MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Mathematical Problems in the Kinetic Theory of Gases

8.5. bis 14.5.1988

The conference was organized by C. **Cercignani** (Milano), H. **Neunzert** (Kaiserslautern) and D.C. **Pack** (Glasgow). There were 28 participants, 25 of whom gave talks.

The contributions ranged over a variety of topics in kinetic theory.

A main subject was the existence of solutions for nonlinear Cauchy problems. Especially the Boltzmann equation, the Enskog equation and the Vlasov-Maxwell system were investigated.

Several other problems, such as hydrodynamic limits for the Boltzmann equation, collision models, inverse problems in linear theory and stationary problems for both linear and nonlinear equations were studied.

A final important topic were numerical particle simulation methods for nonlinear kinetic equations.

Vortragsauszüge:

L. Arkeryd:

Some results for the Boltzmann and Enskog equations

The talk discusses the following three results:

i) for the space homogeneous Boltzmann equation with hard forces, exponential convergence and stability in L^1 under sufficiently high moments;

ii) for the space dependent Boltzmann equation and large L^1 data, the equivalence of Loeb L^1 solution and standard Young measure solutions;

iii) for the Enskog equation with a constant high density factor, wellposedness and regularity globally in the case of bounded velocity and locally in time for unbounded velocities.

N. Bellomo:

Some new results on the Cauchy problem for the Enskog equation

This talk deals with the nonlinear Enskog equation in all space and provides some new results on the analysis of the existence of solutions to the initial value problem in all space as well as on the asymptotic equivalence with the solutions to the same problem for the Boltzmann equation.

A. Palczewski:

The stationary nonlinear Boltzmann equation in unbounded domains

Half-space problems for the steady one-dimensional Boltzmann equation are considered. Two types of boundary conditions are analyzed: specular reflection and the condition with a given distribution function of particles entering the region of interaction. It has been made an attempt to show that these problems possess solutions without discussing the uniqueness of these solutions.

L. Triolo:

<u>Stationary Boltzmann equation for a degenerate gas in a slab: hydrodynamical limit</u>

The following Boltzmann equation (stationary):

$v_x \partial_x f(x,v_x,v_y) =$

$$= \int |v_x - v_{1x}| \{ f(xv_{1x}v_y) f(xv_xv_{1y}) - f(xv_{1x}v_{1y}) f(xv_xv_y) \} dv_{1x}dv_{1y}$$

$x \in [-L,L]$

with the B.C. (reflected, mass conserving):

$$f(\mp L, v_xv_y) = \mp H_{\mp}(v_xv_y) \int v_x f(\mp L, v_x, v_y) dv_x dv_y, \quad v_x \gtrless 0$$

is seen to be equivalent to a linear one, of the type

$v \partial_x h(xv) = \int |v-v'| g(v') [h(x,v') - h(x,v)] dv'$, with $g(\cdot)$ given by the B.C. Its properties are studied.

V. Boffi:

<u>Solving the Boltzmann system for a gas mixture via the relevant conservation system</u>

It is shown that - upon appropriate hypotheses on the collision frequencies, the scattering probability distributions and the initial data - the nonlinear integro-partial differential Boltzmann system for the distribution functions $f_1, f_2, \ldots, f_N$ of the N gases of a given mixture can be converted into a nonlinear conservation system for the

$\rho_1, \rho_2, \ldots, \rho_N$, with $\rho_j(\bar{x},t) = \int_{R_3} f_j(\bar{x},\bar{v},t)d\bar{v}$. The problems of solving this latter systems for the ρ_j's, and then reconstructing the sought distribution functions $f_1, f_2, \ldots, f_N$, are discussed for different physical situations. The hyperbolicity of the conservation system is also commented on physical ground.

S. Kawashima:

The Navier-Stokes equation in the discrete kinetic theory

We investigate the Navier-Stokes equation which is formally derived from the discrete Boltzmann equation as the second order approximation of the Chapman-Enskog expansion.

First, we obtain an explicit form of the Navier-Stokes equation without any particular assumptions. Next, we show that if there exists a "hydrodynamical basis" of the space of summational invariants, then our Navier-Stokes equation can be transformed into a symmetric system of hyperbolic-parabolic type. Consequently, the associated Cauchy problem is well posed on a short time interval. Finally, it is shown that the "stability condition" for the original discrete Boltzmann equation guarantees the global existence of solutions of the Navier-Stokes equation.

M. Pulvirenti:

Kinetic Limit for Stochastic Particle Systems

The outstanding problem of deriving the kinetic (Boltzmann) and hydrodynamical equations (Euler, Navier-Stokes) starting from the Newton law, is far to be achieved, basically for the difficulty of knowing the long time behavior of Hamiltonian systems.

We investigate the same questions in a one dimensional model of stochastic particles leading, in the equivalent to the Boltzmann-Grad limit, to the Carleman equation

$$(\partial_t + v\partial_x) f(x,v) = f(x,-v)^2 - f(x,v)^2 \qquad *)$$

where $x \in [0,1]$ + periodic boundary conditions and $v = \pm 1$.

We prove, globally in time, the convergence of the one particle distribution function of our model, to the solution of *) under suitable assumptions. The hydrodynamical limit is more difficult and it is still an open problem.

W. Greenberg:

A Review of One Dimensional Stationary Problems

Various methods of solving one dimensional stationary boundary value problems are examined. The eigenfunction expansion method of Case and the resolvent integration method of Larsen-Habetler are claimed to be equivalent and have been the most widely utilized. The diagonalization method introduced by Hangelbroek allows for functional analytic techniques not available to earlier analysis. Recent interest is in convolution equations techniques, which allow an algebraic approach to the problem of bisemigroup construction. Typical bisemigroup perturbation results are presented, both in Hilbert and Banach space settings. These are relevant to the solution of the abstract transport equation with operator coefficients.

P.F. Zweifel:

Orthogonality Methods for Singular Integral Equations

Classical methods for solving singular integral equations, as discussed, for example, in the book of Muskhilišvili, involve a Hilbert transform and the solution of a Riemann-Hilbert problem. A considerable simplification is provided by generalizing the orthogonality relation introduced by Kuscer et al. in transport theory to general singular integral equations. In addition to simplicity and elegance, this approach has the virtue that it classifies the rather misterious "endpoint condition" on the X-function introduced in an ad hoc manner by Case; they are now seen to arise naturally as conditions for the existence of certain contour integrals. For equations on closed contours one finds additional benefits, for example one can solve problems with zeroes on the integration contour and problems with fractional (even irrational) index by a simple limiting procedure.

K. Dreßler:

Inverse Problems in Linear Kinetic Theory

Inverse problems for a class of linear kinetic equations are investigated. One wants to identify the scattering kernel of a transport equation (corresponding to the structure of a back-ground medium) by observing the albedo-part of the solution operator for a direct (initial-) boundary value problem. In order to do that we derive a constructive method for solving direct half space and slab problems and prove a factorization theorem for the solutions.

Using that we investigate stationary inverse problems with respect to well-posedness (e.g. reduce them to classical ill-posed problems such as integral equations of first kind). In the time dependent case we show that a quite general inverse problem is well posed and solve it constructively.

D. Butler:

Improved Chapman-Enskog Approximation

The Chapman-Enskog approximation scheme is not uniformly convergent because of its behaviour for high velocity molecules. A consequence is that the series of continuum approximations (Euler, Navier-Stokes, Burnett ...) derived by the Chapman-Enskog scheme is only asymptotically convergent.
A modification of the Chapman-Enskog procedure, in which the local Maxwellian, used as the lowest order approximation, is allowed to have parameters which are "slowly varying" functions of velocity is proposed.

R. Esposito:

Statistical Solutions of Boltzmann Equation and Boltzmann Hierarchy

The understanding of the Boltzmann Hierarchy is an intermediate necessary step in the attempt to prove the validity of the Boltzmann Equation. On the other hand, Boltzmann Hierarchy has an intrinsic interest, because it represents the equation for the moments of the statistical solutions of the Boltzmann Equation. It turns out that this interpretation allows to construct solutions to the Boltzmann Hierarchy at least when solutions to the Boltzmann Equation are available. It is possible to deal with the near equilibrium situation, using the individual theorem of Ukai to prove existence and estimates for the solutions to Boltzmann Hierarchy and Lanford's theorem to get uniqueness locally in time and then extend it globally. This approach does not work in the case of spatially homogeneous Boltzmann Equation because Lanford's theorem cannot be used. In this case a method based on an approximate dynamical evolution is worked out, which allows to prove uniqueness of the statistical solution of the Boltzmann Equation and therefore provides existence and uniqueness for the spatially homogeneous Boltzmann Hierarchy.

K. Nanbu:

Stochastic solution method of the Holway model equation for diatomic gas

Rarefied flows of monatomic gas have been calculated successfully by use of direct simulation Monte Carlo method based on the Boltzmann equation. In simulating diatomic gas flows one has had recourse to some heuristic assumptions such as phenomenological model, together with the simulation method for monatomic gas. The result obtained by means of such a patched procedure is not a solution of any kinetic equation. Here is presented a stochastic solution method of the B-G-K type equation for diatomic gas proposed by Holway. The method is applied to the analysis of shock structure of diatomic gas and is shown to work well.

H. Babovsky:

Low Discrepancy Methods for Solving the Boltzmann Equation

Monte Carlo Simulation Methods play an important role for the numerical evaluation of the spatially inhomogeneous Boltzmann equation. They are mostly intuitively motivated and intended to imitate the behaviour of gas particles in a reduced particle system. We investigate the mathematical structure behind one of these schemes (Nanbu's) and prove convergence for it when the number of particles increases to infinity. Since Monte Carlo methods are based on random numbers, fluctuation errors are high. The convergence proof indicates how to replace the stochastic game by a regular scheme (Low Discrepancy methods). We show some examples in the homogeneous case. Finally, we report on results we obtained recently for the calculation of the reentry phase of the European space shuttle Hermes.

J. Wick:

Point approximation for collision terms occuring in semi-conductor problems

The equations

$$\partial_t f + v \cdot \partial_x f + E \partial_v f = \int P(v,v')f(t,x,v')dv' - C(v)f(t,x,v)$$

$$\mathrm{div} E = \int f(t,x,v)dv, \quad \mathrm{rot} E = 0$$

are one model for semi-conductors in the electro-static case. If the collision term is zero, we have the well-known Vlasov-Poisson system, for which the point approximation is well established. In order to extend this method for the semi-conductor case we consider the space homogeneous problem restricted to 1D, which reads as

(1) $$\partial_t f(t,v) = \int P(v,v')f(t,v')dv' - C(v)f(t,v)$$

and try to approximate $f(t,v)dv = \mu_t(v)$ by

$$\mu_t^N(v) = \frac{1}{N}\sum_{i=1}^{N} \delta(v - v_i(t)).$$

We start with the time-discretization of (1)

$$f_{n+1}(v) = (1-\Delta t C(v))f_n(v) + \Delta t \int P(v,v')f_n(v')dv'$$

where Δt is the time-step and $f_n(v) = f(n\Delta t, v)$.

We define the approximation by

$$\frac{2i-1}{2N} = \int_{-\infty}^{v_i^P} f_o(v)dv \quad , \qquad \mu_0^N = \frac{1}{N}\sum_{i=1}^{N}\delta(v-v_i^o)$$

and from $\nu_0^N := (1-\Delta t C(v))\mu_0^N + \Delta t \int P(v,v')\mu_0^N(dv')$ we compute

$$\frac{i}{N} = \int_{-\infty}^{\hat v_i} \nu_0^N(dv), \quad i=1(1)N-1, \quad \hat v_0 = -\infty, \quad \hat v_N = \infty$$

and set $$\mu_1^N = \frac{1}{N}\sum_{i=1}^{N}\delta(v-v_i^1); \quad v_i^1 := N \int_{\hat v_{i-1}}^{\hat v_i} v \, \nu_0^N(dv)$$

which can be continued.

The approximation converges under slightly weak conditions on P and C weakly to the solution in every finite time interval, if $\Delta t \to 0$ and $N\Delta t \to \infty$.

The algorithm is highly vectorizable and gives with N=127 very good results for the master equation. Since the computation of the v_i^n is difficult to extend in higher dimensions we present another approach based on Lagrangian coordinates, for which the numerical results agree with them quoted above.

H.D. Victory:

On the Convergence of Particle Methods for Vlasov-Poisson Systems

For Vlasov-Poisson systems, particle methods are numerical techniques which simulate the behavior of a plasma by a large set of charged superparticles which obey the classical laws of electrostatics. The trajectories of these charged particles are then followed. We give estimates for the errors incurred for a "semidiscrete" approximation to the underlying Vlasov-Poisson system, by first superimposing a rectangular grid or mesh on all of phase space and then replacing the initial continuous distribution of charges or masses by discrete charges or masses located at the centroid of each grid cell. Our analysis, on one hand, generalizes that of G.H. Cottet and P.A. Raviart (SIAM J. Numer. Anal. 21 (1984), pp. 52-76) to higher-dimensional Vlasov-Poisson systems, and, on the other, those fundamental results of Ole Hald (SIAM J. Numer. Anal. 16 (1979), 726-755) and of J.T. Beale and A. Majda (Math. Comp. 39 (1982), 1-52) on vortex methods for two and three-dimensional Euler equations, to particle-in-cell methods for multidimensional Vlasov-Poisson settings.

C. Cercignani:

Existence of L^1 solutions for the 3-d Enskog equation

Recently existence theorems on the Enskog equation in 1 and 2 dimensions have been given by myself and Arkeryd. Here the problem of existence in three dimensions is attacked. The result obtained so far is a global existence theorem for data small in the L^1-norm. A result of this kind is not available for the Boltzmann equation; indeed, if such results were available in that case one could presumably remove the restriction to small data by exploiting the locality in the space variables. The proof is similar to the one used by Tartar for discrete velocity models in one space dimension; it is also the first extension, to the best of my knowledge, of that technique to problems in more than one space dimension. A deeper result in the direction of removing the small data restriction is presently sought for; the main difficulty lies in the nonlocal nature of the collision term not only in velocity but also in space variables.

R. Illner:

The number of collisions in Sinai's billiard in $\mathbb{R}^3$

I present a simple proof that the number of collisions in Sinai's billiard in all space is finite.

I. Kuščer:

On Collision Models for the Non-Linear Boltzmann Equation

For Monte Carlo simulation of rarefied gas flows one needs a suitable model for the intermolecular differential scattering cross section. Whereas for monatomic gases models are easy to construct, rotating molecules present a more difficult task. If the collisional redistribution of energy is pictured as resulting from diffusion in the space of rotational energies, a cross section can be obtained that obeys detailed balance. Via the simplest Chapman-Cowling approximation the model parameters are fitted to the known values of the viscosity and either of the thermal conductivity or the volume viscosity.

F. Golse:

<u>Velocity averaging techniques in kinetic theory and their applications</u>

The basic example of what I call "velocity averaging techniques" is as follows: assume that $f \equiv f(x,\Omega) \in L^2(\mathbb{R}^3 \times S^2)$ and that $\Omega \cdot \nabla_x f \in L^2(\mathbb{R}^3 \times S^2)$. Then the velocity average $\tilde{f}(x) = \int f \frac{d\Omega}{|S^2|}$ belongs to the Sobolev space $H_x^{1/2}$. Applications of these results will be given, among which

- Rosseland approximation for Radiative Transfer Equations;
- the use of velocity averages in the global existence proof for the Boltzmann equation by P.L. Lions and di Perna;
- the homogenization of transport equations.

C. Bardos:

<u>Diffusion approximation for a free gas with a stochastic boundary</u>

This is a report on a joint work with H. Babovsky and T. Platkowsky. One considers a free gas (no collisions) in a tube (or a slab) arbitrarily long and of a thickness of the order ε (ε will go to zero). Then one shows that if the boundary of the tube creates some stochastic effects the solution behaves like the solution of a diffusion equation. The proof relies on classical analysis and asymptotic expansion.

G. Toscani:

On the discrete velocity models with initial data in $L_1^+(\mathbb{R})$

In a recent paper R. Illner showed that the Broadwell model, one of the simplest discrete velocity models of the Boltzmann equation, has a global mild solution for initial data in $L^+_1(\mathbb{R})$ with small L_1-norm. In addition, if these initial values have finite entropy, he showed that a global solution exists, independently of the size of the L_1-norm, when the Boltzmann H-theorem holds. Even if the H-theorem seems reasonable to expect, as a consequence of the kinetic equations, R. Illner did not give a rigorous proof of it, but only a semi-formal discussion.

In this paper we prove that the program can be completed, and so the H-theorem holds, when the initial data $\phi_i(x)$ are such that $(1+|x|^\alpha)\phi_i(x)$ is in $L^+_1(\mathbb{R})$, for some $\alpha>0$, and $\sum_i \int_{-\infty}^{\infty} \phi_i \log \phi_i \, dx$ is in L_1. This allows us to prove the global existence and uniqueness of a mild solution.

R. Glassey:

Existence Theorems for Vlasov-Maxwell

"The initial-value problem for the Vlasov-Maxwell System: A survey".

On $\mathbb{R}^3$ we consider the initial-value problem for

$$\partial_t f+\hat{v}\cdot\nabla_x f+(E+\hat{v}\times B)\cdot\nabla_v f = 0$$

$$\partial_t E = \operatorname{curl} B-j \qquad \nabla\cdot E = \rho$$

$$\partial_t B = -\operatorname{curl} E \qquad \nabla\cdot B = 0$$

where $p = \int f \, dv$, $j = \int \hat{v} f \, dv$, $\hat{v} = \frac{v}{\sqrt{1+|v|^2}}$.

Smooth initial values f_o, E_o, B_o with compact support are prescribed, which satisfy the consistency conditions $\nabla \cdot B_o = 0$, etc. A general sufficient condition for classical existence for all t is given in terms of an a priori estimate on the v-support of f. This estimate can be made when all of the data functions are small in an appropriate sense, and in the "nearly neutral" case.

W. Strauss:

On the existence of smooth solutions of Vlasov equations with collisions

This is a report on work in progress in which R. Glassey and I are generalizing our previous work on the relativistic Vlasov-Maxwell system to allow Boltzmann-type collision terms (with appropriate collision kernels).

Theorem: Assuming (1) initial data in $C^2{}_c$ and (2) the a priori estimate $0 \le f_\alpha(t,x,v) \le be^{-a|v|}$ holds uniformly for x, v and bounded t.

Then there is a unique C^1-solution for all x, v and $t < \infty$.

We are now working on verifying (2) in case the data are close to the relativistic Maxwellian.

J. Batt:

New solutions and results for the Vlasov-Poisson system (VPS)

New results have been obtained in the following three directions:

1. Investigation of the "locally isotopic" solutions which

are of the form

$$f(t,x,v) = \Phi(w(t,x) + \frac{(v-Ax)^2}{2})$$

$$U(t,x) = w(t,x) + \frac{(Ax)^2}{2} ,$$

where f is the distribution function, U the potential and Φ: $\mathbb{R} \to [0,\infty)$ and the antisymmetric 3×3 matrix A are assumed to be given. The function w has to satisfy the nonlinear elliptic equation

$$\Delta w + \lambda = h_\Phi(w) \qquad \lambda \geq 0$$

for t=0. Varying the parameters Φ, A and w one can prove the existence of time-periodic solutions of the VPS without symmetry and cylindrically symmetric stationary solutions (joint work with H. Berestycki, P. Degond and B. Perthame, to appear in Arch. Rat. Mech. Anal.).

2. Investigation of the stationary spherically symmetric solutions which correspond to the solutions of the generalized Emden-Fowler equation (in connection with numerical experiments of Henon's with respect to the stability of these solutions) (joint work with K. Pfaffelmoser, to appear in Math. Meth. in the Appl. Sci.).

3. Existence of C^1-stationary solutions of the relativistic VPS with compact support (to appear in the Proceedings of the Marcel-Großmann meeting on General Relativity, Perth, Australia 1988).

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