MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Gewöhnliche Differentialgleichungen

26.3. bis 1.4.1989

The meeting was organized by H.W. Knobloch (Würzburg), J. Mawhin (Louvain-la-Neuve) and K. Schmitt (Salt Lake City).

With the now established tradition of stressing one or two special topics during each meeting, the organizers had decided, during the 1987 meeting, to emphasize the various aspects of bifurcation theory, a topics which is central both in the dynamical and the functional-analytic aspects of the theory and applications of ordinary differential equations. It is also clear how much bifurcation theory is paving the way to the recent and exciting developments in the study of chaotic behavior of dynamical systems.

Bifurcation theory, or the study of how the solution set of an equation is varying with the parameters entering the equation, continues to be developed along the classical powerful techniques like the Liapunov-Schmidt method, the theory of integral manifolds and that of differentiable dynamical systems. Of great importance is the role played by the presence of symmetries in the equations, and much progress has been done in this direction, in connection with now classical problems like Hopf bifurcation and the existence of subharmonics. Bifurcation techniques, coupled with the use of analytical computing techniques, remain also one of the ways of attacking a famous question like Hilbert's 16th problem.

The chaotic behavior of the solutions of ordinary differential equations or dynamical systems is now better understood in almost periodic systems, due to the use of analytical techniques like dichotomy theory, and in some differential-delay equations, via an extension of the shadowing lemma to maps which are not necessarily diffeomorphisms. Also the analysis of simple economic processes with lags leads to simple one loop feedback systems having chaotic behavior. Such a behavior can be avoided if the dynamical systems possesses some special structure, as it is the case for the monotone dynamical systems whose importance in applications to biology or population dynamics has not to be emphasized. In a similar direction, progress is made in the understanding of the asymptotic behavior of solutions of infinite-dynamical systems, in particular those coming from reaction-diffusion equations, in their approximation and in their applications, like in combustion theory.

The study of the existence and multiplicity of solutions of boundary-value problems, including periodic ones, is evidently related to bifurcation theory. Topological methods remain a fundamental tool in this approach and new continuation theorems are proposed, which also permit to deal with differential equations on various types of manifolds, differential equations with impulses or with singularities. The important question of stability of periodic solutions of nonautonomous equations is also treated successfully by degree methods and new approaches, like the use of the time-map and the co-area formula, are introduced to obtain the indispensable a priori bounds. Progress has been reported also on the existence and multiplicity of solutions of Ambrosetti-Prodi type problems and equations of various orders with jumping nonlinearities, including their use in providing convincing explanations in famous accidents for bridges or ships. Striking progress was reported also for equations with periodic nonlinearities, where a sensitive dependence on the number of space dimensions was proved in Dirichlet problems and a relative Lusternik-Schnirelmann category theory was introduced for problems including pendulum equations and Arnold conjecture. The existence theory for boundary value problems on the whole real line, including the case of bounded solutions, is a delicate question for which the Conley index appears to be a promising tool.

The development of any nonlinear theory generally depends upon a good knowledge of the associated linear problems which, in addition, have their own interest. Progresses in boundary value and indefinite eigenvalues problems for linear equations have been reported and several lectures on nonlinear boundary value problems dependent upon a very careful analysis of an associated linear equation.

The development of the theory of ordinary differential equations cannot be dissociated from that of its applications, many of which have already been mentioned above. A great impetus still comes from control theory, for which generalizations of the concept of classical differential equations like the quasi-differential equations and the differential inclusions play an important role.

The conference was attended by 42 mathematicians who came from 14 different countries. 33 of the participants presented lectures many of which were followed by interesting and lively discussions which will stimulate further work. Many less formal meetings and discussions also took place and some joint work will very likely come out of those fruitful meetings.

The institute's administration and staff have, as usual, greatly contributed to the success of the meeting by their discrete and efficient work which continues to make Oberwolfach a unique place in the mathematical (and other) world.

Abstracts

J. Bebernes : *Differential equations related to combustion theory.* Induction period models for thermal reactions are developed in a unified manner using activation energy asymptotics. Two models are analyzed. For the reactive-diffusion model, solutions become unbounded in finite time and blow up at a single point for certain geometries. As the explosion time is approached, a precise description of the asymptotic profile of a solution is obtained.

V. Blagodatskikh : *Differential inclusions, optimal control theory.* For differential equations $x' = f(x)$, the solution $x(t,x_0)$ is continuously differentiable on initial value x_0, if the function $f(x)$ is smooth. The solution $x_\varepsilon(t)$ with initial value $x_0 + \varepsilon h$ can be represented in the form $x_\varepsilon(t) = x(t) + \varepsilon\delta x(t) + o(\varepsilon)$, where the variation $\delta x(t)$ is the solution of the linear system $\delta x' = (\partial f(x(t))/\partial x)\delta x$, $\delta x(t_0) = h$. (1)

The similar result is obtained for the differential inclusion $x' \in F(x)$. Here, instead of the linear system (1), we have also a differential inclusion $\delta x' \in P(x(t), \delta x)$, $\delta x(t_0) = x_0$. This property is very useful in optimal control theory. For more details, see : A.F. Filippov, V. Blagodatskikh, Differential inclusions and optimal control, Proc. Steklov Math. Inst., 1986, n° 4.

A. Capietto : *Continuation theorems for non-autonomous differential equations.* We deal with the periodic boundary value problem

$$x' = F(t,x), \quad (1)$$

$$x(0) = x(T). \quad (2)$$

Let C in R^m be a closed Euclidian neighbourhood retract, which is flow-invariant with respect to (1). For example, the set C may be a manifold, a convex set or the space R^m itself. Let G in C be a bounded set, open relatively to C.

Two results, based on slightly different continuation procedures, ensuring the existence of solutions to (1)-(2) lying in the closure of G are provided. As for the applications, we deal with the case when the nonlinear field F splits as $F(t,x) = g(x) + e(t,x)$.

Dang Dinh Ang : *Nonlinear pseudoparabolic equations.* The talk is based on the work "Nonlinear pseudoparabolic equations" by D.D. Ang and T. Thanh. Results on uniqueness and global existence of solutions of initial and boundary value problems for the nonlinear pseudoparabolic equation

$u_t + g(u) - (\partial/\partial x)f(u) - (\partial/\partial x)F(u,u_x,x,t) - (\partial/\partial x)[b(x,t)u_{xt}] = H(x,t)$, $0 < x < 1$, $t > 0$, with nonhomogeneous boundary conditions are given. A salient feature is that F and its partial derivatives are allowed to be unbounded. In the special case $b = \alpha^2$, it is proved that, as $\alpha \to 0$, the corresponding solution $u^\alpha \to u^0$, the solution corresponding to $\alpha = 0$ on a sufficiently small interval. A result on the asymptotic behavior of the solution is given for $t \to \infty$.

W. Eberhard : *The distribution of the eigenvalues of a class of indefinite eigenvalue problems.* For eigenvalue problems $l(y) = \lambda r(x)y$, $U_\nu(y) = 0$, $1 \le \nu \le n$ with a piecewise continuous indefinite weight function r, we derive asymptotic formulas for the distribution of the eigenvalues λ_k. Assuming that the boundary conditions are regular (i.e. a generalization of the well known Birkhoff-regularity for the definite case $r(x) = 1$) it will be proved that for $n = 2\mu$ there exist two sequences $\lambda_k^{(j)}$ of eigenvalues with the asymptotic behavior $\lambda_k^{(j)} = \pm(-1)^{n/2}(k\pi/R_\pm)^n[1 + O(1/k)]$ (+ for j=1, - for j=2) where $R_\pm = \int_0^1 (r_\pm(t))^{1/n}dt$, $r_\pm(t) = \max\{\pm r(t),0\}$. An analogous formula holds in the case $n = 2\mu - 1$.

L. Erbe and W. Krawcewicz : *Boundary value problems for systems with impulses.* We consider boundary value problems for systems of second order equations or inclusions with impulses of the form

(I) $y'' = f(t,y,y')$ or $y'' \in F(t,y,y')$

subject to

(II) $y(t_k^+) = I_k(y(t_k))$, $y'(t_k^+) = N_k(y(t_k),y'(t_k))$, $k = 1,\dots,m$.

and the boundary conditions

(III) $G_i(y^*) = 0$, $i = 0, 1$,

where (i) $f : [a_0,a_1] \times R^{2n} \to R^n$, $F : [a_0,a_1] \times R^{2n} \to 2^{R^n}$

(ii) $I_k : R^n \to R^n$ is a homeomorphism, $k = 1,\dots,m$,

(iii) $N_k : R^n \times R^n \to R^n$ is continuous, $k = 1,\dots,m$,

and $y^* = (y(a_0),y'(a_0),y(a_1),y'(a_1))$, $G_i : R^{4n} \to R^n$ is continuous, $i = 0,1$,

$a_0 = t_0 < t_1 < \dots < t_m < t_{m+1} = a_1$.

We seek solutions of (I),(II),(III) which are piecewise C^1 with points of discontinuity t_k, $k = 1,\dots,m$ of the first type for y, y' at which they are left continuous.

The technique used is an adaptation of the topological transversality method to systems with impulses.

W.N. Everitt : *Quasi-differential equations and control theory.* The linear problems

$$x'(t) = A(t)x(t) + B(t)u(t) \quad (t \in I) \quad (*)$$

and non-linear control problems

$$x'(t) = f(t,x,(t),u(t)) \quad (t \in I) \quad (**)$$

are linked with the linear ordinary quasi-differential equation $y_A^{[n]} = \varphi$ (on I). Sufficient conditions are given for the controllability of (*) and (**) when the matrices A and B, respectively $(\partial f/\partial x)(t,0,0)$ and $(\partial f/\partial u)(t,0,0)$, are such that the linear problems reduce to a quasi-linear differential equation.

M. Fiebig-Wittmaack : *Ein nichtlineares skalares Neumannsches Randwertproblem 2. Ordnung.* Wir betrachten das Problem

$$u''(x) + f(u(x)) = g(x)\,, \quad x \in [0,\pi]$$
$$(*)$$
$$u'(0) = u'(\pi) = 0\,,$$

mit $f \in C^2(R,R)$ und $g \in C^2([0,\pi],R)$. Für Funktionen f und g, die durch eine Beschränktheitsbedingung gekoppelt sind, erhalten wir Aussagen über das qualitative Verhalten von Lösungen von (*). Für eine Duffinggleichung Können wir unter gewissen Einschränkungen die exakte Anzahl von Lösungen bestimmen und Bifurkationsdiagramme erstellen.

B. Fiedler : *The Poincaré-Bendixson theorem for scalar reaction-diffusion equations.* (Joint work with J. Mallet-Paret, Brown University). The classical Poincaré-Bendixson theorem is proved for PDE's of the form

(1) $u_t = u_{xx} + f(x,u,u_x)$, $x \in S^1$, $f \in C^2$.

Specifically, the ω-limit set of any bounded trajectory consists of a single periodic solution or of trajectories connecting equilibria. Moreover, the ω-limit set embeds into the plane. This is surprising because the semiflow (1) is genuinely infinite-dimensional.

A. Fonda : *Topological and variational methods for boundary value problems.* Existence results are presented for equations of second and third order with nonlinearities of "jumping" type. Moreover, a situation of double resonance is studied under Landesman-Lazer type conditions at both sides.

G. Freiling : *Expansion theorems for indefinite eigenvalue problems.* We consider boundary eigenvalue problems of the form

(1) $l(y) = y^{(n)} + \Sigma_2^n\, f_\nu(x)y^{(n-\nu)} = \lambda r(x)y\,, \quad x \in [0,1]\,,$

(2) $U_\nu(y) = 0\,,\ 1 \le \nu \le n.$

It is assumed that r is a step-function and that the boundary conditions (2) are regular.

By using detailed asymptotic estimates for the Green's function of (1), (2) , we obtain results on the pointwise convergence and on the uniform convergence of the corresponding eigenfunction expansions generalizing the classical results for regular definite eigenvalue problems.

G.B. Gustafson : *Boundary value dependence in linear differential equations.* Consider the differential equation L : $y^{(n)} + p_{n-1}(t)y^{(n-1)} + \dots + p_0(t)y = 0$ and Nicoletti boundary conditions B : for distinct points $t_0,\dots,t_k$ and integers $n_0, \dots, n_k$ with $n_0+\dots+n_k = n$, the function y satisfies $y^{(i)}(t_j) = 0$, $i = 0,\dots,n_j-1$, $j = 1,\dots,k$. Define the kernel by $K(B) = \{y : Ly = 0;\ By = 0\}$. The following continuous dependence problem is considered : Is there a conditions which gives : $K(B) \neq \{0\}$ implies $\lim_{n\to\infty} C_n = B$ with $K(C_n) \neq \{0\}$? This is a kind a continuous dependence on B. The answer is given by the following :

Theorem. If dim $K(B) = 1$ and $y \in K(B)$ and $y^{(n_i)}(t_i) = 0$ for all but one index i, then there is a sequence of simple-zero boundary operators $\{C_n\}$ such that : (1) dim $K(C_n) = 1$, (2) $\lim_{n\to\infty} C_n = B$, (3) $\lim_{n\to\infty} K(C_n) = K(B)$.

Also discussed in the talk is a result for testing the uniqueness of a linear two-point BVP. The kind of result is illustrated by : the BVP $(D^6 + D^5)y = f(x)$, $y = y' = y'' = y''' = 0$ at $x = a$, $y = y' = 0$ at $x = b$, has a unique solution if and only if the following IVP has a solution with $z(b) \neq 0$: $D^7(D+1)^5 z = 0$, $z^{(i)}(a) = 0$ for $i = 0,\dots,7$, $z^{(8)}(a) = 14$, $z^{(9)}(a) = -42$, $z^{(10)}(a) = 90$, $z^{(11)}(a) = -165$.

P. Habets : *Periodic solutions of some Liénard and Duffing equations with singularities*. Consider the boundary value problem

$$u'' + (d/dt)\nabla f(u) + \nabla g(u) = h(t)\ , \quad u(0) = u(T),\ u'(0) = u'(T), \qquad (1)$$

where $u \in R^n$, under the general assumption that g becomes infinite as $u \to 0$. Our approach to problem (1) is based on degree theoretical methods and covers both the attractive and repulsive potentials. A key assumption to get a priori bounds is a condition similar to the "strong force" condition introduced by Gordon (Trans. AMS 204 (1975)). This bounds the solutions away from the singularity. Our approach applies in case of complete dissipation, i.e. in case the matrix $\partial^2 f/\partial u^2$ is positive or negative definite, as well as in the conservative case where $f = 0$. At last, similar results can be obtained to deal with a finite number of singularities. This work is joint with L. Sanchez.

G. Harris : *Semilinear elliptic boundary value problems of Ambrosetti-Prodi type*. Mainly, I would discusss an example for which boundary data has a different effect on the multiple solution structure of the differential equation than does the forcing data. In particular, the solution number for this example never exceeds two, regardless of the location of the asymptotic "slopes" of the nonlinearity within the spectrum of the operator -u" with zero Dirichlet boundary conditions.

S. Invernizzi : *Problems in some ODE models in mathematical economics*. C. Sparrow (J. Theor. Biol. 83 (1980)) showed that chaotic behaviour can occur in simple one loop feedback systems of finite dimensions, and he compared such systems with a family of 1-dimensional difference equations of the type discussed by May and Oster. Here we show first that the analysis of simple economic processes with lags leads to the same differential systems and to the same problem of comparison with a discrete equation. Moreover, the study of the distribution of lags in the future in the simple case of an output lagged on demand, suggests that the discrete case is approximated very slowly by a continuous system, so that in general the possible chaotic behaviour of the difference equation may be reflected into the continuous system only for very large dimension of the latter, according to Sparrow's results. Finally, we present numerical estimates of the Lyapunov exponents and Hausdorff dimension of a possible strange attractor of Rössler-Band type present in a feedback system modelling a demand/output process.

F. Kappel : *Approximation of infinite dimensional systems*. After a short discussion of various versions of the so called Trotter-Kato theorem for C_0-semigroups, we develop a general structure for the approximation of delay equations by ordinary differential systems. Main feature of the general scheme is the choice of two sequences (U^N) and (V^N) of finite-dimensional subspaces in the state space of the abstract Cauchy problem with isomorphisms $\Phi^N : U^N \to V^N$. If U^N is in dom A (A the infinitesimal generator of the semigroup to be approximated), we can define $A^N = A(\Phi^N)^{-1}$ in order to get approximating ODE systems. Finally we give a short description of a family of approximation schemes constructed along the guidelines given in the general theory.

H. Kielhöfer : *Hopf bifurcation with an eigenvalue zero - an analytical approach.* We consider a family of evolution equation in Hilbert space

$$du/dt + A(\lambda)u + F(\lambda,u) = 0$$

which is of parabolic type and depends on a real parameter λ. At a critical value λ_0 the linear operator $A(\lambda)$ has finitely many resonant eigenvalues on the imaginary axis, including the eigenvalue 0. We prove a local and global bifurcation theorem for stationnary or periodic solutions by embedding the differential equation into an abstract class of two-parameter equivariant problems which have a resonance property. The main tool is an orbit index which consists of a pair of common Brouwer indices. It turns out that this abstract class allows generically cascades of period doublings which are not possible for differential equations. Finally, in the generic global case, we can replace the continuously varying period by the minimal period.

R. Lauterbach : *Bifurcation and dynamics for problems with spherical symmetry.* In the beginning we recall the basic concepts of equivariant bifurcation theory. We assume that a compact Lie group acts absolutely irreducibly on a finite dimensional real vector space V. Let $f : V \times R \to V$ be equivariant with respect to its first variable. We consider the ordinary differential equation

$$x' = f(x,\lambda).$$

We assume that the given trivial solution looses stability at $\lambda = 0$. Then we are interested in the following questions :

1) what are the stationnary solutions, i.e. what are the solutions of $f(x,\lambda) = 0$, especially we are interested in the symmetries of generic stationnary solutions.

2) what are the dimensions of the unstable manifolds of the stationnary solutions ?

3) what can we say about heteroclinic connections between these solutions ?

4) are there any other (flow-)invariant sets which are contained in small neighborhoods of the bifurcation point.

We investigate those questions for the natural representations of O(3). For the first two questions there are partial answers for all representations. The last two questions can be solved for representations with dim $V < 7$. Our results are closely connected to results by Fiedler and Mischaikow and to results of Chossat and Melbourne.

N. Lloyd : *Bifurcation of limit cycles in two-dimensional systems.* Conditions for a critical point to be a centre are not only of intrinsic interest but are also required to determine the number of limit cycles which can bifurcate. By systematically searching for invariant algebraic curves, such criteria can be obtained. The technique will be illustrated by means of a certain class of cubic systems, and the bifurcation of limit cycles in this case fully discussed.

P.J. McKenna : *Suspension bridges and ships.* Recent work by the speaker (with A.C. Lazer) has demonstrated a new principle for nonlinear diferential equations with asymmetric nonlinearities and large one-sided forcing terms : the greater the asymmetry, the more likely the presence of large-scale oscillatory behavior. Applications are given to destructive oscillations in bridges and ships.

R. Ortega : *Stability of periodic solutions of nonautonomous equations.* Some connections between the topological index of a nondegenerate periodic solution and asymptotic stability are studied. From these results a complete stability study is obtained for a forced second order equation of the kind

$$x'' + cx' + g(x) = p(t), \quad c > 0,$$

assuming that g is strictly convex and satisfies an additional condition of Ambrosetti-Prodi type. The stability of the solution is related to the non-existence of second order subharmonics.

K. Palmer : *Chaos in almost periodic systems.* Let $x' = f(t,x)$ be an almost periodic differential equation with ahyperbolic almost periodic solution u(t) and another hyperbolic solution v(t) satisfying $|v(t) - u(t)| \to 0$ as $|t| \to \infty$. It is shown that the solutions of such an equation exhibit chaotic behaviour.

L. Sanchez : *Boundary value problems for some nonlinear beam equations.* We derive properties of the first eigenfunction for the linear beam equation

$$u^{(4)} - a(x)u = \lambda u$$

with simply-supported ends ($u(0) = u''(0) = u(\pi) = u''(\pi) = 0$) or with hinged ends ($u(0) = u'(0) = u(\pi) = u'(\pi) = 0$). We use these properties to give a variational treatment of the existence of solutions for the nonlinear beam equation at resonance under the above boundary conditions.

R. Schaaf : *Asymptotics for branches of semilinear elliptic problems.* (Joint work with K. Schmitt). Nonlinear perturbations of elliptic problems at resonance

$$(1) \quad Lu + g(u) = h \quad (\Omega \text{ in } R^n),\ u = 0 \text{ on } \partial\Omega,$$

with $N(L) = \text{span } \phi, \phi > 0$, orthogonal to h and $g(u)/u \to 0$ for $|u| \to \infty$ can be embedded into parameter dependent problems

$$Lu + g(u) = h + \mu\phi \quad \text{on } \Omega,\ u = 0 \text{ on } \partial\Omega,$$

which always have a solution branch bifurcating from "$u = \pm \infty$". The number of sign changes of μ along this branch is a lower bound for the number of solutions to (1).

For the special case of g being periodic with zero mean value and Ω being convex, the behavior of the branch depends critically on the space dimension n : one always get an infinite number of sign changes for n = 1,2,3. For $n \geq 5$ and "almost all g", there are only finitely many changes of sign, and for n = 4 there is a necessary and sufficient condition for oscillation.

J. Scheurle : *Interaction of steady state and Hopf bifurcations under the presence of a Z_2-symmetry.* Bifurcations in a two-parameter family of Z_2-symmetric vector fields are analyzed. At criticality, the vector field has a pair of complex conjugate purely imaginary eigenvalues as well as a simple zero eigenvalue. Particular attention is paid to the bifurcation of invariant 2-tori and their stability properties. Explicit computable conditions are given. For example, our results are relavant for the Couette-Taylor problem of hydrodynamics.

K.R. Schneider : *Bifurcation of closed orbits from invariant manifolds.* We are concerned with the bifurcation of a periodic solution from an invariant set of a two-dimensional system $dx/dt = f(x,\mu)$ where μ is a one-dimensional parameter. In the first part of the lecture we derive conditions guaranteeing the existence of a global Hopf branch (in the sense for $\mu \geq \mu_0$

or $\mu \leq \mu_0$). The main tool is the theory of turning vector fields. The second part of the talk is devoted to the bifurcation of a limit cycle from an invariant set consisting of continua of equilibrium points and regular trajectories. Such a case arises when the system under consideration is singularly perturbed. Using the theory of integral manifolds we generalize a theorem of Pontrjagin-Miscenko-Rosov on the existence of relaxation oscillations.

H.L. Smith : *Remarks on monotone dynamical systems theory*. In this joint work with H. Thiema (Tampa), we synthesize ideas of Hirsch and Matano to create a somewhat streamlined and improved theory of monotone dynamical systems. More precisely, we are able to obtain some improvements in the results of Hirsch while using the weaker notion of strongly order preserving semiflows due to Matano. A monotone dynamical system is a dynamical system on a metric space with partial order relation having the property that the order is preserved by the forward flow. Hirsch shows that"almost all" points have omega limit sets consisting entirely of equilibria. Our corresponding result should prove easier to aply to infinite dimensional systems (e.g. functional differential equations and parabolic partial differential equations) than Hirsch's.

A. Vanderbauwhede : *Subharmonic branching in time-reversible systems*. We study the branching of subharmonic solutions at a symmetric periodic solution of an autonomous time-reversible system. Symmetric here means invariant under time-reversal. We show that generically each such symmetric periodic solution belongs to a one-parameter family of similar periodic solutions. Along such family one can generically meet solutions which have multipliers which are roots of unity. We show that at such solutions branching of subharmonic solutions will generically occur. We study in particular the case of period-doubling branchings. Our approach uses a Liapunov-Schmidt method. We also briefly discuss a different approach via normal form theory. Our results suggest that in certain régions one may encounter cascades of subharmonic branchings, leading to complicated and in certain cases maybe chaotic behaviour.

P. Volkmann : *Existenz eines Zweiges positiver Lösungen für ein Problem auf der reellen Geraden*. (Gemeinsame Arbeit mit Alice Chaljub-Simon). Mit $0 < \delta < c$ und einem Parameter $\lambda > 0$ werden für (*) $-y'' + c^2 y = \lambda f(x,y)$ Lösungen $y : R \to (0,\infty)$ im Banachraum $C^0_\delta = \{y \mid y : R \to R$, stetig, $\|y\| = \sup_R |y(x)| e^{\delta |x|} < \infty\}$ gesucht. Satz : A) In (*) sei $f : R \times [0,\infty) \to [0,\infty)$ stetig, und für $r > 0$ gelte $|f(x,y) - f(x,z)| \leq L_r(z - y)$, falls $0 \leq y \leq z \leq re^{-\delta |x|}$. Es sei noch $f(x,0) \leq \text{const.} e^{-\delta |x|}$, $f(x,0) = 0$. Dann existiert $\lambda_0 > 0$ und eine in C^0_δ stetig von λ abhängige Familie y_λ $(0 < \lambda \leq \lambda_0)$ von positiven Lösungen von (*); $\lim_{\lambda \to 0} y_\lambda = 0$ (in C^0_δ). B) Gilt ferner $f(x,y) \geq \beta e^{-\delta |x|} + \gamma(x) y$ $(x \in R,\ y \geq 0)$ mit $\beta > 0$ und $\gamma : R \to [0,\infty)$ stetig, $\gamma(x) \not\equiv 0$, so hat (*) für grosse λ keine Lösungen $y : R \to [0,\infty)$ in C^0_δ.

H.O. Walther : *Hyperbolic sets for C^1-maps*. (Joint work with H. Steinlein). The notion of hyperbolic sets and structures, which are familiar from diffeomorphisms (in finite dimensions), are generalized so that applications to arbitrary C^1-maps $f : U \to L$, U an open subset of a Banach space L, become possible. (C^1-maps which are not invertible arise as Poincaré maps and as time-1-maps in infinite dimensional dynamical systems). The shadowing lemma is extended to maps f with a (generalized) hyperbolic set. Hyperbolic structures are established along homoclinic loops, under a suitable transversality condition.

Both permit a new proof of a result of Hale and Lin on symbolic dynamics for all doubly infinite trajectories in a certain neighbourhood of a homoclinic loop. As an application of the theory, a Poincaré map without continuous inverse is presented. This map describes chaotic motions close to heteroclinic connections between hyperbolic periodic orbits of an autonomous differential delay equation for feedback on the circle.

J. Ward : *Bounded solutions of non-autonomous ordinary differential equations.* Let $f,g : R^N \times R \to R^N$ be continuous, $(x,t) \to f(x,t)$, $g(x,t)$, uniformly almost periodic in t. Assume also

(H_1) there exists $0 < p < 1$ such that $f(\lambda x,t) = \lambda^p f(x,t)$ for each $\lambda > 0$ and $(x,t) \in R^N \times R$.

(H_2) $g(x,t)/|x|^p \to 0$ as $|x| \to \infty$.

A homotopy method is used to establish the existence of a bounded solution to

$$(*) \quad x' = f(x,t) + g(x,t)$$

under additional assumptions. Equation (*) is related to

$$(**) \quad x' = f^*(x)$$

where $f^*(x)$ is the average of $f(x,t)$. It is assumed that $x = 0$ is the only bounded solution of (**). A Conley index associated with (**) is assumed to be non-zero.

If $p > 1$, (*) may be related via homotopy to a family of autonomous problems, and bounded solutions may again exist with an assumption of non-trivial Conley index.

M. Willem : *Relative category and periodic solutions of Hamiltonian systems.* The talk is devoted to the existence of periodic solutions of asymptotically linear Hamiltonian systems. The applications include a simple proof of Arnold conjecture, a Poincaré-Birkhoff type theorem and the forced pendulum equation. The main tools are an estimate on the number of critical points of a functional by using relative category and an estimate of relative category by relative cuplength.

F. Zanolin : *Boundary value problems for nonlinear second-order ordinary differential equations.* The periodic and the two-point BVP's for the second-order scalar nonlinear differential equation (of Duffing's type)

$$x'' + g(x) = p(t)$$

are examined.

The solvability is ensured by the introduction of some "nonresonance" conditions involving the interaction of $2G(x)/x^2$ or $g(x)/x$ with the first two eigenvalues of the associated linear problem ($G(x) = \int_0^x g(s)ds$). Proofs are based on continuation methods and estimates for the "time-map".

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