lambda \min_{x\in\Lambda\setminus\{0\}} |x|^2/\sqrt{\det\Lambda}$, Λ a lattice in $\mathbb{R}^n$.

Theorem 2: *Letting $\nu_n = \sup_\Lambda \lambda_1(\mathbb{R}^n/\Lambda)\mathrm{vol}(\mathbb{R}^n/\Lambda)^{2/n}$, we get $\nu_n = 4\pi^2\mu_n$ and the estimate*

$$\nu_n \leq \frac{(n+2)^{1+\frac{2}{n}}}{n}\mathrm{vol}(S^{n-1})^{\frac{2}{n}}\left(j_{\frac{n}{2}-1}\right)^{2-\frac{4}{n}}\,,$$

where $j_{\frac{n}{2}-1}$ is the first zero of the $(\frac{n}{2}-1)$-th Bessel function.

This estimate is one of the best known today, cf. the book of Conway and Shane, Springer, 1988.

G. COURTOIS

Eigenvalues below 1/4 of degenerating Riemannian surfaces
(joint work with B. Colbois)

A degenerating family $\{S_\epsilon\}_{\epsilon\in]0,1]}$ of Riemann surfaces (i.e. of constant curvature -1) is defined as follows:
i) All the S_ϵ's have the same combinatorial structure, i.e. are built from the same number of hyperbolic parts with the same combinatorial way to glue them together.
ii) A fixed subset of the closed geodesics which bound the parts of the decomposition of S_ϵ is chosen to have length going to zero when ϵ goes to zero. The other geodesics of the decomposition by parts have length staying fixed.
It is then possible to associate a limit Riemann surface S with S_ϵ which is non-compact of finite volume and eventually non-connected.
If we denote by $\lambda_1^\epsilon < \lambda_2^\epsilon \leq \cdots \leq \lambda_N^\epsilon < 1/4$ the eigenvalues of S_ϵ below 1/4 and by $\mu_1 \leq \mu_2 \leq \cdots \leq \mu_M < 1/4$ those of S below 1/4 we have

Theorem *For ϵ small enough*

i) $N \geq M$ and $\lambda_k^\epsilon \to \mu_k$, $k = 1, 2, \cdots, M$
ii) if $N > M$ then $\lambda_k^\epsilon \to 1/4$, $k = M+1, \cdots, N$.

(This work is to be published in Comm. Math. Helv. in 1988).

D. BÄTTIG

Toroidal compactification of the Bloch manifold for the two-dimensional discrete Laplace operator

Consider the spectral problem for the two-dimensional discrete Laplacian with a periodic potential. Using a toroidal embedding one shows that this situation defines spectral problems for the energy level infinity, namely by discrete one-dimenisonal Laplace operators with averaged potentials.

A. GRIGIS

On the Spectrum of Polynomial Hill's equation

Consider Hill's operator $P = -\frac{d^2}{dx^2} + V(x)$ on $L^2(\mathrm{R})$ with V a trigonometric polynomial. The band structure of the spectrum is well-known when V is real-valued. If V is a Hardy-Fegan potential (i.e. has only positive Fourier coefficients) P is isospectral to $-\frac{d^2}{dx^2}$ but can have a nilpotent part when restricted to periodic eigenspace. When V is a general complex polynomial there is a real-analytic curve in $\mathbb{C}$ on which the spectrum is located and a band structrue can be defined. We study the asymptotics of the width of the gaps and the size of the reflexion coefficients when energy E tends to ∞. We use Floquet theory and the complex WKB method. We find a class of V's for which the gaps are all open when E is large. Part of this work is published in Ann. ENS 1987 and the other was inspired to us by V. Guillemin and A. Uribe.

R. HELFFER

Semi-classical Analysis for Harper's equation
(joint work with J. Sjöstrand)

In solid state physics the Harper's (or almost Mathieu) equation appears naturally:

$$\ell^2(\mathbb{Z}) \ni (u_n) \mapsto (H_\theta u)_n = \frac{1}{2}(u_{n+1} + u_{n-1}) + \cos(2\pi(\alpha n + \theta))u_n$$

where $\alpha \in \mathbb{R}$, $\theta \in \mathbb{R}$. We study the properties of the spectrum $\bigcup_\theta \mathrm{Sp} H_\theta$. This problem appears to be equivalent to the study of the spectrum of the pseudodifferential operator on $L^2(\mathbb{R}) : \cos(2\pi\alpha D_x) + \cos x$.
When α is small, the semi-classical analysis is well adopted to the study of this problem and we get in particular that for $\alpha \notin \mathbb{Q}$,

$$\alpha = \frac{1|}{|q_0} + \frac{1|}{|q_1} + \cdots + \frac{1|}{|q_j} + \cdots \qquad \text{with } q_j \geq c_0 (c_0 \text{ big enough}),$$

the spectrum is a Cantor set. This gives a partial answer to the "Ten Martinis" problem of M. Kac.

C. GORDON

Isospectral deformations of Riemannian metrics and potentials

We first discuss (i) the construction of continuous families of isospectral closed Riemannian manifolds using representation theoretic methods and (ii) Sunada's trace formula method for constructing pairs of isospectral Riemannian manifolds with a common finite covering. We then find a common context for viewing both (i) and (ii). Next using a modification of the techniques of (i) and (ii), we construct continuous families of isospectral potentials for the Schrödinger operator on certain nilmanifolds.

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