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MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 5/1989

Analytical and topological theory of semigroups

29.1. bis 4.2.1989

The conference was organized by K. H. HOFMANN (Darmstadt), J. D. LAWSON (Baton Rouge), and J. S. PYM (Sheffield). Participants came from the USA (19), the FRG (12), Canada (4), Denmark (3), France (3), the UK (2), Austria (1), India (1), the Netherlands (1), and the USSR (1). The diversity of the participants' origins was matched by the diversity of their mathematical interests. The conference drew researchers whose primary field of mathematical activity ranged through such varied mathematical disciplines as Lie theory, topological algebra, harmonic and functional analysis, representation theory, probability theory, and algebraic geometry.

In order to accomodate this variety of interests, the organizers built the conference around a large number (16) of one hour survey lectures which were planned well in advance. They served the purpose of making the broad audience aware of major research trends, of pointing out the major developments and the current state of the art, and of suggesting open problems and future lines of research in the area covered by the conference. The common thread was semigroups, but semigroups in analytic, topological, Lie theoretical and related contexts. Manuscripts of the surveys were solicited prior to the conference, and plans were solidified at the meeting for the publication of a collection of the complete set. This monograph should become a useful source both for reference and for open problems in the discipline—more so than a standard proceedings volume. Other contributors presenting original research at the conference were encouraged to submit articles to the specialized journal in the subject.

Several recent developments and results were reported at the conference. There were survey talks (3) and papers (8) in the recently emerged Lie theory of semigroups, further the structure of topological semigroups with jointly, separately, and one-sidedly continuous multiplication, and differentiable semigroups. Other areas receiving broad coverage were functional and harmonic analysis in their relations to semigroup theory. An elegant semigroup-theoretical proof of van der Waerden's Theorem on arithmetic progressions was presented. The recently published solution of Hilbert's Fifth Problem in the semigroup context was reported on. A new approach to the classical theory of algebraic groups via their Zariski-closures, algebraic semigroups, was presented. Of special interest were reports on the cross connections of semigroup theory with such applied disciplines as control- and systems-theory, stochastics, theoretical physics, and theoretical computer science, and with other mathematical disciplines such as combinatorial number theory and representation theory.

A book table was set up from the Institute's well-furnished library display-

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ing some twenty-four books on the topological and analytical theory of semigroups that have appeared over the last 25 years. Many of the authors were among the participants.

Contributions by several young participants bode well for the productivity and creativity of the area in the future.

The excellent personal computer facilities coupled with laser printing capability were a great help for the organizers in their planning and TEX printing of the daily programs.

Abstracts

The abstracts of the contributions to the conference are divided into the following disciplines

[1] Applications to algebraic geometry, computer science, group theory, number theory, systems theory, topology,

[2] Functional analysis,

[3] Lie theory,

[4] Probability and measure theory,

[5] Semigroups with one-sided or separate continuity,

[6] Topological algebra, topological semigroup, order theory,

[1] Applications to algebraic geometry, computer science, number theory, systems theory, topology

[1.1] N. HINDMAN. *The semigroup $\beta\mathbb{N}$ and its applications to number theory*

The operations + and · on $\mathbb{N}$ extend to its STONE-ČECH-compactification $\beta\mathbb{N}$, making it a compact left topological semigroup. We discuss the history of the application of these operations to results in RAMSEY theory (combinatorial number theory) including some very recent proofs of VAN DER WAERDEN'S Theorem on arithmetic progressions.

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[1.2] I. KUPKA. *Semigroups in control theory*

The object is the presentation of some applications or potential applications of semigroups to the theory of systems. We discuss the theory of accessibility where such applications have been most prominent. We continue with realization theory where we feel semigroup theory could help. Other domains of interest in the with applications of semigroup theory are local controllability and optimal control.

[1.3] K. D. MAGILL, JR. *Trends and directions in the investigation of congruences on the semigroup $S(X)$ of continuous self-maps of a space X*

For "most" spaces there at most three congruences ρ for which $S(X)/\rho$ is isomorphic to $S(Y)$ for some generated space Y. The existence of a largest proper and a smallest proper congruence is investigated. The semigroups of a number of spaces, including all Euclidean n-cells, have a largest proper congruence while the semigroup of many local dendrites with finite branch number do not. On the other hand, it is rare for a semigroup $S(X)$ to fail to have a smallest proper congruence although there are examples. The partially ordered set $\mathrm{Con}_c(S(X))$ of all continuum congruences on $S(X)$ is studied. If X is a local dentrite with finite branch number, then $\mathrm{Con}_c(S(X))$ is order isomorphic to a certain partially ordered set of collections of subcontinua of X on which $\mathcal{A} \leq \mathcal{B}$ for two such collections means that that each $B \in \mathcal{B}$ is the union of copies of subcontinua from $\mathcal{A}$. This fact is used to obtain e.g. a characterization of those local dendrites with finite branch number for which $\mathrm{Con}_c(S(X))$ is a lattice. Further information on $\mathrm{Con}_c(S(X))$ is provided. Finally those congruences on $S(X)$ which commute with the equivalence relation identifying two mutually inverse maps are completely determined for a great many spaces X. It turns out that there are two such congruences if X is connected and six if it is not.

[1.4] J. E. PIN. *The profinite and the p-adic topologgy for the free monoid*

The profinite topology for the free group was introduced by M. HALL and was extended by REUTENAUER to the case of free monoids to be the initial topology making all monoid morphisms into finite discrete *groups* continuous. In the same way the p-adic topology is defined by replacing "*groups*" by "p-*groups*". One restricts one's attention to "simple" subsets of the free monoid and tries to determine their properties in relation to these topologies: Are they open or closed? Can one compute their closure? The "simple" sets we have in mind are the *recognizable* (or *regular*) sets of automata theory. These sets are completely described by a finite monoid, called the *syntactic monoid* of the set. We show that certain topological properties of a recognizable set are reflected by some simple algebraic properties of its syntactic monoid. We discuss our conjecture that the converse is true and its possible applications. (Details will appear in J. of Algebra in "Topologies for the free monoid".)

[1.5] L. RENNER. *Algebraic varieties and semigroups*

M. S. PUTCHA and I developed the theory of *linear algebraic semigroups* over the past eight years. The most interesting objects among these are the *irreducible monoids*. The major results in their theory include (1) a characterization of regular elements, (2) a numerical classification of normal monoids with reductive group of units, (3) a classification of normal (completely) regular monoids

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with solvable unit groups, (4) a generalization of the (group theoretic) BRUHAT decomposition to reductive monoids, (5) a determination of the conjugacy classes in reductive monoids, generalizing the classical JORDAN normal form of an endomorphism. Related topics include the equivariant embedding problem for spherical homogeneous spaces.

[1.6] C. TERP *Maximal compact subgroups in locally compact groups via invariant cones*

Maximal compact subgroups in locally compact connected groups are commonly established through some fixed point argument, but not through an argument using Zorn's Lemma. Contrary to the mere existence of maximal compact subgroups, the inductivity of the set of all compact subgroups is inherited by all closed subgroups. We use the fact that a compact group, acting linearly on a convex closed pointed cone, has a fixed point in the algebraic interior of the cone (for invariant cones cf. also [2.4], [3.2], [3.5], [3.9]), and we show that in a locally compact group G the partially ordered set of compact subgroups is inductive if and only if the totally disconnected locally compact group G/G_0 has this property.

[2] Functional analysis

(See also BAKER [4.1], HILGERT [3.4], MISLOVE [6.4])

[2.1] C. BERG. *Positive definite and related functions on semigroups*

This survey discusses the theory of positive definite and related functions on abelian semigroups with involution. Special emphasis was placed on developments since the appearence of the book by C. BERG, J. P. R. CHRISTENSEN and C. U. RESSEL on "Harmonic analysis on semigroups" (Springer-Verlag Heidelberg etc., 1984). In the integral representation of positive definite functions on S we had earlier focused on RADON measures μ on S^* defined on the BOREL σ-algebra $\mathcal{B}(S)$. It turns out to be fruitful to consider measures μ on the smallest σ-algebra $\mathcal{A}(S^*)$ rendering the evaluations $\rho \mapsto \rho(s): S^* \to \mathbb{C}$ measurable. The notions of BISGAARD and RESSEL of semiperfect and perfect semigroups are discussed. Here a semigroup S is *semiperfect* if every positive function φ on S is a moment function: $\varphi(s) = \int_{S^*} \rho(s) d\mu(\rho)$ for some μ on $\mathcal{A}(S^*)$.

[2.2] C. CHOU. *Weakly almost periodic functions on groups*

Let G be an infinite discrete group. (1) For $E \subseteq G$, let $\overline{E}$ denote the closure of E in the weak almost periodic compactification G^w. Set $\widehat{E} = \overline{E} \setminus G$. A subset E of G is called a T-set, respectively, R_W-set, if $x\widehat{E} \cap y\widehat{E} = \emptyset$ for $x \neq y$ in G, respectively, if χ_E is weakly almost periodic and $\overline{E} \cong \beta E$. All T-sets are R_W-sets, and R_W-sets were studied by W. RUDIN and W. A. F. RUPPERT. We show that every G contains an R_W-set D which is not a finite union of T-sets; hence there exists an $\omega \in \overline{D} \cong \beta D$ such that ω is not strongly G-discrete. *Question:* If E is a T-set and $\omega \in E$, is $\alpha \mapsto \alpha\omega: G^w \to G^w\omega$ a homeomorphism? (2) Let $\mathrm{DWAP}(G) \subseteq \mathrm{WAP}(G)$ denote the set of all bounded

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functions on G whose double orbit $\{{}_xf_y \mid x, y \in G\}$ is relatively weakly compact. In general, $\mathrm{DWAP}(G) \neq \mathrm{WAP}(G)$. *Conjecture:* $\mathrm{DWAP}(G) = \mathrm{WAP}(G)$ iff G is an extension of an abelian by a finite group.

[2.3] J. DUNCAN. *Topological theories for inverse semigroups and their representation*

A representation theory of inverse semigroups into the set of partial isometries of a Hilbert space calls for an investigation of star semigroups S of such operators. For example, this gives easily the list of monogenic ones. The natural topology is the weak operator topology with separately continuous multiplication and continuous involution. For a topological semigroup S, it is not clear how to define the C^* algebra $C^*(S)$ of S in such a way that it reduces to the traditional C^*-algebra of a locally compact group. The key problem is to achieve the definition for a semilattice. A definition is given for some classes, but the "naturality" of the definition remains in question.

[2.4] J. FARAUT. *Analysis on ordered symmetric spaces*

Let S be a closed semigroup in a locally compact group G. Then $H = S \cap S^{-1}$ is a closed subgroup, and S defines an invariant ordering on the homogeneous space $X = G/H$. A causal kernel is a function $K: X \times X \to \mathbf{R}$ vanishing outside $\{(x,y) \mid y \leq x\}$. The VOLTERRA-algebra $V(X)^\natural$ is the space of invariant causal kernels, equipped with the composition product of kernels. If there exists an involution $x \mapsto x^\#$ of S such that (i) $(xy)^\# = y^\# x^\#$, (ii) $(\forall x \in H)\, x^\# = x^{-1}$, (iii) $(\forall x \in S)\, x^\# \in HxH$, then $V(X)^\natural$ is commutative. *Example.* $G = \mathrm{Sl}(2,\mathbf{C})$, $H = \mathrm{Sl}(2,\mathbf{R})$, $S = \exp(i{\cdot}C)H$ with an invariant cone C in the Lie algebra $\mathrm{sl}(2,\mathbf{R})$.

[2.5] A. M. LAU. *Amenability of semigroups*

A historical introduction into the investigation of amenability of discrete and semitopological semigroups is presented, and an extensive summary is given of recent developments and open problems.

[2.6] A. L. T. PATERSON. *Representation theory for inverse semigroups*

The motivation for the study of such a theory arises from operator algebras (such as the CUNTZ algebras) which are generated by inverse semigroup representation. The regular representation of such a semigroup S is faithful, so that the representations of S separate points. The theory depends on developing a "twisted" disintegration theory based on $C^*(E)$, where E is the idempotent semilattice of S. This leads to a quasi-invariant measure on the filter completion X of E with respect to a natural action of S on X in terms of partial $1-1$ maps. Associated with this set-up is a natural groupoid G whose elements consist of suitable pairs (s,x), $s \in S$, $x \in X$, and the representation theories of S and G essentially coincide. This allows the well-developed representation theory of groupoids to be applied to give information about inverse semigroup representations.

[2.7] H.L. VASUDEVA. *Multiplier spaces on* $[0,1]$ *and their preduals*

Let I be the real unit interval $[0,1]$ with its structure of a compact topological semigroup given by the maximum operation. Further let $L^p(I)$, $1 \leq p \leq \infty$

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denote the L^p-spaces with respect to LEBESGUE measure and $C(I)$ the Banach algebra of continuous functions on I with the maximum norm. The Banach modules $\mathrm{Hom}_{C(I)}(L^p(I), L^q(I))$, are characterized for $p \leq q$.

[3] Lie Theory

(See also BROWN [6.1], BROWN and HILDEBRANT [6.2], FARAUT [2.4], KUPKA [1.3], SKRYAGO [6.6])

[3.1] M. ANDERSON. *Strong differentiability in semigroups*

In 1938, GARRET BIRKHOFF showed that a local semigroup with identity with an identity neighborhood homeomorphic to a Banach space and with a strongly differentiable multiplication at **1** is a Lie group. For topological semigroups this is not true. Strong differentiability at **1**, therefore, appears to be a powerful condition. Indeed, it implies the existence of local one-parameter subsemigroups in a local differentiable semigroup on an admissible set, provided certain additional conditions are satisfied (e.g., local compactness in the case of finite dimensional manifolds or the existence of a strongly differentiable arc at the identiy in the Banach manifold case). Strong differentiability the identity **1** of a local semigroup also implies that a smooth boundary is a local group.

[3.2] N. DÖRR. *Invariant orders on Lie groups*

A semigroup S in a Lie group G satisfying $gSg^{-1} \subseteq S$ defines a group quasiorder via $x \leq y$ iff $y \in Sx$. *Question:* Are the intervals $D_{ab} = aS \cap Sb^{-1}$ compact? (Cf. [2.4].) We discuss two possibly typical examples for which this is not the case: (1) Let G be the universal covering group of $\mathrm{Sl}(2, \mathbf{R})$. The CARTAN-KILLING-form on $\mathrm{sl}(2, \mathbf{R})$ is Lorentzian and determines two opposite invariant cones, each of which generates a subsemigroup S of G which containsa whole half-space; as a consequence, some intervals D_{ab} fail to be compact. (2) Let V be a $2n$-dimensional Hilbert space with a fixed non-degenerate skew symmetric automorphism d. The Lie algebra $\mathfrak{g} = \mathbf{R} \times V \times \mathbf{R}$ with the Lie bracket $[(r, v, z), (r', v', z')] = (0, r \cdot dv' - r' \cdot dv, \langle dv | v' \rangle)$ and the invariant Lorentzian form $q((r, v, z), (r', v', z')) = rz' + r'z + \langle v | v' \rangle$ supports pointed generating invariant Lorentzian cones. The one determined by the forward light cone W of q generates an invariant semigroup $S = \langle \exp W \rangle$ in the corresponding simply connected Lie group $G = \mathbf{R} \times V \times \mathbf{R}$ with the multiplication $(r, v, z), (r', v', z') = (r + r', v + e^{r \cdot d} v', z + z' + \frac{1}{2} \langle dv | e^{r \cdot d} v' \rangle)$. Now S contains the whole half space $[2\pi, \infty] \times V \times \mathbf{R}$. Once more, there are noncompact intervals.

[3.4] J. HILGERT. *Applications of Lie semigroups in analysis*

Let G be a Lie group and $\pi: G \to \mathcal{U}(\mathcal{H})$ a unitary representation. Consider analytic extensions $\widetilde{\pi}: \Gamma \to \mathcal{C}(\mathcal{H})$ from a complex manifold Γ with SHILOV boundary G which is at the same time a semigroup into a the semigroup of contraction operators on $\mathcal{H}$. Examples of this type of analytic extnesions have been considered (i) by KRAMER, MOSHINSKY, SELIGMAN, BRUNET (1973-85) for $G = \mathrm{Sp}(n, \mathbf{C}) \cap \mathrm{U}(n, n)$

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and the (projective) representation π associated to the CCR, as a computational device in nuclear physics, (ii) by HOWE (1987) for $G = \mathrm{Mp}(n, \mathbf{R})$, the metaplectic group, and the metaplectic representation π, as a computational device to prove estimates for symbols of pseudodifferential operators, (iii) by GELFAND-GINDIKIN, OL'SHANSKĬI, STANTON (1977-85), for G hermitian symmetric and π in the holomorphic discrete series, in order to construct HARDY-spaces on which the holomorphic discrete series can be realized in a uniform fashion. It turns out that the first two examples are essentially the same. G. I. OL'SHANSKĬI gave a general construction of the analytic extension for simple groups. The methods used are flexible and can employed for other groups as well.

[3.4] A. EGGERT. *Semialgebras in reductive Lie algebras*

Let L be a finite dimensional real Lie algebra. A wedge (or closed convex cone) W in L is called a *semialgebra* if there is a CAMPBELL-HAUSDORFF-neighborhood B such that $(W \cap B) * (W \cap B) \subseteq W$. Using LAWSON's Theorem on tangent hyperplane subalgebras we show the following *Theorem.* Let W be a generating semialgebra in a reductive Lie algebra L. Then there are ideals $S_1, \ldots, S_k$ $(k = 0, 1, \ldots)$ all of which are isomorphic to $\mathrm{sl}(2, \mathbf{R})$ and one ideal L^* of L such that (i) $L = S_1 \oplus \cdots \oplus S_k \oplus L^*$, (ii) $E = (W \cap S_1) \oplus \cdots \oplus (W \cap S_k) \oplus (W \cap L^*)$, (iii) All $W \cap S_j$ are generating semialgebras in $S_j \cong \mathrm{sl}(2, \mathbf{R})$ and $W \cap L^*$ is an invariant wedge in L^*. This result classifies all semialgebras in reductive Lie algebras, since the semialgebras in $\mathrm{sl}(2, \mathbf{R})$ are well known and for a classification of invariant cones one is referred to the monograph mentioned in [3.5] below.

[3.5] K. H. HOFMANN. *Lie semigroup theory*

For a general Lie theory of semigroups three tasks have to be mastered: The infinitesimal theory , the local theory, and the global theory. We present an introduction to the infinitesimal theory. With a (local or global) subsemigroup S of a Lie group G one associates a convex cone $W = \mathbf{L}(S)$ in the Lie algebra $\mathfrak{g}$ of G. The characteristic condition reads $e^{\mathrm{ad}\, x} W \subseteq W$ for all $x \in \mathfrak{g}$ with $x, -x \in W$. This gives the concept of a *Lie wedge*, one of the possible generalizations of a Lie algebra. Also discussed are *Lie semialgebras* and *invariant cones* and their characterizations. The details will appear in the monograph "HILGERT, J., K. H. HOFMANN, and J. D. LAWSON, Lie groups, convex cones, and semigroups, Oxford University Press, 664+38 pp., July 1989".

[3.6] J. P. HOLMES. *Differentiable semigroups*

A *differentiable semigroup* is a topological semigroup S on a C^1-Banach manifold with a continuously differentiable multiplication. *Theorem.* If C is a component of the set $E(S)$ of idempotents of S then there is an open neighborhood U of C so that there is a C^1-retraction $\Gamma\colon U \xrightarrow{\text{onto}} C$ so that $x\Gamma(x) = \Gamma(x)x$ is in the maximal subgroup $H(\Gamma(x))$ containing the idempotent $\Gamma(x)$ for all $x \in U$. This result allows us to conclude that C is a C^1-submanifold and that each $e \in E(S)$ is contained in some paragroup R such that $R \cap E(S)$ is a neighborhood of e in $E(S)$. The examples include the multiplicative semigroups of Banach algebras.

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[3.7] J. D. LAWSON. *Embedding semigroups into Lie groups*

Let S be a cancellative topological semigroup on a connected Euclidean manifold. One can make sense locally of left quotient sets and obtain a local group which is locally euclidean and which admits in a natural way a local right action on S. By a result of JACOBY, the local group is locally isomorphic to a simply connected Lie group $\widetilde{G}(S)$. For the product $S \times \widetilde{G}(S)$, there exists a topology finer that the product topology on $S \times \widetilde{G}(S)$ such that in the sequence $S \leftarrow S \times \widetilde{G}(S) \rightarrow \widetilde{G}(S)$ the left hand map is a covering projection and the right hand map is a local homeomorphism. The analytic structure on $\widetilde{G}(S)$ pulls back to S to make it an analytic semigroup. One component $\widehat{S}$ of $S \times \widetilde{G}(S)$ is a subsemigroup and the restriction $\widehat{S} \rightarrow S$ remains a covering projection. The group $\widetilde{G}(S)$ acts as deck transformatuions $g(s,h) = (s, gh)$, and the subgroup leaving $\widehat{S}$ invariant is a countable central subgroup G_S. Then $S \rightarrow \widetilde{G}(S)/G_S \stackrel{\text{def}}{=} G(S)$ is the free group on the semigroup S and $G(S)$ is a Lie group iff S is algebraically embeddable in a group.

[3.8] K. H. NEEB. *globality in the Lie theory of semigroups*

With a subsemigroup S of a Lie group G we associate a Lie wedge $\mathbf{L}(S) = \{x \in \mathfrak{g} \mid \exp \mathbb{R}^+ \cdot x \subseteq \overline{S}\}$. (Cf. [3.5]) There are Lie wedges W in the Lie algebra $\mathfrak{g}$ of certain Lie groups G which do not arise in this fashion. Lie wedges which do are called *global in* G. Our objective is the characterization of global Lie wedges. We describe the tool of covering homomorphisms of connected Lie groups and how it is applied to the problem of globality. In particular, we classify the global Lie wedges in Lie groups whose Lie algebra is compact.

[3.9] K.-H. SPINDLER. *Classification of invariant cones*

The existence of an invariant cone W in a real Lie algebra L imposes restrictions on the structure of L. In L there is a compactly embedded CARTAN-algebra H. The fine structure of the root decomposition of L with respect to H is discussed. An invariant pointed generating cone W in L is uniquely determined by its intersection $C = W \cap H$. A cone C is the trace of an invariant cone in L if and only if it is invariant under the Weyl group and a certain set of rank one operators. The existence of such cones C can be determined from an enriched root diagram. The final goal is a complete classification by completely geometrical and combinatorial means in the spirit of the classification of simple complex Lie algebras.

[3.10] W. WEISS. *Embedding local semigroups into global ones*

For every Lie wedge W in a finite dimensional Lie algebra $\mathfrak{g}$ there exists a local semigroup S with $\mathbf{L}(S) = W$ (cf. [3.5]). Beginning with dimension 3 one can exhibit Lie wedges W for which there is no subsemigroup S of a Lie group with $\mathbf{L}(S) = W$. We give two constructions for *any* pointed cone W in a Lie algebra $\mathfrak{g}$ showing that topological semigroups S exist which are locally embeddable into a Lie group G with $\mathbf{L}(G) = \mathfrak{g}$ and satisfy $\mathbf{L}(S) = W$. One construction is a free one based the adjoint functor theorem, the other is an analytic one based on causal paths in G with respect to the left invariant causal structure given on G through the transport of W.

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[4] Probability and measure theory

(See also MISLOVE [6.4])

[4.1] J. W. BAKER. *Measure algebras on semigroups*

We survey some developments in the field of measure algebras since ca. 1980. Some of the most significant developments concerened multipliers, ARENS regularity and biduals, and weighted measure algebras. The subject of multipliers in its most intersting aspect is covered by VASUDEVA (see [2.6]) so that we concentrate on results on ARENS regularity and biduals of $L^1(G)$ and $M(S,\omega)$ and on some works on representations of foundation semigroups and their measure algebras.

[4.3] W. HAZOD. *Stability and self-decomposability*

Stable, semistable and self-decomposable probabilities on $\mathbf{R}^d$ can be characterized as the possible limit distributions of suitably normalized sums of independent random variables, or, on the other hand, by certain functional equations for their FOURIER transforms. The latter can be understood as relations of the corresponding convolution semigroups. It is possible to generalize the second concept to probabilities on locally compact groups and it turns out that it is essentially sufficient to consider simply connected Lie groups. For this class of groups we obtain a description of the collections of all possible stable, semistable, respectively, self-decomposable measures. Moreover, again in analogy to to the vector space case, on nilpotent Lie groups, stable and semistable measures can be characterized as possible limit distributions. (Cf.S. NOKEL, Doctoral Dissertation, Universität Dortmund.) The proofs are based on the special structure of the semigroup $M^1(G)$; probability is not involved.

[4.4] H. HEYER. *The embeddability of infinitely divisible probability measures into continuous convolution semigroups*

In the classical probabilistic set-up, the problem of embeddability is already present in the work of LÉVY and was studied in the frame work of locally compact groups by K. R. PARTHASARATY, RANGA RAO and VARADHAN as well as many others; the monograph "H. HEYER, Probability distributions on locally compact groups, Springer-Verlag Heidelberg etc., 1977" is a source of reference. The embeddability problem has recently been extended to the context of hypergroups. For a hypergroup X, the following result—mainly due to M. VOIT, 1988—is discussed in the relation to its classical predecessors: (i) If $\widehat{X}$ is arcwise connected, then any infinitely divisible probability measure μ on X can be embedded in a continuous convolution semigroup $(\mu_t)_{t>0}$ in $M^1(X)$ with $\mu_s * \mu_t = \mu_{s+t}$ for all $s,t > 0$ and vague-$\lim_{t\to 0}\mu_t = \varepsilon_e$ such that $\mu_1 = \mu$. In the case that X is hermitian this statement remains true if, in addition, $\widehat{X}$ has no proper compact hypersubgroup. The report concludes with an outlook on related research in the context of connected Lie groups.

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[4.5] A. MUKHERJEA. *Probability theory in semigroups*

It is shown that using semigroup methods we can describe the weak convergence of the sequence of convolution interates μ^n of a probability measure on the semigroup of finite dimensional matrices. Discrete time versions of the voter model (as discussed by LIGGETT in his book on "Interacting particle systems") and the contact process (also discussed in this book) can be treated using semigroup methods. Certain results and examples in these contexts are presented.

[5] Semigroups with one-sided or separate continuity

[5.1] J. F. Berglund. *Semigroup compactifications*

A *semigroup compactification* of a semitopological semigroup S is a pair (X,ψ) with a compact right topological semigroup X and a continuous homomorphism $\psi: S \to X$. A P-compactification is a compactification with property P. A necessary and sufficient condition (save some technical details) for a universal P-compactification to exist is that the property is preserved under the formation of subdirect products. With this theorem it is easy to see that a universal connected compactification, for instance, does not necessarily exist. On the other hand, many universal compactifications exist including those defined by identities and implications.

[5.2] P. MILNES. *Distal functions and* ELLIS-*groups*

For a HAUSDORFF compact right topological group G let $\Lambda(G) = \{t \in G \mid s \mapsto st,\ ts$ are both continuous$\}$. From a result of ELLIS (1957) we know that G is a topological group if $\Lambda(G) = G$. The results on non-topological G include the structure theorem of NAMIOKA (1972) for the case that $\Lambda(G)$ is dense, and the structure theorem of RUPPERT (1975) for the case when the right translations $s \mapsto st$ are even equicontinuous. The two classes of examples intersect precisely in the class of topological compact groups. We describe an example of a HAUSDORFF compact group falling into neither category.

[5.3] J. PYM. *Compact semigroups with one sided continuity*

Semigroups with one-sided continuity appear naturally in the theory of transformation semigroups as solutions to many universal mapping problems. A key example is the STONE- ČECH-compacrtification βS of a discrete semigroup S, or, more specially, $\beta\mathbb{N}$. One topic of the survey is a technique for obtaining in a simple way strong algebraic results on $\beta\mathbb{N}$ and other semigroups not superficially similar, for example that these contain copies of the free group on 2^c generators. The algebraic structure underlying this has been observed (PAPAZYAN) to correspond to a set of distinct finite sums $\mathrm{FS}\langle x_n\rangle = \{x_{i_1} + x_{i_2} + \cdots + x_{i_n} \mid i_1 < i_2 < \cdots < i_m\}$, where $\langle x_n\rangle$ is a sequence in the semigroup. If S is cancellative, then any neighborhood V in βS of any idempotent e in $\beta S \setminus S$ contains a set of distinct finite sums in S (VAN DOUWEN, HINDMAN) so that V actually contains a free group on 2^c generators.

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[5.4] W. A. F. RUPPERT. *Compact semitopological semigroups*

This is an overview of the theory of compact semigroups on which multiplication is continuous in each variable separately. We comment on main trends, tools and results, including semitopological semigroups on compact manifolds, weak almost compactifications of semisimple Lie groups versus abelian groups and other topics. (As a reference see W. A. F. RUPPERT, Compact semitopological semigroup: An intrinsic theory, Lecture Notes in Mathematics **1079**, Springer Verlag Heidelberg etc., 1984)

[5.5] J.-P. TROALLIC. *Semigroupes affines semitopologiques compacts*

Ce travail a pour but de présenter quelques aspects de la théorie des semigroups affines semitopologiques compacts. Dans les sections 4 et 5, deux types de problèmes sont abordés quit ont trait, les uns à l'existence de points de continuité à gauche pout des actions séparément affines et séparément continus de semigroups affines semitopologiques compacts (section 4), les autres à l'extrémalité des points de continuité à gauche obtenus (section 5). Ces deux sections ont ceci en commun qu'elles s'appuient l'une et l'autre sur une variante d'un résultat de NAMIOKA concernent la dentabilité. Cette variante est l'outil principal de ce travail; elle joue un rôle comparable à celui que joue le théorème de point fixe de RYLL-NARDZEWSKI dans l'approche originale de la prèsque-périodicité faible. Les sections 6,7 et 8 sont consacrées à des applications des résultats établis dans les sections 4 et 5.

[6] Topological algebra, topological semigroups, order theory

(See also PIN [1.4])

[6.1] D. R. BROWN. *Semigroups on n-cells, unique divisibility, and matrices*

Let S be a semigroup of a compact n-cell with minimal ideal K, and suppose that S is uniquely divisible with $E = \{1\} \cup K$ and with trivial groups. Also assume $xS \subseteq Sx$ for all $x \in S$ and cancellation on $S \setminus K$. For any idempotent $e \in E$ let let $C(e)$ denote the *core* $\{x \in S \mid xe = ex = e\}$. The possibilities for the dimension of the core are given by $\dim C(e) + \dim K = n$ for $k = 0, \ldots, n-1$. Various theorems for particular values of k are discussed. The aim is to show that portion $S \setminus K$ outside the minimal ideal is embedded into a metabelian Lie group.

[6.2] D. R. BROWN and J. A. HILDEBRANT. *Embedding compact t-semigroups into compact uniquely divisible semigroups*

A *t-semigroup* is a semigroup whose subgroups are singleton. The class of compact commutative semigroups with are embeddable into compact uniquely divisible semigroups includes according to our present knowledge: (1) semigroups which have a totally disconnected semilattice continuous homomorphic image whose point inverses are power ideal semigroups, (2) t-semigroups in which the down set of each idempotent has a neighborhood in its core which is contained in the image of the map $x \mapsto x^2$, (3) t-semigroups in which each idempotent has a finite down set and a core (see [6.1]) satisfying a suitable condition, (4) semigroups containing a

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cancellative element in their divisor. It is proved that for a compact n-dimensional power-cancellative commutative topological semigroup S the subsemigroup $S\setminus\{0\}$ is embeddable into a cone in $\mathbf{R}^n$ if each element of $S\setminus\{0\}$ is divisible.

[6.3] G. GIERZ and A. STRALKA. *Sublattices of* $\mathbf{R}^n$

Let $n \in \mathbf{N}$ be a positive integer. A family $\alpha_{ij}: [0,1] \to [0,1]$, $1 \le i, j \le n$ of upper semicontinuous monotone functions satisfying (1) $\alpha_{ij}(1) = 1$ for all i, j, (2) $\alpha_{ii}(x) = x$ for all i and x, (3) $\alpha_{ij} \circ \alpha_{jk} \ge \alpha_{ik}$ for all $i, j\, k$ is called an *inf-seam*. If $(\alpha_{ij})_{1\le i,j\le n}$ is an inf-seam, then $L = \{(x_1, \ldots, x_N) \in [0,1]^n \mid x_j \le \alpha_{ij}(x_j)$ for all $i, j\}$ is a closed connected sublattice containing $\mathbf{0}$ and $\mathbf{1}$. Conversely, every closed connected sublattice of $[0,1]^n$ containing $\mathbf{0}$ and $\mathbf{1}$ is of this form. *Applications.* A sublattice $L \subseteq \mathbf{R}^n$ is *full* if it is compact and the interior of L is connected and dense in L. A point $x \in \partial L$ is *a* C_1*-point*, if there is a neighborhood U of x and a continuous function $\varphi: U \to \mathbf{R}$ such that (1) $\partial L \cap U = \{p \in \mathbf{R}^n \mid \varphi(p) = 0\}$ and (2) $\operatorname{grad}\varphi(x) \ne 0$. *Theorem.* If L is full, then $\{x \in \partial L \mid x$ is a C_1-point$\}$ is a dense G_δ in ∂L. We call a full sublattice $L \subseteq \mathbf{R}^n$ a *lattice sphere* if Prime(L) = Coprime(L). *Theorem.* Up to isomorphy, there is exactly one lattice sphere in dimensionl 1, 2, and 3. If the dimension is at least 4, then there are uncountably many pairwise non-isomorphic lattice spheres in each dimension.

[6.4] M. W. MISLOVE. *History and applications of compact semilattices*

We trace the development of the structure theory of compact semilattices and their apparatus. The development of the notion of a LAWSON *semilattice* is described, and the structure theory of these semilattices elucidated. Applications of objects—also known as *continuous lattices* to general topology are described, where they arise as the open set lattice of locally compact sober spaces. But we also give applications to harmonic analysis. In this context, we have the following *Theorem.* For a locally compact semilattice the following two conditions are equivalent: (1) The algebra $M(S)$ of all finite regular Borel measures is symmetric. (2) S has compactly finite breadth, that is, for all compact subsets $K \subseteq S$ there is a finite subset $F \subseteq K$ with $\inf F = \inf K$. (3) S contain no copy of $2^{\mathbf{N}}$. If these conditions hold, then $\Delta M(S)$ is the filter semilattice of the discrete semilattice S_d. Thus the idempotent measures are direct sums of point masses, and the invertible measures are exactly the exponential measures.

[6.5] A. AND J. SELDEN. *The continuous extended bicyclic semigroups*

We discuss closures of certain semigroups in locally compact topological inverse semigroups in which inversion is continuous. First we provide the background information on the free inverse semigroup with one generator and on the closures of the discrete and continuous bicyclic semigroups, as well as on the closures of the discrete and continuous extended bicyclic semigroups. We then list some examples of topological inverse semigroups on the plane to provide a setting for the following application of the previously mentioned results: If S is a topological inverse semigroup on the plane containing no non-trivial groups and whose idempotents form a line, then S is a continuous extended bicyclic semigroup.

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[6.6] A. M. SKRYAGO. *Matrix representations of regular semigroups*

Let S be a compact regular semigroup. The following questions have to be discussed: (1) Do the finite dimensional representations separate the points? (2) Does a representation of a subgroup of S extend to a representation of S? (3) How are all representations of S to be classified? Regarding question (1) we have the following *Theorem.* S is representeably if and only if each connected component is contained in a $\mathcal{D}$-class and any maximal simple CLIFFORD-subsemigroup is a rectangular band of groups. Regarding (3) we propose the *Theorem.* A representation with 0-simple image is a tensor product of a group representation and a representation of an $\mathcal{H}$-trivial 0-simple semigroup. We describe various results concerning the extension problem (2).

[6.7] H.-J. WEINERT. *Extensions of topological semigroups by right quotients*

Let $(S, \cdot, \mathcal{S})$ a topological semigroups with topology $\mathcal{S}$ and let $(T, \cdot) = Q_r(S, \Sigma) = \{ad^{-1} \mid a \in S, d \in \Sigma\}$ a semigroup of right quotients of S with respect to a subsemigroup Σ of S. The problem is to describe all topologies $\mathcal{T}$ on T such that $(T, \cdot, \mathcal{T})$ is a topological semigroup with $\mathcal{T}|S \subseteq \mathcal{S}$, and to describe conditions under which a subgroup G of $(T, \cdot)$ is a topological group $(G, \cdot, \mathcal{T}|G)$. This problem is completely solved for the special case that $\mathcal{T}|S = \mathcal{S}$ and $S \in \mathcal{T}$. In general, suitable topologies $\mathfrak{s}$ on Σ provide us with a base $\{U\Omega^{-1} \mid U \in \mathcal{S}, \Omega \in \mathfrak{s}\}$ for such a topology. We discuss generalisations. In that context, cf. J. K. LUEDEMAN, A topological semigroup of quotients, Studia Sci. Math. Hung. **14**(1979), 77–82.

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