MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 3/1989

Mathematische Spieltheorie

15.1. bis 21.1.1989

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This was the third meeting on Mathematical Game Theory in Oberwolfach and similar to previous occasions, participants came from various countries of Europe, the United States, Russia, India, Japan and the Middle East. Lectures dealing with progress in certain areas of research were dominating the meeting. But there were also informal research groups being formed and the (almost traditional) meeting on "Open Problems" took place on Thursday evening.

A certain emphasis was put upon cooperative game theory, mainly with respect to the characteristic (or coalitional) function representing games with and without side–payments. Topics were ranging from the discussion of the structure of certain classes of games to the introduction of new solution concepts as well as to the presentation of new results concerning well known solution concepts (like the nucleolus, the von Neumann–Morgenstern solution, and the Shapley–value). Also the question of how to formalize the formation of coalitions in a cooperative set up was being discussed.

One of the main stream research problems is, generally speaking, the question of how to implement cooperation. More formally: how can the pareto points of a one–shot–game be sustained by appropriately defined Nash–equilibria (subgame perfect

Nash–equilibria, perfect Nash–equilibria, etc.) of the corresponding supergame (the repeated version of the one–shot–game together with a proper evaluation of the stream of payoffs). Generally, the implementation of cooperative concepts by means of the Nash–equilibria of suitably defined non–cooperative games was also one of the main topics of the meeting.

Results were also presented concerning the proper definition of perfectness/ subgame perfectness and various concepts that are derived from these ideas or have been developed parallel to them within the framework of dynamic games.

Some special topics related to of stationary and non–stationary strategies in stochastic games were treated as well as foundations in combinatorics and topology which are closely connected to questions arising in Game Theory. Finally there was a group of talks that could be labelled "applications" in the sense that methods developed in Game Theory were applied to economically motivated models like certain markets with incomplete information.

Maybe the unifying aspect of the meeting was the abundance of cross references and relationships between various fields of Game Theory. Cooperation and strategic aspects nowadays are very much related. The ideas of classifying or selecting certain (Nash) equilibria by concepts of "robustness" or "sequentiality" is also related to the question of how to implement cooperation. All this again appears in most "applications" that Game Theory is now concerned with. These facts were reflected by the talks and discussions that took place on this conference.

Vortragsauszüge

E. van Damme (and W.Güth): **Equilibrium Selection in the Spence Signaling Model**

We consider a simple version of the Spence job market signaling model of which the data are as follows

Type	Productivity	Education Cost	Probability
0	0	y	$1-\lambda$
1	1	y/2	λ

The rules of the game are
- The worker (player 1) learns his type
- The worker chooses an education level y
- Two identical firms (the players 2 and 3) observe y, from that infer something about the worker's type and then simultaneously offer wages ($w_2(y)$, $w_3(y)$).
- the worker chooses a firm
- The payoff to a worker of type A who gets the wage w after an investment y is $w - y/(t+1)$; a firm has zero payoff if it does not effect the worker, the profit is A–w if the firm attacks the type A worker with the wage w.

Aim of the paper is to find the Harsanyi/Selten solution of this game. It turns out that this solution is the equilibrium proposed by Charles Wilson, i.e. if $\lambda < 1/2$, the types separate (type A chooses $y = 1$) and gets wage $w = t$) if $\lambda \geq \frac{1}{2}$ the solution is pooling at $y = 0$ (hence $w = \lambda$). The critical element of the proof is that Harsanyi/Selten give preference to primitive equilibria (i.e. ones with minimal support) of the ϵ–perturbed game. If $\lambda > \frac{1}{2}$ only the pooling equilibrium turns out to be primitive. For $\lambda \leq \frac{1}{2}$ there are many equilibria and the solution is determined by risk dominance.

W.Güth: Majority Voting in the Condorcet Paradox as a Problem of Equilibrium Selection

Voting by simple majority is often viewed as undesirable since it can lead to cyclical majority decisions (Condorcet paradox). In general, there can be no transitive social ordering of alternatives based on majority decisions. Here, we do not follow the welfare theoretic attempt to derive a transitive social ordering but rather consider the situation as a game where agents select among alternatives by majority decisions. Of course, the phenomenon of cyclical majorities entails the fact that such a game has more than just one equilibrium point. But by applying the theory of equilibrium selection, one nevertheless can solve the game uniquely and thereby determine a unique public decision.

To illustrate our approach, we consider the most simple form for the so–called Condorcet paradox with three alternatives and three agents. It is assumed that agents assign cardinal utilities to alternatives including the status quo which results if none of the three proposals is accepted. It is an interesting fact that the set of uniformly perfect equilibrium points depends crucially on cardinal utilities although they always imply the same cyclical majorities. Furthermore, the status quo will only sustain in degenerate cases. In other words: the uniquely determined alternative is Pareto–optimal with probability 1. This indicates that the application of equilibrium selection to majority voting offers new ways to derive mechanisms of social choice. Since agents choose among alternatives and not among preference profiles, etc., such mechanisms are, in our view, much more in line with actual democratic decision processes.

A.Ostmann: Simple Games: On Order and Symmetry

As a starting point Post's classification of boolean functions was applied to the class of (monotonic simple) games; call this class V. Some basic facts on the Post–classes of games are shown. Let $[p] := [v \in V$, property p holds for v$]$ and denote duality by *.
<u>Proposition</u> i, i = 1,2 : Inclusions and intersections can be seen from the diagram i.

diagram 1

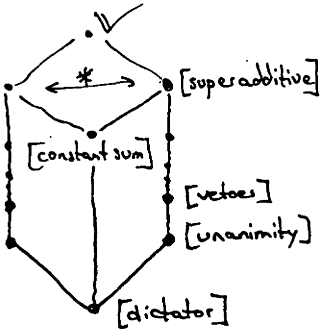
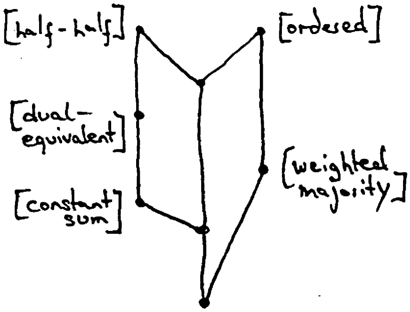


diagram 2



Examples were given that separate these sets. Next the automorphism group of a game and the (sharp) τ–transitivity of a game was introduced.

There are few highly transitive games not weighted majority. Note: [transitive and ordered] $\subset$ [weighted maj.]. The games (sharply) τ–transitive not weighted majority can be constructed by group theoretical tools. There exist only 13 games of this kind for $\tau \geq 4$ (they are connected to Witt–designs, Mathiew–groups, cf. p.8 of the abstract book of the "Mathematisches Forschungsinstitut") and they can be constructed by means of the PSL (2,11).

B.Peleg: **Voting by Count and Account**

Let $N = \{1,\dots,n\}$ be the set of taxpayers in a community and let w^i be the tax paid by $i \in N$. When the community has to elect an officeholder from a set of candidates several majority rules may be applicable. We may consider the symmetric simple game $(n, [\frac{n}{2}]+1) = u$ (voting by count), or the weighted majority game $v = [w^1,\dots,w^n]$ where S wins if $2 \sum_{i \in S} w^i > \sum_{i \in N} w^i$ – voting by account. A third possibility is to use the product $u \cdot v$–voting by count and account, which was the rule in Jewish communities in Europe during the last three hundred years or more. We prove that the Shapley value of $u \cdot v$–Lorenz–dominates that of v.

J.Potters: **The Structure of the Set of Perfect Equilibria of Bimatrix Games**

The set of Nash equilibria of a bimatrix game differs considerably from the equilibrium set of a matrix (zero–sum) game: it is not convex, in general, not a product set and there is no value. This was the reason to define the Nash component of a bimatrix game as a maximal convex subset of the equilibrium set E(A,B).

The following results have been obtained for Nash components.

1. Nash components have a product structure: $N = N_I \times N_{II}$ where $N_I = n_I(N)$ $N_{II} = n_{II}(N)$.
2. Every subset $P = P_I \times P_{II} \subset E(A,B)$ is subset of at least one Nash component.
3. The equilibrium set is a finite and irredundant union of the Nash components.
4. The dimension relation $\# C_I(N_I) \cdot \dim N_I = \operatorname{rank} B_{|C_I(N_I) \times B_{II}(N_I)}$ and a similar relation for N_{II}; here

   $$C_I(B_I) = \{i \mid p_i > 0 \ \text{ for } \underline{\text{some}}\ p \in N_I\}$$

   $$B_I(N_{II}) = \{i \mid e_i A_q \geq e_k A_q \ \text{ for } \underline{\text{all}}\ q \in N_{II} \text{ and } k = 1,\ldots,m\}$$

5. If $C_I \subset X_I$ (strategy set of player I) and $C_{II} \subset X_{II}$ are given, there is at most one Nash component $N_I \times N_{II}$ with $C_I(N_I) = C_I$ $C_{II}(N_{II}) = C_{II}$.

There are more results of Heuer, Milham, Winkels and Jansen (1974–1980). We introduced the what we called Selten components as maximal convex subsets of PF(A,B), the set of perfect equilibria and proved that the properties 1,3,4 and 5 remain valid for Selten components.

Moreover is $S = S_I \times S_{II}$ is a Selten component and N a Nash component containing S (always existing by 2) then

6. $$S_I = N_I \cap \Delta_{C(S_I)} \quad S_{II} = N_{II} \cap \Delta_{C(S_{II})}$$

Property 2 is no longer true for Selten components and also we cannot do the same for the set of proper equilibria.

J.H.Heijmans: **Discriminatory von Neumann–Morgenstern Solutions**

In the context of cooperative games with side–payments, a discriminatory set is a collection of imputations representing the scenario where some players (the discriminated players) receive a fixed amount, and the group of remaining players (the bargainers) can split the rest in any way they like. These discriminatory sets appear frequently as von Neumann–Morgenstern solutions or as building blocks of vN–M solutions. The best known examples are the monotone simple games: every minimal winning coalition has a corresponding discriminatory vN–M solution that assigns 0 to each player outside the minimal winning coalition.

For arbitrary (0,1)–games this paper studies those discriminatory sets that are vN–M solutions. It turns out that the bargainers in any discriminatory vN–M solution form a minimal vital coalition (vital in the sense of Gillies) and the total amount available for the bargainers is smaller than or equal to the worth of the minimal vital coalition. Minimal vital coalitions are most easily described for (0,1)–games as minimal non–trivial coalitions with positive worth. Another result is that in case a discriminatory vN–M solution exists that assigns a positive amount to a discriminated player, then the Core of the game must be empty.

The main result of the paper is an effective characterization to determine whether or not a proposed discriminatory set is a vN–M solution. Besides the above mentioned requirements regarding the group of bargaining players, the result also involves domination requirements for a finite set of competing discriminatory sets. These competing discriminatory sets have the same collection of discriminated players but now some of those players have lost their original allocations to the bargainers, so the competing discriminatory sets are more attractive to the bargainers than the original one. The domination requirement on a competing discriminatory set will not be fulfilled if and only if the Core of a certain attractive reduced game (for the set of bargaining players) is nonempty. The reduced game is very similar to the well–known Davis–Maschler reduced game.

W.Leininger (and M.Hellwig): **The Existence of Markov–Perfect Equilibria for Infinite–Action Games of Perfect Information**

Many economic models specify decision variables (like prices or quantities) as continuous (rather than discrete) variables. Game–theoretic analysis of such models is thus required to consider infinite–action games.

The present paper investigates a broad class of perfect information games with infinite action spaces for which existence of <u>subgame–perfect</u> equilibrium in general history dependent strategies has been established before. It amends those games by a Markovian state–structure and poses the general question under what additional assumptions existence of a subgame–perfect–equilibrium sustained by Markov–strategies, that <u>only</u> condition on the present state and <u>not</u> on the entire past history, exists. Such equilibria are called <u>Markov–perfect.</u>

The answer to this question is given by set of assumptions on the dynamic structure of the game and the payoff functions of players which is sufficient to ensure existence. They are also shown to be necessary in the sense that dropping any single one then leads to the emergence of counter examples which do <u>not</u> possess a Markov–perfect equilibrium.

N.N.Vorob'ev: **A Game–Theoretical Version of the Maximum Principle with Discrete Partially Ordered Time**

1. Let us have a partial non–cooperative game

$$\Gamma = < I, \{\mathfrak{x}_i\}_{i\in I}, \bar{\mathfrak{x}}, \{H_i\}_{i\in I} >$$

where $I = \{1,\ldots,n\}$, $\bar{\mathfrak{x}} \subset \mathfrak{x} = \prod_{i\in I} \mathfrak{x}_i$ and $H_i : \bar{\mathfrak{x}} \to R_1$

Metrics in all $\mathfrak{x}_i$ produce metrics in all $\mathfrak{x}^i = \prod_{j\neq i} \mathfrak{x}_j$ and in $\mathfrak{x}$ as well as (Hausdorffian) metrics in all $2^{\mathfrak{x}_i}$. For $x^i \in \mathfrak{x}^i$ we set $Z_i(x^i) = \{x_i : (x^i, x_i) \in \bar{\mathfrak{x}}\} \in 2^{\mathfrak{x}_i}$ and label a $x^* \in \bar{\mathfrak{x}}$

as equilibrium of Γ iff

$$H_i(x^*) = \max_{x_i \in Z_i(x^{*i})} H_i(x^{*i}, x_i).$$

The set of all equilibria of Γ is denoted as $\mathscr{E}(\Gamma)$.

Theorem: Let in the partial game Γ all $\mathfrak{x}_i$ be convex compact subsets of linear topological spaces, let $\bar{\mathfrak{x}}$ also be convex and compact in $\mathfrak{x}$; assume that all correspondences Z_i are continuous, and all M_i are continuous in x and quasi concave in x_i. Then the game Γ has equilibria ($\mathscr{E}(\Gamma) \neq \emptyset$).

2. The game Γ is said to be a production game if there is a fixed structurized resource $b = (b_1,\dots,b_n) \in R^n$ which allows for a structurized production $x = (x_1,\dots,x_n) \in R^n$ under some fixed restriction $A(x) \leq c$; let $\bar{\mathfrak{x}}$ denote a set of such x.

3. Let us have a finite orientated graph $\mathscr{G} = \langle J,G \rangle$ without loops such that to every vertex $j \in J$ there is assigned a production game Γ^j with the set of players I. These games Γ^j are supposed to be coordinated in a natural manner: the parts x^{k_j} of production x_i^k of Γ^k ($k \in G_j^{-1}$) are identified with the parts of resources $b_i^{k_j}$, their gathering over $k \in G_j^{-1}$ gives (together with the outside resources) the resource b_i^j and the game Γ^j defined by $b^i = (b_1^j,\dots,b_n^j)$ leads to some production $x^j = (x_1^j,\dots,x_n^j) \in \bar{\mathfrak{x}}^j$ which is realized by the player of games Γ^l ($l \in \Gamma_j$) as well as outside of the graph $\mathscr{G}$.

Uniting all games Γ^j for $j \in M \subset J$ we obtain a production game Γ^m which corresponds to the subgraph $\mathscr{G}^M = \langle M, G_M \rangle$:

$$\Gamma^M = \langle I, \{\mathfrak{x}_i^M\}_{i \in I}, \bar{\mathfrak{x}}^M, \{H_i^M\}_{i \in I} \rangle$$

If $M_1 \subset M$, the strategies x_i^M and their n-tuple x^M in the game Γ^M have, as their natural projections, strategies x_i^M and their n-tuple x^{M_1}.

Theorem 1^{o}: If $x^{*M} \in \mathcal{C}(\Gamma^{M})$ and $M_1 \subset M$, then $x^{*M_1} \in \mathcal{C}(\Gamma^{M_1})$.

2^{o}: If $x^{*M} \in \bar{\Gamma}^{M}$, $M = M_1 \cup M_2$ $(M_1 \cap M_2 = \emptyset)$, $x^{*M_1} \in \mathcal{C}(\Gamma^{M_1})$ and $x^{*M_2} \in \mathcal{C}(\Gamma^{M_2})$ then $x^{*M} \in \mathcal{C}(\Gamma^{M})$.

E.Yanovskaja: **On a Definition of Excess in the Games Without Sidepayments**

An axiomatic characterization of excess relation on the set of pairs of coalitions and individually rational payoff vectors in n–person games without sidepayments is given. This relation allows for defining a unique nucleolus of n–person cooperative game without sidepayments. For the sidepayment cooperative games the normalized excess functions are the utility functions representing this relation.

M.Maschler (and B.Peleg): **On Coalition Formation**

Given a society who believes in a certain solution concept, say the nucleolus, there still is a strategic aspect while playing the game, because once a coalition forms, the game changes. So coalitions may want to rush into forming coalitions and others prefer to wait; some players may want to leave the arena of negotiation for a while, while others would rather prefer them to stay. Thus, some players may be willing to pay others to encourage them to form, or to leave or just to stay. How can one treat such situations systematically? The research presented here offers a solution to this problem. Examples were given, which show that the suggested solution does indeed give intuitive prescriptions.

H.Peters: **Self–Optimality and Efficiency**

In a game model where each player has private information about her type, a vector of reported types is called self–optimal if it would constitute a Nash equilibrium given that the reported types were the true types. We explore the relationship between the self–optimality concept and the incentive compatibility concept. We apply the self–optimality concept to a utility distortion game in the context of bargaining and obtain a characterization of efficient Nash equilibria.

T.Ichiishi (and A.Idzik): **Theorems on Closed Coverings of a Simplex and Their Applications to Cooperative Game Theory**

Let N, K be finite sets such that $N \subset K$, let $A := ((a_{ij}))_{i \in N, j \in K}$ be a $(\#N) \times (\#K)$ matrix such that $a_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$ for all $j \in N$, and let
$\Delta^T :=$ convex hull of $\{\text{column } j \text{ of } A\}_{j \in T}$ for each $T \subset N$.

Theorem: Assume that $c \in \Delta^N$ and that the set $\{x \in \mathbb{R}^K_+ \mid Ax = c\}$ is bounded. Let $\{C^j\}_{j \in K}$ be a family of closed subsets of Δ^N such that

$$\forall T \in 2^N : \Delta^T \subset \cup \{C^j \mid j \in K, a^j \in \text{affine hull of } \Delta^T\}.$$

Then there exists $x \in \mathbb{R}^K_+$ such that $Ax = c$ and $\cap \{C^j \mid x_j > 0\} \neq \emptyset$.

This theorem, its dual result, its extension are established. These theorems unify many of the theorems of the Knaster–Kuratowski–Mazurkievicz type, including those of Shapley (1973), Scarf (1967), Fan (1968), Gale (1984), and Ichiishi (1988). Applications to cooperative game theory are also given.

O.N.Bondareva: **Equilibrium Prices in the One–Product Market**

A model of an n–person market as follows. Let $I = M \cup N = \{1,2,\ldots,n\}$ be the set of players where $M = \{1,2,\ldots,m\}$ is the set of producers and $N = \{m-1\ldots,n\}$ is the set of consumers of one indivisible product. Each $i \in M$ produces a_i units of this and each $j \in N$ demands for b_j units. The utility of one unit is u_i for producer i and is w_j for consumer j. Enumerate the players so that $i_1 < i_2$ iff $u_{i_1} \leq u_{i_2}$ and $j_1 < j_2$ iff $w_{j_1} \leq w_{j_2}$.

A distribution is a vector $(\xi,\eta) : \xi = (\xi_1,\dots,\xi_m)$, $\eta = (\eta_{m+1},\dots,\eta_n)$ where $0 \le \xi_i \le a_i$, $i \in M$, $0 \le \eta_j \le b_j$, $j \in N$, $\xi(M) = \eta(N)$ $(x(S) = \sum_{i \in S} x_i)$. Denote the set of all distributions $D(I)$, for $S \subset I$ put $D(S) = \{(\xi,\eta) \in D(I)\colon \xi_i = 0,\ i \notin S,\ \eta_j = 0,\ j \notin S\}$. For distribution $(\xi,\eta) \in D(S)$ the coalition S realizes a profit $v(S,\xi,\eta) = \sum_{j \in S \cap N} w_j \eta_j - \sum_{i \in S \cap M} u_i \xi_i$. Put

$$v(S) = \max_{(\xi,\eta) \in D(S)} v(S,\xi,\eta),\ S \subset I.$$

Function v describes the super–additive cooperative game $\Gamma = \langle I,v \rangle$ with a nonempty core (class G_i). The distribution $(\overline{\xi},\overline{\eta}) \in D(I)$ is optimal iff $v(I,\overline{\xi},\overline{\eta}) = v(I)$. The pair (i_0,j_0) is marginal iff $\overline{\xi}_i = 0$ for $i > i_0$ and $\overline{\eta}_j = 0$ for $j < j_0$.

If p is the price of one unit of the product, then coalition S realizes the profit $x(S,p,\xi,\eta) = \sum_{i \in S \cap M} (p-u_i)\,\xi_i + \sum_{j \in S \cap N} (w_j-p)\,\eta_j$ in distribution (ξ,η).

The price p is called an equilibrium price iff there exists such (ξ,η) that $x(S,p,\xi,\eta) \ge v(S)$, $S \subset I$. For any game $\Gamma \in G_i$ and $u_{i_0} \le \underline{p} \le \overline{p} < w_{j_0}$, any $p \in [\underline{p},\overline{p}]$ is equilibrium price with $(\overline{\xi},\overline{\eta})$ being optimal. For a game with all different u_i, w_j the optimal $(\overline{\xi},\overline{\eta})$ as well as the marginal pair is unique.

Another extremal class with $u_i = 0$ and $w_j = 1$ is denoted by G_0. Any game $\Gamma \in G_0$ satisfies $v(S) = \min(a(S), b(S))$, v is the extension of the Shapley game:
$v(S) = \min(S \cap M, S \cap N)$ where all $a_i = b_j = 1$. For $\Gamma \in G_0$ with "small" deficit and monopoly the Shapley value and nucleolus are expressed with a_i, b_j.
It is possible to extend this notion to many production and dynamic models. The main differences of these models from known ones are the nonconvexity of utility functions and the existence only a coalition equilibrium.

S.K.Chakrabarti: **On the Existence of Equilibria in a Class of Discrete–Time Dynamic Games with Imperfect Information**

We examine the question of existence of subgame perfect equilibrium points in discrete–time dynamic games with infinite–action spaces which allow players to move simultaneously at each period. The previous literature on discrete–time dynamic games (or infinite extensive games) gave us results for games with perfect information; a situation in which simultaneous moves are ruled out. We show that when one restricts oneself to just continuity assumptions on the feasible action correspondences and the payoff functions we can find examples of games which do not have perfect, and therefore, sequential equilibrium points. We then show that if we allow for behavior strategies, and if one defines behavior strategies in the right way, then the behavior strategies define probability distributions over outcomes and one can then define the associated payoffs from the behavior strategies as the expected payoffs, even when the action spaces are infinite. The result is obtained by approximating the original game by finite–action games in an appropriate way and using the equilibrium strategy combination of the finite game to define the ϵ–perfect equilibrium point of the original game.

If restrictions are placed on the strategy space then one can guarantee the existence of subgame perfect equilibrium points.

S.Muto: **Resale–Proofness and Coalition–Proof Nash Equilibria in an Information Trading Game**

Information is freely replicatable. Thus, in trading information, a possibility of resales <of replicas> seems to be unavoidable unless resales are legally prohibited. A notion of resale–proofness was proposed by Nakayama, Quintas and Muto: it characterizes an information sharing pattern in which resales are never carried out even if they are freely allowed. In this paper, it is shown that, in an information trading game, resale–proof information sharing patterns are attained as equilibrium outcomes of the game: the perfectly coalition–proof Nash equilibrium due to Bernheim, Peleg, and Whinston.

W.Albers: **An Aspiration Approach to Bargaining Games**

This approach is based on observations in experimental bargaining chains. Considerations are restricted to 1–step games ($N = \{1,...,n\}$, $v : P(N) \rightarrow \mathbb{R}_+$, $v(\emptyset) = 0$, $[v(S), v(T) > 0 \Rightarrow S \cap T \neq \emptyset]$, $v(\{i\}) = 0$ (all i)).

A state (x,S) is $x \in \mathbb{R}^n_+$, $S \subseteq N$, s.t. $x(S) = v(S)$ and $x_i = 0$ ($i \in N \backslash S$). – A bargaining chain is a sequence $(x^1,S^1),...,(x^n,S^n)$ of states where each dominates the preceding one and where the payoff of each player $i \in S^t$ is at least as high as his aspiration $a^t_i := \max_{\tau<t} x^{\tau}_i$. A save bargaining chain is defined by recursion:

(1) a maximal bargaining chain (i.e. a bargaining chain which cannot be any more extended) is safe for all players in $\mathbb{N}$.

(2) within the recursion we have: axiom 1: if there is a reasonable domination, then one of them will be performed. (A domination to a state (x^{T+1}, S^{T+1}) is reasonable, if the chain up to the new state is safe for all players in $S^T \cap S^{T+1}$). axiom 2: an unreasonable domination is not performed. (A domination to a next state (x^{T+1}, S^{T+1}) is unreasonable, if there is a subsequent reasonable domination to (x^{T+2}, S^{T+2}) s.t. $S^{T+1} - S^{T+2} \neq \emptyset$, i.e. one of the dominating players is punished.) concluding the recursion: a bargaining chain is safe for player j if every domination to (x^{T+1}, S^{T+1}) with $j \notin S^{T+1}$ is unreasonable.

A stable state (x^1,S^1) is a safe bargaining chain of length 1 which is safe for all players in S^1.

Examples are given, a comparison to the bargaining set approach is made. Refinements of this concept are necessary to explain experimental results in detail, namely: Aspirations depend on the "bloc" a player is in – players can develop "reciprocal loyalty" – dominations are only performed, when they give a minimal improvement $\Delta > 0$ – dominations with zero–improvements are possible, when the new state is "socially desirable" – no–coalition states can be entered by breaking the coalition when the expected value of the braeker is afterwards higher than his value x^t, before breaking –. The payoff structure of a selected state has to meet conditions of "prominence" – the behavior can deviate from the prediction when "ultimatum situations" arise.

S.Sorin (and R.J.Aumann): **Cooperation and Bounded Recall**

A two person game has common interests if there is a simple payoff pair z that strongly Pareto dominates all other payoff pairs.

Assume such a game is repeated many times and that each player attaches a small but positive probability to the other playing some fixed strategy with bounded recall rather than playing to maximize his payoff.

The resulting supergame has an equilibrium in pure strategies, and the payoffs to all such equilibria are close to optimum (ie. to z).

J.A.Filar: **Weighted Reward Criteria in Markov Decision Processes and Stochastic Games**

We introduce a parameterized family of Markov Decision Processes (competitive, or non–competitive) called "weighted MDP's". The boundary points of this family are the now classical discounted and limiting average models. It is demonstrated that even in the noncompetitive case optimal policies may fail to exist. In this case an algorithm is given which constructs an ϵ–optimal "ultimately stationary" markov policy for any $\epsilon > 0$.

In the antagonistic competitive MDP's the reward criterion is either a convex combination of two discounted objectives, or of one discounted and one limiting average reward objective. In both cases we establish the existence of the game–theoretic value–vector, and supply a description of ϵ–optimal non–stationary strategies.

K.Vrieze: **Easy Initial States in Stochastic Games**

Discrete time dynamic games are played as follows: At each period the players have to choose an action out of an available–probably state dependent action set. The simultaneously chosen actions jointly determine rewards to the players and a transition distribution according to which the next state is selected.

The infinite horizon model is considered under the limiting average criterion. Strategies at each period may generally depend on the history up to that period. Stationary strategies only take care of the state in which the system is arrived.

It is shown that in the zero–sum case for both players there exist non–empty subsets of states which are "easy" in the sense that the players can guarantee the value by using stationary strategies. For the general sum case this result can be extended as follows: each of the players has a nonempty subset of states, which are almost easy for the players in the following sense: starting in a state belonging to such a set the players can play ϵ–equilibrium wise by using appropriate stationary strategies as long as they do not detect a deviation of one of the other players: if they do detect a deviation than they have to switch to behavioral ϵ–optimal punishment strategies.

T.Parthasarathy (and C.Olech, G.Ravindran): **N–matrices and univalence**

An N–matrix is a square matrix with real entries whose principal minors are negative. This concept was introduced by Inada in connection with production matrix and Stolper–Samuelson condition. Our purpose is two–fold: (i) To characterize N–matrices (ii) To prove new univalence results. It is known that the inverse of an N–matrix is an almost P–matrix. We prove among other results the following univalence result: If F is a C^1 differentiable map from $\mathbb{R}^n$ to $\mathbb{R}^n$ with its Jacobian an almost P–matrix (inverse of an N–matrix) for every $x \in \mathbb{R}^n$ then F is globally one to one in $\mathbb{R}^n$. Our proof of this depends on the K–K–M theorem.

T.Driessen: **The Coincidence of the Prenucleolus and the ENSC–Solution**

Let (N,v) be a cooperative n–person game in characteristic function form. The smallest contribution of coalition $S \subset N$ with respect to the formation of (n–1)–person coalitions in the n–person game v is defined to be

$$m^v(S) := \min \left[v(N-\{j\}) - v(N-\{j\}-S) \mid j \in N-S \right] \quad \text{for all } S \subset N,\ S \neq N,$$

$$m^v(N) := v(N).$$

Let the set U(v) consist of efficient payoff vectors that give rise only to payoffs not greater than the relevant smallest contributions for all coalitions containing at most n–2 players. To be exact,

$$U(v) := \{x \in \mathbb{R}^n \mid \sum_{i \in N} x_i = v(N) \text{ and } \sum_{i \in S} x_i \leq m^v(S)$$

$$\text{for all } S \subset N \text{ with } 1 \leq |S| \leq n-2\}.$$

The interrelationships between the set U(v) and several solution concepts (e.g., the prekernel and the prenucleolus) are studied. The main results are as follows.
Firstly, an efficient payoff vector $x \in \mathbb{R}^n$ belongs to the set U(v) if and only if the maximal excesses at x are determined by the (n–1)–person coalitions. Thus,

$$x \in U(v) \;\text{ iff }\; e^v(S,x) \leq e^v(N-\{i\}, x)$$

$$\text{for all } i \in N \text{ and all } S \subset N-\{i\},\ S \neq \emptyset.$$

Secondly, the part of the set U(v) inside the prekernel consists of at most one efficient payoff vector which equals the so–called ENSC–solution. The egalitarian nonseparable contribution (ENSC–) solution for the n–person game v is defined to be

$$ENSC_i(v) := SC_i(v) + n^{-1}NSC(v) \;\text{ for all } i \in N, \text{ where}$$
$$SC_i(v) := v(N) - v(N-\{i\}) \text{ and } NSC(v) := v(N) - \sum_{j \in N} SC_j(v).$$

Thirdly, the prenucleolus is included in the set U(v) if and only if the ENSC–solution belongs to the set U(v). Furthermore, each of the two equivalent conditions is sufficient for the coincidence of the prenucleolus concept and the ENSC–method.

I.Dragan: **The Compensatory Bargaining Set of a Cooperative 2–Person Game with Side Payments**

The Aumann/Maschler definition of a bargaining set relies upon a stability principle imposed to the payoffs in this set: an admissible payoff belongs to a bargaining set if, for every objection against this payoff, if any, there is a counter objection. Two modifications of the stability principle have been discussed in earlier papers of the author (Dragan, 1985, 1987, 1988). The present paper is considering another modification: an objection is valid only if the players who intend to move to new coalitions agree upon a prior commitment, namely that of compensating all partners who join the venture, in case of failure due to a subsequent move. The mathematical description of the model is given in the first section, where the new stability principle and the corresponding "compensatory" bargaining set $\mathcal{M}_{\mathcal{C}}$ is defined. A feasibility theorem for the existence of a flow in a bipartite network associated to a payoff and two partial coalition structures is derived in the second section from a similar theorem by D.Gale (1957). The result is used in the third section for proving a combinatorial characterization of the core payoff belonging to the compensatory bargaining set. In the last section, in the set of such payoffs $\mathcal{M}_{\mathcal{C}}\,\mathcal{C}(G)$ for a 2–person game the subset of $\mathcal{M}_{\mathcal{C}}\,\mathcal{C}(G)$ consisting of coalitionally rational payoffs is found. This subset is compared with the bargaining set M of Aumann/Maschler (1964) for the same game, in order to illustrate the potential abilities of the new model by a comparison with a well known one.

S.Zamir (and R.Avenhaus): **Safeguard Games**

Safeguard problems are situations in which player I (an inspector) tries to detect "illegal" actions of player O (an operator). He does so on the basis of observations of random variables, the distribution of which depends on player O's actions. We propose a multi–stage extensive form game to model the sequential inspection problems. Under appropriate restrictions on the payoffs and the strategy set of the operator we prove the existence of a unique Nash equilibrium. In a variant of the game in which the inspector has the possibility to commit himself publically to a certain strategy there is again a unique Nash equilibrium which may be called the "commitment equilibrium" in which the inspector strategy is the same as before, but the deviation probability of the operator is zero and hence the inspector's payoff is higher than in the Nash equilibrium of the game without commitment. Therefore, this may also be called a deterrence equilibrium.

For application to pollution control and nuclear material safeguards, the inspector's equilibrium strategy is shown to be the statistical test commonly used in these contexts.

Stef H. Tijs: Properness, Balancedness and the Nucleolus

We define the nucleolus of a continuous convex map $F : X \rightarrow \mathbb{R}^m$ on a compact set X. As special cases we obtain known notions as nucleolus, prenucleolus and weighted nucleolus of a TU–game with (or without) coalition structure. Also the nucleolus of a matrix game turns out to be an interesting special case. It appears that the nucleolus of a matrix game coincides with the set of Dresher optimal strategy pairs of the game. This implies that the nucleolus consists precisely of the proper equilibria of the matrix game. To each (0,1)–normalized TU–game one can construct a matrix game – the excess game – such that the nucleolus of the TU–game coincides with the unique proper optimal strategy of player 2 in the excess game. Also for other nucleoli for TU–games a suitable matrix game can be constructed where the respective nucleolus is related to the nucleolus of the matrix game.

A balancedness condition is given characterizing nucleolus elements of a matrix game. It is shown that this balancedness result implies again the known balancedness characterizations of E.Kohlberg, A.I.Sobolev, G.Owen and E.Wallmeier.

R.Holzman: The Nucleus and the Problem of Strong Implementation

The problem of strong implementation is to determine those social choice correspondences that can be obtained as the strong equilibrium correspondence of a game form. We introduce the notion of the nucleus of an effectivity function. Under certain conditions, it yields the smallest implementable social choice correspondence having that effectivity functions. We contrast it with the core, which yields the largest one (as shown earlier by Moulin and Peleg), and upon that the smaller solution should be preferred when available.

D.Pallaschke: Quasi–differentiable Functions in Optimization Theory

According to V.Demyanov and A.Rubinow, a function $f : U \rightarrow \mathbb{R}$, $U \subseteq \mathbb{R}^n$ open, $x_0 \in U$, is said to be quasi–differentiable, if its directional derivative in x_0, as a function of the direction, can be represented as a difference of two sublinear functions. We consider the directional derivative as a periodic function on the (n–1)– Sphere. The development into a Fourier–Series leads to an approximation by differences of sublinear functions. The norm in which this Fourier–Series converges is used to classify the degree of differentiability. This technique is also used for higher order derivatives and overcomes the typical discontinuities which appear in non–smooth analysis.

M.Wooders: **Large Games are Market Games**

We show that large finite games in coalitional form (games with "many", but a finite number of players) are approximately market games. To model large games we use the notion of a pregame, which enables us to describe the worth of any group of players as a function of the attributes (or "types") of the members of the group. From the pregame, which is required to satisfy only mild conditions, we construct a premarket – a space of characteristics of goods and a continuous, concave, 1–homogeneous utility function. We show that the worth of any sufficiently large coalition in any game derived from the pregame is close to the worth of the corresponding coalition (with the same player set) in the market game derived from the premarket.

We also show that games in coalitional form with a continuum of players and finite coalitions (the Kaneko–Wooders model) are equivalent to Mas–Colell differentiated commodities market games.

D.Schmeidler (and Edi Karni): **Fixed Preferences and Changing Tastes (The Economics of Fashion)**

The phenomenon that is colloquially referred to as "fashion" exists to some extent in the consumption of many goods and services, as well as in other aspects of human activity. To focus the attention on the main issue, we restrict our discussion to pure fashion phenomena, that is the variation over time in the share of a particular brand name or product design at the expense of other brand names or designs of the same good in the market at large or among a specific group of customers. For example, the increase in recent years in the market share of Reebok at the expense of Nike or the complete replacement of the miniskirt by the midiskirt and maxiskirt in the 1970s.

The main idea is that the consumption of many commodities is, in part, a social activity. Therefore, to capture the social aspects of consumption behavior, the standard definition of a commodity, which includes its physical attributes, delivery date, location, and – in the case of contigent commodities – the state of nature, must be extended to include the commodity's social attributes. We claim that the observed patterns of change in the consumption of standard commodities (e.g., Nike sneakers) is consistent with constant preferences over the space of extended commodities.

In the present paper we implement these ideas in a dynamic game model that is reminiscent of an overlapping generations economy. Consider a game that evolves through countably many periods, $Z = \{..., -2, -1, 0, 1, 2 ...,\}$, without a first or a last period. In each period there is a continuum of players. Every player participates in the game during finitely many consecutive periods. At the outset of each period in which he is in the game each player must select a move from a finite set of moves. The selection of moves by all the players that are in the game in a given period is done simultaneously.

The payoff to any given player depends on the sequence of his own moves during the periods in which he is in the game and on finitely many statistics (linear functionals) defined on the moves of all the other players during the same period. In this model any finite set of players is negligible in so far as the relevant statistics are concerned.

The sequence of moves of all the players define a play of the game. A play of the game is an equilibrium play if no player can increase his utility by switching unilaterally to another sequence of moves. Existence of an equilibrium play is proved. An example with a cyclic equilibrium play is induced. It demonstrates the application of the overlapping generations game to model the changes in assumption due to fashion.

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