Mathematisches Forschungsinstitut Oberwolfach

Tagungsbericht 18/1988

Kombinatorik geordneter Mengen

24.4 bis 30.4.1988

Tagungsleitung: M. Aigner (Berlin) und R. Wille (Darmstadt).
Im Mittelpunkt des Interesses stand die Theorie endlicher geordneter Mengen. Die Themen lassen sich in etwa in folgende Teilbereiche gliedern:

- Dimension und andere Parameter geordneter Mengen.
- Diagramme geordneter Mengen.
- Theorie extremaler Mengen.
- "cutsets" und "fibres".
- Strukturtheorie von Verbänden.
- Spernertheorie
- Komplexitätsuntersuchungen.
- Anwendungen.

Zusätzlich fand eine "Problemsitzung" statt, in der offene Probleme vorgestellt wurden.

Vortragsauszüge:

Thomas Andreae

Fibres in Ordered Sets and weakly 2-colorable Graphs

Call a subset of an ordered set a *fibre* if it meets every maximal antichain. Z. Lonc and I. Rival proved several instances of the conjecture that, in an ordered set P without splitting elements, there is a subset F such that both F and $P - F$ are fibres. Reformulated in terms of the incomparability graph of P, this conjecture reads as follows: any incomparability graph is *weakly 2-colorable*, i.e., its vertices can be colored red and blue such that there exists no monochromatic maximal clique of size ≥ 2. We exhibit classes of graphs other than incomparability graphs that are weakly 2-colorable, and investigate related questions concerning clique-transversal sets of graphs.

(Joint work with M. Schughart, Berlin, and Z. Tuza, Budapest)

H.-J. Bandelt

Diagrams, Orientations, and Varieties

One of the central problems in the theory of ordered sets is to describe the orientations of the covering graph of an ordered set. We show that the particular operation called "inversion", together with the classical constructions of retraction and product provide a context for the classification of all such orientations.

(Joint work with Ivan Rival)

Kenneth P. Bogart

Discrete Representation Theory for Semiorders

Semiorders may be characterized as ordered sets which do not contain as restrictions either the sum of two two-element chains or the sum of a point and a three-element chain. We give similar characterizations, using lists of forbidden restrictions, of semiorders representable with intervals of length w (and with the order relation $(a, b) < (c, d)$ if $b \leq c$). A semiorder is representable by intervals of length one if it contains no restriction which is the sum of a point and a one element chain. The family whose absense characterizes posets representable by intervals of length w may be constructed from the corresponding family for length $w-1$ by applying the following construction to each minimal element of each member of the family. Select the minimal element x. Replace it with two incomparable elements x_1 and x_2 each less than all y with $x < y$. Introduce a new element z under x_1 and all elements above x in the cannonical linear extension of the semiorder. These examples are distinct and the number of them is the catalan number $c(w)$. These results follow from the following theorem: A semiorder is representable by intervals of length w if and only if the weighted digraph whose vertices are the vertices of the semiorder and whose edges g from x to y if $x > y$ or if x and y are incomparable and whose weights on $x > y$ edges are $-w$ and on incomparability edges are $w-1$ has no cycles of negative weight. The interval representation may be found in a natural way by applying a minimum distance algorithm to this weighted digraph.

Graham Brightwell

Some results on correlations

The aim of the talk is to present some (fairly) recent results on correlation in posets. For a and b incomparable elements of a finite poset $(X,<)$, define $P(a \prec b)$ to be the propotion of linear extentions of $(X,<)$ with a below b. The relation $a \prec b$ & $c \prec d$ are *(positively) correlated* with respect to $(X,<)$ if $P(a \prec b)P(c \prec d) \leq P(a \prec b \;\&\; c \prec d)$. We are particularly interested in 'universal' results : $a \prec b$ & $c \prec d$ are positively correlated with respect to every $(X,<)$ in a given class, e.g. the subposets of a fixed poset, or the extensions of a fixed poset.

Walter Deuber

On Shelah's proof that the van der Waerden function is primitive recursive

Ein klassischer Satz von van der Waerden besagt, dass zu $k, r \in \mathrm{N}$ eine kleinste Zahl $w(k,r)$ existiert mit der Eigenschaft, dass zu jeder Zerlegung von $\{1,\ldots,w(k,r)\}$ in r Klassen in mindestens einer dieser Klassen eine arithmetische Progression mit k Termen sich befindet.

Während bisherige obere Schranken stets Ackermannqualität hatten, konnte Shelah kürzlich zeigen, dass w eine primitive rekursive Funktion ist.

Dwight Duffus

Fibres in Ordered Sets

A *fibre* in an ordered set X is a subset F of points of X such that $F \cap A \neq \emptyset$ for all maximal antichains A of X.

Motivated by graph theoretic results, Andreae and Aigner asked if every finite ordered set X with no 1-element maximal antichains must have a fibre F satisfying $|F| \leq \frac{1}{2}|X|$. Lonc and Rival asked for more: under the same hypothesis, is there some $F \subseteq X$ such that both F and $X - F$ are fibres?

This question is easily translated to one for the hypergraph $H(X) = (X, \varepsilon)$ whose edges are the maximal antichain of X: is $H(X)$ 2-colorable?

In this talk we present some observations concerning maximal 2-element antichains which support a positive answer to these questions.

Vincent Duquenne

The Core of finite lattices

Motivated by some practical questions in data analysis in Psychology (description of Experimental Designs built on sublattices of 2-permuting partitions, language for describing their statistics ...; analysis of dependencies between attributes in Formal Concept Analysis), as well as a need for generalization the celebrated Birkhoff's theorem which exhibits any finite distributive lattice L as isomorphic to the (order-)filter lattice of its set $M(L)$ of meet-irreducible elements, the following is proved: Let L be a finite lattice; $x \in L$ is said to be $\bigwedge$-*essential* if there exists an order filter $X \subset [x)$ with $\bigwedge X = x$ and $X \cup \{x\}$ a proper sublattice of $[x)$. Let call $K_\wedge(L) := M(L) \cup \{x \in L | \; x \text{ is } \bigwedge\text{−essential}\}$ the $\bigwedge$-*core of* L.

Theorem 1: the filter lattice of the the ***partial*** $\wedge$-***semi-lattice*** constructed on $P \subseteq L$ is isomorphic to L iff $P \supseteq K_\wedge(L)$.

Theorem 2: the $\bigwedge$-core is the union of the converses of the core's factors, for a subdirect product.

Theorem 3: Let L be modular; x is $\wedge$-essential iff the sublattice generated by the covers of x is a covering M_n.

The $\bigwedge$-core and dually the $\bigvee$-core of a geometric lattice are also characterized.

Paul Edelmann

Tableaux and chains in a new partial order of S_n

We define a new partial order on S_n by lettering $\sigma \leq \tau$ if τ can be gotten from σ by a sequence of adjacent transpositions moving a left-right maximum to the left. This is a subposet of the weak order of S_n. We show that this poset has the property that every interval is a distributive lattice, and can explicitly compute the poset of join-irreducibles in the principal ideals. This allows us to compute the number of maximal chains in certain of these principal ideals.

Konrad Engel

On the number of independent sets in an $m \times n$ lattice

Let $Z_{m,n} := \{(i,j) : 1 \leq i \leq m, 1 \leq j \leq n\}$ and $\kappa_{m,n}$ be the number of subsets of $Z_{m,n}$ with the property: there are no $(i_1, j_1), (i_2, j_2) \in A$ with $|i_1 - i_2| + |j_1 - j_2| = 1$. We prove several inequalities for the numbers $\kappa_{m,n}$ and show that

$$1.503 \leq \lim_{m\to\infty} \lim_{n\to\infty} \kappa_{m,n}^{1/mn} = \lim_{n\to\infty} \kappa_{n,n}^{1/n^2} \leq 1.514.$$

We conjecture that $\kappa_{m,2k}^2 \geq \kappa_{m,2k-2}\kappa_{m,2k+2}$ holds for all positive integers m and k which implies $\lim_{n\to\infty} \kappa_{n,n}^{1/n^2} = 1.50304808...$

Zoltan Füredi

Minimal Cutsets of the Boolean Lattice

$\mathcal{C} \subset \mathbf{B}_n$ is a *cutset* if it intersects all maximal chains $\mathcal{L}$ (i.e chains of the form $\emptyset \subset L_1 \subset L_2 \subset \cdots \subset L_{n-1} \subset L_n = [n]$) of $\mathbf{B}_n$. A cutset $\mathcal{C}$ is *minimal* if for every $C \in \mathcal{C}$ there is a maximal chain avoiding $\mathcal{C} \setminus \{C\}$. For example, the whole k-th level is a minimal cutset. However, there are much larger minimal cutsets, e.g., the following family

$$\{C \subset [n] : |C \cap \{1,2\}| = 1\}$$

has size 2^{n-1}. Denote the maximal size of a minimal cutset of $\mathbf{B}_n$ by $c(n)$. It is easy to see that $c(n+1) \geq 2c(n)$, so $\lim_{n\to\infty} c(n)2^{-n}$ exists. Ko-Wei Leih gave a construction $c(6) \geq 33$. Here we give an almost explicit construction proving that

$$\lim c(n)/2^n = 1.$$

(This was a joint work with J.R. Griggs and D.J. Kleitman.)

Gerhard Gierz

The bandwidth problem for partially ordered sets

Let P be a finite poset, and let $f : P \longrightarrow \{1, \ldots, |P|\}$ be a linear extension. Define

$$\begin{aligned} bw(f) &= \max\{f(y) - f(x) : x \text{ is a lower neighbor of } y\} \\ bw(P) &= \min_f bw(f) \end{aligned}$$

Conjecture: If L is a distributive lattice, then $w(L) \leq bw(L) \leq 3/2w(L)$.
Theorem. If L is a distributive lattice of breadth 3, then $bw(L) \leq w(L) + 1 + \sqrt{w(L) - 1}$.
Theorem. If L is a distributive lattice of breadth ≤ 4, then $bw(L) \leq 3/2w(L)$.
(This is joint work with F. Hergert.)

Daniel Grieser

Complexity of families of sets

We consider the following problem: Given a family $\mathcal{P}$ of subsets of some finite set T, determine the complexity $c(\mathcal{P})$, which is defined as the minimal number of tests necessary to decide if an imaginary set $H \subseteq T$ is in $\mathcal{P}$ or not, a test being a question "Is $x \in H$?" for some $x \in T$. This kind of question was first discussed by Holt, Reingolt and Rosenberg 1973 in the special case where $\mathcal{P}$ is a graph property. While most $\mathcal{P}$ have the maximal complexity $t = |T|$, there are $\mathcal{P}$'s with low complexity. A well known theorem states that $c(\mathcal{P}) \leq t - k$ implies $\mathcal{P}$ to be the disjoint union of intervals of length $\geq k$. We prove that a weak inversion of this is true: If $\mathcal{P}$ is the disjoint union of $r > 1$ intervals of length $\geq k$, then $c(\mathcal{P}) \leq 2k \ln r$. In the course of the proof we establish an interesting connection to a probelm concerning edge coverings of a complete graph by bipartite graphs.

Jerrold R. Griggs

Towers of Powers and Bruhat Order

A recent paper of Brunson deals with an interesting partial order on the symmetric group, S_n, which arises from comparing permutations of iterated exponentials. For $\sigma \in S_n$ and $x = (x_1, \ldots, x_n) \in \mathbb{R}^n$, let $T(\sigma(x)) = T(x_{\sigma(1)}, \ldots, x_{\sigma(n)})$ denote the tower of iterated exponential $x_{\sigma(1)}^{x_{\sigma(2)}^{\cdot^{\cdot^{x_{\sigma(n)}}}}}$. For $\sigma, \tau \in S_n$ we have the partial ordering $T_n = (S_n, T)$, where $\sigma \leq_T \tau$ if and only if $T(\sigma(x)) \leq T(\tau(x))$ for all $x_n \geq \cdots \geq x_1 \geq e$. It turns out that for $n \leq 4$, this ordering, call it $T_n(e)$, is isomorphic to the dual of another interesting poset on S_n, Bruhat order. We prove that this is false for $n \geq 5$. It turns out to be essential to determine completly the poset $A_n(e)$ of towers of size n in just two symbols a and b, where for $w, w' \in \{a, b\}^n$, $w \leq_A w'$ if and only if $T(w) \leq T(w')$ for all $b \geq a \geq e$. More generally, let $T_n(c)$ and $A_n(c)$ be the posets above where the lower bound e is replaced by any $c \geq 0$. We prove that for all n $T_n(3,6)$ and $A_n(3,6)$ are chains in reverse lexicographic order. For $n = 3$ we determine all posets $A_n(c)$. There are eight different posets.

Hans-Dietrich O.F.Gronau

Minimal proper Sperner families

Let R be a finite set, $|R| = r$. A family $F \subseteq 2^R$ is called a Sperner family, if $X \not\subseteq Y$ for all $X, Y \in F$. A Sperner family $F = \{X_1, X_2, \ldots, X_k\}$ is called *proper*, if for every $x \in R$ the family $F(x) = \{X \setminus \{x\} : X \in F\}$ is not a Sperner family or $|F(x)| < |F|$. Obviously, maximal Sperner families are proper. But what is the minimum size of a proper Sperner family on R? The main result in attacking this problem is the following one:
Fix the size k of the Sperner family F and ask for the maximum size $r(k)$ of R such that there exists a proper Sperner family F on R.
Theorem:

$$r(k) = \begin{cases} 2k-2 & \text{if } 2 \leq k \leq 7, \\ \left\lfloor \frac{k^2}{4} \right\rfloor & \text{if } k \geq 7. \end{cases}$$

This and related results for further proper families (e.g. t-wise intersecting Sperner families) are presented.

Martin Grötschel

On an ordering problem in manufacturing

A practical optimization problem that comes up in a number of flexible manufacturing systems is the following. Let a complete digraph $D_n = (V, A_n)$ on n nodes and costs $c_e, e \in A_n$, be given. (The nodes correspond to machines. The costs include the costs of moving an object from one machine to another and setting up the machines.) Moreover, an acyclic digraph $D = (V, A)$ on the n nodes is given that describes precedence relations among the machines, i.e., if $(i, j) \in A$ then the object has to be processed on machine i before it can be processed on machine j. The task is to find a hamiltonian path H in D_n that satisfies all precedence relations and that has total cost $c(H)$ as small as possible. This problem is called sequential ordering problem in the flexible-manufacturing literature.

We indicate that it can be viewed as an "intersection" of the asymmetric traveling salesman problem and the linear ordering problem. We give several integer programming formulations of the problem and demonstrate how the corresponding LP-relaxations can be solved in polynomial time by providing polynomial time separation algorithms for certain classes of valid inequalities. Preliminary computational experience is reported with a cutting plane code for the solution of the sequential ordering problem. This code is based on the results mentioned above.

M. Habib

Mechanisms and algorithms for multiple inheritance in object oriented systems.

This talk presents a joint study with R. Ducournau (Inria) about inheritance algorithms. They are the kernel of object oriented systems, where one central problem is: "given an object determine its inheritance". Using simple inheritance (inheritance graph is a tree) is very easy, but when multiple inheritance is allowed conflicts occur. We present some of the principles which are necessary to build good inheritance algorithms. We show that depth-first greedy (also called super greedy and invented by O.Pretzel last Oberwolfach meeting 1985) linear extensions play a great role in all the inheritance mechanisms we propose and compare to known ones.

Jeff Kahn

Fourier analysis of a problem on finite sets

For $X \subseteq \{0,1\}^n$ let E_i be the set of edges from X to $\overline{X}$, and set $\alpha_i = |E_i|/2^{n-1}$. Set $f(n) = \min_{|X|=2^{n-1}} \max_i \{\alpha_i\}$. The question of bounds for $f(n)$ was raised (in the context of computer science) by Ben-Or and Linial, who showed

$$1/n \leq f(n) \leq \log n/n,$$

and conjectured that the upper bound is close to the truth. We prove this conjecture.

Theorem $f(n) = \Omega(\log n/n)$.

The proof uses techniques of Fourier analysis on Z_2^n, and has implications for random walks on the cube and distance distributions in subsets of the cube.

(Joint with G. Kalai and N. Linial)

G.O.H. Katona

The poset of closures

With G. Burosch and J. Demetrovics, we introduced the poset of closures as a model of changing databases:

$\mathcal{L}_1 \leq \mathcal{L}_2$ iff $\mathcal{L}_2(A) \supseteq \mathcal{L}_1(A)$ for any subset A.

We investigate the following questions: number of elements, number of elements with fixed ranks (the poset has a rank function), the min (max) number of upper (lower) immediate neighbours at a fixed rank.

David Kelly

Planar Ordered Sets

We call an ordered set L a *pseudolattice* if $L \dot{\cup} \{0,1\}$ is a lattice. Henceforth, all ordered sets are finite.

Theorem If a pseudolattice L is a subposet of a planar ordered set, then L is also planar.

In other words, nonplanar pseudolattices are obstructions to planarity. Observe that the ordered set $P = 0 < \{a,b\} < c < \{d,e\} < 1$ is planar, but the subposet $P - \{c\}$ is nonplanar.

Hal Kierstead

A polynomial approximation algorithm for Dynamic Storage Allocation

Chrobak and Slusarek proposed a polynomial algorithm which produces a feasible solution S for Dynamic Storage Allocation. They defined a parameter ω^* and showed that $\omega^* \leq v(Opt) \leq v(S) \leq 2\varphi(\omega^*)$ where $v(T)$ is the value of a solution T, Opt is the optimal solution, and $\varphi(x)$ is the number of colors required by the greedy algorithm to color an interval graph with clique size x, in the worst case. They conjectured that $\varphi(x)$ is linear in x, and thus their algorithm had a constant performance ratio. This conjecture was made independently by Woodal in 1973.

We show that $\varphi(x) \leq 40x$.

Daniel Kleitman

Some Applications of Order Theoretic Methods

We address the question: how large can a collection C of divisors of a square free integer N be, if whenever $A, B \in C$, and $A|B$ then $A \not\equiv B \bmod p$?

We show that, when each j the number of prime factors of N congruent to $j \bmod p$ is the same as the number congruent to $1/j$, and the number congruent to -1 is even, and N has n prime factors, than an upper bound is $\binom{n}{[\frac{n}{2}]} + \binom{n}{[\frac{n}{2}]+1}$.

Let $|S| = n$. We consider ordered pairs of subsets of S of equal size, ordered by inclusion in each component. We describe how to construct α pairs of size k that are covered by fewest pairs of size $k+1$, for any appropriate α; though there is no canonical ordering of such pairs so that the answer is an initial segment.

Peter Luksch

Finite modular lattices finitely generated by an ordered set

Free modular lattices are of central interest in lattice theory. In particular, one considers $FM(P)$, the modular lattice freely generated by an ordered set P.

If the width of P is two, $FM(P)$ becomes distributive and hence is isomorphic to $FD(P)$ the free distributive lattice generated by P. In this case we state a recursive structural formula for $FD(P)$ which can be used to obtain a reasonable line diagramm. Our basic idea is to study a decomposition by a congruence relation which has congruence classes isomorphic to a direct product $FD(Q_1) \times FD(Q_2)$ for some $Q_1, Q_2 \subseteq P$. Then a structural formula for $FD(P)$ can be described which uses the knoweldge of some $FD(Q)$ for proper subsets Q of P.

For the modular lattice $FM(\underline{1}+\underline{1}+\underline{n})$ freely generated by two single elements and an n-element chain we state a recursive counting formula. This answers Problem 44 in Birkhoff (Lattice Theorie, Amer. Mat. Soc. (1967)) which asks one to determine $FM(\underline{1}+\underline{1}+\underline{n})$. Therefore we study subdirect products of copies of D_2 and M_3 via their scaffoldings. In this way we obtain a deeper understanding of the structure of $FM(\underline{1}+\underline{1}+\underline{n})$.

Rolf H. Möhring

Partial Orders of Interval Dimension Two and a Channel Routing Problem

It was shown by Dagan, Golumbic and Pinter (DAM, to appear) that certain VLSI channel routing problems can be modeled as the intersection of two interval orders, i.e. by partial orders of interval dimension two or less (idim(P) ≤ 2). We derive a polynomial algorithm that tests whether a partial order P has idim(P) ≤ 2, and, if so, finds two associated interval orders. The algorithm uses properties of the set Q of all downsets $D(u) = \{v \in P | v <_P u\}$ of P ordered by inclusion. In particular, dim(Q) $\leq$ idim(P). The polynomial algorithm solves an open problem of Yannakakis (SIAM J.Alg.Disc.Math.) about the complexity of interval dimension two.

J. Nešetřil

Hasse diagram is an (undirected) covering graph of a poset. Hasse diagrams are known to be difficult to characterize. We present two paradoxical facts supporting this fact. In particular, we give a construction of a poset P_n with high chromatic Hasse diagram and with dimension 2.

Oliver Pretzel

Removing Monotone Cycles from Graph Orientations

Given a graph G give each cycle C a reference direction of traversal. For an orientation R of G an edge e of C is a forward edge if R orients it in the reference direction. Otherwise e is a backward edge. C is monotonic if all its edges are forward or all are backward. C is k-good if it has at least k forward edges and k

backward edges. R is k-good if all cycles are k-good. (R is 1-good iff it is acyclic and 2-good if it makes G into the diagram of an ordered set).

Theorem 1 (Mosesian 1972) If G has given ≥ 4 and an orientation in which every cycle is monotone or 2-good, then G has a 2-good orientation.

Weak Generalization If G has given $\geq 2k$ and an orientation in which every cycle is monotone or k-good, then G has a k-good orientation.

Strong Generalization If G has given $\geq 2k$ and an orientation in which every cycle is either k-good or not $(k-1)$-good, then G has a k-good orientation.

The weak generalization is proved by a method that gives a new proof of Theorem 1.

A counterexample to the Strong generalization found by Dale Youngs is presented.

Hans Jürgen Prömel

Boolean lattices, combinatorial spaces, and Ramsey theory

In this talk we discuss two extensions of Hales-Jewett's theorem on combinatorial spaces with particular emphasis on the special case of Boolean lattices.

The first one is a ordering version of Hales-Jewett's result, describing all natural orders on combinatorial spaces. A characterization of all these natural orders was first given in (Nešetřil, Prömel, Rödl, Voigt, J.Comb.Th(A) 40, 1985). Here we present a new simplified approach (Prömel, 1988 to appear in Discr.Math.). The second result we discuss is a "sparse" version of Hales-Jewett's theorem which is a joint result with B.Voigt and will appear in Trans. of the Amer. Math. Soc.

Klaus Reuter

Order Dimension via Ferrers Relations

A survey of my work on some problems about order dimension will be given.

1. How small can a lattice of order dimension n be? (joint with B.Ganter, P.Nevermann, J.Stahl)

2. It is known that

$$\max\{\dim P, \dim Q\} \leq \dim P \times \dim Q \leq \dim P + \dim Q$$

Can the bounds be improved?

3. Given a convex polytope P. Is it true that

$$\text{order}\dim(\text{face lattice}(P)) = 1 + \text{affine}\dim(P)?$$

4. Does the removal of a critical pair of an ordered set always decreases the dimension by at most one?

The more general concept of Ferrers relations and formal concept analysis are used to get some new insight in these problems (the answer to 3. and 4. is "no").

Ivan Rival

Dimension Invariance for Lattice Subdivisions

Inspired by a conjecture of M. Habib, we (Lee, Liu, Nowakowski and Rival) show that the dimension problem for $\underset{\sim}{N}$-free ordered sets is NP-complete. The proof is based on this fact. For any finite lattice L,

$$\text{dimension}(L) = \text{dimension}(\text{subdivision}(L)),$$

where subdivision(L) is the lattice obtained from L by adding one vertex along every edge with the obvious comparabilities.

Alexander Rutkowski

A (disconnected) variety of results concerning the fixed point property

1. Let, for $a, b \in P$, $x \notin P$, $P(a,b,x)$ be a poset with the order determined by the order of P and inequalities $x < a$, $x < b$.

   Thm. If P and $\{a,b\}^*$ have the FPP, $\{a,b\}_* \neq \emptyset$ then $P(a,b,x)$ has the FPP. If $P \times Q$ and $\{a,b\}^*$ have the FPP,$\{a,b\}_* \neq \emptyset$ then $P(a,b,x) \times Q$ has the FPP.

2. Let M_P stand for $Min_P \cup Max_P$.

   Thm. If M_P has the FPP then P has the FPP.

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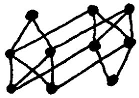
3. (with A. Klimczuk). If P is the poset shown in the figure and n is a positive integer then P^n has the FPP.



4. Let P be a connected, crown-free poset with only finite chains. If every infinite normal fence (each maximal point is a supremum of two adjacent minimal points and conversely) contains an infinite subset which is up-(or down-) bounded then P has the FPP.

Norbert Sauer

Cut-sets in partially ordered sets

$F \subset P$ is a cut-set for $x \in P$ if $F \cup \{x\}$ intersects with all maximal chains of the partially ordered set P and all of the elements in F are unrelated with x. The smallest cardinal κ such that all $x \in P$ have an cut set of cardinality less than or equal to κ is the cut-set number of P. The problem of relating the cut-set number with other order invariants such as width and length has stimulated significant activity. In the infinite case bounds for the cut-set number are closely related to bounds for Δ-systems and hence many of the infinite problems have been resolved recently. In the finite case the corresponding problems seem to be much more challenging but possibly also more interesting as they might constitute a step down in difficulty from the Δ-system problem.

James H. Schmerl

Incidence Algebras

For a poset P and a commutative ring R with 1, let $I(P, R)$ be the incidence algebra over R. Incidence algebras were introduced by Rota. The general problem considered is for which P and R does $I(P, R)$ uniquely determine P. Some results (including all previously known ones) concerning this problem are implied by the following theorem and its proof.

Theorem: For posets P and Q, the following are equivalent:

(1) $P \equiv_{\infty,\omega} Q$

(2) there is R such that $I(P,R) \cong I(Q,R)$.

The equivalence relation in (1) is well-known from model theory; countable posets are characterized by it.

(Joint work with M. Parmenter and E. Spiegel)

Dietmar Schweigert

Pareto extensions for spanning-tree-problems with several objectives

For a spanning-tree-problem with n objectives the weights of the edges are n-tuples. Therefore we have in general a partial order and not a linear order of edges. A linear extension of this partial order of edges is called a Pareto extension if the algorithm of Kruskal (resp. Prim) produces an efficient solution. We present bounds on the number of efficient solutions and study furthermore Pareto extensions given by preference functions.

M.M. Syslo

Bounds on the page number

The page number of a poset has been defined by R. Nowakowski (Ottawa, June 1987) and, for a poset P, is equal $p(P)$, to the minimum number of pages on which the covering edges of the diagram of P can be drawn without intersecting each other provided the elements of P are on the spine in a topological order (i.e., their order formes a linear extension).

It is easy to show that p is not a comparability invariant but it is a diagram invariant for $p = 1$ and for some other classes of posets. If $s = s(L, P)$ denotes the number of jumps in a linear extension L of P then

$$\left\lceil \frac{m-n+1}{s} \right\rceil + 1 \leq p(P),$$

where $n = |P|$ and m is the number of covering relations. On the other hand

$$p(P) \leq c(P),$$

where $c(P)$ is the covering number of the diagram of P. We use these estimates to calculate p for some posets.

William T. Trotter

An Improved Bound for the Dimension of Interval Orders

A poset P is called an interval order if there is an assigment $x \mapsto I_x$ where each I_x is an interval on the real line $\mathbf{R}$ so that $x < y$ in $P \Leftrightarrow I_x \lhd I_y$ on $\mathbf{R}$, i.e., every point of I_x is less than all points in I_y. It is well known that the dimension of an interval order in which the length of the longest chain has n points is bounded as a function of n regardless of the total number of points in P. Rabinovitch proved $\dim(P) < c \log_2 n$ while Bogart, Trotter, and Rabinovitch show that $\dim(P) \geq c_1 \log\log n$. In this paper, we show that if P is an interval order of length n, then $\dim(P) \leq c_2 \log\log n$. We expect that the correct answer is $(1 + o(1)) \log\log n$.

(Joint work with Z. Füredi, and V. Rödl).

Douglas B. West

The Interval Inclusion Number of a Partially Ordered Set

A *containment representation* of a poset P is a map f such that $x < y$ in P if and only if $f(x) \subset f(y)$. We introduce the *interval inclusion number* (or *interval number*) $i(P)$ as the smallest t such that P has a containtment representation in which each $f(x)$ is the union of at most t intervals. Trivially, $i(P) = 1$ if and only if $dim P = 2$. Posets with $i(P) = 2$ include the standard n-dimensional poset and all interval orders; i.e., posets of arbitrarily high dimension. In general, $i(P) \leq \lceil \dim P/2 \rceil$, with equality for Boolean algebras. For lexicographic composition, $\dim(Q) = 2k + 1$ and $i(P) = k$ imply $i(P[Q]) = k + 1$. This and $i(B_{2k}) = k$ imply that testing $i(P) \leq k$ for fixed k is NP-complete. The maximum value of $i(P)$ for n-element posets remains unknown, but $i(P) = \Theta(|P|/log|P|)$ for almost every poset. Concerning removal theorems, $i(P - x) \geq i(P) - 1$ when x is a maximal or minimal element, and in general $i(P - x) \geq i(P)/2$.

(Joint work with Thomas Madej)

Rudolf Wille

On the skeletons of free distributive lattices

The aim is to understand the structure of free distributive lattices via their skeletons. The skeleton $S(L)$ of a finite distributive lattice L consists of all maximal Boolean intervals of L ordered by their lower (or equivalently upper) bounds; $S(L)$

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is again a lattice. To analyse the skeletons of the free bounded distributive lattices $FBD(n)$ with n generators, methods of formal concept analysis are helpful. As key we use the basic fact that $FBD(n)$ is isomorphic to the concept lattice $\underline{\mathfrak{B}}(B_n, B_n, \not\geq)$ and $S(FBD(n)) \cong \underline{\mathfrak{B}}(B_n, B_n, \not>)$ where B_n is the Boolean lattice with n atoms.

Theorem: The maximal Boolean intervals containing $n-1$ of the generators generate in $S(FBD(n))$ a 0-1-sublattice isomorphic to $FBD(n-1)$; if $n \leq 5$, $S(FBD(n))$ is the union of these n sublattices.

Corollary: $|S(FBD(5))| = 386$

Günter M. Ziegler

Topology of oriented matroids

We show that the face lattice of an oriented matroid (as axiomatized by Edmonds, Mandel and Fukuda) is the face lattice of a regular CW-sphere.

For this we use Björner's characterization of the face lattice of shellable CW-spheres, to show that every linear extension of the poset of regions (as studied by Edelman) induces a recursive coatom ordering of the face lattice.

Our method leads to a new proof of the Folkman-Lawrence Representation Theorem: every oriented matroid arises from an arrangement of pseudo-hemispheres on a sphere. Moreover, such arangements as well as their hemispheres and intersections of hemispheres (supercells) are always shellable. This sharpens Mandal's result that oriented matroids arise from constructible (hence PL-) spheres.

(Joint work with Anders Björner, Stockholm)

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