MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 7/1989

Mehrdimensionale konstruktive Funktionentheorie

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The conference has been organized by C.K. Chui (College Station), W. Schempp (Siegen), and K. Zeller (Tübingen). The purpose of the conference was to bring together researchers interested in various aspects of the theory and applications of multivariate approximation. The talks that were delivered reflected well the multiple interest in the audience. There have been lectures on multivariate splines, finite elements, multivariate interpolation, multivariate rational approximation, optimal recovery, Boolean methods, spherical functions, and multivariate discrete Fourier transforms. Moreover, the application of multivariate approximation theory to signal processing has been dealt with. The talks have been followed by lively discussions and a useful exchange of ideas has taken place.

ABSTRACTS

Peter Alfeld:

Multivariate Splines

Splines (i.e., smooth piecewise polynomial functions) are used universally throughout problems involving functions on one variable. It is natural to contemplate the use of similar functions in the case of several variables. However, problems that are trivial for one independent variable turn out to be extremely difficult in the case of two or more variables. In this talk, some unsolved problems concerning multivariate splines will be described and some new results will be given.

Marc Atteia:

Approximation Theory

Let Ω be an open subset of $\mathbb{R}^n$ and $\mathcal{C}$ a regular triangulation of $\overline{\Omega}$. Let us denote by $|\mathcal{C}|$ the simplicid complex associate to $\mathcal{C}$. We define a mapping u from $|\mathcal{C}|$ to an analytic simplicid complex $\mathcal{C}^*$ of multidimensional functions on Ω such that, to each elementary operation on $|\mathcal{C}|$ corresponds an operation on $\mathcal{C}^*$.
Thus, we obtain, by an unique process, for example: Finite elements, spline polynomial functions or classical fractals. Easily, the dual process gives different kinds of box splines.

Günter Baszenski:

An FFT Scheme for Boolean Sums of Trigonometric Operators

It is known that bivariate functions from a Korobov space can be well approximated by Fourier partial sums of hyperbolic type. For practical computations it is desirable to use discret rather than transfinite Fourier coefficients. We construct a pseudohyperbolic trigonometric Boolean sum operator which is interpolatory, yields asymptotically the same error bounds as the hyperbolic Fourier partial sum operator and whose coefficients can be efficiently computed by a FFT scheme.

Rick Beatson:

Constrained Interpolation

An algorithm for bivariate interpolation by quadratic splines is discussed. This is related to the algorithm of B. and Z. Ziegler which appeared in SIAM J. Numerical Analysis in 1985. An $O(\delta^2)$ estimate of the uniform error holds in general, but this can be improved in special cases. In particular at a point where $\frac{\partial f}{\partial x} > 0$ and $\frac{\partial f}{\partial y} > 0$. the error is $O(\delta^3)$. Also if the behaviour in the neighbourhood of the zeros of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ is restricted in a certain way the global error is $O(\delta^{8/3})$.

Louis T. Billera:

Homological Methods for Multivariate Splines

For a triangulated d-manifold $\Delta \subset \mathbb{R}^d$, let $\mathbb{P}^r_m$ be the sheat of real vector spaces over Δ defined by $\mathbb{P}^r_m(\sigma) = P_m/I^{r+1}_\sigma$ for $\sigma \in \Delta$, where P_m is the space of all polynomials in d variables of degree $\le$ m, and $I_\sigma = \{f : f|_\sigma = 0\}$. If $H_*(\mathbb{P}^r_m)$ denotes homology with coefficients in $\mathbb{P}^r_m$, then $H_d(\mathbb{P}^r_m) = S^r_m(\Delta)$, the space of all C^r piecewise polynomials over Δ of degree at most m. For d=z, we use this homology theorem to derive lower bounds for dim $S^r_m(\Delta)$ as well as the generic dimension of $S^1_m(\Delta)$.

Carl de Boor:

Existence of a Local and Stable Basis for Certain Bivariate pp Spaces

Recent work, by Dong, Chui and Lai, Ibrahim and Schumaker, has provided explicit local bases for $\Pi^\rho_{k,\Delta}$ (:= piecewise polynomials of degree $\le$ k on a triangulation Δ and in $C^\rho(\mathbb{R}^2)$) and certain useful subspaces ("supersplines") where $k \ge 3\rho+2$.
An earlier claim that the material in *de Boor and Höllig*, Math. Z. 197 (1988), 343-363, contains enough information to infer the existence of a local and stable basis for these spaces is substen-

tiated. It is also pointed out that the taste of constructing such a basis reduces to the local problem of finding a suitable basis for $\Pi^{\rho}_{2\rho,\Delta_0}$, with Δ_0 the partition formed by all the triangles in Δ having a vertex in common.

Martin Buhmann:

Asymptotic Expansions of Hankel Transforms

Using the theory of asymptotic expansions of Hankel integrals, we establish sufficient conditions on a radial basis function $\phi:\mathbb{R}_{\geq 0} \to \mathbb{R}$ to admit cardinal interpolation on the grid $\mathbb{Z}^n$. More specifically, we aim to find a fundamental function $C:\mathbb{R}^n \to \mathbb{R}$ that is a linear combination of basis functions $\{\phi(\|x-j\|_2) \mid x \in \mathbb{R}^n\}$, $j \in \mathbb{Z}^n$. Our sufficient conditions depend on a derivative of $\phi(\sqrt{\cdot}):\mathbb{R}_{>0} \to \mathbb{R}$ being "multiply monotonic" and having prescribed asymptotic behaviour near 0 and for large argument.

H.G. Burchard:

Multivariate Splines: Polynomial Degree and Approximation Order

Consistent families of polynomials (Appell sequences) have a property, $D^{\beta}h_{\alpha}=h_{\alpha-\beta}$, that leads to a Taylor expansion $h_{\alpha} = \sum_{\gamma\in E} v_{\gamma}\, m_{\alpha-\gamma}$, in terms of monomials $m_{\beta} = x^{\beta}/\beta!$, or a translation representation $h_{\alpha} = \sum_{j\in\Delta} a_j m_{\alpha}(\cdot-j)$. The latter is evidently available only if $\gamma \in E \Rightarrow \gamma \geq 0$, in which case the family $\{h_{\alpha}\}$ is said to be strongly consistent. Families $\{h_{1,\alpha}\}_{1\in I,\alpha\in\Gamma}$ are LMG if

$$m_{\alpha} = \sum_{1\in I} \sum_{j\in\mathbb{Z}^s} h_{1,\alpha}(j)\, \varphi_1(\cdot-j),\ \alpha \in \Gamma$$

relative to a set $\Phi = \{\varphi_1 : 1\in I\}$ of basic functions ("splines"). If $\{h_{1,\alpha}\}$ is strongly consistent then approximation order $\partial\Gamma$ can be achieved, $\partial\Gamma = \{\alpha \in \mathbb{Z}^s_+ : \alpha \notin \Gamma,\ \exists\ j \in \{1,\dots,s\},\ \alpha - e^j \in \Gamma\}$: $\|u-u_{\eta}\|_{L^p} \leq H_u \max\{\eta^{\beta} : \beta \in \partial\Gamma\}$, $\quad u_{\eta}(x) = \sum_{1\in I} \sum_{j\in\mathbb{Z}^s} \tau^{\eta}_1(j)\, \varphi_1(x/\eta-j)$, with local support of the coeffiecients $c^{\eta}_1(j)$. If Γ is concave then the converse holds. However, examples exist, where higher degree LMG formulas with non-strongly consistent $\{h_{1;\alpha}\}$ occur, for the case of C^1-cubic Boxsplines, for one. Note $\max\{\eta^\beta : \beta\in\partial\Gamma\}$ depends only on the concave part of Γ, as $\eta = (\eta_1,\dots,\eta_s) \to 0$.

HAN-LIN CHEN, CHARLES K. CHUI, CHARLES A. MICCHELLI

On Optimal Recovery for Periodic Functions

We studied the following problem: Does the ratio sampling error and n-width converge (or diverge) when n tends to infinity? To be more precise, the problem to be considered is that under which conditions the ratio

$$(1)\qquad \frac{\inf_{A_n} D(F,\Delta_n)}{i_n(F)} = \frac{\inf_{A_n}\ \sup_{f\in F} \|f-A_n(f(x_1),\dots,f(x_n))\|_2}{\inf_{A_n,I_n}\ \sup_{f\in F} \|f-A_n(I_n f)\|_2}$$

converges when n tends to infinity?

where I_n: any continuous linear mapping from F into R^n.

A_n: any mapping from R^n into F.

$\{x_j\}_1^N \in Q_2$, Q_2 unit square.

F : certain unit square.

Δ_n: $\{x_j\}_1^n$.

Theorem 1 states that the ratio (1) is finite if n tends to infinite. On the other hand, the theorem states that the optimal sampling point set is not always optimal. We listed many examples.

TIAN-PING CHEN:

Generalized Bochner-Riesz Means of Fourier Integral

Suppose that $f(x) \in L_2(R^n)$, $\hat{f}(x)$ is it's Fourier Transform. Generalized Bochner-Riesz Means is defined as

$$(B_R^{\delta,\lambda}(f))\hat{}\,(x) = \hat{f}(x)(1-|Rx|^\lambda)_+^\delta$$

which is equivalent to

$$B_R^{\delta,\lambda}(f;x) = \int \hat{f}(u)\left(1-\frac{|u|^\lambda}{R^\lambda}\right)^\delta e^{iux}du$$

We say $f(x) \in B^\gamma$, the Bessel Potential Space of order γ, if both $f(x)$ and $|x|^\gamma\cdot\hat{f}(x)$ belong to $L_2(R^n)$.

The main result of this paper is

Theorem. If $f(x) \in B^\gamma$, $\alpha > 0$, then

$$B_R^{\delta,\lambda}(f;x) - f(x) = o(R^{-\gamma}), \quad \text{if } \lambda > \gamma \text{ a.e.,}$$

$$B_R^{\delta,\lambda}(f;x) - f(x) = O(R^{-\gamma}), \quad \text{if } \lambda = \gamma \text{ a.e..}$$

CHARLES K. CHUI:

Rational Approximation in Signal Processing and System Theory

Rational (matrix-valued) functions not only provide important approximation tools, but also represent realizable models of digital and analog filters and feedback control systems. However, since stability is an essential issue in these applications, the poles of the rational approximants must be restricted to certain regions such as the unit disk $|z|<1$ for digital systems and the half-plane Re $s<0$ for analog systems. Hence, best approximation in the Hankel (operator) norm is very appropriate, since boundedness of the finite rank Hankel approximants is equivalent to stability of the corresponding rational approximations. Recent development based on the AAK theory is surveyed in this lecture. In particular, results related to the four-block problem and minimum-norm interpolation problem are discussed.

ZBIGNIEW CIESIELSKI:

An algorithm for best approximating algebraic polynomial in L^p over a simplex

The problem of finding the best approximating polynomial in question of degree m in dimension d has been reduced to a sequence of minimalization problems for convex functions over R^s with $s = \binom{m+d}{d}$. An algorithm for constructing a sequence of polynomials of degree m and approaching the best approximating polynomial to given $f \in L^p$, $f \in C$ in case $p = \infty$ is presented.

LOTHAR COLLATZ:

Some Applications of Multivariate Rational Approximation to Differential Equations

Rational approximation can sometimes be better as polynomial approximation in the neighbourhood of certain types of singularities. We refer in this paper about experiments and calculations (mostly during the last year) for problems with unbounded domains and corners, edges a.o., especially for the Laplace equation in two and in three dimensions. It is often possible in these cases to give easy calculable lower and upper bounds for the wanted solutions. Numerical results are given.

FRANZ-JÜRGEN DELVOS:

Boolean Lattice Rules

Lattices rules are important methods in multidimensional numerical quadrature. We apply Boolean methods of multivariate interpolation to construct Boolean sums of lattice rules. Using the duality theory of Boolean algebras we derive remainder formulas for Boolean lattice rules. Appling this to functions in a Korobov space we can show that Boolean lattice rules form good cubature rules in the sense of Korobov.

HARVEY DIAMOND:

Characterization and Calculation of Quasi-Interpolants

We work in the space $\mathcal{S}(\Phi) = \{s|s(x) = \sum c_j\Phi(x-j)\}$ where $x \in \mathbb{R}^s$, $j \in \mathbb{R}^s$, $j \in \mathbb{Z}^s$ and Φ is a compactly supported function. We assume that the scaled space $\mathcal{S}_h(\Phi) = \{s(\cdot/h)\}$ has approximation degree n, with $\Pi_n \subset \mathcal{S}(\Phi)$ (Π_n = polynomials of total degree $\le n$).

Let $\mathcal{Q}$ denote the set of quasi-interpolation operators Q of the form

$$Qf := \sum \lambda f(\cdot+j)\Phi(x-j)$$

where λ is a local linear functional; and let $\Lambda_{\mathcal{Q}}$ denote the set of λ's corresponding to $Q \in \mathcal{Q}$. Then we prove that

$$\lambda_1,\lambda_2 \in \Lambda_{\mathcal{Q}} \Rightarrow \lambda_1(p) = \lambda_2(p) \quad \forall p \in \Pi_n.$$

Likewise, if $\lambda_1 \in \Lambda_{\mathcal{Q}}$ and $\lambda_2(p) = \lambda_1(p)$, $\forall p \in \Pi_n$ then $\lambda_2 \in \Lambda_{\mathcal{Q}}$.

Applications of the above result include:

a) A formulation of the problem of "minimally supported" λ's in $\Lambda_{\mathcal{Q}}$.

b) More general quasi-interpolants of the form $\sum \lambda_j f(\cdot+j)\Phi(x-j)$, $\lambda_j \in \Lambda_{\mathcal{Q}}$ which can be used for

i) uniform approximation (via quasi-interpolation) on compact domains (no information on function outside)

ii) Approximation of functions from scattered data

We also have the result that

$\lambda \in \Lambda_{\mathcal{J}} \Leftrightarrow \sum f(j)\, \lambda\Phi(\cdot+x-j)$ is a quasi-interpolant for the space $\mathcal{S}(\psi)$ where $\Psi = \lambda\Phi(\cdot+x)$.

We can produce functions Ψ with desired properties of smoothness/ support size by choosing appropriate $\lambda \in \Lambda_{\mathcal{J}}$.

DINH DUNG:

Multdimensional Band-Limited Functions: Generalization of the Sampling Theorem, $L_q([-T,T]^n)$-Approximation by Finite Sampling Sums and ε-Dimension

Taking the information sense, discovered by Kotelnikov and Shannon, of the well-known Whittaker-Kotelnikov-Shannon sampling Theorem as the basic idea Kolmogorov and Tikhomirov introduced the ε-entropy per "length unit" and the mean ε-dimension. These quantities were studied by Tikhomirov, Dinh Dung, Magaril-Il'jaev and others.

This talk deals with a new approach to the study of ε-entropy and ε-dimension in the space $L_q([-T,T]^n)$ of L_p-bounded sets of functions band-limited to a subset G of $\mathbb{R}^n$ $(1 \le p,q \le \infty)$ and the set of smooth functions. In this study the $L_q([-T,T]^n)$-approximation by finite sampling sums plays a central rôle. We shall be concerned with some multidimensional generalizations of the sampling theorem and analogues of Marcinkiewicz' Theorem for band-limited functions.

HANS G. FEICHTINGER:

Stable Reconstruction of Band-Limited Functions on R^m from Irregularly Distributed Sampling Values

It is the purpose of this talk to explain irregular variants of the famous Whittaker-Shannon theorem (or it's generalizations) in the following sense: Any band-limited function f on $\mathbb{R}^m$ can be reconstructed completely from any family of sampling values, as long as the density of the sampling points is high enough (in dependence of the bandwidth of f).

The recovering of f is contructive, i.e. through an iterative approximation procedure. The resulting approximations converge to the limit f in the "appropriate" norm, i.e., in the any weighted L^p-norm $\| \ \|_{p,w}$ (for $1 \le p < \infty$) for which one has f L^p_w , i.e. $\|f\|_{p,w} < \infty$, as well as uniformly over compact sets. Furthermore stability results showing the robustness of the procedure (and resulting estimates for aliasing and jitter errors) can be given.

WILLI FREEDEN:

Spherical (Vector) Splines

Spherical splines are defined by use of well-known properties for spherical harmonics and the concept of Green's surface functions with respect to iterated Beltrami derivatives and (invariant) pseudodifferential operators. Natural spherical splines are used to interpolate data discretely given on the unit sphere. An a-priori estimate in spherical spline interpolation is given dependent on the spacing of the data. Spherical spline interpolation is discussed for the problem of determining the external gravitational potential and the figure of the earth (i.e. the geoidal surface).

Finally vector spherical harmonics and Green's tensors on the sphere are introduced to generalize the scalar theory to the vectorial case.

M. GASCA:

Some Applications of a Multivariate Horner's Algorithm

A generalized Horner's algorithm, with interesting applications to multivariate polynomials has been recently obtained by the authors. In this talk we apply it to evaluate efficiently some particular expressions of multivariate polynomials and their derivatives, with special emphasis in Lagrange form of interpolating polynomials.

H. GONSKA:

Simultaneous Approximation by n-th Order Blending Operators

The method of n-th order blending was introduced by Delvos and Posdorf and constitutes a generalization of Gordon's discrete blending approach. In our talk we shall investigate the degree of approximation of bivariate functions given on a rectangle by generalized n-th order blending operators. Our aim is to give a fuller description than is available in the literature by using mixed moduli of smoothness of higher orders as introduced by Marchaud. The results include certain permanence principles which explain how generalized n-th order blending operators inherit quantitative properties from their univariate building blocks. Moreover, for univariate spline interpolation we prove an extension of a theorem due to Swartz and Vargha which includes our previous generalization of the Sharma-Meir theorem on the degree of approximation by cubic spline interpolators.

G. HEINDL:

Some Results on Quadratic Splines of three (and more) Variables

The talk is devoted to the following question:

Given prescribed function values and gradients at the vertices of a simplicial complex K, is it possible to construct a refinement K' of K such that there is a unique quadratic C^1-spline function with respect to K', interpolating the given data?

It is shown how to solve that problem in the 3-dimensional case, when K satisfies a certain geometric condition. The underlying concept is independent of the number of variables.

KLAUS HÖLLIG:

Box-Spline Tilings

Let f be an analytic function of d variables with $|f(x+j)| \to \infty$ as $\mathbb{Z}^d \ni j \to \infty$ for almost every x. Then, the translates of the set

$$\Omega_f := \{x : |f(x)| < |f(x+j)|,\ j \in \mathbb{Z}^d \setminus 0\}$$

form a tiling of $\mathbb{R}^d$. More precisely:

(i) $\bar{\Omega} \cap j + \Omega = \emptyset,\ j \neq 0;$

(ii) measure $(\mathbb{R}^d \setminus \bigcup_j j + \Omega) = 0.$

Already for rather simple choices of f the set Ω_f has a rather interesting structure. We discuss in detail the particular case

$$f(x) := (x\cdot\xi)(x\cdot\eta),\ x \in \mathbb{R}^2,$$

with $\xi,\eta \in \mathbb{Z}^2$ which arises in the characterization of functions of exponential type as limits of box-splines.

YING-SHENG HU:

Variation-Diminishing Operators

The phenomenon of the good shape preserving property and low convergence rate in Bernstein-type approximation is investigated by iterates.

Theorem 1 in the paper shows that the iterate process of any variation diminishing approximation on C[0,1] reproducing the linears is convergent, and the limiting function is linear or piecewisely linear. As a result, we found that the first eigenvalue with modulus less than 1 of a variation diminishing operator is a quantity which characterizes the contraction phynomenon mentioned above.

The concept of asymptotic variation diminishing operators introduced in the paper is crucial in characterizing Bernstein-type variation diminishing operator, and it is hopefully useful in studying variation diminishing property and convexity for higher dimension approximation.

KURT JETTER:

Methoden der Fourier-Transformation bei periodischer kardinaler Interpolation

Abminderungsfaktoren spielen bei der numerischen Fourier-Analyse eine entscheidene Rolle; hierbei werden die (äquadistant vorgegebenen) Daten zunächst einem Operator vom Faltungstyp unterworfen. Dieses Vorgehen, das im univariaten Fall inzwischen als

Standardtechnik bezeichnet werden kann, wurde erst kürzlich von Gutknecht und von ter Morsche auf den mehrdimensionalen Fall übertragen. Dabei kommen kardinale Interpolationsoperatoren auf der Basis von Translaten sog. Box-Splines zum Einsatz. Der Vortrag greift diese Fragestellung auf, ergänzt die bisher vorliegenden Ergebnisse und diskutiert einen Algorithmus, der z.B. in der graphischen Bildverarbeitung eingesetzt werden kann. Der Diskretisierungsfehler kann hier leicht kontrolliert werden. Beispiele zeigen die Effizienz des Algorithmus.

RONG-QING JIA

A Dual Basis for the Integer Translates of an Exponential Box Spline

Let $X = (x^1,\dots,x^n) \subset \mathbb{R}^s\setminus\{0\}$ and $\mu \in \mathbb{C}^n$. The exponential box spline $C_\mu(\cdot|X)$ is the linear functional on $C(\mathbb{R}^s)$ given by

$$\phi \longmapsto \int_{[0,1]^n} e^{-\mu\cdot u}\phi\Big(\sum_{j=1}^n x^j u_j\Big)du, \qquad \phi \in C(\mathbb{R}^s).$$

When X spans $\mathbb{R}^s$, $C_\mu(\cdot|X)$ is a piecewise exponential polynomial function. In this paper we construct a dual basis $(\lambda_\alpha)_{\alpha\in\mathbb{Z}^s}$ for the integer translates $C_\mu(\cdot-\beta|X)$, $\beta \in \mathbb{Z}^s$:

$$\lambda_\alpha C_\mu(\cdot-\beta|X) = \delta_{\alpha\beta}\,,$$

when those translates are linearly independent. The dual basis is shown to be unique in a certain sense. Our construction is based on a systematic study of the space $G_\mu(X)$ which consists of all the polynomials p such that $p(D)C_\mu(\cdot|X)$ is a bounded function.

BURKHARD LENZE:

On Multidimensional Lebesgue-Stieltjes Convolution Operators

Let $(D_p)_{p\geq p_0}$ be a family of onedimensional, continuous, non-negative, and even kernel functions with integral value 1 which satisfy for each $\epsilon > 0$ the approximate identity condition

$$\lim_{p\to\infty} \int_{|t|>\epsilon} D_p(t)\,dt = 0.$$

Moreover, we consider the family of integral kernel functions

$(I_p)_{p\geq p_o}$ generated by $(D_p)_{p\geq p_o}$ via

$$I_p(x) := \int_{-\infty}^{x} D_p(t)dt \ , \ x \in \mathbb{R} \ , \ p \geq p_o .$$

In some formal analogy of the n-dimensional radial convolution operators $(\Lambda_p)_{p\geq p_o}$,

$$\Lambda_p(f)(x) := \left(\int_{\mathbb{R}^n} D_p[\{ \sum_{k=1}^{n} (t_k)^2\}^{\frac{1}{2}}]dt \right)^{-1} \int_{\mathbb{R}^n} D_p[\{ \sum_{k=1}^{n} (t_k-x_k)^2\}^{\frac{1}{2}}]f(t)dt \ ,$$

$x \in \mathbb{R}^n$, which we assume to be well-defined on a properly chosen subset of $BV(\mathbb{R}^n)$, we introduce the n-dimensional hyperbolic Lebesgue-Stieltjes convolution operators $(\Omega_p)_{p\geq p_0}$,

$$\Omega_p(f)(x) := (-1)^n \, 2^{1-n} \int_{\mathbb{R}^n} I_p\left(\prod_{k=1}^{n} (t_k-x_k) \right) df(t), \ x \in \mathbb{R}^n,$$

which we show to be well-defined and useful on a properly chosen subset of $BV(\mathbb{R}^n)$.

WILLIAM LIGHT:

Radial Basis Functions Using the l_1 and l_∞ Norms

An interesting subspace of $C(\mathbb{R}^n)$ is obtained by considering m distinct translates of the function $f \in C(\mathbb{R}^n)$ defined by $f(x)=\|x\|$ or, more generally, $f(x) = \phi(\|x\|)$. Such a subspace is spanned by the functions $\{\|\cdot-x_j\|\}_{j=1}^{m}$ where $x_1,\ldots,x_m$ are distinct points in $\mathbb{R}^n$ and $\|\cdot\|$ can be any norm. The case for the Euclidean norm has been studied thoroughly recently by a number of authors. We will consider the case when the norm is the l_1-norm, usually on $\mathbb{R}^2$.

G.G. LORENTZ:

Bivariate Hermite Interpolation and Applications to Algebraic Geometry

The (p,n) "Hermite interpolation" in $\mathbb{R}^2$ concerns polynomials P of total degree n. If some m interpolation knots $Z:(x_i,y_i)$, $i = 1,\ldots,m$ are given, we want to find a P with prescribed values of all its partial derivatives of orders k , $0\leq k\leq p$, at the knots. This requires that $m=(n+1)(n+2)/(p+1)(p+2)$. With purely geometric means (shifts of triangles) we prove: For $p = 0,1,2,3$ and arbitrary n, all Hermite interpolation problems (with the exception of p = 1, n = 2 and p = 1, n = 4) are almost surely solvable. This means that they are solvable for all positions of $Z \subseteq \mathbb{R}^{2m}$, except for a set of Z of measure zero in $\mathbb{R}^{2m}$ and of first category. There are applications to Hirschowitz' problems of Algebraic Geometry.

W.R. Madych:

Polyharmonic Cardinal Splines

Polyharmonic cardinal splines are distributions which are annihilated by iterates of the Laplacian in the complement of a lattice in Euclidean n-space and satisfy certain continuity conditions. Here, we review some of their properties which are remarkably similar to the well-known properties of the univariate cardinal splines of odd degree as considered by I.J. Schoenberg, Cardinal Spline Interpolation, CBMS Vol. 12, SIAM, Philadelphia, 1973.

H. Michael Möller

On the Reconstruction of Multivariate Rational Functions by Interpolation

A typical problem for algorithms which use exact arithmetic is the intermediate coefficient swell. Sometimes the algorithm runs out of memory even if it is known that the result has coefficients of moderate size. The algorithm can be applied to specified values of parameters. This gives the values of the final coefficients at several points.

In most applications, the coefficients of the result are rational functions of the parameters. If the values of these coefficients are known at sufficiently many points, the reconstruction of the coefficients by rational interpolation (r.i.) is successful. A special complication is in this context that in gerneral, there is no information on bounds for the degrees of numerators and denominators of the final coefficients.

First we discuss the concept of rational reconstruction at two univariate r.i. methods, namely Stoer's method and a method attributed to Thiele. Then we present a new bivariate r.i. method and compare it to a method of Siemaszko and one of Kutchminskaya, Cuyt, and Verdonk with respect to our purpose.

H.G. TER MORSCHE

The Role of Exponential Eigen Splines in Translation-Invariant Periodic Spline Spaces

Spaces of spline functions defined on uniform meshes often possess the property of being translation-invariant: In the univariate case this means that translation over the mesh size does not change the underlying spline spaces; in more dimensions the invariance property concerns translations in serveral directions depending on the mesh-shape. In these spaces the so-called exponential eigen splines, i.e., eigen functions with respect to appropriate shift operators, play a fundamental role: In particular for "periodic subspaces" consisting of (multi) periodic functions. For instance, these subspaces are spanned by the corresponding periodic exponential eigen splines.
The problem of computing the dimension of periodic spline spaces is therefore equivalent to counting the total number of independent periodic exponential eigen splines. A survey of results with respect to this problem will be presented in this talk. With respect to interpolation of periodic data at uniform node configurations which are translation-invariant in the same way, the exponential eigen spline will again be at great help.

JULIAN MUSIELAK:

Approximation mittels Riemannschen Summen in verallgemeinerten Orliczräumen $L^{\varphi}([0,1]^m)$

Für die Riemannschen Summen einer Funktion $f:Q=[0,1]^m \subset \mathbb{R}^m \to \mathbb{R}$,

$$R_n(f,y) = \frac{1}{|n|} \sum_{k=0}^{n-1} f\left(\frac{y+k}{n}\right),$$

$$y=(y_1,\ldots,y_m) \in Q\ , \quad k=(k_1,\ldots,k_m) \in \mathbb{N}^m\ , \quad n=(n_1,\ldots,n_m) \in \mathbb{N}^m\ ,$$

$$\frac{y+k}{n} = \left(\frac{y_1+k_1}{n_1},\ldots,\frac{y_m+k_m}{n_m}\right), \quad |n| = n_1 n_2 \ldots n_m,$$

wird der Approximationsfehler

$$\Omega_n(f) = \int_Q \varphi(t, |R_n(f,t) - \int_Q f(x)dx|)\, dt$$

abgeschätzt, wobei $\varphi: Q\times\mathbb{R}_+ \to \mathbb{R}_+$ konvex in der zweiten Veränderlichen ist und entsprechende Bedingungen erfüllt. Für ein von der ersten Veränderlichen unabhängiges φ bekommt man

$$\Omega_n(f) \le \omega_\varphi(2^m f, \frac{1}{|n|}) \quad \text{für } n \in \mathbb{N}^m,$$

wobei ω_φ das φ-Integralstetigkeitsmodul von f ist.
Im Falle $m = 1$ und $\varphi(u) = |u|^p$, $p \ge 1$, erhalten wir ein Resultat von M. Yu. Fominykh (1985).

R.J. NESSEL:

Some Negative Results in Multivariate Approximation

The aim of this paper, which represents joint work with Norbert Kirchhoff, is to apply a general nonlinear uniform boundedness principle with rates, obtained previously, in connection with some negative results, concerned with the approximation on the N-dimensional Euclidean space $\mathbb{R}^N$ by (nontrivial) convolution processes $\{T_n\}$ of Fejér's type. More precisely, we are interested in the pointwise sharpness of well-known uniform estimates $\|T_n f\|_c = O_f(n^{-\alpha})$ for elements f, belonging to some Lipschitz class in, e.g., the space $C_{2\pi}$ of functions f, defined and continuous on $\mathbb{R}^N$, 2π-periodic in each variable. The main result states that there exist counterexamples f_α in that Lipschitz class for which $\limsup_{n\to\infty} |T_n f_\alpha(x)| n^\alpha \ge 1$ holds true simultaneously for each $x \in \mathbb{R}^N$. Whereas the existence of complex-valued functions f_α is rather easily established, the real-valued situation is somewhat more involved. The problem is also discussed in the space $C_o(\mathbb{R}^N)$ of functions, continuous on $\mathbb{R}^N$ and vanishing at infinity. Explicit applications are mentioned in connection with the Bochner-Riesz means in $C_{2\pi}$ and the Cauchy-Poisson integral in C_o.

HELMUT NIENHAUS:

A Trivariate Boolean Cubature Scheme

The objective of the talk is the numerical integration of smooth periodic functions in three dimensions. Using parametric extensions of the univariate trapezoidal rule we construct a trivariate cubature rule of interpolatory type that is related to the concept of discrete blending function interpolation. Besides an explicit representation formula we derive an error estimation for functions of the trivariate Korobov space E_3^{α}. We show that the error is of the order $O\left[\frac{\ln(N)^{2(\alpha+1)}}{N^{\alpha}}\right]$, where N is the number of evaluation points.

M.J.D. POWELL:

Singular p-Norm Distance Matrices

The given work was done without my assistances by my student B.J.C. Baxter. Let $x_1, x_2, \ldots, x_m$ be distinct points in $\mathbb{R}^d$ where $m \geq 2$ and where d is any positive integer. Then the $m \times m$ matrix A with elements $A_{ij} = \|x_i - x_j\|_p$ is a p-norm distance matrix. It is known already that such matrices are always nonsingular for $1 < p \leq 2$, and this paper proves that singularity can occur for any $p > 2$. We let d be the even integer 2n and we let the points $\{x_i\}$ have the components $(\pm1\ \pm1 \ldots \pm1\ 0\ 0 \ldots 0)$ or $(0\ 0 \ldots 0\ \pm1\ \pm1 \ldots \pm1)$, the number of zero components being n which implies $m = 2^{n+1}$. We ask whether a nonzero vector of the form $(a\ a \ldots a\ b\ b \ldots b)$ can be in the null space of A. We find that p has to satisfy a single nonlinear equation that depends on the Bernstein polynomial approximation of degree n to the function $\{f(\theta) = \theta^{1/p} : 0 \leq \theta \leq 1\}$ at $\theta = \frac{1}{2}$. It follows from properties of these approximations that A can be singular for p arbitrarily close to 2. Further, by scaling some of the vectors $\{x_i\}$, it can be shown that singularity is possible for all larger values of p, which completes the proof.

J.B. PROLLA:

Slice-Products and Bivariate Polynomial Approximation on Rectangles

Let $\{n_k\}$ and $\{m_j\}$ be two sequences of non-negative integers. Let G,H and W be the closed linear span of the sets

$$\{s^{n_k};\ k = 1,2,3,\ldots\},\quad \{t^{m_j};\ j = 1,2,3,\ldots\}$$

and

$$\{s^{n_k} t^{m_j};\ k, j = 1,2\ 3,\dots\}$$

in C(S), C(T) and C(S × T), respectively, where S and T are closed and bounded intervals in $\mathbb{R}$. We give conditions under which W is equal to the slice-product G#H. Recall that a function f∈C(S × T) belongs to G#H, by definition, if and only if, for every pair (s,t) ∈ S×T, the sections f_t and f_s belong to G and H respectively, where f_t is the mapping $x \in S \mapsto f(x,t)$ and f_s is the mapping $y \in T \to f(s,y)$.

LOUISE A. RAPHAEL:

Shape-Preserving Quasi-Interpolation and Interpolation by Box Spline Surfaces

This paper is devoted to the study of shape-preserving approximation and interpolation of functions by box spline surfaces on three and four directional meshes. The properties of positivity, monotonicity, and convexity are considered. A characterization of the grid spacing is given which guarantees the preservation of these properties for functions in certain Lipschitz classes.
This is a joint work with Charles Chui and Harvey Diamond.

M. REIMER:

Berechnung günstiger Interpolationsknoten für die Vollkugel B^r

Als günstig erweisen sich Interpolationsknoten, für welche die Lagrangeelemente sämtlich von minimaler Maximumnorm Eins sind. Zu ihrer Berechnung haben wir schon früher einen Algorithmus angegeben, der aber im Einzelfall noch der Konkretisierung bedarf, die wir hier für die Polynominterpolation auf der Vollkugel beschreiben. Für die ersten Polynomgrade und die Vollkugel B^3 haben wir die Knoten und die zugehörige Kubaturformel berechnet und an Beispielen getestet. Für die Lebesgue-Konstanten können sehr günstige Schranken angegeben werden.

S.D. RIEMENSCHNEIDER:

Problems in Multivariate Spline Interpolation

The problem of cardinal Hermite interpolation is discussed. The general problem begins with q compactly supported functions φ_ν, $\nu=1,\ldots,q$, on $\mathbb{R}^s$, and q differential operators T_ν, $\nu=1,\ldots,q$. The question is whether there is a function of the type

$$S = \sum_{\nu=1}^{q} \sum_{j\in\mathbb{R}^s} a_\nu(j)\varphi_\nu(\cdot-j)$$

that satisfies the interpolation conditions

$$T_\nu S(k) = f_\nu(k), \quad \forall\ k \in \mathbb{R}^s \ \text{ and } \ \nu = 1,\ldots,q,$$

for specified data $f_1,\ldots,f_q$. A general solution to the problem is given, but this solution admits fundamental solutions of power growth. In the case when $\varphi_\nu = T_\nu\varphi$, φ is a box spline with certain restrictives, then a bounded $L^2(\mathbb{R}^s)$ fundamental solution is found if T_ν are successive directional derivatives. The open problem is whether there is a nice class of box splines for which Hermite cardinal interpolation of this type is correct; i.e., there is a unique solution (bounded) for bounded data.

AMOS RON:

Multivariate Splines on Regular Grids

We introduce here a map $H \to H_\downarrow$, that assigns to every finite-dimensional space of entire functions (in s variables) a corresponding space of polynomials of the same dimension. This map is dual to another map $I \to I_\uparrow$, which assigns to every ideal I which has a finite codimension in the space Π of all polynomials on $\mathbb{C}^s$ a homogeneous ideal I of the same codimension. The duality is expressed by the fact that

$$I\perp_\downarrow = I_\uparrow\perp,$$

with $I\perp$ the kernel of I, i.e.,

$$I\perp = \{f \in \mathcal{D}'(\mathbb{R}^s); p(D)f = 0, \forall\ p \in I\}.$$

The duality allows us to compute in certain cases the kernels of homogeneous ideals in terms of kernels of their non-homogeneous counterpart.

These observations are important in the analysis of the space H of all exponentials in a box spline space. In particular, we provide an algorithm for the construction a basis for H, and give an elementary derivation of its dimension.

These results are joint with Carl de Boor.

Paul Sablonnière:

Bernstein Quasi-Interpolants

Bernstein quasi-interpolants are linear operators of the form $B_n f = \sum_i \mu_i(f) b_i^n$, where $\{b_i^n\}$ is a Bernstein basis of polynomials in one or several variables, and $\{\mu_i\}$ is a family of linear forms involving values of f and its derivatives up to some limited order. We discuss some examples of such operators illustrating a general method of construction and give some applications to the approximation of curves and surfaces.

R. Schaback:

Adding Corners Sometimes Works

As a counterpart to Carl de Boor's paper "Cutting Corners Always Works" the problem of constructive subdivision to generate interpolating curves or surfaces is considered (see also the lecture given by Prof. Utreras).

Two types of subdivision algorihms are considered and convergence of a class of rather elementary subdivision methods is proved. More elaborate algorithms involve first derivatives and practically yield nice-looking surfaces. However, the theoretical investigation of these algorithms still is incomplete.

Walter Schempp:

Interpolation and Parallel Two-Dimensional Data Compression

Starting with the Paley-Wiener theorem which forms the classical result for information-preserving sequential bandwidth compression, and its Stone-von Neumann-Segal analogue for the real Heisenberg nilpotent Lie group which is at the basis of holographic reciprocity, the lecture points out a unified approach to parallel two-dimensional data compression by holographic image processing. Brief descriptions of hardware implementations are also included.

XIE-CHANG SHEN:

Complex Interpolatory Approximation

The object of this paper is to introduce some recent results on complex interpolating approximation obtained in China such as the mean convergence of Lagrange interpolating polynomials based on the Fejer's nodes in the domain the boundary which belongs to $C^{1+\delta}$, $\delta > 0$; the mean convergence of Hermite interpolating polynomials based on the Fejer's nodes in the two kind of domains of the complex plane; the mean convergence of $(0,m_1,m_2,\ldots m_q)$ Birkhoff interpolation based on the roots of unity; the convergence and divergence problems of the interpolating rational functions in the unit disk.

JOACHIM STÖCKLER:

Periodic Spline Functions on Regular Partitions

Periodic box-splines are very useful for interpolation and approximation of smooth periodic functions. The translates of a periodic box-spline are linearly dependent, unless a quite restrictive condition on the directions of the box-spline is satisfied. Using the four-directional box-spline as an example we introduce different ways of modifying the periodic box-spline in order to overcome these restrictions. In the case of even multiplicities of all the directions we obtain a variational characterization for the modified box-spline.

MANFRED TASCHE:

Index Transforms for Multidimensional DFT's

Index transforms of m-dimensional arrays into n-dimensional arrays play a significant role in many fast algorithms of multivariate discrete Fourier transforms (DFT's) (e.g. prime factor algorithm, Winograd algorithm). By an index transform of the input data, the m-dimensional DFT can be transfered into an n-dimensional DFT of "short lengths" (n m). Then by efficient algorithms for the one-dimensional DFT's of short length, the n-dimensional DFT is computed.

C.S. Burrus (1977), H.J. Nussbaumer (1981) and J. Hekrdla (1987) dealt with properties of index transforms. In a joint paper with G. Steidl, the nature of index transforms is explored using group-theoretical ideas. We solve open problems posed recently by J. Hekrdla.

FLORENCIO UTRERAS:

Interpolatory Subdivision Schemes for Surface Modelling

We introduce a subdivision algorithm for surface generation. The method uses interpolation and iterative knot insertion. Starting from a grided surface in parametric form we introduce additional points using a curve generation algorithm in the 3-D space along the mesh curves. The process is repeated until the required visual effect is achieved or screen precision is attained. The method is local and is shown to be "shape" preserving along mesh curves. We present convergence results and several numerical examples.

WANG KUN-YANG:

Strong Uniform Approximation by Bochner-Riesz Means

For a continuous periodic function f of n variables, the Bochner-Riesz means are denoted by $S_R^{\alpha}(f;x)$. The special value $\alpha_o = \frac{n-1}{2}$ of the index is called critical index. The estimate of the order of the strong uniform approximation of f by its Bochner-Riesz means with the critical or lower index is given. The result is the following

Theorem. If $\frac{n}{2} - 1 < \alpha \le \frac{n-1}{2}$ then for any continuous periodic function f of n variables,

$$\left\| \frac{1}{R} \int_0^R \left| S_{\gamma}^{\alpha}(f) - f \right|^2 dr \right\| \le c(n,\alpha) \frac{1}{R} \int_0^R \omega_2(f;\tfrac{1}{\gamma})^2 dr$$

where ω_2 denotes the modulus of continuity of order 2.

JOSEPH D. WARD:

Bivariate Cardinal Interpolation by Shifted Box Splines on a 3-direction Mesh

In this joint work with J. Stöckler and C. Chui, we show that bivariate cardinal interpolation by shifted box splines on a 3-direction mesh is correct if the shifts lie in the appropriate subset of the 1/2 square as described by Sivakumar. This result gives an appropriate generalization of the univariate result obtained independently by Micchelli and de Boor-Schoenberg. The result is obtained by a careful analysis of the "symbol" of the interpolation operator.

Berichterstatter: Walter Schempp

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