MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 24/1988

Orders and their applications

29. Mai – 4. Juni 1988

Die diesjährige Tagung wurde von mir geplant bezüglich der einzuladenden Wissenschaftler und der Vortragenden. Durch den Tod meiner Mutter zwei Tage vor Beginn konnte ich sehr zu meinem Bedauern nicht an der Tagung teilnehmen. In dieser Situation ist Karl Gruenberg (Queen Mary College, London) eingesprungen und hat die Leitung der Tagung vor Ort übernommen. Er hat der Tagung ohne meine Präsenz aber mit häufigem Telefonkontakt mit mir zu einem Erfolg verholfen. Ich möchte ihm hiermit meinen besonderen Dank aussprechen. Dank gilt auch den anderen Tagungsteilnehmern, die zu einem guten Gelingen beigetragen haben.

Die mathematischen Inhalte der Vorträge entstammten den folgenden Gebieten:

1. **Einheiten in Gruppenringen**
   Hier sind insbesondere hervorzuheben die Ergebnisse von
   Al Weiss: Die Konjungiertheit der endlichen Untergruppen der Einheitengruppe in p–adischen Gruppenringen von p–Gruppen,
   J. Ritter–S.K. Sehgal: Die natürliche Konstruktion einer Untergruppe von endlichem Index in der Einheitengruppe des ganzzahligen Gruppenringes von nilpotenten Gruppen (mit einigen durch Kürzung bedingten Ausnahmen),
   W. Kimmerle, der eine alte Vermutung von R. Brauer bewiesen hat, daß nämlich die Charaktertafel einer endlichen Gruppe G bestimmt, ob G zu einer vorgegebenen Primzahl p abelsche p–Sylowgruppen besitzt. Diese sind dann bis auf Isomorphie bestimmt,
   L.L. Scott–K.W. Roggenkamp: Es wurde über ein Gegenbeispiel einer Vermutung von Zassenhaus berichtet, daß nämlich Automorphismen des ganzzahligen Gruppenringes modulo Gruppenautomorphismen zentral sind. Ebenso wurde über ein positives Ergebnis der Zassenhaus–Vermutung berichtet, nämlich für endliche Gruppen G mit $C_G(O_p(G)) \leq O_p(G)$ für eine Primzahl p.

   Weitere Vorträge auf diesem Gebiet wurden von Ch. Bessenrodt, Z. Marcinak, F. Röhl, und H. Zassenhaus gehalten.

2. **Endlicher Darstellungstyp von Ordnungen und globale Dimension**
   Hier ist besonders der Bericht von H. Fujita hervorzuheben, der ein Gegenbeispiel zu einer Vermutung von Tarsy über die globale Dimension von Schur'schen Ord–

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nungen konstruiert hat (diese Vermutung war in speziellen Einzelfällen schon bewiesen worden),
S. Koenig hat die Auslander-Reiten Köcher von Bäckströmordnungen vom zahmen Typ beschrieben, Th. Weichert gab eine Liste der kritischen minimalen einfach zusammenhängenden unendlichen sockelprojektiven Algebren (dies korrespondiert zu den Gitterdarstellungen von lokalen Ordnungen).
In diesem Zusammenhang sind noch die Vorträge von E. Kirkman, L. Klinger, W. Rump, D. Simson und A. Wiedemann zu nennen, die sich mit dem Darstellungstyp spezieller Ordnungen bzw. mit globalen Dimensionsfragen beschäftigten.

3. Arithmetische Fragen und Zusammenhänge mit der Zahlentheorie (Galois-Moduln, Stickelberger-Moduln und optimale Einbettungen) wurden in den Vorträgen von J. Brzezinski, A. Fröhlich und L. McCulloh behandelt.

4. In den Vorträgen von M. Auslander, L. Le Bruyn, E. Dieterich, I. Reiten und M. Van den Bergh ging es um Ordnungen über höherdimensionalen Ringen: Analoga zu Auslander-Reiten Sequenzen, Kurvensingularitäten, Rationalitätsprobleme, graduierte Ringe und deren Vervollständigungen sowie über Invariantenringe.

5. J.A. Green gab einen Übersichtsvortrag über Darstellungsringe , K. Gruenberg und A. Jones trugen über kohomologische Fragen bei Gruppenringen vor, A. O. Kuku über die Fragen der endlichen Erzeugbarkeit höherer K-Gruppen.

6. E. Kleinert berichtete über mehrfache Pullback-Konstruktionen für Ordnungen, R. Guralnick und L.Levy über Präsentationen von Moduln, T.Y.Lam über Gleichungen in p-Gruppen.

Dem Andenken an Irving Reiner war der abendliche Vortrag von W. Gustafson über das wissenschaftliche Werk von I. Reiner gewidmet.

VORTRAGSAUSZÜGE

M. AUSLANDER: Cohen–Macaulay approximations

Let R be a complete local Cohen–Macaulay ring having a dualizing module. Buchweitz and I have shown the following. For each finitely generated R–module there is a unique (up to isomorphism) exact sequence, called a Cohen–Macaulay approximation of C,

$$0 \to Y_C \to X_C \to C \to 0$$

satisfying the following:

a) X_C is a Cohen–Macaulay module

b) inj.dim.(Y_C) is finite and

c) no indecomposable summand of X_C is contained in Y_C

The lecture was devoted to showing various consequences of the existence and properties of Cohen–Macaulay approximations including a way of computing the multiplicity of hypersurfaces.

CH. BESSENRODT: Some new invariants for blocks

Let G be a finite group, $\mathbb{Z}_p$ the p–adic integers. In 1986, Scott asked whether the defect group $\delta(B)$ of a p–block B of $\mathbb{Z}_pG$ is determined up to conjugation and "normalisation" by the block, independently of the group G ; weakening this, Alperin asked whether at least the isomorphism type of $\delta(B)$ is determined by B.

Using some new cohomological invariants, we can give a contribution to this question even for more general coefficient rings A such as complete discrete valuation rings with residue field of characteristic p , or even fields of characteristic p . For certain classes of p-groups D , including in particular abelian p-groups, the invariants for AD and for blocks B of AG with D as a defect group coincide. In the abelian case, the invariants for AD determine the isomorphism type of D . Thus, if we know in advance that the defect group is abelian, we can determine its isomorphism type from the block.

J. BRZEZINSKI: Optimal embeddings of orders

Let R be a Dedekind ring with quotient field K, and let A be a central simple K-algebra. Let Λ be an R-order in A and let $\Lambda_1 = \Lambda, \Lambda_2, \ldots, \Lambda_t$ represent all isomorphism classes in the genus of Λ. Let S be an R-order in a commutative separable K-algebra L. An R-embedding $\phi\colon S \longrightarrow \Lambda$ is optimal if $\Lambda/\phi(S)$ is R-projective. If A is a quaternion algebra, K is global, Λ an intersection of two maximal orders and $\mathrm{rank}_R S = 2$, then Eichler's formula says that

$$\Sigma_{i=1}^{t} H(\Lambda_i) e_{\Lambda_i^*}(S,\Lambda_i) = h(S) e_{U(\Lambda)}(S,\Lambda),$$

where $H(\Lambda_i)$ is the two-sided class number of Λ_i, $e_{\Lambda_i^*}(S,\Lambda_i)$ is the number of optimal embeddings $\phi\colon S \longrightarrow \Lambda_i$ modulo the natural action of the unit group Λ_i^* on them, h(S) the class number of S, and $e_{U(\Lambda)}(S,\Lambda)$ is the number of local optimal embeddings $\phi = (\phi_p)$, $\phi_p : S_p \longrightarrow \Lambda_p$ modulo the natural action of $U(\Lambda) = \Pi_p \Lambda_p^*$, $p \in \mathrm{Spec}\, R$.
We give a general version of the above equality for arbitrary Λ and (with some modifications) for arbitrary S, as a special case of a still more general formula for optimal embeddings of lattices with tensor structure. The result follows from purely combinatorial

considerations concerning transitive actions of groups on pairs of sets and relations invariant with respect to these actions. We also present several applications.

E. DIETERICH: Einige zahme Kurvensingularitäten

Sei k ein algebraisch abgeschlossener Körper der Charakteristik 0, $C \subset \mathfrak{A}^n(k)$ eine affin-algebraische Kurve mit singulärem Punkt $o \in C$, und sei $\Lambda = \hat{\mathcal{O}}_{C,o}$ der vollständige lokale Ring der Singularität (C,o).

Problem: Wie kann man diejenigen Kurvensingularitäten $\hat{\mathcal{O}}_{C,o}$, die von zahmem Typ sind, charakterisieren? Und wie sieht die Lösung des Klassifikationsproblems für ihre jeweiligen Gitterkategorien aus?

Diese Fragen beantworte ich für eine spezielle Klasse von Kurvensingularitäten, nämlich für $\mathcal{C} = \{\Lambda = \hat{\mathcal{O}}_{C,o} \mid b = 4 \text{ und } \mathfrak{F} \supset \mathrm{rad}^2\,\Omega\}$, wobei b die Anzahl der Zweige von (C,o) ist, Ω die Normalisierung von Λ, und $\mathfrak{F}$ der Führer von Ω nach Λ: in $\mathcal{C}$ gibt es $6 + 1 \cdot \infty$ analytische Isomorphieklassen zahmer Singularitäten; davon sind 3 domestiziert, die unendliche Serie ist nicht domestiziert von endlichem Wachstum (tubulär vom Röhrentyp (2,2,2,2)), und 3 sind von unendlichem Wachstum.

A. FRÖHLICH: Factorizability and Galois modules

Γ a finite group. A homomorphism f from the Burnside ring B_Γ into an abelien group A is factorizable if it factorizes as:

$$\begin{array}{ccc} B_\Gamma & \longrightarrow & R_\Gamma(\mathbb{Q}) \\ & {\scriptstyle f}\searrow & \downarrow \\ & & A \end{array}$$

(rational character ring).

The use of this notion in Galois module theory was outlined.

H. FUJITA: **Tiled orders of finite global dimension**

We introduce a projective link between maximal ideals of an arbitrary ring with identity, with respect to which an idealizer preserves being of finite global dimension. Let D be a local Dedekind domain with the quotient ring K. When $2 \leq n \leq 5$, every tiled D-order of finite global dimension in $(K)_n$ is obtained by iterating the idealizers w.r.t. projective links from a hereditary order. If $n \geq 6$ then there exists a tiled D-order in $(K)_n$ which does not have the above property. This is also a counterexample to Tarsy's conjecture. Using the above result, a list of the representatives of isomorphism classes of tiled D-orders of finite global dimension in $(K)_n$ is obtained where n = 4, 5.

J. A. GREEN: **Representation rings**

The representation rings a'(kG), a(RG), A(kG), A(RG) have been studied since about 1962; Irving Reiner and his pupils made important contributions by constructing non-zero nilpotent elements in these rings in suitable cases. Reiner, with Hannula & Ralley also used the bilinear form $\Omega(V,W) = \dim \mathrm{Hom}_{kG}(V,W)$ on a(kG) to give an especially short proof of the semisimplicity of $a(kC_q)$. More recently, Benson & Parker have used Auslander-Reiten sequences in further study of the form Ω.

K. W. GRUENBERG: **Resolutions of periodic lattices**

Let A be a periodic lattice over $\mathbb{Z}G$ (G a finite group). This means $\mathrm{Ext}^{n+q}_{\mathbb{Z}G}(A,-)$ is naturally equivalent to $\mathrm{Ext}^{n}_{\mathbb{Z}G}(A,-)$ for all $n \geq 1$. The minimum such q is the projective period of A.

1.) A is periodic if, and only if, A has a periodic projective resolution; and A also has a periodic free resolution of period some multiple of q. What is the relation between the free and the projective periods?

2.) Two minimal projective resolutions with the same rank sequences are in the same genus as augmented complexes over A.

This fails for minimal free resolutions and, moreover, a periodic A need not have a periodic minimal free resolution. However, if $\mathbb{Z}G$ allows cancellation, all minimal free resolutions lie in one genus; and if $\mathbb{Z}G$ is not a summand of the periodic A, then A has a periodic minimal free resolution.

3.) For given A, we may define a sequence $\sigma_n(A)$ of invariants of A: these are elements in various factor groups of the projective class group of $\mathbb{Z}G$. They generalize the Swan obstruction for $\mathbb{Z}$ of projective period q: $\sigma_{q-1}(\mathbb{Z}) = 0$ if, and only if, q is also a free period. We show that if A has projective period q, then q is essentially also a free period if, and only if, $\sigma_{q-1}(A) = 0$.

R. GURALNICK: Uniqueness of presentations of modules

Let Γ be a (possibly noncommutative) pid. It is well known that any matrix order Γ is equivalent to a diagonal matrix where each term is a total divisor of the next. In the commutative case, these diagonal terms (up to units) are also a complete set of invariants for the equivalence class of the matrix. Nakayama lamented the fact that this is not true if Γ is noncommutative. We shall discuss:

Theorem: (G., Levy, Odenthal) If $B, A \in \Gamma_{m \times n}$ have rank ≥ 2, then A + B are equivalent $\Leftrightarrow$ coker A $\cong$ coker B.

Given $A \in \Gamma_{m \times n}$, one has a presentation $\Gamma^n \to \Gamma^n / A\Gamma^m$. The theorem above can be stated in terms of uniqueness of presentations and this guise can be extended to orders which are not pid's.

W. GUSTAFSON: The representation ring of a group of prime order

Let G be a group of prime order p. We first develop the classification of $\mathbb{Z}G$-lattices, using a method that avoids extensive matrix manipulations. We then calculate the representation ring $a(\mathbb{Z}G)$. Additively, $a(\mathbb{Z}G) \cong \mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z} \oplus \mathscr{C}$, where $\mathscr{C}$ is the ideal classgroup of $\mathbb{Z}[\xi_p]$, $\xi_p = e^{2\pi i/p}$. We then calculate the multiplication in $a(\mathbb{Z}G)$.

W. GUSTAFSON: The mathematical work of Irving Reiner

We review Reiner's work on quadratic forms, classical groups and orders, emphasizing those topics that have most influenced the development of integral representation theory.

A. JONES: Lattices with a condition on exponent $\hat{\mathrm{Ext}}^0_{RG}(M,M)$

Joint work with Jon Carlson: Let R be a complete discrete valuation ring with maximal ideal ΠR, G finite; M,N RG-lattices. Let $\underline{\mathrm{Hom}}(M,N)$ be homomorphisms modulo projectives. For $\alpha \in \mathrm{Hom}_{RG}(M,N)$ let $\exp \alpha \simeq \Pi^a$ if $\Pi^a \cdot \alpha$ factors through projectives but $\Pi^{a-1} \cdot \alpha$ does not. Let $\exp M = \exp \mathrm{id}_M$.

For M with $\exp M \simeq \Pi^a$ and almost split sequence $0 \longrightarrow \Omega M \xrightarrow{\alpha} E \xrightarrow{\beta} M \longrightarrow 0$, The following are equivalent:

1. $E = \Omega(M/(\Pi^{a-1}M))$

2. socle $\underline{\text{Hom}}(M,M) = \Pi^{a-1}\underline{\text{Hom}}(M,M)$,

3. exp $\alpha <$ exp M,

4. exp $\beta <$ exp M,

5. exp $E' <$ M,

6. If γ: N $\longrightarrow$ M is not split epi, exp $\gamma <$ exp M.

7. If γ: M $\longrightarrow$ N is not split mono, exp $\gamma <$ exp M.

The condition is conserved under Green correspondence and taking sources for absolutely indecomposable lattices. Jacques Thévenaz has shown that the absolutely indecomposables that satisfy this condition are the Knörr lattices.

W. KIMMERLE: Hall subgroups, isomorphic integral group rings and a question of R. Brauer

Let G be a finite group and let $\mathbb{Z}G$ be its integral group ring.

Theorem 1 (joint work with R. Sandling): $\mathbb{Z}G$ determines hamiltonian (in particular abelian) Hall subgroups of G up to isomorphism.

Theorem 2: The character table of G determines abelian Sylow subgroups up to isomorphism.

The proof of both results is based on the earlier result that $\mathbb{Z}G$ determines the chief series of G (joint paper with R. Lyons and R. Sandling). Theorem 2 answers an old question of R. Brauer (Reps. of finite groups, lectures on modern math., Vol. I, pp. 133–175, problem 12, 1963). The chief series result holds even with respect to character tables. All results are proved making use of the classification of the finite simple groups.

E. E. KIRKMAN: Tiled orders of finite global dimension

Let D be a DVR with maximal ideal (x), and let Λ be an order of finite global dimension $\Lambda \subseteq M_n(D)$, the ring of n by n matrices over D. Tarsy conjectured that gldim $\Lambda \leq n-1$. I will summarize what is known about the conjecture, including my work showing that the conjecture is true when Λ is a tiled (Schurian) order which contains the ideal $M_n((x))$. I will show how orders of finite global dimension are related to Artin algebras of finite global dimension. I will summarize some of what is known about the valued quiver of Λ when gldim $\Lambda = 2$, and I will suggest some questions about the structure of the valued quiver of Λ when Λ has finite global dimension.

E. KLEINERT: Orders and multiple pullbacks

To every semisimple order Λ there is associated a canonical overorder $\Lambda^{\sim}$ which can be characterized as the unique minimal overorder of Λ which is a multiple pullback. Several basic properties of $\Lambda^{\sim}$ are established; the question is treated when $\Lambda = \Lambda^{\sim}$.

L. KLINGLER: Integral group rings of finite representation type

I study integral representations of groups all of whose Sylow subgroups are cyclic (of at most prime squared order).I view the integral group ring as an "iterated pullback" of hereditary coordinate rings and describe the lattice theory of the group ring in terms of lattices over these hereditary coordinate rings.

At the moment, I am focussing on groups G of square-free order. I determine, for example, which such groups have the property that every (left) $\mathbb{Z}G$-lattice is isomorphic to a (left) ideal of $\mathbb{Z}G$.

S. KÖNIG: Zahme und wilde verallgemeinerte Bäckströmordnungen und sockelprojektive Kategorien

Ausgehend von einem Klassifikationssatz, der sockelprojektive Kategorien (SK) und damit (nach Ergebnissen von Ringel und Roggenkamp) auch verallgemeinerte Bäckströmordnungen (VBO) in die drei Klassen "endlicher Darstellungstyp", "zahm" und "wild" einteilt, werden die Auslander-Reiten-Köcher (ARK) der SK unendlichen Typs untersucht. Als zentral erweist sich dabei der Begriff "reduzibel": Der Graph γ einer solchen SK ist genau dann reduzibel, wenn ein einfacher präprojektiver, aber nicht projektiver γ-Modul existiert. SK zu reduziblen Graphen unterscheiden sich nur in der präprojektiven Komponente von den SK der zugehörigen nicht reduziblen Graphen. Die ARK der zahmen VBO und SK werden vollständig bestimmt; im wilden Fall wird die Struktur der Komponenten angegeben.
Bei wilden erblichen Algebren wird die Verteilung der sockelprojektiven Moduln auf die regulären Komponenten des ARK untersucht; dabei werden alle möglichen Anordungen dieser Moduln bestimmt, wobei wiederum zwischen reduzibel und nicht reduzibel zu unterscheiden ist.

A. O. KUKU: Some finiteness results in the higher K-theory of orders and group rings

Let R be the ring of integers in an algebraic number field F, Λ an R-order in a semi-simple F-algebra Σ. It is well known that for $n = 0, 1$, $K_n(\Lambda)$, $G_n(\Lambda)$ are finitely generated Abelian groups and $SK_n(\Lambda)$, $SG_n(\Lambda)$ are finite groups. However, answers to questions as to whether or not such finiteness results hold for $n \geq 2$ have been outstanding for some time. We now answer these questions positively.

Theorem I: For $n \geq 1$

(i) $K_n(\Lambda)$ is a finitely generated Abelian group

(ii) $SK_n(\Lambda)$ is a finite group

(iii) $SK_n(\hat{\Lambda}_{\mathfrak{p}})$ is finite (or zero) for any prime ideal $\mathfrak{p}$ of R

Theorem II: For all $n \geq 1$

(i) $G_n(\Lambda)$ is a finitely generated Abelian group

(ii) $SG_{2n-1}(\Lambda)$ is finite; $SG_{2n-1}(\Lambda_{\mathfrak{p}})$, $SG_{2n-1}(\Lambda_{\mathfrak{p}})$ are finite groups of order relatively prime to the prime p lying below $\mathfrak{p}$

(iii) $SG_{2n}(\Lambda), = SG_{2n}(\Lambda_{\mathfrak{p}}) = SG_{2n}(\hat{\Lambda}_{\mathfrak{p}}) = 0$

Theorem III: If π is a finite group, then for all $n \geq 1$ $G_{4n+3}(\mathbb{Z}\pi)$, $G_{4n+3}(\mathbb{Z}_p\pi)$ are finite groups.

T. Y. LAM: On the number of solutions of $x^{p^k} = a$ in a p–group

This work is a contribution toward the enumeration problem in group theory. The following result is obtained, among others: Let G be a finite group, and $H \subseteq G$ be a p–elementary abelian normal subgroup of order p^r. Then for any central element $a \in Z(G)$ and any integer $k \geq 1$, the number of solutions of the equation $x^{p^k} = a$ in G is divisible by $p^{r-[r/p^k]}$. Variations of the methods used for proving this theorem also lead to new proofs of a theorem of Kulakoff, and a theorem of Huppert and Berkovich. This talk is probably not in the main-stream of the theory of orders, but has some relations with the modular representations of finite groups.

L. LE BRUYN: The rationality problem for the n-subspace problem;

Vector bundles and the Jacobian conjecture

GL(N) acts on the product of Grassmannians $GR_n = \overset{n}{\underset{i=1}{X}} \text{Grass}(n_i,N)$.If this action has stable points one can construct a nice quotient variety: $GR_n/GL(N)$. One of the main open problems is whether this quotient is (stably) rational. We relate this to the problem of matrix invariants and thereby show

(a) stable rationality if $g.c.d.(n_1,..,n_n,N) \leq 4$

(b) retract rationality if $g.c.d.(n_i;N)$ is squarefree

(c) the desingularization of $GR_n/GL(N)$ has trivial Brauer group.

Presently I am trying to prove stable rationality in general by considering finite dimensional representations of the preprojective algebra of the path algebra of the n-subspace quiver.

L. LEVY: Non-uniqueness of presentations of modules

(joint work with R. Guralnick)

We extend elementary divisor theory by studying equivalence classes of presentations of Λ-modules, where Λ belongs to a class of rings that includes global orders and coordinate rings of affine curves. Here $f,g: P \twoheadrightarrow U$ are called **equivalent** if $g = \alpha \cdot f \cdot \beta$ for automorphisms β and α of P and U, respectively.

The set $\mathcal{P}$ of all equivalence classes of presentations: $P \twoheadrightarrow U$ (P, U fixed) has a natural group structure, and this group can be either finite or infinite. Two sample properties of $\mathcal{P}$ are:

(i) When Λ is a global order, $\mathcal{P}$ is a finite group whose order has a bound independent of P and U.

(ii) When Λ is a commutative (geometric) ring and U has finite length, $\mathcal{P}$ is a (possibly infinite) torsion group of finite exponent, i.e. there exists an n such that for any two $f,g \in P$, $\oplus^n f$ is equivalent to $\oplus^n g$.

Z. MARCINIAK: Units in the integral group ring of D_∞

This is a report on a joint work with M. Mirowicz. We completely determine the structure of the group of units in $\mathbb{F}_2 D_\infty$, where D_∞ is the infinite dihedral group. We prove that the group of normalized units in the integral group ring $\mathbb{Z}D_\infty$ cannot be finitely generated. For a Bieberbach group Γ we prove that all finite subgroups in $U(\mathbb{Z}\Gamma)$ can be found in $U(\mathbb{Z}G)$ where G is a point group of Γ.

L. McCULLOH: On Stickelberger modules for group rings

For a finite group G, a $\mathbb{Q}$-bilinear map $<\,,\,> : \mathbb{Q}R_G \times \mathbb{Q}G \longrightarrow \mathbb{Q}$ (where $\mathbb{Q}R_G$ is the $\mathbb{Q}$-span of the virtual character ring R_G) is defined as follows: For a character χ of degree one, and $s \in G$, $<x,s>$ is defined by $0 \leq <x,s> < 1$ and $\chi(s) = e^{2\pi i <\chi,s>}$. For an arbitrary character χ, $\mathrm{res}^G_{<s>}\chi$ is a sum of characters of degree one of $<s>$ and we put $<\chi,s> = <\mathrm{res}^G_{<s>}\chi,s>$. A Stickelberger map $\Theta_G : R_G \longrightarrow c(\mathbb{Q}G)$ (center of $\mathbb{Q}G$) is

defined by $\Theta_G(\chi) = \sum_{s \in G} <x,s> \cdot s$, and Stickelbergers module $S_G = \mathbb{Z}G \cap \Theta_G(R_G)$.
Among numerous relations between S_G and the class group $Cl(\mathbb{Z}G)$, we can show $[c(\mathbb{Z}G)^- : S_G{}^-] = |Cl(\mathbb{Z}G)^-|$ when G is abelian of type $(p^n,\dots,p^n)$ or non-abelian of order p^3 (p odd prime). (The "minus" parts are with respect to the canonical involution $s \longrightarrow s^{-1}$ of G.)

I. REITEN: Graded rings and their completions

Let k be a field and $T = k[X_1,\dots,X_n]$ a $\mathbb{Z}$-graded ring with $\deg X_i > 0$, $j: T \longrightarrow \Lambda$ a $\mathbb{Z}$-graded T-algebra, such that Λ is a finitely generated free T-module and gl. dim $\Lambda_{\mathfrak{p}} = \dim T_{\mathfrak{p}}$ when $\mathfrak{p}$ is a nonmaximal prime ideal in T. Completing at $\mathfrak{m} = (X_1,\dots,X_n)$, we prove that the category $CM(\mathrm{gr}\ \Lambda)_0$ of finitely generated graded C.M. modules with degree 0 maps has only a finite number of indecomposable objects if and only if $CM(\hat{\Lambda})$ does. We also prove that almost split sequences in $CM(\mathrm{gr}\ \Lambda)_0$ go to almost split sequences in $CM(\hat{\Lambda})$ under completion.

There are generalzations to other groups than $\mathbb{Z}$ (Then almost split sequences may have to be replaced by direct sums of almost split sequences). The talk will be based on joint work with M. Auslander.

J. RITTER: Construction of units in integral group rings of finite nilpotent groups

This is the second part of the presentation of a joint paper with S. K.Sehgal. Proofs are given for the three main lemmas, namely:

1. Let T and T' be two absolutely irreducible representations of the non-abelian p-group G, p odd. Assume that T is faithful. Then there is a maximal subgroup M of G and an element $a \in M$ of order p such that $T = \mathrm{ind}_M^G T_M$, $T_M(a) = 1$, $T_M(a^b)$ has no eigen-value 1 for any $b \notin M$, and either $T'(a) = 1$ or $T'(a)$ has no eigen-value 1 or $T' = \mathrm{ind}_M^G T'_M$.

2. Each projection T of $\mathbb{Q}G$ onto a Wedderburn component maps $\mathbb{Z}G$ into the matrices over the ring of integers $\mathcal{O}$ in the centre field.

3a. The group $< T(1+(a-1)\cdot b\cdot \hat{a}) \mid a, b \in G >$ is of finite index in $SL(\mathcal{O})$, provided T is non-abelian. Here $\hat{a} = 1 + a + a^2 + \ldots + a^{\mathrm{ord}(a)-1}$; $1 + (a-1)\cdot b\cdot \hat{a}$ is, by definition a Bicyclic unit.

3b. If T and T' are two different non-abelian projections, then there is a product c of Bicyclic units with $T'(c) = 1$, $T(c) =$ non-central.

F. ROEHL: Group rings of p-groups over fields of characteristic p

In connection to the modular isomorphism problem, the following question is of interest: If V is a set of words, G a finite p-group and $(\Delta G, \cdot)$ the circle group of the augmentation ideal ΔG of $\mathbb{F}_pG$, under what conditions does the verbal subgroup V(G) have the following property

$$* \qquad 1 + V(\Delta G, \circ) = V(G).$$

If gr G denotes the graded Lie-p-algebra associated to G with respect to its modular dimension subgroup series, one has $\mathrm{gr}(\Delta G, \cdot) \xrightarrow{\simeq} \mathrm{gr}\, G \oplus C$, a Lie-p-ideal. This shows that an object closely related to G always admits a complement in an object rather close to the group of normalized units of $\mathbb{F}_pG$, and moreover, that modular dimension subgroups have (*). Also for certain other verbal subgroups it is possible to use the above decomposition in order to show that they satisfy (*).

W. RUMP: A stability theorem for representation–finite orders

Stability properties have been observed for representation–finite structures such as posets, quivers, and finite dimensional algebras. For example, Drozd showed that the indecomposables of representation–finite posets are in "general position"; Gabriel proved that finite representation type is "open"; and Roiter et al. showed that representation–finite algebras possess multiplicative bases. The general stability problem for a group which operates on a set may be stated thus: characterize the orbits by discrete invariants! In our talk we shall consider the set $\mathscr{I}_\Lambda$ of irreducible representations of an order Λ in a simple algebra A, the elements of $\mathscr{I}_\Lambda$ belonging to a fixed simple A–module. Then $\mathscr{I}_\Lambda$ is a lattice on which the unit group $G = D^\times$ of the skewfield part D of A operates, and the isomorphism classes of irreducibles in $\mathscr{I}_\Lambda$ coincide with the orbits on $\mathscr{I}_\Lambda$. Our theorem states that $\mathscr{I}_\Lambda$ is G–stable for representation–finite Λ.

L. L. SCOTT: On a conjecture of Zassenhaus on finite group rings

Zassenhaus had conjectured that, whenever $\mathbb{Z}G = \mathbb{Z}H$ as augmented $\mathbb{Z}$–algebra, for finite groups G,H, then G is conjugate to H in $\mathbb{Q}G$. Klaus Roggenkamp and I have obtained many positive results on this conjecture. However, we now believe we have a counter–example G of order $2^6 \cdot 3^2 \cdot 5$.

The lecture discusses many details of the example, including its raison d'être in terms of central automorphisms and a 1–cohomology–style obstruction theory.

L.L.SCOTT: The isomorphism problem: Defect groups, the $\mathbb{Z}^*$ theorem, and philosophical remarks

This is joint work with Klaus Roggenkamp. I discuss briefly the ingredients of our theorem which establishes the Zassenhaus conjecture (and thus a positive answer to the isomorphism problem) for finite groups G satisfying $C_G(O_p(G)) \leq O_p(G)$ for some prime p. These ingredients include a Green correspondence theory for automorphisms of blocks stabilizing a defect group, a study of Coleman's theory of normalizers in unit groups of p-subgroups of G, and Weiss's new results on permutation modules.

I also discuss an application of these permutation module methods to give a positive answer, in the case of a cyclic, T.I. set Sylow p-subgroup,to the question of conjucacy of defect groups in blocks, for the principle block, with the second defect group the image of the Sylow group (or, rather, its projection on the principal block B_o under an augmentation preserving automorphism of B_o.

It is mentioned that the general defect group conjugacy problem for principal block defect groups implies the $\mathbb{Z}^*$ theorem for $p > 3$, through a reduction of G. Robinson.

S.K. SEHGAL: Construction of units in integral group rings of finite nilpotent groups I

Let $U = U\mathbb{Z}G$ be the group of units of the integral group ring $\mathbb{Z}G$. In this talk we give a set of generators of a subgroup of finite index in U, if G is a finite nilpotent group with

$$* \qquad \mathbb{Q}G = \Sigma\oplus (K_i)_{n_i \times n_i}, \ K_i \text{ fields, } K_i \neq \mathbb{Q} \text{ or } \mathbb{Q}(i) \text{ if } n_i = 2.$$

Let $|G| = n$, $\phi(n) = m$. Let $a \in G$ and $(i,o(a)) = 1$. Then

$$u = (1+a+\ldots+a^{i-1})^m+((1-i^m)/o(a))\cdot \hat{a}\ , \ \hat{a} = 1+a+\ldots+a^{o(a)-1}$$

is a unit of $\mathbb{Z}$<a>. We call the units u , obtained by varying $a \in G$ and i , the Bass cyclic units of $\mathbb{Z}G$ and denote by B_1 the group generated by them.

Theorem of Bass: If $G = A$ is abelian then $(U\mathbb{Z}A : B_1) < \infty$.

We define for $a,b \in G$, $u_{a,b} = 1+(a-1)ba\hat{}$ the bicyclic units of $\mathbb{Z}G$ by varying $a,b \in G$. Let $B_2 = < u_{a,b} | a,b \in G >$, $B = < B_1, B_2 >$.

Theorem: If G is nilpotent, satisfying (*), then $(U\mathbb{Z}G : B) < \infty$.

We give an example of a 2-group for which $(U\mathbb{Z}G : B) = \infty$.

The proof of the theorem is reduced to three statements which are proved by J. Ritter in a second talk. This work is joint work of the speaker with J. Ritter.

D. SIMSON: Matrix problems with applications for the classification problems of modules

A semiperfect ring R is a right peak ring if $soc(R_R)$ is essential in R and $soc(R_R) \simeq P_*^t$ for $t < \infty$ and a simple projective module P_* . Our main interest is to classify indecomposables in $mod_{sp}(R)$ – the category of f.g. socle projective right R-modules. For any such ring R one can associate an order Λ such that $lat(\Lambda)$ and $mod_{sp}(R)$ are representation equivalent.

If R is artinian and schurian (i.e. eRe is a field for all primitive idempotens $e \in R$) we associate to R a valued poset $(I_R{}^*,d)$. We prove that $mod_{sp}(R)$ is of finite type, iff $(I_R{}^*,d)$ does not contain a list of 12 extended Dynkin diagrams and two forms

$$\bullet \xrightarrow{(d,d')} \bullet, \quad \bullet \xrightarrow{(d,d')} \bullet \xleftarrow{(e,e')} \bullet, \quad d \cdot d', \; e \cdot e' \geq 2.$$

Moreover we present a list of 30 right peak rings R_i, $i \leq 30$, and a list of 82 indecomposable R_i–modules X_j, $j \leq 82$ such that if $mod_{sp}(R)$ is of finite type, R–schurian and X is an indecomposable in $mod_{sp}(R)$ then there exists i,j such that $X \simeq T_i(X_j)$, where

$$T_i\colon mod_{sp}(R_i) \longrightarrow mod_{sp}(R) \text{ is a tensor product type functor.}$$

Applications to the classification of indecomposable lattices will be discussed.

M. VAN DEN BERGH: Cohen Macaulayness of invariant modules.

If R is a regular ring and G is a reductive group acting rationally on R then R^G is Cohen Macauley by the famous Hochster Roberts theorem. However if F is free R,G module then F^G is not necessarily Cohen Macauley. We get some partial results in the case that $G = SL_2$ and in particular we recouver L. Le Bruyn's result that the trace ring of generic matrices is CM.

Th. WEICHERT: The representation type of algebras with projective socle

Let S be a k-algebra with projective socle over an algebraically closed field k .
All such algebras occur in the representation theory of lattices over orders. Denote the category of socle-projective S-modules by $\mathcal{F}(S)$. We should decide wether one given algebra S is of finite representation type with respect to $\mathcal{F}(S)$ or not. Ringel and Roggenkamp defined reduction of S which does not change the $\mathcal{F}$-representation type. Ifan hereditary algebra cannot be further reduced, then S is $\mathcal{F}$-representation finite iff the diagramm of S is Dynkin. For simply connected algebras with projective socle we define a strong-reduction-process which can change the $\mathcal{F}$-representation type of S and give a list of ca. 300 $\mathcal{F}$-critical algebras such that a socle projective simply connected algebra is $\mathcal{F}$-representation infinite iff it can be strong-reduced to one of these algebras or it contains a socle chain.

A. WEISS: Torsion units in $\mathbb{Z}G$ via permutation lattices

Given finite groups H, G the goal is to classify the group of homomorphisms $\phi: H \longrightarrow U_1(\mathbb{Z}G)$, with U_1 the augmentation 1 units, up to conjugation by units of $\mathbb{Z}G$. The 'double action' construction associates to ϕ a lattice $M(\phi)$ for the group $H \times G$, which classifies ϕ. In the spirit of integral representation theory it is then natural to place homomorphisms ϕ, ϕ' in the same genus precisely when they are conjugate by p-adic units for all p and to emphasize the tentative

Genus conjecture: Every ϕ has a group homomorphism $\sigma: H \longrightarrow G$ in its genus.

This sharper version of a conjecture of Zassenhaus holds for p-group G and yields very complete information in that case. More generally it is perhaps too optimistic but is never the less suggestive as a model of the goal.

A. WIEDEMANN: Some Auslander orders of finite lattice type

Let R be a complete discrete valuation ring, Λ a connected R-order of finite lattice type. Let $A(\Lambda)$ be the Auslander order of Λ. If $A(\Lambda)$ is again of finite lattice type, then denote by $A^2(\Lambda)$ the Auslander order of $A(\Lambda)$ etc. We give some answers to the following questions of M. Auslander:

1.) When is $A(\Lambda)$ again of finite lattice type?

2.) When exist $A^i(\Lambda)$ for al $i \in N$?

The fact that 2.) holds possitively for an artin algebra A iff A is semisimple gives the answer to 2.) which is essentially due to C. Munroe.

Theorem 1: For Λ all $A^i(\Lambda)$ exist iff Λ is a Bäckström order with associated graph a disjoint union of Dynkin diagrams of types $\mathbb{A}_2$, $\mathbb{A}_3$, $\mathbb{B}_2$, $\mathbb{C}_2$.

To question 1.):

Theorem 2: If Λ is a generalized Bäckström order with associated graph $G(\Lambda)$, then $A(\Lambda)$ is of finite lattice type iff $G(\Lambda)$ is a disjoint union of graphs of the form

Theorem 3: If Λ is a local order, $R \supset R/\mathrm{Rad}R$, then $A(\Lambda)$ is of finite lattice type iff Λ (is Bass and) has at most one nonhereditary proper over order.

H. ZASSENHAUS: Brauer Invariance II

This report on joint work with Sudarshan S. Sehgal and Surinder K. Sehgal expands and gives detailed proof for the result partially announced 4 years ago. The finite group G is said to be **Brauer invariant** if $\mathbb{Z}G = \mathbb{Z}H$, $H \subset U_1(G)$, $H \simeq G$ implies that H is conjugate to G under the inner automorphism group of $\mathbb{Q}G$. It is **Higman invariant** if $\mathbb{Z}G = \mathbb{Z}H$ implies $G \simeq H$ (s. G. D. Higman's thesis, article of R. Sandling). G is an A-group if every Sylow group is abelian.

It is shown that every solvable A-group is Brauer-Higman invariant.

Full use of the maximal over orders of the group orders involved is the main tool. Michler's question regarding arbitrary Sylow tower groups is still open.

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Gefördert durch
DFG Deutsche Forschungsgemeinschaft