MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 11/1989

Mathematische Stochastik

12.3. bis 18.3.1989

Die Tagung fand unter der Leitung von Herrn Prof. Dr. P. Gaenssler (München) und Herrn Prof. Dr. W. Stute (Giessen) statt.

Ziel dieser Tagung war es, das breite Spektrum der Forschung auf dem Gebiet der Mathematischen Stochastik sichtbar zu machen. Es wurden insgesamt 40 Vorträge gehalten, davon 5 mit Übersichtscharakter.

Trotz der Vielfalt der Themen wurde bei der Organisation der Tagung versucht, folgende Schwerpunkte zu setzen: Kurvenschätzungen, Grenzwertsätze der Wahrscheinlichkeitstheorie, Grundlagen empirischer Prozesse, spezielle stochastische Prozesse.

Die Diskussionen im Anschluß an die Vorträge sowie am Rande der Tagung waren intensiv und haben zu einem regen Gedankenaustausch geführt.

Vortragsauszüge

BINGHAM, N.H.

On the work of A.N. Kolmogorov on strong limit theorems

The historical setting is briefly reviewed, from J. Bernoulli's Ars Conjectandi (1713), through to Hilbert's 6th problem (1900), the work of Borel, Cantelli and others. Next, the weak law is considered (Khinchin, Lévy, Kolmogorov) and random series (three-series theorem, Kolmogorov inequalities,-).
Kolmogorov's work on the strong law is considered (Comptes Rendues (1930), Grundbegriffe der Wahrscheinlichkeitsrechnung (1933)), together with its generalisations (ergodic theorem, martingale convergence theorem, L_p version, Banach spaces,...).
Next, Kolmogorov's law of the iterated logarithm (1929) is discussed, and its applications, e.g., the Hartman-Wintner LIL (1941).
Finally, we discuss randomness and computational complexity. The basic references for this section are the papers of Kolmogorov & Uspensky (Tashkent 1986/Th. Prob. Appl. 1987 and Vovk (Th. Prob. Appl. 1987).
Connections with von Mises' theory of collectives are briefly mentioned.

BOLTHAUSEN, E.

Improved asymptotics for the Wiener sausage

The Wiener sausage of dimension d is defined as $W_t^\varepsilon(\beta) = \bigcup_{s\le t} (\beta_s + B_\varepsilon)$ where β is a d-dimensional Brownian motion and B_ε is the centered ball with radius ε.
Donsker and Varadhan proved that

$$\lim_{t\to\infty} t^{-d/(2+d)} \log E(e^{-\nu|W_t^\varepsilon(\beta)|}) = -\nu^{2/(2+d)} c(d) \quad .$$

This remains true if ε depends on t in the form $\varepsilon(t) = t^{-a}$ for $a < 2/(d^2-4)$ $(d\ge 2)$ but becomes false for $a = 2/(d^2-4)$. This is shown by a refinement of results on large deviations in the total variation topology.

CSÖRGÖ, S.

Convergence of infinitely divisible laws

A purely probabilistic proof of an equivalent form of Gnedenko's classical criterion of convergence of infinitely divisible laws will be sketched.

DAVIES, P.L.

Improving S-estimators

It may be postulated that a good robust estimator should be (1) globally robust (high breakdown point), (2) locally robust (Fréchet differentiable), (3) efficient at the assumed model and (4) exhibit no pathologies away from the assumed model. The above properties (1)-(3) can be attained by using Rousseeuw's minimum volume estimator to obtain a high breakdown point and then a two-step M-estimator to improve its local properties. By using a smooth S-estimator property (4) can also be satisfied but smooth S-estimators are not practical as they cannot be calculated.

DEHEUVELS, P.

Pointwise Bahadur-Kiefer theorems

Let $\alpha_n(t)$ be the uniform empirical process and $\beta_n(t)$ the uniform quantile process. Bahadur (1966) initiated the study of the process $R_n(t) = \alpha_n(t) + \beta_n(t)$. Kiefer (1967) proved that, for any fixed $0 < t < 1$,

$$\limsup_{n \to \infty} \pm R_n(t) / \left\{ 2^{+5/4} 3^{-3/4} (t-(1-t))^{1/4} n^{-1/4} (\log\log n)^{3/4} \right\} = \pm 1 \quad \text{a.s.} \quad .$$

We extend this result to the case where $t = t_n \downarrow 0$ together with $nt_n/\log\log n \uparrow \infty$. Similar but distinct results hold when $nt_n/\log\log n \to c$ where $0 < c \leq \infty$. These results are related to the following theorem due to Einmahl and Mason (1989):

$$\limsup_{n \to \infty} \pm \left\{ \sup_{0 \leq t \leq t_n} |R_n(t_n)| \right\} / \left\{ 2^{1/4} t_n^{1/4} n^{-1/4} (\log\log n)^{1/4} \sqrt{2\log\log n + \log(nt_n)} \right\} = 1 \quad \text{a.s.} ,$$

where here $t_n \downarrow 0$, $nt_n \uparrow \infty$ and $nt_n/\log\log n \to \infty$, with $\frac{\log(nt_n)}{\log\log n} \to \infty$, through functional laws of the iterated logarithm. As far as uniform laws are concerned, we have (Deheuvels and Mason (1988))

$$\lim_{n\to\infty} \left[\sup_{0\le t\le 1} |R_n(t)|\right] / \left\{ \left[\sup_{0\le t\le 1} |\alpha_n(t)|^{1/2}\right] n^{-1/4} (\log n)^{1/2} \right\} = 1 \quad \text{a.s.}$$

DINGES, H.

<u>On t-distribution</u>

Let $Z_1, Z_2, \ldots, Z_n$ be i.i.d. and

$$X_n = \frac{\frac{1}{n}\sum^{n} Z_j}{\sqrt{\frac{1}{n-1}\sum (Z_j - \bar{Z}_n)^2}} = \frac{L_n}{\sqrt{V_n}} \; .$$

Consider the power function of the t-test applied to a scale-shift-family

$$\Pr_\vartheta(X_n \ge x^*)$$

for fixed n and x^* as a function of the noncentrality parameter ϑ. It turns out that in many situations there exists an asymptotic expansion which yields surprisingly accurate approximations for rather small sample size n:

$$-\frac{1}{\sqrt{n}} P^{-1} \circ \Pr_\vartheta(X_n \ge x^*) = A_0(\vartheta, x^*) + \frac{1}{n} A_1(\vartheta, x^*) + O(n^{-2}) \quad .$$

In the classical case where the Z_j are normally distributed the functions $A_0(\vartheta,x)$ and $A_1(\vartheta,x)$ can be written down explicitely in a form which makes numerical computation possible.

Question: Everybody knows that the sample mean is not a good estimator for the location and the sample variance is even worse as an estimator of the scale. Why do practical people still use t-tests? Do they expect some cancellation of the nonrobustness of nonimator and denominator?

DUDLEY, R.M.

Nonlinear functionals of empirical measures and robustness

For a sample space $(X,\mathcal{A})$ let T be a (non-linear) functional of probability measures on $(X,\mathcal{A})$, Fréchet differentiable for a norm $||\cdot||$. Then for empirical measures P_n of a law P, $T(P_n) = T(P) + \int 1_P d(P_n-P) + o(||P_n-P||)$ for the derivative ("influence") function 1_P. For a family $\mathcal{F}$ of measurable functions on $(X,\mathcal{A})$ let $||\mu|| = ||\mu||_{\mathcal{F}} := \sup_{1\in\mathcal{F}}|\int 1d\mu|$. If $\mathcal{F}$ is a Donsker class for P, i.e. $\sqrt{n}(P_n-P)$ converges in distribution to its Gaussian limit G_P for $||\cdot||_{\mathcal{F}}$, and $\int 1_P^2 dP < \infty$, then $\sqrt{n}(T(P_n)-T(P)) \sim G_P(1_P) + o_P(1)$, $n \to \infty$.
For $\mathcal{F}$ to be universal Donsker (Donsker for all P on $(X,\mathcal{A})$, R.D. A.P. 15, 1987, 1306-1326), $\mathcal{F}$ must up to additive constants be uniformly bounded. Also $||\cdot||_{\mathcal{F}}$ has breakdown point 0 unless $\mathcal{F}$ is uniformly bounded. M-estimates are also considered.

EINMAHL, U.

The Darling-Erdös theorem for sums of i.i.d. random variables

Let $\{X_n\}$ be a sequence of i.i.d. random variables with $EX_1 = 0$, $EX_1^2 = 1$. Darling and Erdös (1956) have shown that one has under the additional assumption of a finite third absolute moment for appropriate sequences $\{a_n\}$, $\{b_n\}$:

$$a_n \max_{1\le k\le n} \sum_1^k X_m/\sqrt{k} - b_n \xrightarrow{\mathcal{D}} E \quad ,$$

where E is an extreme value distribution.
They raised the question whether this result can hold under the sole assumption of a finite second moment. Using a skillful truncation argument due to Feller (1946), we show that one can obtain a general Darling-Erdös type theorem when slightly changing the normalizing sequence $\{\sqrt{k}\}$.
We infer that the Darling-Erdös theorem holds in its classical formulation if and only if $EX^2 1\{|X| \ge t\} = o((LLt)^{-1})$ as $t \to \infty$.
As a by-product we are able to re-prove fundamental results of Feller (1946) dealing with lower and upper class functions in the Hartman-Wintner LIL.

FALK, M.

On the rate of weak convergence of the prepivoted sample quantile

It is shown that the rate of weak convergence of the prepivoted sample q-quantile to the uniform distribution on (0,1) is exactly $O(n^{-1/2})$. Consequently, this is also the level error of confidence intervals for the underlying q-quantile which are derived by bootstrapping the sample q-quantile. In view of the poor rate of convergence of the bootstrap estimate of the distribution of the sample q-quantile, this is an unexpected high accuracy.
A conficence interval of even more practical use is derived by using backward critical points. The resulting confidence interval has the same length as the one derived by ordinary bootstrap but it is distribution free and has higher coverage probability.

FÖLLMER, H.

Large deviations and new versions of the Shannon-Mc Millan theorem

We review the role of limit theorems for the relative entropy $h(Q|P)$ in proving lower bounds for large deviations, with special emphasis on cases where problems of large deviations create a need for new versions of the Shannon-Mc Millan theorem. For a Gibbs measure P on $\Omega = S^{Z^d}$, large deviations of the empirical field $R_n(w)$ from its spatial ergodic behavior $R_n \to P$ are of the form

$$n^{-d} \log P[R_n \in A] \sim -\inf_{Q \in A \cap M_\rho(\Omega)} h(Q|P) .$$

But in the presence of phase transition, the right side may be 0 even though $P \notin \bar{A}$. This leads to a refined description where volumes n^{-d} are replaced by surface areas, and where a new Shannon-Mc Millan theorem for entropy on surfaces comes in; cf. F., Ort: Astérisque 157-158 (1988). As another example, we consider the behavior of Brownian motion under a large deviation of its quadratic variation. This involves the relative entropy $h(Q|P) = \lim 2^{-n} H_n(Q|P)$ (along dyadic partitions) of measures Q on C[0,1] with respect to Wiener measure, and a corresponding Shannon-Mc Millan theorem.

GILL, R.D.

Estimation of a Markov process model with data on "occurences but no exposures"

Consider a continuous time homogeneous, discrete state space Markov process and suppose that for n independent copies of the process and a fixed time interval [0,1] say, the total number of moves between each pair of states is observed and the initial distribution over the states. This kind of data has to be analysed in demography but so far only ad hoc estimators with various undesirable properties could be derived. I would like to discuss a new estimator based in fact on the ancient method of moments (!) and describe results and open problems concerning its statistical properties, using fixed point and homotopy theory.
Ref: Gill (1986) Scand. J. Statist. 13, 113-134.

GÖTZE, F.

A Berry-Esseen theorem for general sampling statistics

A Berry-Esseen theorem is proved for sequences of multidimensional general permutation statistics which are asymptotically normal. The rate of convergence to a multivariate normal distribution is bounded uniformly on the system of convex sets by the third moment of the linear projection of the statistic and the expected size of the non linear part and the first difference of the rank statistic. The result applies to general sampling statistics and independent observations and is joint work with E. Bolthausen, Berlin.

HÄRDLE, W.

Bootstrap simultaneous error bars for nonparametric regression

Simultaneous error bars are constructed for nonparametric kernel estimates of regression functions. The method is based on the bootstrap, where resampling is done from a suitably estimated residual distribution. The error bars are seen to give asymptotically correct coverage probabilities uniformly over any number of gridpoints. Applications to an economic problem are given and comparison to both pointwise and Bonferroni-type bars are presented.

HAEUSLER, E.

A law of the iterated logarithm for modulus trimmed sums

Let $X_1, X_2, \ldots$ be a sequence of independent random variables with a common (continuous) distribution function F. For $0 \leq k \leq n - 1$ let $S_n(k)$ denote the "modulus trimmed sum" formed when the k summands largest in absolute value are excluded from the partial sum $X_1 + \ldots + X_n$. When F is stochastically compact and symmetric about 0 and k_n are positive integers with $k_n/\log\log n \to \infty$ and $k_n/n \to 0$, the law of the iterated logarithm is shown to hold for the trimmed sums $S_n(k_n)$. This result, which is joint work with D.M. Mason, answers a question posed by P.S. Griffin, Probab. Th. Rel. Fields 77, 241-270 (1988).

HENZE, N.

Some peculiar boundary phenomena for extremes of r^{th} nearest neighbour links

Let $D_{n,r}$ denote the largest r^{th} nearest neighbour link for n points drawn independently and uniformly from the unit d-cube C_d. We show that according as $r < d$ or $r > d$, the limiting behaviour of $D_{n,r}$, as $n \to \infty$, is determined by the two-dimensional 'faces' respectively one-dimensional 'edges' of the boundary of C_d. If $d = r$, a certain 'balance' between edges and faces occurs. In case of a d-dimensional ball (instead of a cube) the boundary dominates the asymptotic behaviour of $D_{n,r}$ if $d \geq 3$ or if $d = 2$, $r \geq 3$.

IMKELLER, P.

Malliavin's calculus and stochastic equations with anticipating integrands

There are many problems in mathematics and mathematical physics which require the consideration of stochastic differential equations the boundary data of which are non-adapted with respect to the driving noise. The most widely used notion of stochastic integral needed in this context was designed by Skorokhod. It owes its success to the fact that it has a beautiful characterization as the adjoint operator of the derivative in Malliavin's infinite dimensional calculus. Skorokhod integral processes have non-trivial quadratic variation, a fact which makes notions such as occupation densities look useful for their study. In lack of a substitute for Tanaka's formula which describes local times in Ito's calculus we propose a different approach for deciding on the existence of occupation densities. It is given by the classical martingale approach of the Radon-Nikodym theorem.

JAGERS, P.

Branching processes as Markov fields

The natural Markov structure for populations growth is not in real time but rather in genetics:
Newborns inherit types from their mothers - given those they are independent of the history of their earlier ancestry. This leads to Markov fields on the space of sets of individuals, partially ordered by descents.
This Markov property implies branching, i.e. the conditional independence of disjoint daughter populations. It also turns out that the process must have a strong Markov branching property, on a type of predetermined random sets of individuals.
The expected evolution in real time is caught by Markov Renewal Theory. The actual evolution can be analyzed by an intrinsic, set indexed martingale and classical limit theory for sums of independent random variables. As a result exponential growth and stable, ultimate population composition follow.

JANSSEN, P.

Resampling from centered data in the two-sample problem

Bootstrap and permutation approximations to the distribution of U-statistics are shown to be valid when the resampling is from residuals in the two-sample problem. The motivation for using residuals comes from testing for homogeneity of scale in the presence of nuisance location parameters. New asymptotic results for U-statistics with estimated parameters are key tools in the proofs.

JOHANSEN, S.

Statistical analysis of cointegration of non stationary time series

We consider a non stationary vector autoregressive process which is integrated of order 1, and generated by i.i.d. Gaussian errors. We then derive the maximum likelihood estimator of the space of cointegration vectors and the likelihood ratio test of the hypothesis that it has a given number of dimensions. Further we test linear hypotheses about the cointegration vectors. The asymptotic distribution of these test statistics are found and one is described by a natural multivariate version of the usual test for a unit root in an autoregressive process, and the other by a χ^2 test.

JUREČKOVÁ, J.

Second order asymptotic distribution of M-estimators

Let $Y_1,\dots,Y_n$ be independent random variables, $Y_i \sim F(y-\underset{\sim}{x}_i'\beta)$, $i=1,\dots,n$, where $\beta \in R^p$ is the parameter of interest, $\underset{\sim}{x}_i = (x_{i1},\dots,x_{ip}) \in R^p$, $i=1,\dots,n$ are given vectors, $\underset{\sim}{X}_n = \begin{bmatrix} \underset{\sim}{x}_1 \\ \vdots \\ \underset{\sim}{x}_n \end{bmatrix}$. An M-estimator $\underset{\sim}{M}_n$ of β is defined as a solution of

$$\sum_{i=1}^{n} \rho\left[\frac{Y_i-\underset{\sim}{x}_i'\underset{\sim}{t}}{S_n}\right] := \min_{\underset{\sim}{t}\in R^p},$$

where $S_n = S_n(Y_1,\dots,Y_n)$ is translation invariant and scale equivariant, $\sqrt{n}(S_n-S(F)) = O_p(1)$; ρ is abs. continuous and such that $h(t) = \int \rho\left[\frac{x-t}{Y}\right]dF(x)$ has a unique min. at $t = 0$. The asymptotic study of $\underset{\sim}{M}_n$ is based on the process

$$\underset{\sim}{S}_n(\underset{\sim}{t},u) = n^{-1/2}\sum_{1}^{n} \underset{\sim}{x}_i\left[\psi((Y_i-\underset{\sim}{x}_i'\beta-n^{-1/2}\underset{\sim}{x}_i'\underset{\sim}{t})/Se^{n^{-1/2}u}) - \psi((Y_i-\underset{\sim}{x}_i'\beta)/S)\right],$$

$$\psi = \rho', \ (\underset{\sim}{t},u) \in R^p \times R^1 .$$

We shall show some limiting properties of $\underset{\sim}{S}_n(\underset{\sim}{t},u)$ and their applications to the asymptotic theory of M-estimates. We shall also discuss some open problems.

KRENGEL, U.

A minimax theorem for the secretary problem

The secretary problem with a random number $N \leq n$ of applicants is considered. Let T_n denote the set of "reasonable" randomized stop rules $t = (q_1,\dots,q_n)$ with $q_i \in [0,1]$. If the process has not been stopped before time i and the relative rank of the i-th applicant is 1, then t stops with probability q_i. Let $V(t|\vec{p})$ be the probability that the t-th secretary is the best, given $\vec{p}$ is the distribution of N. It is shown that

$$\sup_{t\in T_n} \inf_{\vec{p}} V(t|\vec{p}) = \inf_{\vec{p}} \sup_{t\in T_n} V(t|\vec{p}) \quad .$$

(The usual convexity conditions used in minimax theorems are not satisfied.) The least favourable $\vec{p}$ and the optimal t are determined.

(joint work with T. HILL)

LIERO, H.

Limit theorems for adaptive regression estimates of kernel type

Let (X,Y) be a bivariate random vector. Adaptive kernel estimates of the regression function $r(t) = \mathbb{E}(Y|X=t)$ can be written in the form $r_n(t) = \sum_1^n Y_i K((t-X_i)/A_n(t))/\sum_1^n K((t-X_i)/A_n(t))$, where (X_i,Y_i), $i=1,\dots,n$ is a sample of i.i.d. r.v.'s, K is a kernel function and $A_n(t) = A_n(t;(X_1,Y_1),\dots,(X_n,Y_n))$ is a sequence of bandwidths depending on the data and t. It is shown that the estimate $r_n(t)$ is asymptotically normal (at a fixed point t) and that the distribution of a weighted integrated squared error of r_n (properly normalized) tends to the standard normal distribution.

On the basis of these limit theorems optimality properties of adaptive estimates r_n are investigated and connections to the optimality of the Nadaraya-Watson-estimate are discussed.

MANTEIGA, W.G.

Nonparametric estimation and applications to parametric estimation in regression theory

In my talk a general view of applications of nonparametric estimation to parametric estimation in regression theory will be given. Correlation model, Regression model, Sequential model, the model with dependent data and the model with random bandwidth are reviewed. Promising results in the model with censored data and considerations about fitting the window with simulation results are also introduced.

MARRON, J.S.

Automatic smoothing parameter selection: a survey

For nonparametric curve estimation, choice of smooting parameter, i.e. bandwidth, is crucial to the performance of the estimator. A review is given of traditional data based methods for this, including cross-validation and plug-in techniques. Bound results on how well a bandwidth can be selected provide insight into which methods of comparison of selectors are most appropriate. Also discussed are very recent methods with root n rates of convergence. The ideas are illustrated with an example.

MASON, D.M.

A universal LIL

Let $X_1, X_2, ...$ be a sequence of non-negative independent random variables with common non-degenerate distribution function F and corresponding inverse or quantile function Q. For each integer $n \geq 1$ let $S_n = X_1+\ldots+X_n$, $b_n = (\log\log n)/n$ and $a_n = n^{1/2}\sigma(b_n)$, where for $0 < s < 1$

$$\sigma^2(s) = \int_0^{1-s}\int_0^{1-s} (u\wedge v-uv)\,dQ(u)\,dQ(v) \quad .$$

Then for some $-\sqrt{2} \leq K_F \leq 0$

$$\liminf_{n\to\infty} \frac{\left\{S_n - n\int_0^{1-b_n} Q(u)\,du\right\}}{a_n(\log\log n)^{1/2}} = K_F \quad \text{a.s.} \quad .$$

Moreover any value in the interval $[-\sqrt{2},0]$ is attainable by an appropriate choice of F.

MULLER, H.G.

An adaptive nonparametric peak estimator

Kernel estimators for the location and size of a peak of a regression function are considered. The problem addressed is how to estimate local bandwidths from the data. A stochastic process in local bandwidths for location and size of the peak is shown to converge weakly to a Gaussian limit process. This result is applied to establish efficiency of a variety of data-driven local bandwidth selection procedures. The bandwidths for location and size of the peak have to be chosen differently. Simulation results indicate superiority of local over global bandwidth choice.

MULLER-FUNK, U.

Some more remarks on the Cramér-Rao lower bound

In the talk (based on joint work with F. Pukelsheim and H. Witting) two theorems supplementing the Cramér-Rao inequality as stated in Witting (1985), for instance, are presented:

(1) A streamlined version of a result due to Wijsman (1973) is given ("global

attainment is possible iff the underlying class is an exponential family").
(2) Sufficient conditions for L_2-differentiability in the sequential case are stated. As a corollary, the sequential Cramér-Rao-inequality is proved.

PFLUG, G.

Coefficients of ergodicity for discrete Markov chains

Let P be a finite Markovian transition matrix. A coefficient of ergodicity is a real function ρ, such that

$$0 \leq \rho(P) \quad ; \quad \rho(P) = 0 \text{ iff P has identical rows}$$

and

$$\rho(P_1 \cdot P_2) \leq \rho(P_1) \cdot \rho(P_2) \quad .$$

A typical example is Dobrushin's $\delta(P) = \frac{1}{2} \sup_{i,j} \sum_k |p_{ik}-p_{jk}|$.

All coefficients of ergodicity satisfy

$$\delta(P) \geq \beta(P) \quad \text{where } \beta \text{ is the second largest eigenvalue of P} \quad .$$

Theorem. (i) For each $\varepsilon > 0$ there is a coefficient of ergodicity

$$\rho_d(P) < \beta(P) + \varepsilon \quad ,$$

(ii) If P is spectrally bounded, then there is a ρ_d with

$$\rho_d(P) = \beta(P) \quad .$$

Here ρ_d is the coefficient of ergodicity which belongs to the Wasserstein-distance induced by a metric d on S.

POLLARD, D.

Some difficulties with functionals of the empirical measure

The difficulties referred to in the title concern the problems of definition and continuity for functionals used to represent simple statistics such the argmax - the point where a stochastic process achieves its maximum. This will be illustrated by a multidimensional example that generalizes a mode estimator of Chernoff (1964). It will be argued that the estimator, which converges at the unusual $n^{-1/3}$ rate, can be directly analyzed using the new Hoffmann-Jørgensen/Dudley theories for convergence in distribution and almost-sure representation. The conclusion will be that not all asymptotic problems fit comfortably within the framework of functionals on function spaces. The talk will describe joint work with Jean Kim.

PYKE, R.

Sample path behaviour of scanning Brownian processes

The talk surveyed some recent activity on the interface between probability and statistics. Via a directed excursion connecting some very specific topics, the necessity and value of close interaction is emphasized. Beginning with datum X, the result of an experiment, the need for structure of X is stressed; structures can and will be much more complex than common IID. The goal is to interpret reasonably the relations $\delta_X \equiv P_X$, $\delta_{X^{(n)}} \longrightarrow P_X$ if δ_x is unit mass at x, P_X is law of X. Illustrations included stationary sequences ($P_{n,r_n} \xrightarrow{?} P_X$ where P_{n,r_n} is empirical of all r_n-tuples starting at $j=1,2,\dots,n-r_n$). In models about independence, product processes of two types arise; $Z^1 \times \mu^2$, $Z^1 \times Z^2$, in which some or all components are random. When models are not too closely specified (non-parametric), goodness-of-fit methods are valid, so empirical process methods are pertinent. Metrics are needed for goodness-of-fit criteria. Determining-class metrics are described; scanning classes $\mathcal{F}_B = \{B+x:x\in R^d\}$ emphasized and applied. Related (limiting) Brownian Scanning process reviewed, with results about a.s.-characterizations of the scanning set. Slides of simulations were shown for a scanning square and a scanning triangle.

RÉVÉSZ, P.

Random walk in Z^d

Let R(n) be the largest integer for which the integer points of the disc $x^2+y^2 \le (R(n))^2$ is covered by the first n steps of a simple symmetric random walk. A strong theorem is the following:

$$\exp((\log n)^{1/2}(\log\log n)^{-3/4-\varepsilon}) \le R(n) \le \exp(2(\log n)^{1/2}\log\log\log n) \quad \text{a.s.}$$

for all $\varepsilon > 0$ and for all but finitely many n.
We also have

$$\exp(-120z) \leq \liminf_{n\to\infty} \mathbb{P}\left[\frac{(\log R(n))^2}{\log n} > z \right] \leq \limsup_{n\to\infty} \mathbb{P}\left[\frac{(\log R(n))^2}{\log n} > z \right] \leq \exp(-\tfrac{z}{4}).$$

This result suggests the following

CONJECTURE. There exists a $\lambda > 0$ such that

$$\lim_{n\to\infty} \mathbb{P}\left[\frac{(\log R(n))^2}{\log n} > z \right] = \exp(-\lambda z) \quad .$$

In higher dimension ($d \geq 3$) we are interested in the radius $\rho_d(n)$ of the largest ball (not necessarily around the origin) covered by the random walk in time n. A result says

$$(\log n)^{\frac{1}{d-1} - \varepsilon} \leq \rho_d(n) \leq (\log n)^{\frac{1}{d-2} + \varepsilon} \qquad \text{a.s.}$$

for any $\varepsilon > 0$ and for all but finitely many n.

RIEDER, H.

<u>A connection between robust functionals and Huber-Strassen pairs</u>

Given a $\mathbb{R}^d$-valued differentiable functional T with influence curve ψ, and "full" infinitesimal neighborhoods U, we consider the local asymptotic testing problem:

$$H:\ Q \in U, \quad T(Q)'\mathrm{Cov}(\psi)^{-1}T(Q) \leq b^2$$

$$K:\ Q \in U, \quad T(Q)'\mathrm{Cov}(\psi)^{-1}T(Q) \geq c^2$$

and derive the LAM-bound

$$\mathbb{P}[\mathrm{Chi}^2(d,c^2) > c_\alpha(d,b^2)]$$

where $c_\alpha(d,b^2)$ denotes the upper α point of a Chi^2-distribution, d degrees of freedom, noncentrality parameter b^2. We discuss this bound from a robustness viewpoint.
In the one-dimensional, one-sided case

$$H:\ Q \in U, \quad T \leq b \qquad\qquad K:\ Q \in U, \quad T \geq c$$

for which one obtains the LAM-bound

$$\mathcal{P}\left[N(0,1) > u_\alpha - \frac{b-c}{\sqrt{Var(\mathcal{P})}} \right] = LAM(\mathcal{P})$$

one may test capacity-neighborhoods $\mathcal{P}_0$ vs. $\mathcal{P}_1$ (ε-contamination, total variation) using functionals. Maximizing LAM($\mathcal{P}$) subject to $H \subset \mathcal{P}_0$, $K \subset \mathcal{P}_1$ yields an asymptotic version of the Huber-Strassen test based on least favorable pairs for $\mathcal{P}_0$ vs. $\mathcal{P}_1$.

ROOTZÉN, H.

Tail estimation for stochastic processes

There is a need for statistical methods for dependent data, in particular in the presence of clustering of extreme values. The present work, joint with L. de Haan and M.R. Leadbetter, is concerned with one problem in this area. This is to estimate the (small) probabilities of very large values and the distribution of the maximum over very long intervals. The solution hinges on estimating the extremal index θ, where $1/\theta$ is the limiting mean cluster length, and a parameter β defined as the limiting mean height of an exceedance of a high level, given it is nonzero. Both θ and β are estimated by obvious "moment" estimations, i.e. the number of clusters divided by the number of exceedances (this has been studied by T. Hsing) and by the average height of exceedances, the "Hill estimator", respectively. A central limit theorem is obtained under suitable mixing conditions and conditions on the tail of the one-dimensional distribution. The results are applied to water level data from den Helder, Holland.

RUSCHENDORF, L.

Minimal and ideal metrics and the rate of convergence in stable limit theorems

We report on some new results on the construction and characterization of minimal metrics and the applications to limit theorems. Furthermore, we show that the minimal L_p-metrics behave like ideal metrics of order $r > 1$ w.r.t. summation and maxima in separable Banach spaces (lattices) of type $p > 1$. This implies convergence rates in stable limit theorems of index $\alpha \geq 1$, and we obtain especially an improvement of Zolotarev's classical result on the Prohorov distance. Zolotarev's idea of doubly ideal metrics turns out to fail in a formal sense. But the minimal L_p-metrics act as doubly ideal metrics of order $r > 1$. Some applications are given to the stationary distribution in queueing models.

STEINEBACH, J.

On the sample path behaviour of the first passage time process of a Brownian motion with drift

Let $\{W(t);\ t \geq 0\}$ be a standard Wiener process, and consider the Brownian motion with positive drift $\mu \geq 0$ and variance $\sigma^2 > 0$ defined by $X(t) = \mu t + \sigma W(t)$, $t \geq 0$. We will be concerned with the first passage time process $\{M(t);\ t \geq 0\}$ of $\{X(t);\ t \geq 0\}$, i.e. $M(t) = \inf\{s \geq 0:\ X(s) \geq t\}$. Strong limit theorems are established on the behaviour of the sample path modulus of $\{M(t);\ t \geq 0\}$, characterized by the maximal and minimal increments $\Delta^{\pm}(T,K) = \pm\sup_{0\leq t\leq T-K} \pm(M(t+K)-M(t))$ for $0\leq K\leq T$. The case where $K=K_T=O(\log T)$ as $T \to \infty$ is of particular interest here. The results are derived from their corresponding analogues for partial sums of inverse Gaussian random variables, which are developed first.

STRASSER, H.

Global extrapolation of local efficiency

Given an EDF test for goodness of fit it is shown how the second derivative of the asymptotic power function can be employed to obtain global upper bounds for the efficiency. These bounds are least in the class of tests with convex and centrally symmetric acceptance region. The proof is based on a linear optimization lemma for isotonic critical functions.

SWANEPOEL, J.W.H.

On a new goodness of fit test to detect rotating cosmic pulsars of γ-rays

A new goodness of fit test is proposed for the analysis of circular data in order to detect rotating cosmic pulsars of γ-rays. the test is invariant under rotations and consistent for a broad class of alternatives. The limiting distribution of the test statistic is derived under both the null-hypothesis and the above-mentioned alternatives. Also, the test has high Pitman asymptotic relative efficiency with respect to the Greenwood spacings test. Small sample studies indicate that the proposed test performs better than well-known test in the literature, such as Watson's test, with respect to power.

TEUGELS, J.L.

Multivariate Bernoulli distribution

Let $X_1,X_2,\ldots,X_n$ be an arbitrary sequence of random variables taking only the values 0 or 1. Using the concept of Kronecker- or tensor product from multilinear algebra we give a simple representation for the multivariate joint probability $p_{k_1,k_2,\ldots,k_n} = P\left\{ \bigcap_{i=1}^{n} [X_i = k_i] \right\}$ in terms of combined simple and/or centralized moments. Extensions to multivariate binomial distributions are possible. Our representation should provide an alternative to the traditional log-linear model.

VAN DER VAART, A.

An asymptotic representation theorem

Let a net of statistical experiments converge to a limit experiment. Furthermore, assume that a net of statistics possesses a limit distribution under every statistical parameter. Then its set of limit distributions is also the set of distributions of some randomized estimator in the limit experiment.

This provides a simple way of saying that the limit experiment is a 'lower bound' for the converging net of experiments. Moreover, convolution and minimax theorems can be obtained as corollaries.

WELLNER, J.A.

Continuous mapping theorems and the delta method

Suppose that $\{X_n\}_{n\geq 0}$ are arbitrary maps from $(\Omega, A, \mathbb{P})$ to a metric space M satisfying $X_n \Rightarrow X_0$ in the sense of Hoffmann-Jørgensen and that $P(X_0 \in M_0) = 1$, $M_0 \subset M$. If $\{g_n\}$ is a sequence of maps from M to M' satisfying $g_n(x_n) \to g(x)$ for all sequences $x_n \to x \in M_0 \subset M$, then $g_n(X_n) \Rightarrow g(X_0)$. This generalizes a familiar theorem in the theory of weak convergence of measurable functions in separable metric spaces (Billingsley (1968), pp. 33-34).
The extended continuous mapping theorem can be used to establish the validity of the delta method in the Hoffmann-Jørgensen weak convergence theory as follows: if $X_n \equiv a_n(Y_n - y) \Rightarrow X_0$ with $P(X_0 \in M_0) = 1$ in a Banach space M and $a_n \uparrow \infty$, and ν: $M \to M'$ is Hadamard-differentiable tangentially to M_0, then $a_n(\nu(Y_n) - \nu(y)) \Rightarrow \dot{\nu}(X_0)$.
For empirical measures $\mathbb{P}_n$ at i.i.d. X_i's and ν: $\mathcal{P} \subset l^\infty(\mathcal{F}) \to B$, a Banach space, Hadamard-differentiable at P_0 tangentially to $M_0 \equiv UC(\mathcal{F}, \tau) \subset l^\infty(\mathcal{F}) \equiv M$, where $\mathcal{F}$ is a P_0-Donsker class ($\sqrt{n}(\mathbb{P}_n - P_0) \Rightarrow X_0$, a P_0-Brownian bridge), this yields the delta method for Hadamard-differentiable, nonlinear functions ν:

$$\sqrt{n}\,(\nu(\mathbb{P}_n) - \nu(P_0)) \Rightarrow \dot{\nu}(X_0) \quad .$$

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