MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 19/1988

Gruppen und Geometrien

1.5. bis 6.5. 1988

Die Tagung fand unter Leitung von Herrn M.Aschbacher (Pasadena), Herrn D.G.Higman (Ann Arbor), Herrn B.Fischer (Bielefeld) und Herrn F.G.Timmesfeld (Gießen) statt. Im Mittelpunkt des Interesses standen diesesmal Fragen der Untergruppenstruktur von Lie-Typ Gruppen über endlichen und algebraisch abgeschlossenen Körpern im Zusammenhang mit der Operation dieser Gruppen auf assozierten geometrischen Strukturen wie z.B. distanz-transitive Graphen, Diagrammgeometrien und anderen.

Nach Abschluß der lokalen Klassifikation der parabolischen Systeme und der zugehörigen lokal endlichen klassischen Tits Kammersysteme, hat sich das Interesse bei den "Amalgamen" auf allgemeinere Fragestellungen verlagert. Hierbei scheint der Zusammenhang des von F.Timmesfeld eingeführten Graphen auf der Indexmenge eine wesentliche Voraussetzung zu sein. (Sie entspricht dem Zusammenhang des Diagramms.)

Ingsgesamt kann man sagen, daß die Theorie der endlichen Gruppen und zugehörigen geometrischen Strukturen nach einer kurzen Verschnaufpause nach erfolgter Klassifikation einen neuen Aufschwung genommen hat. Hierbei fällt auf, daß sich die Methoden gegenüber denen der Klassifikation völlig geändert haben. Insbesondere dem Zusammenhang zwischen den endlichen und unendlichen Gruppen (Gruppe über dem algebraisch abgeschlossenen Körper, amalgamiertes Produkt) kommt große Bedeutung zu.

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Vortragsauszüge

M. Aschbacher:

The subgroup structure of groups of type E_6

We prove the following result describing the maximal closed subgroups of a universal group $G = E_6(F)$ of type E_6 over a finite or algebraically closed field F:

Theorem. Let H be a proper closed subgroup of G. Then either

1) H stabilizes some member of a set C of natural structures on the 27-dimensional FG-module, or
2) $F^*(H) = LZ(H)$ where L is quasisimple, $C_G(L) = Z(G)$, and one of the following holds:
   (a) $L \in S$, a certain set of subgroups of SL(V),
   (b) some irreducible FL-submodule of V can be written over a proper finite subfield F_o of F, and $H \leq N_G(S)$ for some $S \leq G$, $S \simeq E_6(F_o)$,
   (c) $L \in$ UNK.

Here UNK is a set of about 15 small finite subgroups of SL(V) where existence and uniqueness is left open.

A. Böhmer:

Rank-3-amalgams

A chamber system approach for the study of weak BN-pairs of rank 3 is introduced, which leads to the following result:

Theorem. Let p be a prime and G a primitive rank 3 amalgam of its subgroups P_o, P_1, P_2 over B. For $i \neq j \in \{o,1,2\}$ let $L_i := O^{p'}(P_i)$,

$Q_i := O_p(P_i)$, $P_{ij} := \langle P_i, P_j\rangle$, $L_{ij} := \langle L_i, L_j\rangle$ and for a subgroup U of B $U_{P_{ij}}$ denotes the larges normal subgroup of P_{ij} contained in U. Assume that G satisfies:

1) $L_i/Q_i \simeq (S)L(p^{n_i})$, $(S)U(p^{n_i})$, $Sz(p^{n_i})$ or $Ree(p^{n_i})$

2) $B = \prod_{i\in\{o,1,2\}} (B\cap L_i) = N_{P_i}(S)$, $S \in Syl_p(B)$

3) $P_i \ntrianglerighteq Q_i \cap Q_j \ntrianglelefteq P_j$ for $\{i,j\} \neq \{o,1\}$ and $i \neq j$, but $P_o \trianglerighteq Q_o \cap Q_1 \trianglelefteq P_1$

4) $\Omega_1(Z(S)) \trianglelefteq P_{12}$ and $L_{12}/S_{P_{12}}$ is parabolic isomorphic to $(S)L_3(p^{n_1})$.

Then $p=2$, $|B| = |S| = 2^{10}$, $S_{P_{12}} \simeq 2^{1+6}$, $S_{P_{o1}} \simeq 2^{2+6}$ and in addition one of the following three cases holds:

(a) $\overline{P}_{o2} \simeq L_3(2) \simeq \overline{P}_{12}$, $\overline{P}_{o1} \simeq 3^3(2\times L_2(2))$ and $S_{P_{o2}} \simeq 4^3{\cdot}2$

(b) $\overline{P}_{o2} \simeq L_3(2)$, $\overline{P}_{12} \simeq A_6$, $\overline{P}_{o1} \simeq 3^n(2\times L_2(2))$, $n = 2$ or 3, and $S_{P_{o2}} \simeq 4^3{\cdot}2$

(c) $\overline{P}_{o2} \simeq \hat{\Sigma}_6$ or $L_3(2)\wr 2$, $\overline{P}_{12} \simeq L_3(2)$, $\overline{P}_{o1} \simeq L_2(2)\times L_2(2)$ and $S_{P_{o2}} \simeq 2^6$.

Here $\overline{P}_{ij} := (P_{ij}/C_{P_{ij}}(S_{P_{ij}}))/O_p(P_{ij}/C_{P_{ij}}(S_{P_{ij}}))$.

A.M. Cohen:

Distance-transitive graphs

Suppose G is a finite group acting primitively and distance transitively on a graph Γ and suppose $L = \mathrm{soc}\,G$ is a simple group.

Theorem. If G has a BN-pair with Coxeter system (W,R), then it is known.

Theorem. (joint with Van Bon) If $L \simeq L(n,q)$ then Γ is either complete, a Grassmann graph or known.

Theorem. (joint with Liebeck & Saxl) $L \not\simeq E_8(q)$ for $q \geq 4$.

Theorem. (joint with Van Bon & Cuypers) $L \not\simeq He$.

B. Cooperstein:

The Fifty Six Dimensional Module for Groups of Type E_7

Let V be an eight dimensional vector space over a field K, of characteristic not two, V^* a dual space to V and $S \simeq SL(V)$. Set $\chi = \Lambda^2(V)$, the second exterior product, and $\chi^* = \Lambda^2(V^*)$. χ and χ^* are dual with pairing $(\, , \,) : \chi \times \chi^* \to K$ given by $(u_1 \wedge u_2, w_1^* \wedge w_2^*) = \det(w_i^*(u_j))$. This can be used to get an A-invariant quadratic form, Q, and sympletic form, $< \, , \, >$ on $M = \chi + \chi^*$. It is shown that S fixes pointwise a four-space Y in $K[M]$, the polynomial algebra on M, consisting of 4-homogeneous forms, making use of the exterior algebra Ext(V). A certain collection of 56 one-spaces is identified and in a natural way a graph defined which, it is shown, has automorphism group Weyl (E_7). This is used to define a transformation g_σ in $Sp(< \, , \, >, M)$ which fixes this frame and induces the automorphism σ of this graph. It is shown that the group $E = <S, g_\sigma>$ fixes a one-space $<\mathfrak{J}>$ in Y and that $E \simeq E_7(K)$, a universal group of type E_7 over K, is the isometry group of $\mathfrak{J}$. Then orbits of E on one-spaces of M are enumerated and their stabilizers determined.

G. Hanssens:

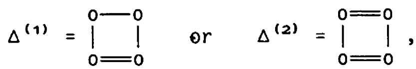
Some remarks on the coodinatization of generalized polygons

Coordinatization has been carried out for projective planes, and has proved to be a valuable tool in understanding and creating such objects. We present a coordinatization theory for generalized quadrangles, that extends to generalized hexagons and 8-gons. It appears that the more elations a GQ has, the nicer its coordinatizing structure becomes. This method might also be useful to give a more elementary proof of Tits' classification of Moufang polygons. A first step in that direction is made.

S. Heiss:

Two sporadic geometries related to the Hoffman-Singelton graph

Let $\Gamma^{(i)}$ $(i = 1,2)$ be a residually connected Tits geometry belonging resp. to the diagram



such that every rank 3 residue belonging to a subdiagram of type C_3 is the sporadic A_7-geometry.

A construction of such geometries will be given. This construction makes use of the Hoffman-Singelton graph Λ and the cocliques of size 15 in Λ.

On the other hand it can be shown, that the incidences of these geometries are consequences of the structure of the rank 3 residues, which proves:

Theorem. Up to isomorphism there exists an unique geometry $\Gamma^{(i)}$ $(i = 1,2)$, which satisfies our assumtions.

C.Y. Ho:

A sufficient condition for an element to belong to a Sylow p-subgroup

Let G be a finite group. Let g and y be two elements in G. We say that g is right-engel with respect to g if there exists an integer n such that

$[x,\underbrace{g,\dots,g}_{n}] = 1$ for all x in $\langle g,y\rangle$.

Conjecture I. Let π be a set of primes and let H be a Hall π-subgroup of G. Suppose g is a π-element of G. If g is right-engel respect to h for all h in H, then g belongs to H.

This would generalize a result of Baer. For a prime p, a p-element g belongs to $O_p(G)$ if and only if g together with any element in its conjugacy class generates a p-subgroup. This leads to the following equivalent versions of Conjecture I:

Conjecture II. Let P be a Sylow p-subgroup and g a p-element of G. If g is right-engel with respect to each element of P, then g belongs to P.

Conjecture III. Let P be a Sylow p-subgroup and g a p-element of G. If g together with any element of P generates a p-subgroup, then p belongs to P.

Völklein and Ho verify that Conjecture III holds for $p \geq 5$ except possibly when $p = 7$ and G involves a simple group of type F_1.

A.A. Ivanov:

Classification of distance-transitive graphs which are s-transitive for $s \geq 2$.

Let Γ be a graph and $G \leq \mathrm{Aut}(\Gamma)$. The pairs (Γ, G) which satisfy the following two conditions are classified:

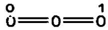
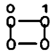
(1) Γ is a distance-transitive graph and G acts distance-transitively on Γ;

(2) for a vertex x of Γ the permutation group $G(x)^{\Gamma(x)}$ contains a normal subgroup which is the group $PSL_n(q)$ in its natural doubly transitive representation of degree $(q^n-1)/q-1)$.

Here $G(x)$ is the stabilizer of x in G, $\Gamma(x)$ is the set of vertices which are adjacent to x and $G(x)^{\Gamma(x)}$ is the permutation group induced by $G(x)$ on $\Gamma(x)$.

W. Kantor:

Asymptotic properties of some GABs

Let $f = \sum_1^6 X_i^2$, $p > 2$, and let Δ be the affine building for $\Omega(f,\mathbb{Q}_p)$ with diagram  if $p \equiv 3(4)$,  if $p \equiv 1(4)$. Then $G = O(f,\mathbb{Z}[\frac{1}{p}])$ is transitive on the set of vertices of type 0 or 1. For each integer $m > 1$, $m \not\equiv 0(p)$ consider $G(m) = \{g \in G \mid g \equiv 1(m)\} \lhd G$, $G/G(m) = O(f,\mathbb{Z}(m))$. This acts on the simplical complex $\Delta/G(m)$, and is transitive on the set of vertices of type 0 or 1.

The "diameter" of $\Delta/G(m)$ can be considered in terms of either the graph of vertices of types 0 or 1, the 1-skeleton of $\Delta/G(m)$, or the chamber graph. For each of these, the diameter is at most $C \log_2 |G/G(m)|$ for some constant C (proved using Kayhdan's Property (T) for G; an explicit estimate for C is unknown).

The "geometric girth" of $\Delta/G(m)$ is the length of a shortest circuit not homotopic to 0, where the circuit can be in the simplical complex or the chamber graph. For either definition, the geometric girth is $\geq C' \log_p m$ for a known constant C'; $C' = 1$ works in the case of the simplical complex. (Here $\log_p m \approx \frac{1}{12} \log_p |G/G(m)|$.)

Similar results hold for other GABs arising from classical affine buildings (class number 1 is required).

P. Kleidman:

Simple subgroups of simple groups

Let G be a finite simple group. We are concerned with finding the maximal subgroups of G. The hardest problem in this situation is to classify the simple subgroups of G. Thus we are led to the question: what are the simple subgroups of the simple groups? Here we address a very special aspect of the problem: when is a sporadic simple group contained in an exceptional group of Lie type. Many interesting examples arise, such as $J_1 < G_2(11)$ (Janko, Coppel), $J_2 < G_2(4)$ (Suzuki, Wales), $J_3 < E_6(4)$ (Kleidman, Aschbacher), $F_{i_{22}} < {}^2E_6(2)$ (Fischer), $M_{12} < E_6(5)$ (Kleidman, Wilson). We settle this question, with a few cases left open, including: $Ru < E_7(5)$? $M_{22} < E_7(5)$? $HS < E_7(5)$?

M.W. Liebeck:

Primitive groups of genus zero

A primitive group of genus zero is a primitive subgroup G of S_n such that

1) $G = \langle x_1,\ldots,x_r\rangle$ with $x_i \neq 1$ and $x_1\ldots x_r = 1$ and
2) if $\mathrm{ind}(x_i) = n - \#\mathrm{orb}\langle x_i\rangle$ then $\sum_1^r \mathrm{ind}(x_i) = 2n-2$.

Such groups arise in the study of monodromy groups and Riemann surfaces. R.Guralnick and J.G.Thompson have obtained strong results when G is affine (i.e. $\mathrm{soc}G \simeq Z_p^k$). In the other case, when $\mathrm{soc}G \simeq L^k$ with L as non-abelian simple group, they have shown that there is a group X with $L \trianglelefteq X \leq \mathrm{Aut}\, L$, a subgroup M of X with $L \nleq M$, and an element $1 \neq g \in X$ such that

$$|g^X \cap M|/|g^X| > \tfrac{1}{85} \qquad (*)$$

They conjecture that here is a number N such that for $q > N$, no group $G(q)$ of Lie type over $GF(q)$ can satisfy (*) for any M,g. I outlined a proof of this conjecture for $G(q)$ of type E_7 or E_8.

G. Lunardon:

On the flocks of $Q^+(3,q)$

A complete characterization of the flocks of $Q^+(3,q)$ is given. As an application, it follows that if q is odd, $q \neq 11, 23, 59$, there exist no maximal exterior sets of $Q^+(2n-1,q)$.

R. Lyons:

Component uniqueness theorems

The general notion of a "uniqueness theorem" and its role in the proof of the classification of finite simple groups was discussed heuristically. The following particular "component-uniqueness theorem" was stated:

Let G be a finite simple group all of whose proper subgroups are "known" simple groups. Let p be a prime, and let M be a maximal subgroup of G with the following properties:

(i) M has a p-component K with $m_p(K) \geq 2$;

(ii) a Sylow p-subgroup Q of $C_M(K/O_{p'}(K))$ has p-rank ≥ 2;

(iii) if $p > 2$, then $m_p(M) \geq 4$.

Then under any one of the following hypotheses, M is p-strongly embedded in G:

(a) $K \ntrianglelefteq M$ (in truth, in this case, an extra hypothesis is required if K is of such an isomorphism type that it has a p-strongly embedded subgroup itself;

(b) for all $x \in Q$ of order p and all $g \in G$, $x^g \in M$ if and only if $g \in M$;

(c) for all $x \in Q$ of order p, $C_G(x) \leq M$, and for all $E \leq Q$ with $E \simeq Z_p \times Z_p$, and all $g \in G$, $E^g \leq M$ if and only if $g \in M$.

Finally, it was asked whether there is a workable geometric or combinatorial way to study the Chevalley groups from the point of view of semisimple subgroups, rather than unipotent subgroups.

Th. Meixner:

Failure of factorization modules for Lie type groups in odd characteristic

A faithful $F_p[G]$-module V is called a failure of factorization module in char p for G, if there is an elementary abelian p-subgroup $1 \neq A \leq G$ satisfying $|A| \geq |V : C_V(A)|$.

The irreducible FF-modules in char 2 for Lie-type groups of char 2 were determined by Cooperstein, while the irreducible FF-module in natural characteristic for rank 2 Lie-type groups are given by Delgado. For higher rank, Thiel treated the $A_n(q)$ and $D_n(q)$ cases.

Theorem. Let G be a finite Lie-type group in odd characteristic with irreducible type and rank ≥ 3, and let V be an irreducible FF-module in natural characteristic for G. Then one of the following holds:

(1) G is of type $A_n(q)$, V is natural, exterior square or dual to one of these.

(2) G is of type $B_n(q)$, $C_n(q)$, ${}^2A_n(q)$ or ${}^2D_n(q)$ and V is the natural module (orthogonal, symplectic, unitary)

(3) G is of type $D_4(q)$ or $D_5(q)$ and V is a spin module

(4) G is of type $B_3(q)$ and V is the spin module.

Part of the proof is the application of a theorem by Premet/Suprunenko classifying quadratic modules.

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H. Van Maldeghem:

Buildings at Infinity

A building at infinity is a building at infinity of a certain affine building.

Theorem 1. The class of projective planes at infinity coincides with the class of projectiv planes coordinatized by some planar ternary ring with valuation. (Non-classical examples possible)

Theorem 2. The class of generalized quadrangles at infinity coincides with the class of generalized quadrangles coordinatized by some quadratic quaternary ring with valuation. (Non-classical examples possible)

Conjecture. The class of generalized polygons at infinity coincides with the class of generalized polygons with valuation.

Proved for generalized n-gons with $n \geq 3$ and $n \neq 6$.

S.Norton:

Presenting 2-lokal subgroups of the monster

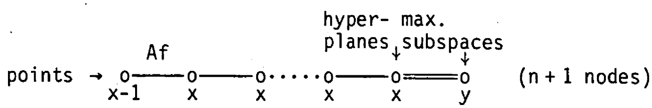
We mention the current state of knowledge regarding the groups presented by Y-diagrams subject to an additional relation. These are orthogonal or trivial if any of the parameters is at least 6.

We then proceed from Y_{555} to the projective plane of order 3 and identify a subgroup with relations that present a 2-local group. In most cases this is isomorphic to a subgroup of the presented Y-group, but in Y_{553} it is $2^{1+25}.2\,Co_1$ (not $2^{1+25}.Co_1$) and in Y_{555} it has a homomorphism onto $2^{1+26}.2^{24}.2\,Co_1$ (not $2^{1+26}.2^{24}.Co_1$). This is evidence that Y_{553} and Y_{555} may not be the monster $(M \times 2)$ and bimonster $(M \,wr\, 2)$ respectively.

A. Pasini:

On a class of geometries related to affine polar spaces

Let Γ be a finite residually connected geometry belonging to the following diagram



and assume that the Intersection Property holds in it. Then there is a constant γ $(1 \leq \gamma \leq x)$ such that, given a maximal subspace u and a point $a \notin u$ but such that $a^{\perp} \cap u \neq \emptyset$, there is exactly one hyperplane w and there are γ maximal subspaces $u_1, \ldots u_\gamma$ such that: for $i = 1, \ldots \gamma$, $u \cap u_i$ is a hyperplane, $w \subseteq u_i$ and w is parallel to $u \cap u_i$ inside u_i; moreover the hyperplanes $u_1 \cap u, \ldots u_\gamma \cap u$ are pairwise parallel and $a^{\perp} \cap u = \bigcup_{i=1}^{\gamma} u \cap u_i$.

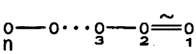
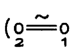
It is known that $\gamma = 1$ if and only if Γ is an affine polar space. I prove that $\gamma = x$ if and only if Γ is either the geometry for the 2-transitive action of $x^{2n} \cdot Sp_{2n}(x)$ or one of the geometries for the 2-transitive action of $Sp_{2n}(2)$.

P. Rowley:

Parabolic systems over GF(2)

Suppose G is a group containing a minimal parabolic system $\{P_1, \ldots, P_n\}$ satisfying P_n^+ and such that for each $i \in I = \{1, \ldots, n\}$

$$P_i/O_2(P_i) \simeq S_3 (\simeq SL_2(2)).$$

This talk concerned those parabolic systems whose diagram Δ is of the form  ( meaning that $\langle P_1, P_2\rangle / O_2(\langle P_1, P_2\rangle) \simeq \hat{S}_6$). Put $S = \bigcap_{i \in I} P_i$, $S_0 = \mathrm{core}_G S$ and $S_{123} = \mathrm{core}_{\langle P_1, P_2, P_3\rangle} S$.

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A proof of the following was outlined.

Theorem. If $\Delta = \underset{5}{o}\!-\!\!-\!\underset{4}{o}\!-\!\!-\!\underset{3}{o}\!-\!\!-\!\underset{2}{o}\overset{\sim}{=\!=}\underset{1}{o}$ and $|S/S_{123}| \neq 2^9$, then $|S/S_0| = 2^{46}$.

J. Saxl:

On subfield subgroups

We outlined a method for studying the action of a group G of Lie type on the set of cosets of a subgroup S of the same type (possibly twisted) over a smaller field. The idea is to work in the corresponding algebraic group. As an illustration, we considered the case where G is $Sp_4(q)$ with q even and S is either $Sz(q)$ or $Sp_4(q^{\frac{1}{2}})$, and the case where $G = E_8(q)$ and $S = E_8(q^{\frac{1}{2}})$.

In the first case, the rank of G on S is equal to the number of conjugacy classes of S and all (resp. all but two) suborbits are self-paired. In the second case we found $\frac{1}{30}(\Phi_{30}(q^{\frac{1}{2}})-1)$ self-paired suborbits of size $\Phi_{15}(q^{\frac{1}{2}})$ and $\frac{1}{30}(\Phi_{15}(q^{\frac{1}{2}})-1)$ self-paired suborbits of size $\Phi_{30}(q^{\frac{1}{2}})$.

R. Scharlau:

On the classification of arithmetic hyperbolic reflection groups

We consider groups W of isometries of n-dimensional hyperbolic space H^n generated by reflections and such that H^n/W is of finite volume. In particular (following Vinberg, Nikulin, Mennicke), we are interested in arithmetic non-compact groups of that kind. That is, W is commensurable to a group $O(f,\mathbb{Z})$ where f is an integral quadratic form of signature $(n,1)$, and isotropic over $\mathbb{Z}$. This means more or less that we are looking for those quadratic forms such that the subgroup $W(f) \triangleleft O(f)$ generated by all reflections preserving f is of finite index. We are particularly interested in the case $n = 3$.

From general results of Nikulin it follows that the list of such f is finite. On the other hand, the list of candidates that come from Nikulin's proof is much t-o large to deal with. This problem is not only a computational one, because there is no procedure known to us which decides for a given f whether $[O(f):W(f)]$ is finite or infinite.

By combining an idea of Vinberg (used in the proof of the fact that $[O(f):W(f)]$ is always infinite if $n \geq 30$) with methods by J. Mennicke involving the genus of a certain plane stabilizer, we hope to produce sufficiently sharp criteria that allow to prove infiniteness in each concrete case where $[O(f):W(f)]$ is not "obviously" finite.

As an example, we have proved that for $f_p = x_0x_1 + x_2^2 + px_3^2$, p prime, $p \equiv 1(4)$ one has $[O(f):W(f)] < \infty$ if and only if $p = 5,13,17$. We have produced a list of about 50 forms such that the $O(f)$ are maximal, pairwise non-conjugate and $[O(f):W(f)] < \infty$. We hope that this list will turn out to be (almost) complete. The proof of this fact will be joint work with F.Grunewald.

J.J. Seidel:

Designs of Strength t

1. A measure ξ in $\mathbb{R}^d$ is said to have strength t if $\int f d\xi = \int f d\xi \circ \varphi$, for all polynomials f of degree $\partial f \leq t$, and all $\varphi \in O(d)$, the orthogonal group.

2. For finite support X on the unit sphere S, weights $w_x = 1$, this amounts to spherical t-design: $\operatorname{Ave}_X f = \operatorname{Ave}_S f$, equivalently $h(X) := \sum_{x \in X} h(x) = 0$ for h harmonic homogeneous, $\partial h \in \{1,2,\ldots,t\}$. Example: $(d,n,t) = (3,12,5)$.

3. For a lattice $Y = \bigcup_{r\in R} Y_r$ (unimodular, integral, even) the condition reads $\sum_{r\in R} w(r)h(Y_r) = 0$, where $R := \{r = (g,y) : y \in Y\}$, with smallest $0 \neq r_0 \in R \subset 2\mathbb{Z}$. By use of theta series it follows that each Y_r is a spherical design of strength $\frac{1}{2}(12\, r_0 - \dim) - 1$, cf. Hecke, Schoeneberg, B.B. Venkov (1984). Examples: $(d,n,r_0,t) = (8,240,2,7)$: E_8, and $(24,2\binom{28}{5},4,11)$: Leech.

4. For finite support Y on p spheres we prove the Fisher inequality
$$|Y| \geq \sum_{i=0}^{2p} \binom{d-1+e-i}{d-1}.$$
Joint work with Neumaier (Indag.Math.) and Delsarte (Lin.Alg.Appl.) to appear.

E. Shult:

Further Characterizations of Lie Incidence Systems

Suppose $\Gamma = (\ , \ )$ is a weak parapolar space with the local pentagon property, such that for each non-incident point-symplection pair (x,S), $x^\perp \cap S$ is empty or contains a line. Then Γ is either a polar space, a metasymplectic space, a Grassman space of type $E_{n,2}$ or a polar Grassman space of type $C_{n,2}$.

Other characterization where the hypothesis on symplecta are replaced by other properties of symplecta, lead to characterizations of all residually connected geometries covered by a building with diagram

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as well as homorphic images of polar Grassman spaces of type $C_{n,d}$, $d \leq n-2$.

St.D. Smith:

Combinatorial & geometric techniques in modular representation theory

Recent results will be surveyed. At this time, I would expect to mention:

-- joint work with A.Ryba, constructing and decomposing projective representations on sporadic geometries.

-- joint work with G. Lehrer, using "parabolics" to decompose induced modules -- for example 1_U^G in the Chevalley case, and analogues in sporadic cases.

G. Stroth:

Quadratic modules for finite simple groups

Part of this is joint work with U.Meierfrankenfeld.

We consider the following situation. Let H be a perfect central extension of a finite simple group and $G \le Aut(H)$. Furthermore let V be a faithful irreducible GF(2)G-module for G and $E \le G$ a four-group such that $[V,E,E] = 1$.

Then we have the following results:

(1) If $H/_{Z(H)}$ is a group of Lie type over a field of odd characteristic, then H is isomorphic to one of the following groups: $L_2(5)$, $L_2(7)$, $L_2(9)$, $3 \cdot L_2(9)$, $U_3(3)$, ${}^2G_2(3)'$, $PSp_4(3)$ or $3 \cdot U_4(3)$.

(2) If $H/_{Z(H)}$ is a sporadic group, then H is one of the following: M_{12} $3 \cdot M_{22}$, M_{24}, C_2, C_1, J_2, $3 \cdot Suz$.

J.G. Thompson:

Fuchsian Groups and Galois Theory

Two topics were discussed:

1. Belyi's theorem concerning covers of $P^1 - \{\infty, 0, 1\}$ leads to the problem of finite groups of genus zero.
2. 4-punctured spheres lead to Fuchsian groups generated by three involutions.

F.G. Timmesfeld:

Classical locally finite Tits chamber systems of rank 3

Some aspects of the proof of the classification of (quasi) parabolic systems of rank 3 and their related Tits chamber systems were discussed. The paper will appear in Journal of Algebra. Since the main theorem has to many cases, the statement cannot be given here.

S.V. Tsaranov:

Monoids of Coxeter type having attractors

Let $M = (m_{ij})_{n\times n}$ be a matrix with positive integer entries such that $m_{ii} = 1$ for all i and such that if $i \neq j$ then either $m_{ij} = m_{ji} \geq 2$ or $\{m_{ij}, m_{ji}\} = \{2s-1, 2s\}$ for some $s \geq 2$.

We define a monoid $F(M)$ with h generators $\{X_i\}_{i\in I}$ and the following relations:

$$X_iX_i = X_i, \quad i \in I;$$

$$\underbrace{X_iX_jX_i\ldots}_{m_{ij}} = \underbrace{\ldots X_iX_jX_i}_{m_{ij}} = \underbrace{X_jX_iX_j\ldots}_{m_{ji}} = \underbrace{\ldots X_jX_iX_j}_{m_{ji}}.$$

We state the following convention. We write $(m_{ij}) = M \leq L = (l_{ij})$ if $m_{ij} \leq l_{ij}$ up to simultaneous permutations of rows and columns. A monoid is called indecomposible if it cannot be presented as a direct product of two proper submonoids.

Definition: A word $X \in F(M)$ is called an attractor if $X = XX_i = X_iX$ for every $i \in I$.

Theorem.

1) An indecomposable monoid F(M) is finite if and only if M is a spherical Coxeter matrix;

2) F(M) has an attractor if and only if either M is a spherical Coxeter matrix or $A_n \leq M \leq A_n'$ where A_n' corresponds to the diagram $o \overset{3,4}{\text{—}} o \cdots o \overset{3,4}{\text{—}} o$ (by usual conventions).

H. Völklein:

Geometric approach to representations of Chevalley groups

Let G be a Chevalley group over the finite field k. With each irreducible kG-module V, Ronan and Smith associated a (geometrically defined) extension module $\tilde{V}$ of V. We give conditions ensuring that the 1-cohomology $H'(G,V^*)$ can be read off from $\tilde{V}$. As an application, we obtain the 1-cohomology of G in its adjoint module.

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