MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 12/1989

Rekursionstheorie

19.3. bis 25.3.1989

Die Tagung fand unter der Leitung von Prof. Dr. Klaus Ambos-Spies (Heidelberg), Prof. Dr. Gert H. Müller (Heidelberg) und Prof. Gerald E. Sacks (Cambridge, MA) statt. Es haben 48 Wissenschaftler aus 12 Ländern teilgenommen. Das Vortragsprogramm war breit gestreut und umfangreich: In 33 Vorträgen wurden aus vielen Teil- und Anwendungsgebieten der Rekursionstheorie neue Ergebnisse vorgestellt. Schwerpunkte bildeten die Untersuchungen zur Struktur der Unlösbarkeitsgrade, die verallgemeinerte Rekursionstheorie, die Anwendungen der Rekursionstheorie in der (deskriptiven) Mengenlehre und die rekursionstheoretische Untersuchung von Fragestellungen aus der Theoretischen Informatik. Außerdem fand eine abendliche Podiumsdiskussion statt, in der Prof. Moschovakis, Prof. Nerode, Prof. Sacks und Prof. Soare, zum Teil in unterschiedlicher Sichtweise, die vitalen Tendenzen in der künftigen Entwicklung der Rekursionstheorie herausgestellt haben.

Ein Tagungsband wird in der Reihe "Springer Lecture Notes in Mathematics" erscheinen.

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Vortragsauszüge

Klaus Ambos-Spies (Heidelberg)
Minimal pairs and complete problems for polynomial reducibilities

We look at the tops of minimal pairs for polynomial many-one reducibility ($\leq^P_m$).

A recursive set A is such a top if A is the direct sum $B \oplus C$ of sets B and C such that B,C $\notin$ PTIME and, for any set D,

$$D \leq^P_m B \text{ and } D \leq^P_m C \Rightarrow D \in \text{PTIME}.$$

We show that the complete sets for most of the standard complexity classes extending exponential time are not the tops of minimal pairs. The corresponding question for NP turns out to be oracle dependent.
(Joint work with S. Homer and R.I. Soare)

Marat M. Arslanov (Kazan)
Turing degrees of sets of Ershov's difference hierarchy

A set $A \subseteq \omega$ is called n-r.e. if $A = \lim_s A_s$ for some recursive sequence $\{A_s\}_{s \in \omega}$ such that for all x, $A_0(x)=0$ and $\text{card}\{s : A_s(x) \neq A_{s+1}(x)\} \leq n$. A Turing degree is called α-r.e. if it contains an α-r.e. set and it is called properly α-r.e. if it is α-r.e. but not β-r.e. for any $\beta < \alpha$. I have discussed the following results due to Ishmuchametov, Selivanov and myself: 1.) For all n>1 there exist n-r.e. degrees **a** < **b** such that **a'** = **0'**, **b'** = **0''**, and there is no (n-1)-r.e. degree **c** between them. 2.) Given any d-r.e.(= 2-r.e.) degrees **a** < **b**, **a** $\notin \mathcal{D}_1$, **a'** = **0'**, there exists a d-r.e. degree **c** such that **a** < **c** < **b** and there is no r.e. degree between **a** and **c**. Here $\mathcal{D}_1$ is the set of all r.e. degrees.- So, this theorem would solve the density problem for d-r.e. degrees if, for example there are no properly d-r.e. degrees whose jumps are not equal to **0'**. But this is not the case: 3.) For every r.e. degree **a** < **0'** there exists a properly d-r.e. degree **b** such that **a** < **b** < **0'**. 4.) For every r.e. degree **a** there exists a properly n-r.e. degree **b** (here n>1 is arbitrary) such that **a'** = **b'**. Furthermore, we have discussed some related results.

Peter Clote (Chestnut Hill)
Boolean functions, invariance groups and parallel complexity

We study the invariance group S(f) of boolean functions $f \in B_n$ (i.e. f: $2^n \rightarrow 2$) on n variables, i.e. the set of all permutations on n elements which leave f invariant. We give necessary and sufficient conditions via Polya's cycle index for a general permutation group to be of the form S(f), for some $f \in B_n$. For cyclic groups $G \leq S_n$

we give an NC-algorithm for determining whether the given group is of the form S(f), for some $f \in B_n$. Also for any sequence $< G_n \leq S_n : n \geq 1 >$ of permutation groups we study the asymptotic behaviour of $| \{ f \in B_n : S(f) = G_n \} |$. For instance, it is shown that asymptotically "almost all" boolean functions have trivial invariance groups. We show the applicability of group theoretic techniques in the study of the parallel complexity of languages. For any language L let L_n be the characteristic function of the set of all strings in L which have length exactly n and let $S_n(L)$ be the invariance group of L_n. We consider the size of the index $| S_n : S_n(L) |$ as a function of n and study the class of languages whose index is polynomial in n. We use the classification results on maximal permutation groups to show that any such language is in NC^1. We also show that the problem of "weight-swapping" (modulo a sequence of groups of polynomial index) is in NC^1. We give the invariance groups of Dyck and palindrome languages, provide an algorithm for testing membership in the invariance group of a regular language, and consider the problem of constructing languages with given invariance group structure.

A. N. Degtev (Tuymen)
<u>The basic reducibilities of the truth-table type</u>
This note is a short survey of the results about relationships between elementary theories of the upper semilattices of the r.e. r-degrees $\mathcal{L}_r$, r-complete sets and r-degrees for the basic reducibilities of truth-table type $r \in \{m\text{-},c\text{-},d\text{-},p\text{-},l\text{-},tt\text{-}\}$.

<u>Recursively combinatorical and selector properties of the subsets of the natural numbers</u>
For a Boolean function β and a subset of $N=\{0,1,2,...\}$ are introduced the notions of β-combinatorical, β-selector and weak β-selector sets. It is proved that for an "admissible" β the class of β-combinatorical sets is equal to some of the following seven classes $\mathbb{M}$, $\mathbb{A}$, $\mathbb{L}$, $\mathbb{C}$, $\mathbb{D}$, $\mathbb{P}$ or $\mathbb{T}$; the class of β-selector sets to some two, and the class of weak β-selector sets to some three classes. All of these classes are defined and several relationships between them are obtained.

Ding Decheng (Heidelberg and Nanjing)
<u>Some results on e-generic sets</u>
The notion of e-genericity has been introduced by Jockusch. An r.e. set A is e-generic if A has a recursive enumeration $\{A_s\}$ such that $< A_0,...,A_s > \in E$ for all s, and for every primitive recursive set $C \subseteq E$, if C is dense along $\{A_s\}$ then $\{A_s\}$ meets C truly. We proved some new results on e-generic sets and their degrees. For instance we show that the degrees of e-generic sets are meet-inaccessible, i.e., if a is e-generic then a is

not contained in the closure of **R**($\nleq$**a**) under joins and meets.

Yuri L. Ershov (Novosibirsk)
<u>Recursion theory on admissible sets</u>
The next results were obtained in Novosibirsk recently:
1.) V. Rudnev constructed an example of an admissible set HF($\mathcal{M}$) such that:
a.) $X \subseteq \omega$ is in Σ iff X is arithmetical; b.) If X,Y are disjoint Σ-sets then there is a Δ-set Z such that $X \leq Z$ and $Z \cap Y = \emptyset$.
2.) Theorem (Ershov): If $S \subseteq \mathbb{A}$ is such a family that $\alpha \in S \Rightarrow$ rk $\alpha \leq 1$, then Cov S - the least admissible set containing S - exists.
3.) If X is an infinite set of the urelements $x_0, x_1, \ldots \in X$, $x_i \neq x_j$ for $i \neq j \in \omega$ then for $S = \{X\} \cup \{ \{x_0, x_1\}, \{x_1, x_2\}, \ldots, \{x_n, x_{n+1}\}, \ldots \} \}$ does not exist Cov S.
4.) The right definitions for the notions Σ-predicates and Σ-functions are given and studied.

Peter Fejer (Boston)
<u>A direct construction of a minimal r.e. tt-degree</u>
We give a direct construction of a minimal recursively enumerable truth table degree. (The existence of such degrees is due to Degtev and Marchenkov by a less direct construction.) Our proof uses ideas from the full approximation construction of a minimal Turing degree below **0'**.
(Joint work with Richard Shore)

Sy D. Friedman (Cambridge, MA)
<u>Recursion theory and class genericity</u>
In this talk I applied ideas from recursion theory to the study of genericity in set theory. For classes $X, Y \subseteq$ ORD we write $X \leq Y$ if X is M_Y-definable where $M_Y = \langle L[Y], \in, Y \rangle$. And $X \equiv Y$ iff $X \leq Y$, $Y \leq X$; Sat(X)= $\{\varphi(\vec{\alpha}) \mid M_X \models \varphi(\vec{\alpha})\}$. Then X<Sat(X). X is <u>Y-low</u> if Sat(<X,Y>) $\leq$ <X, Sat(Y)> and <u>Y-complete</u> if <X,Y> $\geq$ Sat(Y). X is <u>strictly generic</u> over Y if <X,Y> $\equiv$ <G,Y> where G is $\mathcal{P}$-generic over M_Y for a p.o. $\mathcal{P}$ s.t. $\mathcal{P}, \Vdash_{\mathcal{P}}$ for Δ_0 sentences are both M_Y-definable. X is <u>generic</u> over Y if $X \leq$ <G,Y> for some G as above. A useful fact is that X strictly generic over Y $\rightarrow$ X Y-low and X generic over Y $\rightarrow$ X <u>not</u> Y-complete. The <u>Genericity Conjecture</u> states that $0^\# \nleq X \rightarrow X$ Y-generic for some Y such that L[Y] = L.

<u>Theorem</u> (a) There is $R < 0^\#$, $R \subseteq \omega$ such that R is Y-complete for all Y s.t. L[Y]=Y.

So the Genericity Conjecture is false.

(b) There is $R < 0^{\#}$ s.t. R is generic over $\varnothing$ but R is not Y-low for any Y s.t. L[Y]=L. So generic $\not\rightarrow$ strictly generic.

(c) $0^{\#}$ is Y-low for some Y, $0^{\#} \notin M_Y$.

Conjecture $0^{\#}$ is strictly generic over some Y, $0^{\#} \notin M_Y$.

Robin O. Gandy (Oxford)
Sequentially computable functions of finite type and Platek's recursion theory
A notion of sequential computation for hereditarily continuous functionals is given; it may be considered as a game of questions and answers played between a functional Δ and an argument F leading to a value for $\Delta(F)$. R.O. Gandy and G. Pani have shown that functions (over $\omega \cup \{\perp\}$) recursive in Platek's sense are sequential. It is believed that the converse is also true.

Marcia Groszek (Hanover)
Priority arguments and fragments of arithmetic
Theorem (Groszek, Slaman): $I\Sigma_1$ does not suffice to prove that $\leq_T$ is a transitive relation on recursively enumerable sets.
This answers a question of Hájek and Kučera.

Leo Harrington (Berkeley)
The non-density of the d-r.e. degrees
A set is d-r.e. if it is the difference of two r.e. sets.
Theorem (Cooper, Harrington, Lachlan, Lempp, Soare)
There is a d-r.e. set B s.t. $0' \not\leq_T B$, yet for all d-r.e. (or even n-r.e. or even recursively bounded Δ_2^0) sets U, either $0' \leq_T B \cup U$, or $U \leq_T B$.
The proof is quite analogous to that of Lachlan's Nonsplitting Theorem.

Christine Haught (Chicago)
Limitations on initial segment embeddings in the r.e. tt-degrees
We prove that all of the finite initial segments of the r.e. tt-degrees have a least non-zero element. (by Haught and Shore there are nontrivial finite initial segments of the r.e. tt-degrees.)
Theorem If **a** is a non-zero r.e. tt-degree and there are at most finitely many tt-

degrees **b** such that $\mathbf{0} <_{tt} \mathbf{b} \leq_{tt} \mathbf{a}$, then there is a nonzero r.e. tt-degree **c** such that for all **b** such that $\mathbf{0} <_{tt} \mathbf{b} \leq_{tt} \mathbf{a}$, $\mathbf{c} \leq_{tt} \mathbf{b}$.
Corollary (special case) The diamond lattice cannot be embedded in the r.e. tt-degrees as an initial segment.
Corollary (to the proof of the theorem) In fact, if **a** is a non-zero r.e. tt-degree and there are at most finitely many tt-degrees below **a** , then all of the nonzero tt-degrees $\leq$**a** are inside the same Turing degree.
(Joint work with Leo Harrington)

Peter G. Hinman (Ann Arbor)
Jump embeddings in the Turing degrees
A Jump Partial Ordering (JPO) $\mathcal{P} = (\mathbf{P}, \leq_\mathbf{P}, \mathbf{j_P})$ is a partial ordering together with a unary function which is monotone and strictly increasing. We consider which JPO's are embeddable in the Turing degrees with Turing jump $(\mathbf{D}, \leq_\mathrm{T}, \mathrm{j_T})$ either locally (such that $\mathcal{P}$ has a least element which maps to the zero degree **0**.) or globally (without further restriction). One group of local embeddings is stated in terms of the jump trace of a degree **a** : $\mathrm{JTr}(\mathbf{a}) = (h_0, h_1,..., h_{k-1}, h; l, l_{k-1},...,l_1, l_0)$ such that for all i, $\mathbf{0}^{(h_i+i)} \leq \mathbf{a}^{(i)} \leq \mathbf{0}^{(l_i+i)}$ and these are optimal bounds. The elementary properties of the jump operator imply that $h_i \leq h_{i+1} \leq h \leq l \leq l_{i+1} \leq l_i$ and $l_i \leq l_{i+1} +1$.
Theorem 1. Any such sequence with $l\text{-}h \leq 2$ is JTr(**a**) for some degree **a**.
Theorem 2. Every countable linear JPO which is well-founded and sparse (every interval $[p, \mathrm{j_P}(p)]$ is finite) is locally embeddable. Theorem 3. Every countable linear JPO such that the relation $\mathrm{j_P}(p) \leq_\mathbf{P} q$ is well-founded and $\mathrm{j_P}$ is injective is locally embeddable . Theorem 4. Every countable JPO is globally embeddable.
(Joint work with Theodore A. Slaman)

Alexander S. Kechris (Pasadena)
Descriptive Dynamics
We present joint work with R. Dougherty and S. Jackson on the structure of Borel (and other) equivalence relations on the Polish Spaces. Given such an equivalence relation E on X, a (countable) separating family for E is a sequence A_n of sets such that $xEy \Leftrightarrow \forall n\, [\, x \in A_n \Leftrightarrow y \in A_n\,]$. By E_0 we denote the equivalence relation $xE_0y \Leftrightarrow \exists n\, \forall m \geq n\, [\, x(m) = y(m)\,]$ on 2^ω. If E,F are Borel equivalence relations we denote by $E \leq F$ the partial order of Borel embeddability i.e. $E \leq F \Leftrightarrow \exists f$ Borel and 1-1 such that $xEy \Leftrightarrow f(x)Ff(y)$. The following extends a result of Glimm-Effros.
Theorem1. Let E be a Borel equivalence relation with F_δ equivalence classes. Then

exactly one of the following holds: (i) E has a Σ_1^1- separating family or (ii) $E_0 \subseteq E$.

A Borel equivalence relation is countable if every equivalence class is countable. We classify countable Borel equivalence relations in (i) finite, (ii) smooth (i.e. having Borel separating families), (iii) hyperfinite (i.e. increasing unions of finite ones), (iv) amenable.

Theorem 2. If E, F are hyperfinite non-smooth Borel countable equivalence relations then $E \leq F$ and $F \leq E$.

Antonin Kučera (Praha and Ithaca)

On a diagonalization of Σ_1^0-objects and generalizations

A function f is called

(i) a diagonally nonrecursive function (DNR) if $f(x) \neq \varphi_x(x)$ for all x,

(ii) a fixed-point free (FPF) function if $W_x \neq W_{f(x)}$ for all x,

(iii) a *-fixed point free (*-FPF) function if $W_x \neq^* W_{f(x)}$ for all x (where $=^*$ means equality modulo finite sets).

Arslanov proved that **0'** is the only r.e. degree containing an FPF (or a DNR) function. There was an open question whether the situation relativizes to higher levels, too. We answer the question in the negative. More precisely, there is a Σ_2 degree **a** containing a *-FPF function such that the only r.e. degree below **a** is **0** (thus, $\mathbf{0}' \not\leq \mathbf{a}$). The method used to prove this is based on the concept of Σ_1^0 random sets over $\emptyset'$ (i.e. $\emptyset'$-NAP sets).

The second part of the talk dealt with the use of Π_1^0 classes of 0-1 valued DNR functions and self-referential principles for constructions of r.e. sets as an alternative method to the standard priority methods.

Martin Kummer (Karlsruhe)

Computable one-one numberings

Let $\mathcal{W}$ denote the r.e. sets. $S \subseteq \mathcal{W}$ has a computable one-one numbering (an ON) $:\Leftrightarrow$ $\exists B \in \mathcal{W}\ [\ S = \{ W_i \mid i \in B \}\ \wedge\ \forall i,j \in B\ [\ i \neq j\ \rightarrow\ W_i \neq W_j\ ]\ ]$. The proof of several results of the literature concerning ONs (e.g. Friedberg's Theorem, the criterion of Malcev and Wolf, and Khuthoretskij's Theorem on incomparable ONs) can be unified and generalized using the following: Extension Lemma. Given r.e. $L_1, L_2 \subseteq \mathcal{W}$ s.t. (i) $L_1 \cap L_2 = \emptyset$, (ii) L_2 has an ON, (iii) Each finite subset of an element of L_1 has infinitely many supersets in L_2. Then $L_1 \cup L_2$ has an ON. - Furthermore, the proof of this Lemma is in a natural way priority free.

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Limits of this approach appear if S contains $\subseteq$-maximal finite sets. For such cases we have the following Theorem. Given $S \subseteq \mathcal{W}$ r.e. and $S_0 \subseteq S$ canonically enumerable s.t. each finite subset of an element of $S \setminus S_0$ has infinitely many supersets in S_0. Then S has an ON. - If the infinite elements of S are total recursive functions then we can show: Theorem. If L is a set of total recursive functions and S_0 is a canonically enumerable set of finite functions s.t. $L \cup S_0$ is r.e. then $L \cup S_0$ has an ON.

Steffen Lempp (Heidelberg and Madison)

$\mathbf{0}^{(n)}$-priority arguments

A general framework of $\mathbf{0}^{(n)}$-priority arguments has been studied by M. Lerman and myself at the example of the following

Theorem. The existential theory of the r.e. degrees within the language of least and greatest element and n^{th} jump reducibility predicates for all n is decidable.

The proof involves requirements of the form $A^{(n)} \not\leq_T B^{(n)}$ and $C^{(n)} \leq_T D^{(n)}$, so the requirements are easy and uniform for all n and are therefore suitable for studying $\mathbf{0}^{(n)}$-priority arguments. The framework involves a finite sequence of priority trees . A strategy on tree T_{i+1} is decomposed into infinitely many substrategies on T_i, each working on an instance of the T_{i+1}-strategy's requirement.

Alain Louveau (Paris)

On Borel quasi-orders with small antichains

This lecture was devoted to the proof of the following result of Khalid Kada (Paris):

Theorem (K. Kada) Let k be a natural number, and $(X,\leq)$ a Borel quasiorder on some Polish Space X. If every antichain in X has cardinality at most k, then X can be partitioned in k Borel pieces each of which is a chain.

This theorem is the "Borel" analogue of a celebrated result of Dilworth in combinatorics. It also admits a lightface refinement, which in fact is instrumental for proving it.

Wolfgang Maass (Chicago)

The complexity types of computable sets

We analyse the fine structure of time complexity classes for RAM's in particular the equivalence relation $A =_C B$ ("A and B have the same time complexity") $\Leftrightarrow$ (for all time constructible f: $A \in DTIME_{RAM}(f) \Leftrightarrow B \in DTIME_{RAM}(f)$).

The $=_C$ - equivalence class of A is called its complexity type. We prove that every set

X can be partitioned into two sets A and B such that $X =_C A =_C B$, that a complexity type C contains sets A, B which are incomparable with respect to polynomial time reductions if and only if $C \not\subseteq P$, and there is a complexity type C that contains a minimal pair with respect to polynomial time reductions. Furthermore we analyze the fine structure of P with respect to linear time reductions: We show that each complexity type $C \not\subseteq \mathrm{DTIME}(n)$ contains a rich structure of linear time degrees, and that these degree structures are not isomorphic (in particular we characterize those C that have a maximal linear time degree). Our proofs employ finite injury priority arguments, together with a new technique for constructing sets of a given time complexity type.
(Joint work with Theodore A. Slaman)

Yiannis N. Moschovakis (Los Angeles)
Computable concurrent processes
The talk presented a modeling of concurrent computation in which the perception of the situation by each process is represented by a natural game of interaction. Behaviors are partial strategies (for the second player) in the game and processes are non-empty sets of behaviors. The first player in the game represents the collective actions of all the other processes operating in the same environment (the "world"). The usefulness of the modelling derives from the fact that we can give natural, game-theoretic definitions of all the natural operations on processes, including the fair merge operation and (full) recursion. In addition, the model gives a useful framework for program verification for asynchronous, concurrent systems, which on this picture amounts to showing that certain winning strategies exist for various payoffs associated with the basic game. The main mathematical result of the paper is a transfer result between the notion of recursion for process-functions which we use and the classical interpretation of recursion by least fixed points. This and other basic results will appear in the proceedings of the LICS conference to be held in Asilomar in June , 1989. In the proceedings of the conference for this meeting, I will put a follow up of this work which was discussed very briefly in this talk: the model suggests (and gives evidence in favor of) a very natural class of computable processes and functions on processes, a sort of Church´s Thesis for asynchronous, concurrent communication.

Anil Nerode (Ithaca)
Polynomially isolated sets
We develop a P-Time analogue of ISOLS within the Polynomial Equivalence Types (PETS), $N = \{0\}^*$, $\alpha, \beta \subseteq N$ are P-Time equivalent if $\exists$1-1 partial P-Time honest f, $\mathrm{dom}(f) \supseteq \alpha$, $f(\alpha) = \beta$. $<\alpha>$, the PET of α, is $[\, \beta \subseteq N \mid \beta$ P-Time equivalent to $\alpha\,]$.
Operations are $<X> + <Y> = <\{0^{2n} \mid 0^n \in X\} \cup \{0^{2n+1} \mid 0^n \in Y\}>$,

$\langle X\rangle \bullet \langle Y\rangle = \langle \{ T(0^m,0^n) \mid 0^n \in X$ and $0^m \in Y \} \rangle$ where $T(0^m,0^n) = 0^{(1/2)[(m+n)^2+3m+n]}$. A set $A \subseteq N$ is polynomially isolated if for any P-Time monotone map f of finite sets of A to itself, there is a polynomial g such that for all finite $X \subseteq A$, $\bigcup_{n=1}^{\infty} f^n(X)$ is finite with length as a string $\leq$ g(length of X). These are the analogue of Dekker's ISOLS in the PETS. There are infinite P-Time polynomially isolated sets. They are closed under +, •, have +, • cancellation. The study of (recursive) P-Time P-Isolated Sets presents new challenges.
(Joint work with J. B. Remmel)

Wolfram Pohlers (Münster)
The proof theoretic collaps of a cardinal
It is known that admissible ordinals may be regarded as a recursion theoretic analogue of cardinals. We showed that admissible ordinals again may be collapsed to ordinals below ω_1^{CK} . On examples we illustrated why these collapses may be regarded as proof theoretical analogues of their cardinal ancestors.

Helmut Schwichtenberg (München)
Primitive recursion on the continuous functionals
In the context of Scott's notion of an information system define D_{nat} to be the flat domain of the natural numbers and $D_{\vec{\rho}\to\sigma}$ to be $D_{\vec{\rho}} \to D_\sigma$. Let $|D_\rho|$ ($|D_\rho|^{re}$) be the set of all ideals (r.e. ideals). To denote primitive recursive functionals we use terms built up from constants $\{(\vec{u}_i, v_i) \mid i\in I \}$ for finite approximations and recursive operators R by $\lambda\vec{x}.r$ and $t\vec{s}$. Let $|D_\rho|^{pr}$ denote the set of all primitive recursive functionals. A standard type model (e.g. for Bishop's constructive analysis) then is a system $\mathcal{M} = \{ \mathcal{M} \mid \rho \text{ type} \}$ such that $|D_\rho|^{pr} \subseteq \mathcal{M}_\rho \subseteq |D_\rho|$, which is closed against application. Theorem. Any closed term of ground type reduces to a numeral.-
For the proof one has to extend the notion of a term in oreder to take care of situations like $\{(\vec{u}_i,v_i) \mid i\in I \} (\lambda x.s) \vec{t}$.

Richard A. Shore (Ithaca)
The undecidability of the r.e. tt-degrees
We extend the methods of Fejer and Shore for constructing a minimal r.e. tt-degree to embed certain lattices as segments of the r.e. (and all) tt-degrees above a minimal degree.
Theorem. For every partition lattice Π_n of a finite set $\{1,...,n\}$ there are r.e. sets A_0

and A of tt-degrees $\mathbf{a}_0$ and $\mathbf{a}$ such that every tt-degree below $\mathbf{a}$ is r.e., the interval of degrees $[\mathbf{a}_0, \mathbf{a}] \simeq \Pi_n$ and $[\mathbf{a}_0, \mathbf{a}] \cup \{0\}$ is an initial segment of the tt-degrees.

Corollary. The theories of the r.e. tt-degrees and the tt-degrees below $\mathbf{0}'$ are undecidable.

(Joint work with Christine Haught)

Theodore A. Slaman (Chicago)

The polynomial time Turing degrees - automorphisms of generic ideals

Definition. (i) $2^{<\omega}$ is the set of finite binary sequences; if $s \in 2^{<\omega}$ then $|\sigma|$ is the length of σ. (ii) A Turing functional Φ is in PTIME if there is a polynomial φ: N$\rightarrow$N such that for all $\sigma \in 2^{<\omega}$, $\Phi(X,\sigma)$ is computed in less than $\varphi(|\sigma|)$ many steps. (iii) For A, B $\subseteq 2^{<\omega}$, A $\leq_p$ B if there is a $\Phi \in$ PTIME with $\Phi(B) = A$.

Let REC be the collection of recursive subsets of $2^{<\omega}$. Let REC($\leq_p$ A) be the collection of sets that are below A in $\leq_p$.

Question Is there a nontrivial automorphism of <REC, $\leq_p$> ?

Theorem. There is a recursive set A and a bijection f: REC($\leq_p$ A) $\rightarrow$ REC($\leq_p$ A) such that (i) f preserves $\leq_p$. (ii) f preserves the set theoretic operations $\cap, \cup, \subseteq$ modulo finite differences. (iii) There is an X$\in$ REC($\leq_p$ A) such that f(X) $\not\leq_p$ X and X $\not\leq_p$ f(X). (iv) Further, we may ensure that f preserves time complexity relative to A or that f moves some X in DTIME^A(n) to one in DTIME^A(n^2) - DTIME^A(n).

Thus <REC($\leq_p$ A), $\leq_p$> is not always rigid for A$\in$ REC.

(Joint work with Christine Haught)

Robert I. Soare (Chicago and Heidelberg)

Continuity properties of recursively enumerable degrees

Let $\mathcal{R}$ = ($\mathbf{R}$, $\leq$, $\vee$, $\wedge$, $\mathbf{0}$, $\mathbf{0}'$) be the structure of the (Turing) degrees of recursively enumerable (r.e.) sets with least elemnet $\mathbf{0}$ and greatest element $\mathbf{0}'$. A formula $\varphi(x_1,\ldots,x_n)$ in the language L($\leq$, $\vee$, $\wedge$, $\mathbf{0}$, $\mathbf{0}'$) is contiuous at ($\mathbf{a}_1$, $\mathbf{a}_2$,..., $\mathbf{a}_n$); $\mathbf{a}_i \in \mathbf{R}$ for $1 \leq i \leq n$, if there exists an open interval I($\mathbf{a}_i$) containing $\mathbf{a}_i$, for all $1 \leq i \leq n$, such that

$\mathcal{R} \models \varphi(\mathbf{b}_1,\ldots,\mathbf{b}_n)$ for all $\mathbf{b}_i \in$ I($\mathbf{a}_i$).

Theorem 1. Every quantifier-free formula φ(x,y) of two variables is contiuous at every pair (a, b) $\in$ $\mathbf{R}\times\mathbf{R}$ $\mathbf{a}, \mathbf{b} \neq \mathbf{0}, \mathbf{0}'$. (By known results in recursion theory this result cannot be extended to quantifier-free formulas of ≥ 3 variables or to formulas of ≥ 1 quantifiers.) Theorem 1 is an immediate consequence of the new theorems:

Theorem 2. (Harrington, Soare) There is no maximal minimal pair of r.e. degrees;
Theorem 3. (Ambos-Spies, Lachlan, Soare) There is no pair of r.e. degrees **a**, **b** cupping to **0'** and minimal with this property.

Dieter Spreen (Siegen)

On effective topologies: some characterization

Countable topological T_0-spaces (T,τ) with a countable basis on which a relation of strong inclusion is defined are considered and under very natural effectivity assumptions it is shown that up to effective equivalence τ is the greatest Malcev topology on T that is effectively related to τ, where a topology η on T is a Malcev topology, if it has a base of completely enumerable subsets of T. Moreover, it is effectively related to τ, if for $B \in \tau$ and $C \in \eta$ with $B \not\subseteq C$ a witness for this can effectively be found. As examples constructive Scott domains and recursively separable recursive metric spaces are considered. In the finite case one obtains the generalized Rice/Shapiro Theorem which says that the Scott topology on the domain is effectively equivalent to the Ershov topology on it which is generated by all of its completely enumerable subsets. In the second case it follows that up to effective equivalence the metric topology is the greatest Malcev topology on the space such that its lattice of completely enumerable open sets is effectively closed under the pseudocomplement operation given by the Heyting algebra of all open sets of this topology.

Michael Stob (Lexington and Cambridge, MA)

Array nonrecursive sets and multiple permitting arguments

We define a class of r.e. sets, the array nonrecursive (anr) sets. The Turing degrees of anr sets are exactly those below which certain multiple permitting arguments can be performed. We give three natural examples of such arguments from the recursion theory literature. We prove a number of theorems classifying the Turing degrees of anr sets. *(Joint work with R. Downey and C. Jockusch, jr.)*

Stanley S. Wainer (Leeds)

Proofs and programs

This talk was a brief summary of some elementary proof-theoretic ideas which can be used to analyse the logical complexity of natural classes of programs and program transformations. The underlying theme was the strong analogy which exists (and which is now being exploited in computer science) between recursive programs and their formal "specification" proofs. As an illustration, the transformation from "recursive" into "while" programs was considered. In proof-theoretic terms, this corresponds to an increase in "cutrank" and an exponential increase in the ordinal bounds on corresponding infinitary proofs. From this one can obtain "trade-offs" in sub-recursive complexity, generalizing an old theorem of Tait.

Gerd Wechsung (Jena)

Applications of Kolmogorov complexity in computational complexity

Following L. Adleman and L. A. Hemachandra we prove necessary and sufficient conditions in terms of Kolmogorov complexity for collapsing complexity classes like Δ_2^p and NP, PP and NP or Σ_2^p and Δ_2^p . A string y is called Kolmogorov simple relative to x if there exist a c>0 and a z, $|z| \le c + c \log|x|$ s.t. a universal Turing machine U (which is fixed once and forever) outputs y on input z#x within $c + |x|^c$ steps. Adleman's result (1979): "P=NP if and only if any NP-machine has on each x accepted by that machine Kolmogorov simple certificates (= accepting paths) relative to x." may serve as a typical example for the results proved here. The certificates in Adleman's case, however, have to be replaced with the appropriate notions reflecting essential parts of the computation in question. For instance, for Δ_2^p machines one uses guides defined as follows:

$\#y_1\#y_2\# \ldots \#y_k$ is a guide of the Δ_2^p -machine $M^{(SAT)}$ on input x $\Leftrightarrow_{df}$ $M^{(SAT)}$ on input x makes exactly k queries, and for each i = 1, ... , k,

$$y_i = \begin{cases} \text{the empty word} & \text{,if the } i^{th} \text{ query gets a negative answer} \\ \text{a certifcate of a SAT accepting machine on the } i^{th} \text{ query} & \text{,otherwise.} \end{cases}$$

Klaus Weihrauch (Hagen)

Constructivity, computability, and computational complexity: A single approach

A powerful formalism for investigating constructivity for sets not greater than the continuum is presented. It extends ordinary recursion theory and the theory of numberings by a formally similar theory of continuous and computable functions on Baire's space and a theory of representations. The theory is a consequent further development of the "Polish recursive analysis". It admits a natural presentation and interpretation of results obtained in other approaches, and evades foundational problems by exclusive use of classical logic. As an essential feature, continuity can be interpreted as a kind of constructivity. In the talk the formalism will be outlined, illustrated by some applications, and compared with other approaches.

W. Hugh Woodin (Berkeley)

Determinacy and scales

Definition 1) AD: Every $A \subset \omega^\omega$ is determined. 2) Unif: For each $A \subset \mathbb{R}\times\mathbb{R}$ there exists $f: \mathbb{R}\to\mathbb{R}$ such that for all $x\in\mathbb{R}$, if $<x,y>\in A$ for some $y\in\mathbb{R}$ then $<x,f(x)>\in A$.

3) $AD_{\mathbb{R}}$: Every $A \subset \mathbb{R}^\omega$ is determined

Theorem (ZF + AD + DC) Unif $\leftrightarrow$ $AD_{\mathbb{R}}$.

This is proved by showing:

Theorem (ZF + AD + DC) Unif $\rightarrow$ Every $A \subset \omega^\omega$ is λ-Souslin for some ordinal λ.

Corollary (ZF + AD + DC) Assume Unif. Then SCALE($\Sigma_1{}^2$) and if $X \subset P(\mathbb{R})$ is $\Sigma_1{}^2$ then X contains a $\Delta_1{}^2$ subset of $\mathbb{R}$. Further this holds in any inner model containing R.

Question Does AD $\rightarrow$ SCALE($\Sigma_1{}^2$) ?

Dongping Yang (Beijing and Heidelberg)

On the set limit operation

A, B are subsets of N. We say that A is partial set limit of B, $A=\lim_s B$, if $x\in A$ iff there exists t such that for all s >t [$<x,s>\in B$]. When N - lim(B) = lim(N-B), we say the limit is total. An infinite set C is $0^{(n)}$-cohesive if there is no $0^{(n)}$-r.e. set W such that $W\cap C$ and $(N-W)\cap C$ are both infinite. An $0^{(n)}$-r.e. set M is $0^{(n)}$-maximal if N-M is $0^{(n)}$-cohesive. An r.e. set A is an n-quasi cohesive set if $\lim_{s_n}\ldots\lim_{s_1} A$, is an $0^{(n)}$-cohesive set when n is odd and $\lim^n A$ is an $0^{(n)}$maximal set when n is even.

A function g(x) is an n-limit function if there is a recusrsive function $f(x, s_n,\ldots, s_1)$ such that $g(x) = \lim_{s_n} \ldots \lim_{s_1} f(x, s_n, \ldots ,s_1)$. We call f the n-base function of g. g is called n-dominant function if g dominates every total n-limit function.

Theorem. If A is an n-quasi cohesive set and n is odd (even), then the principal function $p_{\lim^n A}$ ($p_{\lim^n (N-A)}$) is n-dominant.

Theorem. An r.e. set A satisfies $0^{(n)} \leq_T A^{(n)}$ iff there are (n-1)-dominat functions g and an (n-1)-base function f of g such that $f\leq_T A$.

Corollary. An r.e. degree **a** is in H_n iff there is an (n-1)-dominant function f and an (n-1)-base function g of f such that the degree of **g** ≤ **a**.

Corollary. The degree of all n-quasi cohesive sets is an element of H_n.

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