Mathematisches Forschungsinstitut Oberwolfach

Tagungsbericht 9/1989

Kombinatorik

26.2. bis 4.3.1989

Die Tagung fand unter der Leitung von Herrn Deuber (Bielefeld), Herrn Jackson (Waterloo) und Herrn Jungnickel (Gießen) statt.

Das Ziel der Tagung war es, einen Überblick über das gesamte Spektrum der Kombinatorik zu geben, die sich immer mehr in der Gefahr befindet, in eine Anzahl spezialisierter Einzelgebiete zu zerfallen. Die Tagungsteilnehmer sollten sich demzufolge in Oberwolfach über Forschungsergebnisse in der diskreten Mathematik auch außerhalb ihres eigenen Spezialgebietes informieren und mit Kollegen darüber diskutieren können.

Um diese Ziele zu erreichen, wurde ein inhaltlich wie auch geographisch ausgewogener Teilnehmerkreis eingeladen. Alle wesentlichen Teilgebiete der Kombinatorik waren vertreten, insbesondere Codierungstheorie, Designtheorie und endliche Geometrie, Graphentheorie und kombinatorische Optimierung, kombinatorische Polytope, partiell geordnete Mengen, Matroidtheorie, Ramsey- und Partitionstheorie und Zähltheorie.Wenngleich die algebraischen Aspekte der Kombinatorik eine gewisse Betonung erfuhren, sind auch die analytischen Aspekte sowie Bezüge zu den Anwendungen vertreten gewesen.

Die Vorträge wurden bewußt nicht zu Teilgebieten zusammengefaßt, um einen möglichst breiten Gedankenaustausch zu fördern, der auch -sowohl durch die rege Teilnahme an den Vorträgen wie durch zahlreiche Einzeldiskussionen- weitgehend erreicht wurde, wie die positive Resonanz, die die Tagung fand, gezeigt hat.

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M. Aigner (with E. Triesch): Degree Sequences of Graphs

Let $w: E(G) \to \{1,\ldots,m\}$ be a weighting of the edges of a graph G; w is admissible if all weighted degrees $w(x) = \Sigma_{e \ni x} w(e)$ are distinct ($x \in V(G)$). The irregularity strength s(G) is the minimum number m for which an admissible weighting is possible. A survey is given on the numbers s(G). In particular:

Theorem 1: Let T be a tree on n vertices, then $s(T) \leqq n-2$ except when T is a star (then $s(T) = n-1$).

Theorem 2: Let G be a connected graph on n vertices. Then $s(G) \leqq n-1$ except for $s(K_3) = 3$.

The method of proof uses partitions of the additive group, and alternatively, results in the geometry of numbers. "Graceful" conjecture: Let T be a tree on n vertices. Then there always is an admissible weighting which uses all the numbers $1,2,\ldots,n-1$.

K.T. Arasu: Difference sets

We present a condition on the intersection numbers of difference sets which follows from a result of Jungnickel and Pott. We apply this condition to rule out several putative (non-abelian) difference sets and to correct erroneous proofs of Lander for the nonexistence of (352, 27, 2) - difference sets in $Z_{11} \oplus Z_8 \oplus (Z_2)^2$ and $Z_{11} \oplus (Z_4)^2 \oplus Z_2$.

A. Beutelspacher: The chromatic index of a finite projective space

The chromatic index of a linear space S is the least number n such that one can colour the lines of S in such a way that any two intersecting lines have different colour. The chromatic index of S is denoted by $\chi(S)$.

The conjecture of Erdös-Faber-Lovász says that in any linear space $\chi(S)$ is at most the numbers of its points. We investigate this problem in the special case where S is a projective space of dimension d.

There are direct constructions (using spreads and parallelisms) and recursive constructions which prove the conjecture

- for d odd if the order is not too small;
- for some even d.

A. Blockhuis: Solution of an extremal problem for sets using resultants of polynomials

We give a short and completely new proof of the following fundamental theorem of Bollobás: Let $A_1,\ldots,A_h$ and $B_1,\ldots,B_h$ be collections of sets with $|A_i| = r$, $|B_i| = s$ $\forall i$ and $A_i \cap B_j = \varnothing$ if and only if $i = j$. Then

$$h \le \binom{r+s}{s}$$

The proof immediately extends to the generalization of this theorem obtained by Frankl, Alon and others. The essential ingredient is to associate to each set A_i (resp. B_i) a polynomial a_i ($= \prod(x-\alpha)$) (resp. b_i) where α runs through A_i, such that the resultant of $(a_i, b_j) = 0$ if and only if $i \neq j$. The bound then follows from a dimension argument.

David Bressoud: OZ and Unimodality

The Ohara-Zeilberger identity (OZ)

$$\begin{bmatrix} n+j \\ j \end{bmatrix} = \sum_{\lambda \vdash j} q^{\sigma(\lambda)-j} \prod_{i=1}^{j} \begin{bmatrix} (n+2)i - L_{i-1} - L_{i+1} \\ \lambda_i - \lambda_{i+1} \end{bmatrix}$$

with

$$\sigma(\lambda) = \lambda_1^2 + \ldots + \lambda_j^2, \quad L_i = \lambda_1 + \ldots + \lambda_i, \quad \begin{bmatrix} n+j \\ j \end{bmatrix} = \begin{cases} \prod_{i=1}^{j} \dfrac{1-q^{n+i}}{1-q^i}, & n \ge 0 \\ 0, & n < 0 \end{cases}$$

implies unimodality of the Gaussian polynomials since each summand is a unimodal polynomial (by inductive hypothesis) with mode at $nj/2$.

OZ is easily proven by demonstrating that

$$\sum_{\substack{\lambda \vdash j \\ l(\lambda) \le k}} q^{\sigma(\lambda)} \left(\prod_{i=1}^{k-1} \begin{bmatrix} ni - L_{i-1} - L_{i+1} \\ \lambda_i - \lambda_{i+1} \end{bmatrix} \right) \begin{bmatrix} n_k - L_{k-1} - L_k \\ \lambda_k \end{bmatrix}$$

is the generating function for partitions with j parts $< n$ such that if f_i = # of parts of size i then $f_{i-1} + f_i \leqq k$ $\forall i$ and $f_{i-1} + f_i = k$ implies

$$f_i + 2\sum_{t>i} f_t + ik \le n_k.$$

A.R. Calderbank: Quasi-symmetric Designs

A quasi-symmetric t-design is a t-design with two block intersection sizes p and q (where $p < q$). We describe algebraic invariants for quasi-symmetric designs that are similar to the Bruck-Ryser-Chowla theorem for symmetric designs. We shall also settle a conjecture of Sane and Shrikhande, by classifying quasi-symmetric 3-designs with $p = 1$: our method is to reduce the classification problem to that of finding all integer points on the elliptic curves $y^2 = x^3 - 11x^2 + 32x$ and $y^2 = x^3 - 4x + 4$.

W. Deuber: Complexity theory for fast growing functions

By establishing the complexity of a Ketonen-Solovay function in the Grzegorcik/Wanier hierarchy it is shown that in Peano arithmetic the totality of such a function is not provable. W. Thumser (Dr. dissertation 1989) also gives good upper bounds for the complexity.

M. Deza: Facets for the complete cut cone

We present results on the facets of the complete cut cone, i.e. the cone C_n of dimension n(n-1)/2 generated by the cuts of the complete graph on n vertices. We describe some operations on facets, in particular, a lifting procedure for constructing facets of C_{n+1} from given facets of the lower dimensional cone C_n. We present several new classes of valid inequalities for C_n and we prove facetness for some subclasses. The elements of the complete cut cone C_n admit the following geometric characterization: they are exactly the semi-metrics on n points which are isometrically embeddable into L^1. The results presented follow from a joint work with M. Laurent.

Z. Füredi: Covering the complete graph by partitions

Let f(D,c) denote the minimum integer n such that every c-colouring of the edges of the complete graph K^m contains a monochromatic, connected subgraph whenever $m > f(D,c)$. If a resolvable block design with c parallel classes and with block sizes $< D$ over n vertices exists then $f(D,c) \geqq n$. Our main tool to investigate f(D,c) is the fractional matching theory of hypergraphs.

C. Godsil: Distance regular antipodal covers of complete graphs

An r-fold cover of the complete graph K^n is obtained by replacing each vertex in it by a set of r vertices, and each edge by an r-matching joining the vertices in the corresponding r-sets. Thus it is a regular graph on rn vertices of valency n-1. A cover G of K^n is called antipodal if any two vertices in the same r-set are at distance three in G, and any two vertices in different r-sets are at distance at most two. Finally, an antipodal cover is distance regular if, for each pair of vertices x and y in G, the number of vertices adjacent to both x and y only depends on the distance between x and y.

We now restrict the word "cover" to mean "distance regular antipodal cover". The cube is a 2-fold cover of K^4, while the line graph of Petersen's graph is a 3-fold cover of K^5. An (n-1)-fold cover of K^n exists if and only if there is a Moore graph of diameter two with valency n-1. Covers of K^n are interesting, in part because they are related to a number of structures arising in finite geometry.

My talk will be a report on recent attempts to obtain a better understanding of this class of graphs, and on a few of the problems remaining to be solved.

I.P. Goulden: Enumeration of tableaux by number of columns

For $(n-3)/2 \leqq m \leqq n-1$, we prove that the number of involutions on {1,2,...,n} whose longest increasing subsequence has length m is

$$(1)\quad \sum_{\substack{i,j\geq 0\\ 2i+j\leq n-m}} (-1)^{n+m+i+j}\binom{i+j}{i}\binom{n}{i+j} Inv(j)$$

where Inv(j) is the number of involutions on {1,2,...,j}, and that the number of permutations on {1,2,...,n} whose longest increasing subsequence has length m is

$$(2)\quad \sum_{\substack{i,j,l\geq 0\\ i+j+l\leq n-m}} (-1)^{i+j}\, l!\binom{i+l}{i}\binom{j+l}{j}\binom{n}{i+l}\binom{n}{j+l}.$$

The proof of (1) uses the Schensted correspondence to express this number as the sum of degrees of all irreducible representations of the symmetric group corresponding to partitions λ with largest part equal to m. This sum is thus the coefficient of $x_1 \ldots x_n$ in $\Sigma s_\lambda(x_1,\ldots,x_n)$, where s_λ is a Schur symmetric function, and the sum is over partitions with largest part m. The Schur function sum is evaluated using an idea of I.G. Macdonald, yielding (1) as well as more complicated formulas for smaller values of m relative to n. The proof of (2) proceeds similarly and involves the sum $\Sigma s_\lambda(x_1,\ldots,x_n)s_\lambda(y_1,\ldots,y_n)$, again restricted to partitions λ with largest part m.

The simple form of (1) and (2) suggests that a nice constructive proof exists, and it is hoped that such a construction would lead to new results in symmetric functions.

C. Greene: Permutations with balanced patterns

We consider balanced nm-staircase tableaux, that is (equivalently) permutations $x_1,\dots,x_n$ of $1,\dots,n$ which have no peaks or valleys in even positions. (A peak is an element x_i such that $x_i > x_{i-1}, x_{i+1}$, and a valley is an element x_i such that $x_i < x_{i-1}, x_{i+1}$.) If b_n is the number of such permutations, for each n, we show that $B_0(x) = \Sigma_n b_{2n} x^{2n}/(2n)! = 1/(1 - x/2^{\frac{1}{2}} \tanh(x/2^{\frac{1}{2}}))$ and $B_1(x) = \Sigma_n b_{2n+1} x^{2n+1}/(2n+1)! = 2^{\frac{1}{2}} \tanh(x/2^{\frac{1}{2}})/(1 - x/2^{\frac{1}{2}} \tanh(x/2^{\frac{1}{2}}))$.

We note that Gessel has considered the related problem of enumerating permutations with no valleys in even positions (peaks allowed). If g_n is the number of such permutations, Gessel obtains $G_0(x) = \Sigma g_{2n} x^{2n}/(2n)! = (\text{sech } x)/(1 - x \tanh x)$, and $G_1(x) = \Sigma g_{2n+1} x^{2n+1}/(2n+1)! = (\tanh x)/(1 - x \tanh x)$. It follows (comparing generating functions) that $g_{2n+1} = 2^n b_{2n+1}$ for all n, a fact for which we have no simple combinatorial explanation.

H. Gronau (with B. Ganter): On two concectures of Demetrovics, Füredi and Katona, concerning partitions

Is it possible to find n partitions of an n-element set whose pairwise intersections are just all atoms of the partition lattice? Demetrovics, Füredi and Katona verified this for all $n \equiv 1$ or 4 (mod 12) by constructing a series of special Mendelsohn Triple Systems. They conjectured that such triple systems exist for all $n \equiv 1$ (mod 3) and that the problem on the partitions has solutions for all $n \geqq 7$. We prove both conjectures, except for finitely many n.

M. Grötschel: Upper Bounds for Block Codes from Polyhedral Theory

Let A(n,d,q) denote the largest size of a block code of words of length n over an alphabet with q letters and minimum (Hamming) distance d. We transform the problem of calculating A(n,d,q) into a stable set problem and use methods of polyhedral theory and linear programming to compute upper bounds for A(n,d,q). This way we can give new interpretations of known bounds and we obtain - in a number of cases - improvements over the best upper bounds known to date. This work is joint with E. Zehendner.

J.W.P. Hirschfeld: Projective spaces of square size

If $|PG(n,q)| = rs$ and T is a Singer cycle, then there are many cases in which the orbits of $<T^r>$ give interesting subsets of the space. When $r = s$, the only possibilities for (n,q,r) with $n>1$ are (3,7,20) and (4,3,11). In the former case, the 20 points of an orbit lie by fours on five skew lines; the lines of the orbits form a regular spread. In the latter case, the 11 points of an orbit lie by fives in 66 solids and form a familiar 4-(11,5,1) design.

D.M. Jackson (with T. Vesentin): A character tbeoretic approach to embeddings of rooted maps

The group algebra of the symmetric group and properties of the irreducible characters are used to derive combinatorial properties of rooted maps in orientable surfaces of arbitrary genus. We show that there exists, for each genus, a correspondence between the set of rooted quadrangulations and a set of rooted maps of all lower genera, with a distinguished subsets of vertices. The theory can be extended to 2-face colourable rooted maps. We show that there is a corresponding correspondence for rooted triangulations of given genus. Both correspondences specialise to Tutte's correspondences for the sphere, but the latter are known not to extend to higher genera. It seems reasonable to expect a combinatorial construction which will account for these facts.

These techniques can be used to examine arbitrary classes of maps of prescribed genus. T

D. Jungnickel: Affine difference sets

We present some recent existence tests for abelian affine difference sets which allow us to prove the prime power conjecture for orders up to 10000. These results follow from various papers of K.T. Arasu, D. Jungnickel and A. Pott.

K.W.J. Kadell: The Selberg-Jack polynomials

Aomoto has recently given a simple proof of an extension of Selberg's integral. We prove the following generalization of Aomoto's theorem. If the integrand of Selberg's integral is multiplied by a Jack symmetric polynomial with $\alpha = 1/k$, then the integral has a certain closed form. Our proof requires Macdonald's extension of the duality of the Schur functions to the Jack symmetric functions and Stanley's extensions of the Pieri formula and the combinatorial representation.

We give alternative proofs of some results of Stanley and Macdonald and conjecture a constant term orthogonality for the Jack symmetric functions. We discuss the extension of our results to the q-case.

A. Kerber: Algebraic combinatorics: The use of finite group actions

The basic tools are the Cauchy-Frobenius and Burnside's lemma, both in constant and in weighted form. They were presented and it was shown how they apply to enumeration of symmetry classes of mappings. Then a redundancy free construction of orbit representations using double cosets in symmetric groups was mentioned as well as the method of Dixon/Wilf for generating orbit representatives uniformly at random was described. Specific applications are the construction of chemical isomers and the evaluation of catalogs of graphs with $p \leqq 10$ points. Emphasize was laid on the fact that these methods apply in many, other cases, too.

D. Kleitmann: Two Colouring Problems

I We show that any 3-hypergraph uniform of degree 3 on n vertices can have its vertices coloured by 3-dimensional 0-1 vectors such that the colours on any edge span the space (joint with Z. Füredi, J. Griggs and R. Holzman). Does this hold for k-hypergraphs with $k = 4, 5$? A. Blockhuis (this meeting) shows that this statement fails in general for $k \geqq 6$. If the hypergraph is further restricted to have a vertex transitive, cyclic symmetry, does this hold for general k? This would prove a conjecture of Graham, Chung, et al. The $k = 3$ case can be proven by a method based on Lovász' proof of Brooks' theorem.

II Any planar graph admits a partition of its vertices into 3 blocks (colour classes) of which two are forests and one an independent set. This is somewhat stronger than the 5-colour theorem. Question: Is this result new? It can be proven by classical methods, appropriately arranged.

B. Korte: Exchange properties and elimination processes

The Gaussian elimination algorithm is besides the Euclid algorithm probabely among the most famous and certainly among the most used algorithms in mathematics. It turns out that its combinatorial backbone, i.e. the sequence of its pivot elements is nothing but a combinatorial exchange structure, namely a special greedoid. This greedoid has neither the interval nor the transposition property thus it seems to have less structure. However, we can give some nice algorithmic, duality and polyhedral results: Gauß greedoids can be characterized by the optimality of the greedy algorithm for linear objective functions; they are closed under an appropriate duality operator (of which matroid duality is a special case). Finally, we give some polyhedral characterizations. For special Gauß greedoids we can linearly describe the convex hull of its characteristic vectors completely and there is some hope to extend these results to general Gauß greedoids. My lecture reports on some earlier results of my student O. Goecke and recent joint work with L. Lovász and R. Schrader.

M. Las Vergnas: Bases and Orientations in Matroids

The structure of oriented matroid abstracts the main combinatorial properties of signed linear dependence over ordered fields. Classical examples include: cycle spaces of directed graphs, configurations of points and (dually) arrangements of hyperplanes in Euclidean spaces, arrangements of pseudolines in the projective plane and generalizations in higher dimensions (this last example being generic by the Folkman-Lawrence Topological Representation Theorem). Oriented matroids provide several ways to encode the different combinatorial types of configurations of points or hyperplanes.

Theorem A (Las Vergnas 1975): The number of acyclic reorientations of an oriented matroid M (or, equivalently, the number of maximal covectors, or the number of regions of the Folkman-Lawrence Representation) is given by the evaluation $t(M;2,0)$ of its Tutte polynomial.

Theorem A generalizes Stanley's theorem (1973) on acyclic orientations of graphs and contains Zaslavski's theorem (1975) on the number of regions of an arrangement of hyperplanes. It can be generalized to oriented matroid perspectives, oriented matroid counterpart of linear applications (Las Vergnas 1977). A further generalization of Theorem A deals with the notion of activities.

Theorem B (Las Vergnas 1982): Denoting by o_{ij} the number of reorientations with activities i, j of an oriented matroid of an ordered set, we have $t(M;\zeta,\eta) = \Sigma_{i,j} 2^{-i-j} o_{ij} \zeta^i \eta^j$.

Comparing Theorem B with

Theorem C (Crapo 1969, generalizing works of Tutte for graphs): Denoting by b_{ij} the number of bases with internal activity i and external activity j of a matroid M on a totally ordered set, we have $t(M;\zeta,\eta) = \Sigma_{i,j} b_{ij} \zeta^i \eta^j$.

we get the equality $o_{ij} = 2^{i+j} b_{ij}$. This equality suggests a question: Is there a natural correspondence between bases and reorientations of an oriented matroid compatible with these equalities for all i, j? Our purpose in the talk is to describe such a correspondence.

A. Lascoux (with M.P. Schützenberger): Permutations are tableaux and tableaux are permutations

A decomposition of a permutation μ is any product $\sigma\sigma'...$ of simple transpositions which is equal to it. Taking the subwords of $\sigma\sigma'...$ produces the permutations smaller than μ for the Ehresmann order (also called strong or Bruhat order). To any permutation μ, we can associate the tableau $K(\mu)$ whose columns are the successive left (reordered) factors of μ written as the word $\mu_1\mu_2$...The Ehresmann order is just the componentwise order on the special tableaux $K(\mu)$ called keys.

Conversely, given any tableau t, pushing successively each of its columns to the right by the jeu de taquin or by Schensted algorithm gives a key $k_+(t)$; symmetrically, we get on the left another key $k_-(t)$ and we have $k_-(t) \leqq t \leqq k_+(t)$. Thus, we can add to the Ehresmann order an edge, labelled by t, joining the vertices $k_-(t)$ and $k_+(t)$. This new order is Eulerian (see Séminaire Lotharingien, Sept. 88) and has many properties generalizing those of the Ehresmann/ Bruhat/strong order, in connection with the geometry of flag varieties (see Minneapolis meeting of combinatorics, June 88, to appear in Springer L.N.).

H. Lefmann: On families with prescribed intersection properties

In this talk combinatorial extremal problems in ranked lattices $(X,\wedge,\vee)$ are considered. In particular, for families $\mathcal{F} \subseteq X$ whose members have pairwise prescribed intersection properties, the maximum cardinality of $\mathcal{F}$ is given for various lattices like powerset-lattices, linear lattices and Graham-Rothschild lattices.

S.C. Milne: Classical Partition Functions and the U(n + 1) Rogers-Selberg Identity

In this talk we show that after suitable specialization the "balanced" side of the U(n + 1) Rogers-Selberg identity gives the generating function for all partitions whose parts differ by at least n + 1. A similar specialization yields the additional condition that the parts must be $\geqq n + 1$. The case n = 1 is the sum side of the pair of classical Rogers-Ramanujan-Schur identities.

This connection between classical partition functions and the U(n + 1) Rogers-

Selberg identity depends upon the identity

$$(1)\quad \sum_{\substack{m_1+\ldots+m_n=m \\ m_i\geq 0}} \left\{ \left(\prod_{1\leq r<s\leq n} (q^{nm_r} - q^{(s-r)+nm_s}) \right) \left(\prod_{i=1}^{n} (q)_{i-1+nm_i} \right)^{-1} q^{(n(n+1)/2)(m_1^2+\ldots+m_n^2)} \right.$$

$$\left. (-1)^{(n-1)m} \left(\prod_{i=1}^{n-1} q^{(i-n)(n+1)m_i} \right) \right\} = \frac{q^{m((n+1)m-n+1)/2}}{(q)_m},$$

where $(A)_m = (1-A)(1-qA)\ldots(1-q^{m-1}A)$.

Our proof of (1) involves using partial fraction techniques, Hall-Littlewood polynomials, Raising operators, q-Kostka matrices, the Cauchy-identity for Schur functions and generating functions for column-strict plane partitions to solve a general q-difference equation. One outcome of this proof is a new class of symmetric functions, analogous to Hall-Littlewood polynomials, that interpolates between Schur functions and complete homogeneous symmetric functions.

A. Pott: A generalization of Mann's theorem on difference sets

The main tools to prove the nonexistence of certain (v, k, λ)-difference sets are multipliers and a theorem due to Mann. There are several proofs of Mann's theorem. We simplify Lander's proof and generalize his results. We obtain new non-existence results even for non-abelian difference sets (joint work with D. Jungnickel). In particular, we obtain:

Theorem: Let D be a (v,k,λ)-difference set in a group G, $H<G$, G/H abelian, $\exp(G/H) = u^*$. Then the following holds: If $p^i \equiv -1 \bmod u^*$ (p prime), then $p^{2j} \| n$ (i.e. $p^{2j+1} \nmid n$) for some j (generalization of Mann's theorem to non-abelian groups).

Corollary: (i) $p^j \leqq |H|$,

(ii) $|G/H| > k \Rightarrow p^{2j} \mid v\lambda$.

For instance, we prove: There exists no abelian (704, 38, 2)-difference set if $\exp(Syl_2 G) \leqq 4$ and no (343, 19, 1)-difference set. The latter result holds for non-abelian groups, too.

H.J. Prömel: The restricted Ramsey theorem for graphs

Apparently P. Erdös was the first to ask whether there exists a graph F such that

$$F \rightarrow (K_3)_2^{K_2}$$

but F has small clique size cl(F), where cl(F) denotes the maximal size of a complete subgraph in F. Answering this question J. Folkman (1970) constructed a graph F with

$$F \rightarrow (K_3)_2^{K_2}$$

and cl(F) = 3. This result was a starting point for Ramsey Theory for graphs and hypergraphs.

One of the key results in this area is the restricted Ramsey theorem for graphs and hypergraphs due to Nešetřil and Rödl (1977, 1983). A hypergraph $(X,\mathcal{L})$ is called irreducible if for any two vertices $x,y \in X$ there exists an edge $E \in \mathcal{L}$ such that $x,y \in E$. Observe that with respect to ordinary graphs cliques are the only irreducible ones. Let $\mathcal{F}$ be a family of irreducible hypergraphs. Then Forb($\mathcal{F}$) denotes the set of all hypergraphs which do not contain any member of $\mathcal{F}$ as an induced subgraph. Let G,H$\in$Forb($\mathcal{F}$). Then Nešetřil and Rödl proved that there exists an F$\in$Forb($\mathcal{F}$) such that

$$F \rightarrow (G)_2^H.$$

The original proofs of this results are quite involved and conceptually not that easy to understand, even in the case of ordinary graphs. The aim of the talk is to present a short and simple proof for the restricted Ramsey theorem for hypergraphs. This proof was obtained jointly with B. Voigt and will appear in J. Comb. Th. (A).

A. Regev: Symmetry for the dual Schensted Knuth correspondence

In a classical paper, Knuth (1970) corresponded matrices of nonnegative integers with two rows array of integers

$$A \leftrightarrow \begin{pmatrix} u_1,\dots,u_n \\ v_1,\dots,v_n \end{pmatrix},$$

then followed by the Schensted algorithm

$$A \leftrightarrow \begin{pmatrix} u_1,\dots,u_n \\ v_1,\dots,v_n \end{pmatrix} \leftrightarrow (P,Q).$$

That correspondence has the symmetry property: If $A \leftrightarrow (P,Q)$ then $A^T \leftrightarrow (Q,P)$ (A^T is the transpose).

In that paper, Knuth also introduced a dual correspondence $A \leftrightarrow (P,Q)$. Here A had to be a 0,1 matrix, and in general, it did not have symmetry: $A^T \leftrightarrow (Q,P)$.

We follow a change in the Knuth dual correspondence that was suggested by Berele and Remmel. With that change, that modified Knuth dual correspondence has the symmetry: $A \leftrightarrow (P,Q)$ and $A^T \leftrightarrow (Q,P)$. As an application we give a bijective proof of a "hook" generalization of an identity of Schur.

A. Rosa: Halving Steiner Triple Systems

When does there exist a Steiner triple system $(V,\mathcal{B})$ of order v (STS(v)) which admits a partition of the set of its triples $\mathcal{B} = \mathcal{B}_1 \cup \mathcal{B}_2$ such that $(V,\mathcal{B}_1)$ and $(V,\mathcal{B}_2)$ are isomorphic hypergraphs? An obvious necessary condition is $b = |\mathcal{B}| \equiv 0 \pmod 2$, i.e. $v \equiv 1,9,13$ or $21 \pmod{24}$. This is not sufficient: We prove that an STS(v) with the above property exists if and only if $v \equiv 1$ or $9 \pmod{24}$. On the other hand, almost all STS's do not have the above property.

We also prove that when b is odd, i.e. when $v \equiv 3,7,15$ or $19 \pmod{24}$, there exists an STS(v) $(V,\mathcal{B})$ and a triple $t \in \mathcal{B}$ such that there exists a partition of $\mathcal{B} \setminus \{t\} = \mathcal{B}_1 \cup \mathcal{B}_2$ with $(V,\mathcal{B}_1) \simeq (V,\mathcal{B}_2)$.

B.E. Sagan: Log concave sequences of symmetric functions and analogs of the Jacobi-Trudi determinant

We define the notion of log concavity for a sequence of polynomials. Next it is shown that various sequences of elementary and homogeneous symmetric functions are log concave. The methods used are lattice path arguments of the type employed by Gessel, Viennot, and others. Finally these results are generalized to $n \times n$ determinants, giving new analogs of the Jacobi-Trudi determinant.

J. Spencer: Threshold functions for extension statements

When Z is the sum of many rare mostly independent events and $E[Z] \sim \mu$ then $\Pr[Z=0] \sim e^{-\mu}$. We call this the Poisson Paradigm. For example, when

$$\binom{n}{4} p^6 = \mu$$

the random graph has no K_4 with probability $e^{-\mu}$. We give a general corrolation inequality that allows one to estimate the probability that no event occurs by the product of the probabilities that each event fails when the pairwise corrolations are small. In particular this allows good estimates for the probability that every pair of vertices is joined by a path of a given length d.

J. Stembridge: Connections between Hall-Littlewood functions and the Rogers-Ramanujan identities

There exist several identities from the theory of Hall-Littlewood functions that can be viewed as multi-variate generalizations of multiple basic

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hypergeometric series (i.e., q-series). Included in this list of q-series that can be generalized are some extensions of the Rogers-Ramanujan identities originally due to G. Andrews and D. Bressoud. The most important part of Hall-Littlewood function theory that is relevant to this development involves the adaptation of a technique of I. Macdonald first used in the proof of MacMahon's and Bender-Knuth's plane partition conjectures.

T. Trotter (with G. Brightwell, K. Reuter): The Order Dimension of Convex Polytopes and Planar Maps

Associate with a convex polytope **M** in $\mathbb{R}^3$ a partially ordered set $\mathbf{P_M}$ consisting of the vertices, edges and faces of **M** partially ordered by inclusion. We want to determine the order dimension (in the Dushnik-Miller sense) of the poset $\mathbf{P_M}$. There are three factors motivating this problem. First, given a graph $\mathbf{G} = (V,E)$, define a poset $\mathbf{Q_G}$ by ordering $V \cup E$ by inclusion. Then Schnyder proved that **G** is planar if and only if $\dim(\mathbf{Q_G}) \leqq 3$. So it is natural to ask what happens if we add the faces to the poset. Second, the problem of determining the dimension of the lattices of faces (of all ranks) of a convex polytope in $\mathbb{R}^n$ can be posed, but for $n \geqq 4$ there is no bound which depends only on n. This is due to the fact that for $n \geqq 4$, there exist cyclic polytopes which have large sets of vertices with each pair belonging to an edge. As a third motivation, if $\dim(\mathbf{P_M})$ is small, then we obtain a useful data structure for representing the vertex/face incidence relation of a polytope. With these comments as background, we prove that if **M** is a convex polytope in $\mathbb{R}^3$, then $\dim(\mathbf{P_M}) = 4$. The upper bound $\dim(\mathbf{P_M}) \leqq 4$ holds whenever **M** is a planar map. Our argument includes a polynomial time algorithm for the coordinatization of $\mathbf{P_M}$.

S.A. Vanstone: Graph Theoretic Codes

Let G be a graph with q edges, p vertices and girth d. It is well known that the cycle space of G gives rise to a binary (q, q-p + 1, d)-code and that these codes are majority logic decodable. Such codes are usually referred to as graph theoretic codes. In this lecture we describe a decoding scheme for graph theoretic codes based on k-regular graphs having a particular type of 1-factorization. We consider a 1-factorization *F* such that there exists an automorphism of G which acts cyclically on the 1-factors of *F*. In addition we would like *F* to have the property that the union of any two of its 1-factors does not contain a 4-cycle. In particular, using these properties we display an efficient method to decode complete graph codes. The algorithm corrects all single and double adjacent errors and all double errors confined to a 1-factor.

B. Voigt: Sparse Ramsey Theory

This talk reports on results from the following three papers: Ramsey theorems for finite graphs I/II, submitted to JCT(B), and A sparse Graham-Rothschild theorem, Trans. AMS (1988). These are joint papers with H.J. Prömel. I concentrate to discuss the following results:

Theorem A: (Sparse graph Ramsey theorem) Given positive integers k,m,r and g there exists a graph G with the following property: the set-system

$$\binom{G}{K_m}_k$$

which has the set of complete K_k-subgraphs of G as vertices and edge

$$\binom{K}{K_k} \text{ for } K \in \binom{G}{K_m}, \text{(where } \binom{G}{H} \text{ is the set of unsolved H}-\text{subgraphs of G)}$$

has chromatic number larger than r and girth larger than g.

This result has been conjectured by J. Spencer, the particular case k = 2 is due to Nešetřil and Rödl.

Theorem B: (Sparse partition theorem for Boolean lattices) Given positive integers $m \neq 1$, r and g there exists a positive integer n and a set $S \subseteq \underline{2}^n$ of points in the n-dimensional Boolean lattice $\underline{2}^n$ such that the set-system which has the points in S as vertices and $\underline{2}^m$ sublattices which are completely contained in S as edges has chromatic number larger than r and girth larger than g.

Theorem C: (Sparse Hales-Jewett theorem) Given a finite set A and positive integers r and g there exists a positive integer n and there exists a set $S \subseteq A^n$ such that the set-system which has the points in S as vertices and generalized combinatorial lines which are completely contained in S as edges has girth larger than g and for every r-colouring of S there exists a (special) combinatorial line in S which is monochromatic.

As a corollary from this we obtain the following resultation of a conjecture of J. Spencer (1975).

Corollary: Given positive integers k,r and g there exists a set S of positive integers such that the set system which has the elements of S as vertices and k-term arithmetic progressions which are contained in S as edges has chromatic number larger than r and girth larger than g.

This is a sparse version of a celebrated theorem of van der Waerden on arithmetic progressions.

D.J.A. Welsh: The Complexity of Colourings and Knots

I relate the Jones polynomial of a knot with the Tutte polynomial of an associated graph and hence with the general Tutte polynomial of a matroid. Thus we prove that

a) determining the Jones polynomial of an alternating knot is #P-hard;
b) evaluating the Jones polynomial of an alternating knot is #P-hard except at a set of special points at which it is already known;
c) determining the Tutte polynomial of graphs is #P-hard except possibly along the special hyperbola $H_\alpha \equiv (x-1)(y-1) = \alpha$;
d) determining the Tutte polynomial at a point (a,b) is no easier than evaluating it along the special hyperbola through (a,b) unless (a,b) is one of the special points (0,0), (1,1), (-1,-1), (-1,0), (0,-1), (i,-i), (-i,i), (j,j^2), (j^2,j), where $j = e^{2\pi i/3}$.

R. Wille: Conceptual measurement and finite structures

The aim of conceptual measurement is to understand the conceptual structure of data sets by comparision with given patterns of concept systems. Our approach to conceptual measurement uses the framework of formal concept analysis (cf. B. Ganter, J. Stahl, R. Wille: Conceptual measurement in many-valued contexts. In: W. Gaul, M. Schrader (eds.): Classification as a tool of research. North-Holland, Amsterdam 1986, 169-176). A scale is defined as a context $\mathbf{S} := (G_S, M_S, I_S)$ with a clear conceptual structure which reflects some meaning. The **S**-measures of an (empirical) context $\mathbf{K} := (G, M, I)$ correspond to V-preserving maps from the concept lattice $\mathfrak{B}(\mathbf{K})$ into the concept lattice $\mathfrak{B}(\mathbf{S})$ respecting objects. Most important in measurement is the problem: By which scales can a given empirical structure be measured? Answers are given by measurability theorems which describe the use of considered finite structures for analyzing data. An example of such a theorem is:

Theorem: A finite context $\mathbf{K}$ admits a full measure into a direct product of one-dimensional ordinal scales if and only if $\mathbf{K} \simeq (P, P, \geqq)$ for some finite ordered set P.

G.M. Ziegler: Posets with maximal Möbius function

Let P be a poset of length l + 1, bounded, of cardinality n + 2. Then the Möbius function of P satisfies

$$|\mu(P)| \leq \max_{r \leq l} \max_{p_i + \dots + p_r = n,\ p_i \geq 1} (p_1 - 1) \dots (p_r - 1)$$

This bound is sharp: for every poset P there is a poset P^* with $|P^*| = |P|$ and $I(P^*) \leqq I(P)$ that achieves the bound. The posets achieving equality are classified. [This solves a problem of R. Stanley]. The analogous problem is solved for graded posets and attacked for finite lattices.

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