MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 10/1989

Partielle Differentialgleichungen

5.3. bis 11.3.1988

Die Tagung fand unter Leitung von Herrn Brüning (Augsburg) und Herrn von Wahl (Bayreuth) statt.

Die thematischen Schwerpunkte lagen in der Theorie der Evolutionsgleichungen, insbesondere der parabolischen Gleichungen, und in der Spektraltheorie. Es wurden jedoch auch interessante Fragen zu partiellen Differentialgleichungen vom hyperbolischen und vom elliptischen Typ vorgestellt. Es erwies sich als besonders nützlich, daß in beiden Spezialdisziplinen auch Übersichtsvorträge angeboten wurden; vor allem die Nichtspezialisten haben davon sehr profitiert. Für die Planung künftiger Tagungen sollte diese positive Erfahrung berücksichtigt werden. Darüberhinaus nutzten fast alle Teilnehmer die Gelegenheit, in Einzelgesprächen weitere Ergebnisse und Projekte auszutauschen.

Ein großer Gewinn war die zahlreiche Beteiligung sowjetischer Mathematiker, während diesmal leider kein Teilnehmer aus den USA angereist war. Besonders bedauert wurde, daß Fritz John wegen Krankheit absagen mußte.

VORTRAGSAUSZÜGE

A. AMANN:

Gobal existence for quasilinear reaction-diffusion systems

We report on existence theorems for classical solutions of quasilinear parabolic systems whose principal parts are in divergence form. In the special case of upper-triangular systems ("chemotaxis systems") it is shown that an L_∞-bound implies global existence.

C. BANDLE:

On the positive solutions of the Emden equations in cones

(joint work with M. Essén)

Let $C = \{(r,\theta) : r = |x| > 0,\ \theta \in \Omega \subset S^{N-1} = \{|x| = 1\}\}$ be a cone in $\mathbb{R}^N$. It is know that the problem $\Delta u + r^\sigma u^p = 0$ in C, $u = 0$ on ∂C with $p > 1$ and $\sigma \in \mathbb{R}$ has solutions of the form $u = r^{-(2+\sigma)/(p-1)}\alpha(\theta)$ for a certain range of $p \in (p^*, p^{**})$. It turns out that for $p \le p^*$ no regular or singular solutions exist. The asymptotic behaviour at the vertex and at infinity can be computed for certain classes of functions by means of a potential theoretical approach.

J. CHABROWSKI

Multiple solutions for the nonlinear Dirichlet problem with L^2-boundary data

The purpose of this talk is to describe the existence of multiple solutions for the Dirichlet problems

$$\begin{aligned} &Lu + b(x)u^+ - a(x)u^- = s\psi_1(x) + h(x) \quad \text{in } Q\,,\\ &u(x) = \varphi(x) \quad \text{on } \partial Q\,, \end{aligned}$$

and

$$\begin{aligned} &Lu + b(x)u^+ - a(x)u^- = h(x) \quad \text{in } Q\,,\\ &u(x) = t\varphi(x) \quad \text{on } \partial Q\,, \end{aligned}$$

where $\varphi \in L^2(\partial Q)$, $h \in L^2(Q)$, s and t are parameters, L is a self-adjoint elliptic operator and ψ_1 is the first eigenfunction of the operator $L + b$. If a and b interact with the spectrum of the operator L then both problems admit multiple solutions for s and t large. Since $\varphi \in L^2(\partial Q)$, these solutions belong to a weighted Sobolev space.

G. DA PRATO

Evolution equations in noncylindrical domains

(joint work with P. Cannarsa and J.P. Zolesésio).

We consider the problem

$$\begin{aligned} &u_t = \Delta u + f(t,x), \quad t \in [0,T], \ x \in \Omega_t \subset \mathbb{R}^N \\ &u(0,x) = u_0(x), \end{aligned} \tag{1}$$

where Ω_t depends (smoothly) on t. We reduce (1) to an abstract problem by setting $H = L^2(\mathbb{R}^N)$,

$$D(A(t)) = \{u : u|_{\Omega_t} \in H^2(\Omega_t) \cap H_0^1(\Omega_t), u|_{\Omega_t^c} \in H^2(\Omega_t^c) \cap H_0^1(\Omega_t^c)\},$$

$\Omega_t^c = \mathbb{R}^N \setminus \Omega_t$, and

$$A(t)u = z,$$

where $\int_{\mathbb{R}^N} z(x)\varphi(x)dx = \int_{\mathbb{R}^N} u(x)\Delta\varphi(x)dx$ for all $\varphi \in C_0^\infty(\mathbb{R}^N)$ such that $\varphi = 0$ on the boundary Γ_t of Ω_t.

Problem (1) reduces to

$$u'(t) = A(t)u(t), \quad u(0) = u_0. \tag{2}$$

We prove that $\{A(t)\}_{t\in[0,T]}$ fulfills the Kato-Tanabe hypothesis

$$\|(\lambda - A(t))^{-1}\| \le \frac{K}{|\lambda|}, \quad \|\frac{\partial}{\partial t}(\lambda - A(t))^{-1}\| \le \frac{K}{\sqrt{|\lambda|}} \tag{3}$$

and we solve problem (2).

We also consider the damped wave equation

$$\begin{aligned} &u_{tt} = \Delta(u + u_t) + f(t,x), \quad x \in \Omega_t \\ &u + u_t = 0 \text{ on } \Gamma_t \\ &u(0) = u_0, \quad u'(0) = u_1. \end{aligned} \tag{4}$$

P. DEURING

Quasilinear parabolic equations with nonlinear boundary conditions

I consider the following problem:

$$u_t - \sum_{\ell,m=1}^{N} A_{\ell m}(x,t,u(x,t),\nabla u(x,t))u_{x_\ell x_m} = F(x,t,u(x,t),\nabla u(x,t))$$

$$\text{in } \Omega \times (0,T], \tag{P1}$$

$$\sum_{\ell,m=1}^{N} A_{\ell m}(x,t,u(x,t),\nabla u(x,t))n_m(x)u_{x_\ell} = G(x,t,u(x,t),(|D_r u(x,t)|^\gamma)_{1\le r\le N})$$

$$\text{in } \partial\Omega \times [0,T]\,, \qquad (P2)$$

$$u(x,0) = \psi(x) \text{ on } \bar{\Omega}\,. \qquad (P3)$$

Here $\Omega \subset \mathbf{R}^N$ is a bounded domain; n is the outward unit normal to Ω; $\gamma \in (1,\infty)$. Let $(A_{\ell m})$ be strongly elliptic, and assume that $A_{\ell m}$, F, ψ, G satisfy certain (rather low) smoothness conditions. Moreover, assume the following compatibility conditions:

$$\psi_{x_\ell}(x) = 0 = G((x,0,\psi(x),(|\psi_{x_r}(x)|^\gamma)_{1\le r\le N}) \text{ for } x \in \partial\Omega\,,\ 1 \le \ell \le N\,. \qquad (*)$$

Then we can show that a solution to (P1) - (P3) exists, locally in time. We further show by a counterexample that condition $(*)$ may not be replaced by the "natural" compatibility condition. Further counterexamples prove that local continuation and uniqueness are not possible in our context.

J. FLECKINGER

Estimates of the eigenvalues of the Dirichlet Laplacian on a domain with fractal boundary

We consider the following eigenvalue problem:

$$-\Delta u = \lambda u \quad \text{in } \Omega, \quad \text{with } u = 0 \text{ on } \partial\Omega\,.$$

When the boundary is fractal, the Minkowski dimension appears in the remainder term of the Weyl's estimate.

R. HEMPEL

Eigenvalues of the Schrödinger operator $H - \lambda W$ in a spectral gap of H

(joint work with S. Alama and P. Deift)

We consider Schrödinger operators $H = -\Delta + V$, acting in the Hilbert space $L_2(\mathbf{R}^\nu)$, where the bounded, measurable function V is such that the spectrum of H has a **gap** (a,b). We then ask for the eigenvalue branches in the gap of the operator family $H - \lambda W$ (here W is a relatively compact perturbation of H and λ a real coupling constant). Such operators arise in the quantum theory of solids as a model for crystals with localized impurities.

In the present talk, we concentrate on the case $W \geq 0$ and describe the asymptotic distribution of eigenvalue branches of $H \pm \lambda W$, as λ goes to infinity: we define (for E in the gap (a,b))

$$N_{\pm}(\lambda) := \sharp\{0 < \lambda_j < \lambda;\ E \in \sigma(H \mp \lambda_j W)\}, \quad \text{for } \lambda > 0\,,$$

and discuss the relationship between the asymptotic behaviour of $N_{\pm}(\lambda)$, as $\lambda \to \infty$, and the volumina of the related classically allowed regions in phase space. While the semi-classical approximation gives the correct answer for the asymptotics of N_+, the asymptotics of N_- may be determined in some cases by means of the integrated density of states of H and will **not** in general agree with the associated phase space volume.

A. HINZ

Asymptotic behaviour of eigensolutions and the spectrum of Schrödinger operators

Pointwise decay or growth properties of solutions to the equation $-\Delta v + qv = \lambda v$ are closely related to the spectrum of the corresponding Schrödinger operator. Bounds on eigenfunctions, well-known for q_- (the negative part of q) bounded or in K^n, extend to $q_- = o(|x|^2)$, as do estimates for the distance of λ to the essential spectrum, depending on the rate of growth of non-L_2-solutions v. The case $q_- = O(|x|^2)$ appears as a border line with a striking example turning up, the crucial point of which is still an open problem.

L. HÖRMANDER

Non-linear second order hyperbolic differential equations

For non-linear perturbations of the wave equation in $\mathbb{R}^{1+n}$,

$$\Box u = G(u, u', u'')\,, \tag{1}$$

where G is a C^∞ function vanishing of second order at 0 it is now well-known that if $u_j \in C_0^\infty$ then there is a global solution with Cauchy data

$$u = \varepsilon u_0\,, \quad \partial_t u = \varepsilon u_1 \quad \text{when } t = 0\,, \tag{2}$$

for small $\varepsilon > 0$ if $n \geq 5$. When $n = 4$ this is also true if there is no u^2 term, that is, $G(u,0,0) = O(u^3)$ as $u \to 0$; for arbitrary G there is a constant c such that a solution exists for $0 \leq t \leq e^{c/\sqrt{\varepsilon}}$ (unpublished; the square root can probably be eliminated with some additional work). Assume now that $n = 3$. When G is a function of u' and u'' only then John and Klainerman have proved that a solution exists for $t \leq e^{c/\varepsilon}$. A lower bound for c has been given by F. John and the speaker when G is linear in the second derivatives; it agrees with an upper bound established by John in a special case. The lecture is mainly devoted to the extension of this result to fully non-linear equations. The main point is the determination of the lifespan of the solution of an approximating Cauchy problem in $\mathbb{R}^2$,

$$\partial u/\partial t = a(\partial u/\partial x)^2 + 2bu\partial u/\partial x + cu^2\,; \quad u(0,x) = u_0(x)\,. \tag{3}$$

The case $a = c = 0$ (Burgers' inviscid equation) was sufficient for the earlier result.

W. JÄGER

Symmetry breaking for semilinear elliptic equations

Consider the Dirichlet problem

$$\Delta u + \lambda f(u) = 0 \text{ in the } n\text{-dimensional unit ball } B^n, \quad u = 0 \text{ on } \partial B^n\,.$$

Due to the theorem of Gidas, Ni and Nirenberg positive solutions are always radially symmetric. The question if symmetry breaking bifurcation from branches of radially symmetric solutions (changing sign) occurs was studied by Pospiech, Smoller and Wasserman using techniques of local bifurcation with symmetry groups. In a joint paper with K. Schmitt a result of Pospiech obtained by global arguments could be improved to nonlinearities f which are asymptotically linear at infinity, positive in 0 and whose primitive has exactly one minimum. Consider a connected component of fully symmetric solutions in functionspace and a bounded domain B in the parameter-functionspace and assume that there is a door D to this box B through which this component enters and leaves the box. Assume that on the door there are only fully symmetric solutions and that the degree can be computed on the door and does not vanish. Then there must be symmetry breaking bifurcation inside the box. This idea is made precise and is applied to the special boundary value problem. Bifurcation from infinity is used to study the branches of radially symmetric solutions, on which symmetry breaking bifurcation takes place.

B. KAWOHL

A family of torsional creep problems

Subject of the lecture is the study of solutions to the problem

$$-\Delta_p u \equiv -\text{div}(|\nabla u|^{p-2}\nabla u) = 1 \quad \text{in } \Omega$$
$$u = 0 \quad \text{on } \partial\Omega$$

as $p \to 1$ or as $p \to \infty$. For $1 < p < \infty$ the problem has a unique solution $u_p \in W_0^{1,p}(\Omega)$. The limiting case $p \to \infty$ models perfectly plastic torsion. It is shown that $\lim_{p\to\infty} u_p(x) = d(x, \partial\Omega)$, the distance function to $\partial\Omega$. The limiting equation is no longer elliptic. The case $p \to 1$ is of independent geometric interest because it leads to interesting free boundary problems. Even if Ω is a ball in $\mathbb{R}^n$ there are surprises: If the ball has radius less than n, u_p tends to zero as p goes to 1; but if the radius is greater than n, u_p blows up to ∞ everywhere in Ω. this phenomenon is linked to the isoperimetric inequality between perimeter and volume of Ω. It can be explained for general domains if one solves the geometric problem: Given Ω find $D \subset \Omega$ such that surface area of D minus volume of D becomes minimal.

V.A. KONDRAT'EV

Asymptotic properties of solutions of the elasticity system

We consider the equation

$$\sum_{|\alpha|\le m} A_\alpha D^\alpha u(x) = f(x) \tag{1}$$

where $X_m \subset X_{m-1} \subset \cdots \subset X_0$ are Hilbert spaces, $A_\alpha : X_{|\alpha|} \to X_0$, $u(x) \in X_m$, $f(x) \in X_0$, $x = (x_1, \cdots, x_n)$ and $D^\alpha = \frac{\partial^{|\alpha|}}{\partial x_1^{\alpha_1} \cdots \partial x_n^{\alpha_n}}$. Suppose that there exists $P(\lambda) = \sum_{|\alpha|=N} a_\alpha \lambda^\alpha$ such that

$$P(\lambda)\Big(\sum_{|\alpha|\le m} (i\lambda)^\alpha A_\alpha\Big) = P(\lambda)R(\lambda)$$

is analytic and bounded for $|\mathrm{Im}\,\lambda| \le C$.
Then

$$\|u(x)\|_{X_m} \le C_1 e^{-C_2|x|} \int_{\mathbb{R}^n} e^{C_2(y)} \|f(y)\|_{X_0} dy\,.$$

A number of questions in the theory of elliptic equations is reduced to the investigation of equation (1). For example, consider the system of elasticity

$$\sum_{i,j,k=1}^n \frac{\partial}{\partial x_i} a_{ij}^{hk} \frac{\partial u_k}{\partial x_j} = f_h\,, \quad h = 1, \cdots, n,\ x \in \Pi = \mathbb{R}^{n-1} \times (0,1)\,.$$

Problem. Find the weak solution u of this system with

$$E(u, \Pi) = \sum_{i,j=1}^n \int_\Pi \Big(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\Big)^2 dx < \infty$$

and

$$\sigma_h(u) = \sum_{i,j,k=1}^n a_{ij}^{hk} \frac{\partial u_k}{\partial x_j} \cos(\overrightarrow{n}, x_i)\big|_{\partial\Pi} = 0$$

Some theorems of uniqueness of solutions are proved.

B.M. LEVITAN

The asymptotics of the Weyl-Titchmarsh m-function on the spectra of perturbed Hill's equation

In the last time there appeared some work in which the asymptotics of the Weyl-Titchmarsh m-function is studied in the angle $0 < \varepsilon < \arg z < \pi - \varepsilon$ ($|z| \to \infty$). A more difficult problem consists in studying the asymptotics of the m-function on the spectra. Of course this is possible in the case that the limit of $m(z)$ for $\operatorname{Im} z \to 0$ exists.

In the report there will be given an account of some results about the asymptotic expansion of the m-function for $\lambda \to +\infty$ ($\operatorname{Im} \lambda = 0$) in the case of perturbed Hill's equation:

$$-y'' + (p(x) + q(x))y = \lambda y\,, \quad (-\infty < x < \infty)\,,$$

where p is a smooth periodic function of period π, and where q is a smooth function satisfying

$$\int_{-\infty}^{\infty} (1 + |x|)|q(x)|dx < \infty\,.$$

S. LUCKHAUS

Pressure jumps for the dam problem

This talk presents a joint work with G. Gilardi (Pavia) on the dam problem in the free boundary formulation. To save writing let all physical constants be 1. Denote by p the pressure, by $-e_z$ the vector of gravity, and by s the relative water saturation of an earth dam Ω. The flow equations can then be written

$$\partial_t s - \nabla(s(\nabla p + e_z)) = 0 \text{ in } \Omega, \quad 0 \le p, 0 \le s \le 1, p \cdot (1-s) \equiv 0$$
$$s(\partial_\nu p + \nu e_z) = 0 \text{ in } \Gamma_N\,, \quad p = p_D \text{ in } \Gamma_D,\ s(\partial_\nu p + \nu e_z) \ge 0 \text{ in } \Gamma_D \cap \{p_D = 0\}$$

where $\partial\Omega$ is the disjoint union of $\Gamma_N, \Gamma_D \in C^2$.

We prove: Under suitable conditions on p_D ($C^{1,\alpha}$ would be enough), $\partial_t p_D$ is a measure and $\partial_t^- p_D$ bounded (locally). In other words: the pressure jumps are always positive as the examples indicate.

A key ingredient in the proof is a lemma on subharmonic function:

p nonnegative, subharmonic in B_ρ implies an estimate

$$\int_{B_\rho} \Delta_p > \frac{c}{\rho} \int_{B_\rho} p \,|\, \{p = 0\} \cap B_\rho|^{1-\frac{1}{n}}\,.$$

This allows to give a rigorous proof of the bound from below for the time derivative of the pressure on the free boundary.

A. LUNARDI

A semigroup approach to parabolic equations in Hölder spaces

We consider a parabolic initial-boundary value problem in $[0,T]\times\bar{\Omega}$, where $\Omega\subset\mathbb{R}^n$ is a bounded open set with regular boundary $\partial\Omega$:

$$\begin{aligned}
&u_t(t,x)=A(t,x,D)u(t,x)+f(t,x)\,, \quad 0\le t\le T\,, \quad x\in\bar{\Omega}\\
&u(0,x)=u_0(x)\,, \quad x\in\bar{\Omega}\\
&B_j(t,x,D)u(t,x)=g_j(t,x)\,, \quad j=1,\cdots,m,\ 0\le t\le T\,, \quad x\in\partial\Omega\,.
\end{aligned}$$

Here $A(t,x,D)$ is an elliptic differential operator of order $2m$, and the $B_j(t,x,D)$, $j=1,\cdots,m$, are boundary differential operators satisfying roots and complementing conditions.
Such problems have been studied recently by means of the theory of analytic semigroups in the space $C(\bar{\Omega})$ adapted to the case of non dense domains and nonhomogeneous boundary conditions.
The classical theory of Solonnikov concerning optimal Hölder regularity in (t,x) has been recovered by Lunardi, Sinestrari and von Wahl. Hölder continuity with respect to the space variable x has been studied by Lunardi, and Hölder continuity with respect to time (in the homoegeneous case) by Sinestrari and von Wahl and by Acquistapace and Terreni.

A.M. MOLCHANOV

Nonlinear Equations of Schrödinger type

We consider the nonlinear eqution

$$-\Delta\psi+V\psi=A\psi\,.$$

Here $A=\lambda$ gives the classical Schrödinger equation, $A=\psi^2$ the "ψ-cube" equation (without parameter) and $A=\int_\Omega K(x,y)\psi^2(y)dy$ the Pekar-Bogolubov polaron-equation.
A particular case, $K(x,y)=a(x)b(y)$, leads to equations

$$\begin{aligned}
&-\Delta\psi+V(x)\psi=Ba(x)\psi\\
&B=\int_\Omega b(y)\psi^2(y)dy\,.
\end{aligned}$$

The first equation reduces to a linear one for ψ and gives a *spectrum* of "eigenvalues" B. Each B determines two solutions ψ.
This case gives a model for *branching* of solutions for the general Pekov-Bogolubov equation.

W. MÜLLER

Some aspects of spectral theory on locally symmetric

The talk is a survey of some recent results on spectral theory for the Laplacian on a locally symmetric space $\Gamma \setminus G/K$ of finite volume. A new approach to the study of the continuous spectrum of the Laplacian via scattering theory is briefly discussed in the talk. The main result is the estimation of the counting function $N(\lambda) = \sharp\{\lambda_i \leq \lambda\}$ of the eigenvalue sequence $0 = \lambda_0 < \lambda_1 \leq \lambda_2 \leq \cdots$ by a polynomial bound in λ. This implies an affirmative answer to the so-called trace class conjecture in the theory of automorphic forms. This means all operators considered in Selberg's trace formula are of trace class when restricted to the point spectrum of the Laplacian. Also the heat operator is of trace class on the point spectrum. This, for example, makes it possible to apply the heat equation method to study index problems on locally symmetric spaces of finite volume.

E. MÜLLER–PFEIFFER

Sturmsche Theorie für elliptische Differentialgleichungen 2. Ordnung

Der bekannte Vergleichssatz von Sturm und Picone für gewöhnliche, selbstadjungierte Differentialgleichungen zweiter Ordnung wird auf selbstadjungierte elliptische Differentialgleichungen verallgemeinert. Dabei sind das Grundgebiet G und die Koeffizienten der Differentialgleichung nicht notwendig beschränkt, und es werden keine Regularitätsforderungen an den Rand ∂G gestellt.

R. RACKE

Asymptotic behaviour of solutions of dissipative systems

(joint work with J. Sjöstrand)

We consider parabolic resp. damped hyperbolic systems of the following form: $u_t + L^k u = 0$ resp. $u_{tt} + L^k u + u_t = 0$, where $L = -\Delta$ or $L = -\partial_i a_{ik}(x)\partial_x$ elliptic with $L = -\Delta$ outside a ball, in an exterior domain $\Omega \subset \mathbb{R}^n$, together with initial and suitable boundary conditions. To get the desired results, namely the decay behaviour of L^q-norms of the solution u, $2 \leq q \leq \infty$, we make an ansatz with generalized Fourier transforms and are led to the study of pointwise estimates of solutions of exterior boundary value problems for the operator L. – As further motivation we discuss the importance of these estimates for corresponding nonlinear systems.

R. REDLINGER

Globale a priori Schranken für parabolische Systeme mit Kreuz-Diffusion

Zunächst wird ein Invarianzsatz für parabolische Systeme der Form

$$u_t = A(t,x,u)Lu + f(t,x,u,u_x) \quad \text{in } G \subset \mathbb{R}^{n+1}$$

unter Randbedingungen 3. Art

$$B(t,x,u)u_\gamma = g(t,x,u) \quad \text{auf } \partial_p G$$

angegeben. Dabei ist γ eine äußere Normale an G, und $L = L(t,x)$ ein gleichmäßig elliptische Operator 2. Ordnung.
Als Anwendung wird gezeigt, daß die Lösung (u,v) des Zwei-Komponenten-Systems $(a_i, \cdots, e_i > 0)$

$$\begin{aligned} u_t &= [(c_1 + d_1 v)u]_{xx} + u(e_1 - a_1 v - b_1 u) \\ v_t &= c_2 v_{xx} + v(e_2 - a_2 u - b_2 v) \end{aligned} \qquad \text{in } t > 0,\ x \in (0,1).$$

mit $u_x = v_x = 0$ für $x \in \{0,1\}$ und nichtnegativen Anfangsbedingungen, im Falle $c_2 < c_1$ global beschränkt ist (in der Maximumnorm).

H. SCHRÖDER

The Riemann-Roch Theorem on algebraic curves

(joint work with J. Brüning and N. Peyerimhoff)

An analytic proof of the following generalization of the Riemann-Roch Theorem was sketched: Let $C \subset \mathbb{C}P^N$ be an algebraic curve, $\pi : S \to C$ the Noether normalization and $\Sigma \subset C$ the singular locus. Given a vectorbundle E of rank k over C, holomorphic over $C \setminus \Sigma$, and equipped with an Hermitean metric which is constant near Σ one has the "twisted" Cauchy-Riemann operator

$$\bar\partial_E : C_0^\infty(E|_{C\setminus\Sigma}) \to C_0^\infty(\Lambda^{0,1}E|_{C\setminus\Sigma}).$$

$\bar\partial_E$ is closable with domain in $L^2(E)$ and range in $L^2(\Lambda^{(0,1)}E)$ where the metric on $C \setminus \Sigma$ comes from restricting the Fubini-Study metric of $\mathbb{C}P^N$. All closed extensions are Fredholm operators and correspond to the subspaces W of the finite dimensional space

$$W_0 = \mathcal{D}(\bar\partial_{E,\max})/\mathcal{D}(\bar\partial_{E,\min}).$$

The corresponding extension $\bar\partial_{E,W}$ has index

$$\operatorname{ind} \bar\partial_{E,W} = \operatorname{ind} \bar\partial_{E,\min} + \dim W,$$

with

$$\text{ind}\, \bar{\partial}_{E,\min} = k\chi(S) + \int_{C\setminus\Sigma} c_1(E)$$

and

$$\dim W_0 = k \sum_{q\in\pi^{-1}(\Sigma)} (n(q)-1)\,.$$

Here $\chi(S) = 1-g$ is the arithmetic genus of the compact Riemann surface S (with g "holes"), and $n(q)$ is the multiplicity of the branch of C which is determined by q.

C.G. SIMADER

Helmholtz decomposition and the weak Neumann problem on L^q

(joint work with H. Sohr)

Let $G \subset \mathbb{R}^N$ denote either a bounded or an exterior domain and

$$D_0^\infty(G) := \{\phi \in C_0^\infty(G)^N \mid \text{div}\,\phi = 0 \text{ in } G\}\,.$$

For $1 < q < \infty$ let $D^q(G) := \overline{D_0^\infty(G)}^{\|\cdot\|_q}$ and

$$G^q(G) := \{\nabla p \mid p \text{ measurable}, \, p \in L^q(G_R)\forall R > 0, \, \nabla p \in L^q(G)\}$$

where $G_R := G \cap \{|x| < R\}$. Then the *Helmholtz decomposition* states that

$$L^q(G) = D^q(G) \oplus G^q(G) \quad (q \neq 2 \text{ direct decomposition}, \, q = 2 \text{ orthogonal}),.$$

Further there is a constant $K > 0$ such that

$$\|u + \nabla p\|_q \geq K(\|u\|_q + \|\nabla p\|_q) \quad \text{for } u \in D^q(G)\,, \; \nabla p \in G^q(G)\,.$$

The equivalence to the weak *Neumann problem* on L^q is indicated: let

$$\hat{\hat{H}}^{1,q}(G) := \{u : G \to \mathbb{R} \mid u \text{ measurable}, \, u \in L^q(G_R)\forall R, \; \nabla u \in L^q(G_R)\}$$

and define $\hat{H}^{1,q}(G) := \hat{\hat{H}}^{1,q}(G)/\mathbb{R}$. Equipped with $\|\nabla.\|_q$, $\hat{H}^{1,q}(G)$ is a reflexive Banach space. Then there is a constant $C > 0$ such that

$$C\|\nabla u\|_q \leq \sup\{< \nabla u, \nabla\phi > \mid \phi \in \hat{H}^{1,q'}(G)\,, \quad \|\nabla\phi\|_{q'} \leq 1\}$$

where $q' = \frac{q}{q-1}$ for $u \in \hat{H}^{1,q}(G)$. Further for $F \in (\hat{H}^{1,q}(G))^*$ there exists a unique $u \in \hat{H}^{1,q}(G)$ such that $F(\phi) =< \nabla u, \nabla\phi >$.

J. SJÖSTRAND

The Schrödinger equation with a constant, weak magnetic field

(joint work with B. Helffer)

This talk is based on recent joint work with B. Helffer, related to earlier works of Avron-Simon, Nencin, Bellissard, Guillot-Ralston-Trubowitz ... and many physicists. Let $e_1, \cdots, e_n$ be a basis in $\mathbb{R}^n$, $\Gamma = \bigoplus_1^n \mathbb{Z} e_j$, $V \in C^\infty(\mathbb{R}^n; \mathbb{R})$ with $V(x+\gamma) = V(x)$, $\forall \gamma \in \Gamma$. Let $b_{j,k} = -b_{k,j}$, $1 \leq j,k \leq n$, be real and constant. With $A_k(x) = \frac{1}{2}\sum_j b_{j,k} x_j$, $B := d(\Sigma A_k dx_k) = \frac{1}{2}\Sigma\Sigma b_{j,k} dx_j \wedge dx_k$, we put $P_{B,V} = \sum_1^n (D_{x_k} + A_k(x))^2 + V(x)$.
Let $E_0(\theta) \leq E_1(\theta \leq \cdots$ be the Floquet eigenvalues of $P_{0,V}$ for $\theta \in \mathbb{R}^{n*}$. If:

$$\sup E_{k-1} < \inf E_k \leq \sup E_k < \inf E_{k+1}\,, \qquad (*)_k$$

for some k, the Peierls substitution in solid state physics says that for energies close to $[\inf E_k, \sup E_k]$ and for $|B|$ small, "$P_{B,V}$ is well described by the pseudodifferential operators $E_k(D_x + A_k(x))$". Our first theorem justifies this, in the sense that it gives a corresppondіng reduction of the study of the spectrum. We can even drop the assumption $(*)_k$. Then the reduced operator is a matrix of pseudodifferential operators.
The second theorem treats the 3-dimensional case. Under suitable assumptions, we find singular oscillations in the density of states measure. These are related to the de Haas-van Alphen effect, explained heuristically by Onsager.

M. STRUWE

Globally regular solutions to the u^5-Klein-Gordon equations

The Cauchy problem for the semi-linear wave eqution in $\mathbb{R}^3 \times \mathbb{R}_+$

$$u_{tt} - \Delta u + u^5 = 0; \quad u\big|_{t=0} = u_0\,, \quad u_t\big|_{t=0} = u_1 \qquad (1)$$

for radially symmetric initial data $u_0(x) = u_0(|x|) \in C^3(\mathbb{R}^3)$, $u_1(x) = u_1(|x|) \in C^2(\mathbb{R}^3)$ is shown to admit a unique, global solution $u(x,t) = u(|x|,t) \in C^2(\mathbb{R}^3 \times \mathbb{R}_+)$.
Essential tools are the local (small time) existence results of Jörgens, the a-priori estimate of Rauch for solutions with small initial energy, and a decay estimates for solutions near a singularity. We heavily exploit invariance of (1) under scaling $u \mapsto u_R(x,t) = R^{1/2}u(Rx, Rt)$. Moreover, the energy inequality is used extensively.
The result suggests that also in the non-symmetric case problem (1) will always admit a global, regular solution.

D. VASIL'EV

The distribution of eigenvalues of boundary value problems

Consider an eigenvalue problem for a self-adjoint elliptic differential operator of order $2m$ on a compact n-dimensional manifold with boundary. Let $N(\lambda)$ denote the eigenvalue distribution function, i.e. the number of eigenvalues λ_k smaller than a given λ. The asymptotic formula

$$N(\lambda) = a\lambda^{n/2m} + o(\lambda^{n/2m}); \quad \lambda \to +\infty, \tag{1}$$

is a well-known classical result. A refined two-term asymptotics

$$N(\lambda) = a\lambda^{n/2m} + b\lambda^{(n-1)/2m} + o(\lambda^{(n-1)/2m}), \quad \lambda \to +\infty, \tag{2}$$

is established by the author; coefficient b takes account of the boundary conditions. Formula (2) holds when a certain geometrical condition is fulfilled. This geometrical condition is formulated in terms of a branching Hamiltonian billiard associated with the differential operator. In this respect periodic, absolutely periodic and deadend trajectories are investigated.

V. VOGELSANG

Über das Spektrum der Maxwellschen Gleichungen

Die Feldgleichungen elektromagnetischer Schwingungen lauten

$$\left.\begin{aligned} \operatorname{rot} a_1 &= i\omega\alpha_2(x)a_2 \\ \operatorname{rot} a_2 &= -i\omega\alpha_1(x)a_1 \end{aligned}\right\} \tag{$*$}$$

mit positiv definiten, hermiteschen Matrizen $\alpha_1(x)$, $\alpha_2(x)$ in einem Außengebiet Ω und einer Zahl $\omega \in \mathbb{R}$, $\omega \neq 0$. Unter einer Bedingung der Form

$$\alpha_j(x) = p_j(|x|)1 + o(r^{-\delta}),\ ;0 < \delta \leq 1 \text{ fest}, \ (|x| \to \infty),\ p_j \geq k_j > 0\,,$$

und entsprechender Bedingung an $\partial_r\alpha_j(x)$ beweisen wir das exponentielle Abklingen der Lösung $a = a_1 + ia_2$ von $(*)$ im L^2-Mittel. Für $\delta = 1/2$ gilt sogar die Freiheit von Punkteigenwerten des Maxwelloperators $(*)$ auf der ganzen reellen Achse. In engem Zusammenhang mit diesem Problem steht das Prinzip der eindeutigen Fortsetzbarkeit, das wir für Ungleichungen der Form

$$|\operatorname{rot} a| + |\operatorname{div}(\alpha a)| \leq cr^{\varepsilon-1}|a|$$

unter Voraussetzung, a klinge bei 0 von unendlicher hoher Ordnung ab, beweisen. Entscheidend für den Beweis sind Integralungleichungen vom Carleman'schen Typ bei ∞ und bei 0. Dazu werden in Analogie zur Diracgleichung die Maxwellschen Gleichungen $(*)$ in Polarkoordinaten dargestellt, die Nebenbedingung $\operatorname{div}\alpha_j a_j = 0$ geeignet verarbeitet und Testfunktionen vom Type $q\partial_r a$ und $q\hat{x}\wedge\partial_r(\alpha a)$ gewählt.

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W. WALTER

Elliptic differential inequalities with an application to gradient bounds

Let L be an elliptic operator defined by

$$Lu : \sum_{i,j=1}^{n} a_{ij}(x)u_{x_i x_j} + \sum_{i=1}^{n} b_i(x)u_{x_i}\,, \quad x \in D\,,$$

where $D \subset \mathbb{R}^n$ is open and bounded, and

$$\alpha\xi^2 \le \xi^T a(x)\xi \le k\xi^2\,, \quad |b_i(x)| \le K\,, \quad -K \le c(x) \text{ in } D\,, \qquad (A)$$

where $\alpha, K > 0$ (no upper bound on c is assumed). The main result is the following theorem, which generalizes a classical theorem on the strong minimum principle.

Theorem: *Assume that D satisfies a uniform interior ball condition, that (A) holds and that there exists a function h satisfying*

$$Lh + ch \le 0 \quad \text{and } h > 0 \text{ in } D\,.$$

Then if

$$Lu + cu \le 0 \quad \text{in } D \quad \text{and } u \ge 0 \text{ on } \partial D\,,$$

either (i) $u = \beta h$ with $\beta < 0$ or (ii) $u \equiv 0$ or (iii) $u > 0$ in D.

Remark In case (i) u is an eigenfunction for $L + c$ corresponding to the first eigenvalue $\lambda_1 = 0$.
Among the applications a theorem on gradient bounds for a nonlinear elliptic equation (1) $F(\varphi, \varphi_x, \varphi_{xx}) = 0$ by Weinberger (1987) is generalized. Here $\varphi_x = \operatorname{grad}\varphi$, $\varphi_{xx} =$ the Hessian, and it is assumed that $(\frac{\partial F}{\partial r_{ij}})$ is positive definite.
Let L be as above with $F = F(z, \varphi, r)$,

$$a_{ij} = \frac{\partial F}{\partial r_{ij}}\,, \quad b_i = \frac{\partial F}{\partial \varphi_i}\,, \quad c = \frac{\partial F}{\partial z}\,, \text{ argument } (\varphi(x), \cdots)\,.$$

Theorem: *If (A) holds, then for $\varphi \in C^1(\bar{D}) \cap C^2(D)$ satisfying (1)*

$$\varphi_\xi \ge 0 \text{ on } \Gamma \quad \text{implies} \quad \varphi_\xi \ge 0 \text{ in } D$$

under each of the following assumptions:
(i) There exists $h \in C^0(\bar{D}) \cap C^2(D)$ satisfying

$$Lh + ch \le 0 \text{ in } D \text{ and } h > 0 \text{ in } D\,.$$

(ii) There exists η with $|\eta| = 1$ such that $\varphi_\eta \ge 0$ in D.
Here $\varphi_\eta = \eta \cdot \varphi_x$ is the directional derivative in the η direction, and φ_ξ is defined analogously.

P. WEIDEMAIER

Boundary value problems for parabolic equations

In the theory of nonlinear heat conduction the following problem occurs: (cf. Potier-Ferry: Arch. f. Rat. Mech. 77, 301–321, (1981))

$$u_t - \frac{\partial a_i}{\partial p_j}(x,t,u,\nabla u)\partial_i\partial_j u = f \quad \text{in } \Omega \times (0,T) =: \Omega_T\,,\ \Omega \subset \mathbb{R}^n\,,$$
$$< a(\xi,t,u\nabla u), n > \; = \psi \quad \text{on } \partial\Omega \times (0,T)$$
$$n := \text{ outward unit normal to } \Omega$$
$$u(0) = \varphi\,.$$

We construct a local (in time) solution in the space

$$W_p^{2,1}(\Omega_T) := \{u \in L^p(\Omega_T) \mid \partial_t^\beta \partial_x^\alpha u \in L^p(\Omega_T) \forall 2\beta + |\alpha| \leq 2\}\,, \quad p > n+2\,,$$

under natural compatibility conditions. The underlying estimates for the linear problem are essentially due to Solonnikov. Problems of the type above were also considered in Acquistapache/Terreni: preprint 205 (1987), Univ. Pisa; these latter authors work in different spaces (Hölder in t, Sobolev in x).

J. WEISSLER

The Cauchy problem for the time dependent nonlinear Schrödinger eqution

(joint work with T. Cazenave)

We consider the Cauchy problem

$$\left.\begin{array}{c} iu_t + \Delta u = f(u) \\ u(0,x) = \varphi(x) \end{array}\right\} (NLS)$$

where $u = u(t,x) \in \mathbb{C}$, $t \geq 0$, $x \in \mathbb{R}^n$. We are primarily concerned with the model case $f(u) = \lambda|u|^\alpha u$, where $\lambda \in \mathbb{R}$, and $\alpha > 0$. For $\alpha < 4/(n-2)$ it has been known for a long time that the problem (NLS) is well-posed in H^1. We show that the problem (NLS) is also well-posed if $\alpha = 4/(n-2)$, and the solution is global if $\|\nabla\varphi\|_2$ is small, independent of the sign of λ. If $\alpha > 4/(n-2)$, the problem (NLS) is well-posed in higher H^s spaces.

M. WIEGNER

A $C_{\alpha,\beta}$-theory for parabolic equations

The well-known $C_{\alpha,\alpha/2}$-Schauder-theory for parabolic equations

$$\frac{\partial u}{\partial t} - \sum_{i,j} a_{ij}(x,t)\frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_i b_i(x,t)\frac{\partial u}{\partial x_i} + c(x,t)u = f(x,t) \text{ on } \Omega \times (0,T) \quad (*)$$

$$u = 0 \quad \text{on } \Sigma_T = \partial\Omega \times (0,T) \cup \bar{\Omega} \times \{0\}$$

does not give the complete picture of the dependence of the regularity of the solution from the data. We assume data from function spaces $C_{\alpha,\beta} - \alpha$ – Hölder-continuous with respect to x and β to time, with $\beta = 0$ included – and give various interior and global estimates for the solution in corresponding spaces. By counterexamples it is shown that additional information on the data along the boundary is needed to prove global Hölder-estimates. On the other hand, we prove that $(*)$ has a classical solution u in $C_{2,1}(\overline{\Omega \times (0,T)})$ (including the boundary) if for some $\alpha > 0$ the data are in $C_{\alpha,0}$, $\partial\Omega \in C_{2+\alpha}$, $f(x,0) = 0$ on $\partial\Omega$, and as the only assumption concerning time-regularity: for $x \in \partial\Omega$ the form $\sum_{i,j} a_{ij}(x,t)\nu_i\nu_j$ (ν the outer normal) is δ-Hölder- continuous with respect to t for some $\delta > 0$.

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