MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 14/1989

Ordinary Differential Equations in the Complex Domain and Special Functions

2.4. bis 8.4.1989

The conference was directed by R. Mennicken (Regensburg) and D. Schmidt (Essen). Main fields of interest were the general theory of singular linear differential equations (especially formal solutions, local actual solutions, connection problems), higher special functions (here especially series expansions of arbitrary functions) and related questions. Three lectures (D. Babbit, A.G. Ramm, J.K. Shaw) were attended together with the participants of the parallel conference on 'Spectral Theory of Ordinary Differential Equations'.

Abstracts

F. M. Arscott

A global analysis of Lamé's equation

Besides its physical importance, Lamé's equation is significant in two other ways: it is the best-known example of a differential equation with doubly-periodic coefficients, and is also a simple Heun-type equation - that is, an equation with four regular singularities in the complex plane.

Despite its long standing in mathematical literature, investigation of Lamé's equation has mostly been piecemeal, with concentration on particular aspects such as single- or double-periodicity, stability, special values of the order, etc.. This talk will describe how the equation can be analysed as a whole, using the techniques of 'multiplicative solution theory' on the equation in algebraic form.

D.Babbit

Chebyshev polynomials and the completeness of the Bethe Ansatz eigenfunctions for the Heisenberg–Ising spin chain

This talk concerns the Plancherel theorem for a family of finite difference Schrödinger operators $H_N(c)$, $N = 2, 3, 4, \ldots$, $c \in \mathbf{R}, c \neq 0$, which arise when the Heisenberg–Ising Hamiltonian $H_N(c)$ is restricted to "N-magnon sectors", N=2,3,.... Here c is a coupling constant. Drastic spectral changes occur when the coupling constant c crosses zeroes of certain Chebyshev polynomials.

W. Balser

A calculation of the Stokes' multipliers for systems of the form $zx' = (zA_0 + A_1)x$

For $A_0 = \mathrm{diag}\{\lambda_1, \cdots, \lambda_n\}$ with distinct $\lambda_1, \cdots, \lambda_n$, and an arbitrary $n \times n$ matrix A_1, we calculate the Stokes' multipliers of the above system in terms of an infinite series. The terms of the series, aside from explicit factors, involve the solutions of a difference equation, which (via Laplace transform) is equivalent to a system of the above type, but of dimension $n-1$. For a certain entry in the Stokes' multipliers, the series converges under a natural restriction upon the configuration of the $\lambda_1, \cdots, \lambda_n$. In general, this entry may be obtained via analytic continuation with respect to $\lambda_1, \cdots, \lambda_n$.

B. L. J. Braaksma

Connection problems for some differential and difference equations

We consider differential equations

(D) $$y'(x) = x^{p-1}A(x)y(x), \quad p = 1, 2, \cdots$$

and difference equations

(Δ) $$y(x+1) = A(x)y(x),$$

where A is analytic at $\infty, A(\infty) = \mathrm{diag}\{\lambda_1, \cdots, \lambda_n\}$, $\lambda_j \neq \lambda_k$ if $j \neq k$ and in case (Δ) also $\lambda_j \neq 0$, $j = 1, \cdots, n$. From the known formal fundamental matrices we construct actual fundamental matrices in sectors bounded by Stokes rays via Laplace integrals $\mathcal{L}\varphi$. Here φ satisfies a convolution equation, φ admits a known series expansion near 0, and has singular points outside 0. From the behavior of φ at these singular points we deduce the connection between fundamental systems in neighbouring sectors. This method is a particular case of Ecalle's theory of resurgent functions and it gives another approach to results obtained by Balser, Jurkat and Lutz and R. Schäfke.

V. Dietrich

Zur Konstruktion von asymptotischen Darstellungen für ein Fundamentalsystem von Lösungen einer linearen Differentialgleichung

Ausgehend von einer linearen Differentialgleichung

(*) $$\vartheta^n w = b_1 \vartheta^{n-1} w + \cdots + b_{n-1} \vartheta w + b_n w \quad (\vartheta := zd/dz)$$

oder der entsprechenden Normalform von Frobenius, lassen sich mit Hilfsmitteln der Computeralgebra asymptotische Darstellungen für ein Fundamentalsystem von Lösungen von (*) bestimmen. Sie haben die allgemeine Darstellung:

$$z^{k/p} \ln(z)^m (1 + 0(1)) \exp(q(z^{1/p}) + \lambda \ln(z^{1/p}))$$

mit $k, m \in \mathrm{N}_0$, $p \in \mathrm{N}$, $\lambda \in \mathrm{C}$ und q ein Polynom ohne konstantes Glied. Der hier zugrundeliegende Algorithmus ist inzwischen implementiert in MAPLE (Version 4.2). Er erlaubt die Verwendung von Parametern, so daß Klassen von Differentialgleichungen geschlossen untersucht werden können. Wesentlich für den Algorithmus ist u. a. eine genaue Untersuchung der Koeffizienten b_j: Es werden die für das Endergebnis relevanten Anteile bestimmt und nur diese für die weitere Rechnung eingesetzt.

R.Gérard

Convergent solutions of singular difference and differential equations

We consider operators on the ring of factorial series which are generated by taking analytic combinations of a finite number of linear lower triangular operators. We seek conditions that guarantee that the resulting operator D is singular regular. This means that if $Dy = f$ is satisfied by a formal factorial series and f is a convergent factorial series, then y must converge, too. The conditions are based on separating the operator into terms of the lowest order plus higher order terms. If certain easily verifiable dominance and non-degeneracy conditions are satisfied we show that the operator has the singular regular property. This result specializes to a classical theorem of Nørlund and also includes generalizations to nonlinear cases on difference equations as well as to other operator equations.

H. Gingold

Approximation of solutions of $y'' = \phi(x, \varepsilon)y$ with several transition points and moving singularities with $\phi(x, \varepsilon)$ meromorphic

Given a second order linear differential equation $y'' = \phi(x, \varepsilon)y$ let $\phi(x, \varepsilon)$ be a meromorphic function of two independent variables x and ε. x is allowed to vary on an interval $[a, b]$ and ε varies on $(0, \varepsilon_0]$. The approximation of its solutions, as well as of their derivatives as $\varepsilon \to 0^+$ are provided for the entire interval $[a, b]$ in the presence of "*several-coalescing transition points*". Approximations on *closed subintervals* of $[a, b]$ are provided. The end points of those subintervals are allowed to coincide with turning points.

Our method avoids analytic continuation, Langer transformations and special functions. The analytic theory of second order linear differential equations can be served by no more then two formulas. In particular, new approximations for special functions are given. Ramifications on reflection and transmission coefficients, the connection problem and oscillation theory are mentioned. Special emphasis will be put on a new mechanism which simultaneously renders global as well as uniformly valid approximations in a half neighbourhood of a turning point.

A. Grigis

On the spectrum of the finite Hill's equation

Let $-\frac{d^2}{dx^2} + V(x)$ be a Hill operator where $V(x) = \sum_{-N}^{N} c_n e^{2inx}$ is a real trigonometric polynomial with period π. If γ_n is the width of the gap in the spectrum located near n^2, then we give an asymptotic formula as n tends to $+\infty$ in the following form: γ_n is the modulus of a sum of N terms which tend to 0 faster than any exponential. These terms are computable and closely related to the location of moving turning points of the equation.

The study of the balance between these N terms is delicate, but it can be shown that under very simple algebraic conditions on the coefficients c_n of V, then $\gamma_n > 0$ for sufficiently large n and all but a finite number of gaps are open.

P. F. Hsieh

Adiabatic invariants of a Hamiltonian system and global analytic simplification of a matrix

This talk is devided into two parts. In Part I, we will prove that if an n by n matrix $A(t)$ is analytic on a finite or infinite interval $[a, b]$ and all of its eigenvalues are real on this interval, it is globally upper-triangularizable in the same interval by an analytic unitary matrix. The result is to be used to discuss the global diagonalization of a Hermitian matrix. In the process of proving the main result, we have to extend the matrix $A(t)$ to an analytic matrix $\tilde{A}(x)$ in a simply connected complex domain D containing $[a, b]$ and devise a constructive algorithm to find the required unitary matrix. More general results are to be given also. In Part II, a result is to be used to show that the magnitudes of all the solutions of a certain Hamiltonian system are adiabatic invariants. (This work was done jointly with H.Gingold.)

C. Hunter

Complex valued periodic solutions of the van der Pol equation

It is well-known that, for each real $\varepsilon > 0$, all solutions of van der Pol's equation

$$\frac{d^2u}{dt^2} - \varepsilon(1 - u^2)\frac{du}{dt} + u = 0, \tag{1}$$

other than the trivial solution $u \equiv 0$, tend to a unique periodic limit cycle as $t \to \infty$, the form of which does depend on ε. Andersen and Geer generated a Fourier expansion of the limit cycle using long (more than 160 terms) power series in ε. In particular, they found the period $T(\varepsilon)$ of the limit cycle to have a complex conjugate pair of branch point singularities $\varepsilon^2 = \mathrm{R}e^{\pm i\beta}$ ($R \sim 3.42, \beta \sim 4\pi/7$). In seeking to understand the significance of these branch points, we tried to track periodic solutions of equation (1) continuously from real values of ε through complex ε and out to the branch points. However, we were unable to do this because the complex periodic solutions so generated become singular along two continuous curves in the complex ε-plane, and these curves separate the branch points from the real ε-axis. The singular complex periodic solutions explain the "moving singularities" of $u(t, \varepsilon)$, for t fixed that Dadfar, Geer and Andersen found in subsequent studies of their long power series in ε.

An asymptotic analysis of the singular periodic solutions for $|\varepsilon|$ large can be given using singular perturbation theory and matched asymptotic expansions. It has a structure similar to that for the non-singular real case, and likewise involves four different regions. (This work was done jointly with M. Tajdari.)

K. Iwasaki

Structure of the moduli space of SL-operators on a Riemann surface and the monodromy preserving deformation

Let M be a compact Riemann surface of genus g and let ξ be a line bundle over M with $c_1(\xi) = 1 - g$. The SL-operators are a certain class of 2^{nd} order Fuchsian differential operators $\mathcal{M}(\xi) \to \mathcal{M}(\xi \oplus \kappa^2)$, where κ is the canonical line bundle over M. Given $m \in \mathbf{N}$ and $\theta_j \in \mathbf{C} \setminus \mathbf{Z}, 1 \le j \le m$, let E be the space of SL-operators with $m + n$ ($n := m + 3g - 3$) ordered singularities such that the j^{th} singularity has the characteristic exponents $\frac{1}{2}(1 \pm \theta_j), 1 \le j \le m$, and the last n singularities are apparent and of "ground state". E is naturally an analytic space of pure dimension $m+2n$. We have the projection $\pi : E \to B \subset M^{m+n}$ which assigns an SL-operator to its singularities. There exists a nonempty Zariski open subset X of B such that $F := \pi^{-1}(X)$ is a complex manifold and $\pi : F \to X$ is a holomorphic affine bundle of rank n. There exists a closed 2-form Ω on F defined canonically. The monodromy preserving deformation determines an Ω-invariant foliation on F.

D. A. Lutz

Singular operators on factorial series

The structure and properties of various types of operators (e.g. differential, difference) acting on the ring of formal factorial series of the form

$$y(x) = \sum_{m=0}^{\infty} y_m m!/x(x+1)\cdots(x+m)$$

will be discussed. The purpose is to provide the background for the subsequent talk (by R. Gérard) in which some joint results on operators having the so-called *singular regular property* will be presented. This part will focus on some basic properties of factorial series, their convergence, operators on factorial series, and dominance relations. A central role is played by *good* operators, which are defined by a certain type of dominance relation.

C. Markett

Convolution structures for eigenfunction expansions associated with singular Sturm-Liouville equations

In studying the harmonic analysis of classical singular eigenfunction expansions, most notably of Jacobi series, a convolution arises in a natural way and plays the same role as ordinary convolution in Fourier analysis. An approach is presented to so-called product formulas of the eigenfunctions of singular Sturm-Liouville equations which are essential for establishing appropriate convolution structures. To this end, partial differential equation techniques are used which generalize Riemann's integration method of solving hyperbolic initial value problems. In particular, various aspects of the corresponding Riemann functions are discussed. Since the approach employs only information given by the Sturm-Liouville problem, it is accessible, in principle, to more general eigenfunction expansions. Typical examples which are investigated in some detail are those related to the Jacobi polynomials and to eigenfunctions associated with the confluent Heun equation.

R. Mennicken

Expansions of analytic functions in series of special functions

The present lecture deals with λ-nonlinear eigenvalue problems of the type

$$D^n u(x) + \sum_{j=0}^{n-1} q_j(x,\lambda) D^j u(x) = 0, \quad u(xe^{2\pi i}) = e^{2\pi i\nu} u(x),$$

where the coefficients $q_j(x,\lambda)$ are holomorphic in x in a ring region around 0 and polynomials in λ. Problems of this kind have been studied in detail by Krimmer, Mennicken and Karl. The main object is the proof of eigenfunction expansions. These expansions are realized in two steps. In the first step the "formal expansions" are established; the main tool is a generalization of a theorem of Keldyš. In the second step the convergence of the formal expansion is proved; this is achieved by a careful analysis of Green's functions.

Various applications yield a series of expansions of analytic functions in terms of Special Functions.

J. P. Ramis

Wild Cauchy theory and applications

It is possible to build a new theory of infinitesimals for the ordinary complex plane: infinitesimal neighbourhoods are a lot larger than those introduced in algebraic geometry ("nilpotent elements" of Grothendieck). Then it is possible to get a very nice "Cauchy theory" in these infinitesimal neighbourhoods (that is "wild Cauchy theory") and in particular a theory of "analytic continuation" along continuous paths. So we get a "wild monodromy" related to a "wild homotopy". Then the dream of a lot of people comes true: it is possible to handle irregular singular equations (linear or not) just like regular singular equations, we get a "perfect" theory of resummation of divergent expansions and an exact theory of asymptotic expansions. As an example of application we get (recent work of C. Mitschi) usual classical Lie groups as Galois differential groups of generalized hypergeometric equations and more surprising it is also possible to get the exceptional group G_2 (for an equation of order 7). So some special functions are related to G_2 just like Airy functions are related to SL_2.

A. G. Ramm

Inverse problems

A method to prove uniqueness theorems and solve numerically some inverse problems is given. Using this method we prove, in particular, that a compactly supported real-valued $q(x) \in L^2(B_a)$, $q(x) = 0$ in $R^3 \setminus B_a, B_a = \{x : |x| \le a\}$, $x \in \mathbf{R}^3$, in the Schrödinger equation $l_q u := [\Delta^2 + K^2 - q(x)]u = 0$ in R^3, is uniquely determined by the scattering amplitude $A(\Theta', \Theta, K)$ known at a fixed $K > 0$ for all $\Theta' \in \tilde{S}_1^2, \Theta \in \tilde{S}_2^2$ where $\tilde{S}_j^1$ are open sets in S^2, $j = 1, 2$, S^2 is the unit sphere in R^3.
Similar results are given for inverse problems of geophysics with surface data given at a fixed frequency. The method that we develop for proving these uniqueness theorems and also for numerical solution is based on the completeness of the set of the products of solutions to PDE.

R. Schäfke

On formal fundamental solutions of irregular singular differential equations depending upon parameters

Consider a system of n equations

$$(1) \qquad z^{s+1}\frac{dy}{dz} = A(\varepsilon, z)y, \quad A(\varepsilon, z) = \sum_{l=0}^{\infty} A_l(\varepsilon)z^l,$$

where s is a nonnegative integer and $A_l(\varepsilon)$ are n by n matrices and are analytic near 0 in several parameters $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_\alpha)$. Babbit and Varadarajan (BV) studied the problem

whether there exist *analytic* formal fundamental solutions (ffs) of (1), i.e. $\hat{Y}(\varepsilon, z) = H(\varepsilon, z)z^{J(\varepsilon)}\exp(Q(\varepsilon, z))$, where now $J(\varepsilon)$ and all coefficients of z-powers in Q, H and H^{-1} are analytic near $\varepsilon = 0$. In general this is not possible. With BV we assume that (1) is *well behaved*, i.e. for sufficiently many ε (1) has a ffs $\hat{Y}(\varepsilon, z) = H_\varepsilon(z)z^{J_\varepsilon}\exp(Q(\varepsilon, z))$, where Q is analytic in U, say, (but no assumption is made on the ε-dependence of $H_\varepsilon, J_\varepsilon$) and the degrees of $q_i(\varepsilon, z) - q_j(\varepsilon, z)$ are independent of ε in U. Then we have
Theorem(BV): *If (1) is well behaved then it has a analytic ffs.*
I give a constructive proof of this theorem using some works of Wagenführer, Lutz–S. and S.–Volkmer on ffs. This method also yields an extension of the above theorem: if all given functions additionally belong to some Henselian subring R of the germs of analytic functions then also the coefficient functions of the ffs can be chosen in R.

W. Schempp

Serial and massively parallel data compression, high resolution image processing, and neural computer architecture for optical pattern recognition

Metaplectic harmonic analysis is well matched to coherent signal processing with square-law signal detection. The metaplectic representation of the symplectic group and its twofold cover arises when the symplectic group acts as a group of outer automorphisms of the irreducible unitary linear representations of the Heisenberg two-step nilpotent Lie group. The holographic transforms are defined as the coefficient functions of the infinite dimensional irreducible unitary linear representations of the Heisenberg group. The talk points out a unified metaplectic approach to coherent signal processing, and specifically, deals with the application of holographic transforms to massively parallel data compression, high resolution image processing, holographic coupling of optical fibers, optical wavefront conjugation, all-optical holographic associative memory, and a neural computer architecture for optical pattern recognition realized by an orientation-selective silicon VLSI retina. In contrast to the usual procedure, the optoelectronic applications give rise to an unexpected spin-off of results in pure mathematics, to wit, new identities for the matching polynomials associated to complete bipartite graphs which put in evidence the combinatorial character of certain identities for theta-null values.

J.K. Shaw

Half–bound states of the perturbed Hill's equation

We consider Hill's equation $-y'' + q(x)y = \lambda y$, $0 \leq x < \infty$, where $q(x)$ is periodic, $q(x+1) = q(x)$, and the corresponding perturbed equation $-y'' + (q(x) + p(x))y = \lambda y$, where $\int_0^\infty (1+x) \mid p(x) \mid dx < \infty$. Both free and perturbed equations have the same essential spectrum, namely the bands associated with $q(x)$, but either may have finitely many eigenvalues for the problem $y(0) = 0$ in the gaps. The ends of gaps are eigenvalues, but may be half–bound states of the perturbed equation; i.e., the solutions satisfying $y(0) = 0$ are bounded but not in $L^2(0, \infty)$. We characterize the half–bound states by studying the behavior of the Titchmarsh–Weyl coefficient at the ends of the gaps.

Y. Sibuya

Asymptotic solutions of a system of linear ordinary differential equations of higher order

Let $\mathcal{D}$ be the ring of differential operators; i. e.

$$\mathcal{D} = \{p = \sum_{k=0}^{N} a_k D^k;\ a_k \in \mathbb{C}\{x\}\}.$$

We assume that the following facts are known:

1. $0 \to A_0(\theta) \hookrightarrow A(\theta) \xrightarrow{J} \mathbb{C}[[x]] \to 0$ is an exact sequence of $\mathcal{D}$-module homomorphisms;

2. $A_0(\theta)$ is a divisible $\mathcal{D}$-module and hence $A_0(\theta)$ is an injective module.

Fact 2. implies that there exists a $\mathcal{D}$-module homomorphism $\mu : A(\theta) \to A_0(\theta)$ such that

$$\mu \mid_{A_0(\theta)} = \mathrm{id}_{A_0(\theta)}.$$

Set

$$\varphi(\hat{f}) = f - \mu(f), \quad \text{where} \quad J(f) = \hat{f}.$$

Then $\varphi(\hat{f})$ is independent of the choice of f, and φ defines a $\mathcal{D}$-module homomorphism: $\mathbb{C}[[x]] \to A(\theta)$. Utilizing φ and μ and the injectivity of $A_0(\theta)$, we can treat various problems of linear systems of ordinary differential equations of higher orders.

H. Volkmer

The expansion of a holomorphic function in a Laplace series

It is well known that a function on the sphere

$$S = \{(x, y, z) \in \mathbf{R}^3 \mid x^2 + y^2 + z^2 = 1\}$$

can be expanded in a series of spherical surface harmonics. We show that if the function is holomorphic on $T_\gamma = \{(x, y, z) \in \mathbf{C}^3 \mid x^2 + y^2 + z^2 = 1, |x|^2 + |y|^2 + |z|^2 < \cosh 2\gamma\}$ $(0 < \gamma \leq \infty)$ then the expansion is uniformly convergent on compact subsets of T_γ. As an application we obtain the expansion of a holomorphic function in a series of products of Lamé polynomials.

E. Wagenführer

Formal series solutions of singular systems of linear differential equations and singular matrix pencils

An n by n system of linear differential equations $xy'(x) = x^{-s}B(x)y(x)$ is considered, in which s is a positive integer and $B(x)$ is a formal power series in x. The problem is the practical evaluation of a formal fundamental solution $Y(x) = H(x)x^J e^{Q(x)}$. The method presented leads to the algebraic treatment of certain singular matrix pencils which are derived from the leading coefficients of $B(x)$. Some algorithms are presented by which singular matrix pencils are reduced to more specific forms. These algorithms are used for evaluating all parts of the formal fundamental solution.

M. Yoshida

Quadratic relations of the hypergeometric functions

Many identities for hypergeometric functions come from "symmetries". A symmetry of a Pfaffian system $du = \omega(\alpha)u$ defined on X $\quad(\alpha \in \mathbf{C}^l$: parameters) is a quadruple $(\varepsilon, \sigma, g, A)$ where $\varepsilon = \pm 1, \sigma \in \mathrm{Aut}(X), g$ is a gauge transformation and $A \in \mathrm{Affin}(l)$ such that

$$\text{if } \varepsilon = 1 \quad \text{then} \quad \{\sigma^*\omega(\alpha)\}[g] = \omega(A\alpha), \quad \omega[g] := g^{-1}\omega g + g^{-1}dg,$$

$$\text{if } \varepsilon = -1 \quad \text{then} \quad \{\sigma^*\{-{}^t\omega(\alpha)\}\}[g] = \omega(A\alpha), \quad -{}^t\omega \text{ is a dual of } \omega.$$

Symmetries form a group. Let G be the projection of the group to Affin(l). A symmetry $(1, \sigma, g, A)$ gives us a linear identity among solutions of the Pfaffian equation. A symmetry $(-1, \sigma, g, A)$ gives us a quadratic identity among solutions of the Pfaffian equation.

Example: For the Pfaffian system corresponding to the Appell-Lauricella hypergeometric function $F(\alpha, \beta_1, \cdots, \beta_n; x_1, \cdots, x_n)$, the group G has a subgroup (which is very likely to be G itself), described as follows: Affine Weyl group of the root system A_{n+2} extended by the involution of A_{n+2}. The involution corresponds to a symmetry of type $(-1, \sigma, g, A)$ and gives a quadratic relation of the said hypergeometric functions.

Berichterstatter: R.Schäfke

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

Tagungsteilnehmer

Prof.Dr. F. M. Arscott
Dept. of Applied Mathematics
University of Winnipeg

Winnipeg, Manitoba R3T 2N2
CANADA

Prof.Dr. R. Gerard
Institut de Mathematiques
Universite Louis Pasteur
7, rue Rene Descartes

F-67084 Strasbourg Cedex

Prof.Dr. D. G. Babbitt
Dept. of Mathematics
University of California
405 Hilgard Avenue

Los Angeles , CA 90024
USA

Prof.Dr. H. Gingold
Department of Mathematics
West Virginia University

Morgantown , WV 26506
USA

Prof.Dr. W. Balser
Abteilung für Mathematik V
der Universität Ulm
Oberer Eselsberg

7900 Ulm

Prof.Dr. A. Grigis
Centre de Mathematiques
Ecole Polytechnique
Plateau de Palaiseau

F-91128 Palaiseau Cedex

Prof.Dr. B. L. J. Braaksma
Mathematisch Instituut
Rijksuniversiteit te Groningen
Postbus 800

NL-9700 AV Groningen

Prof.Dr. P. F. Hsieh
Dept. of Mathematics
Western Michigan University

Kalamazoo , MI 49008-3899
USA

Dr. V. Dietrich
Lehrstuhl II für Mathematik
(für Ingenieure)
der RWTH Aachen
Templergraben 55

5100 Aachen

Prof.Dr. C. Hunter
Department of Mathematics
The Florida State University

Tallahassee , FL 32306
USA

Prof.Dr. K. Iwasaki
Dept. of Mathematics
Faculty of Science
University of Tokyo
Hongo 7 - 3 - 1, Bunkyo-ku

Tokyo 113
JAPAN

Prof.Dr. A. G. Ramm
Department of Mathematics
Kansas State University

Manhattan , KS 66506
USA

Prof. D.A. Lutz
Dept. of Mathematical Sciences
San Diego State University

San Diego , CA 92182-0314
USA

Prof.Dr. A. Sattler
FB 6 - Mathematik
Anwendungsorientierte Mathematik
der GHS Essen
Universitätsstr. 2

4300 Essen

Dr. C. Markett
Lehrstuhl A für Mathematik
der RWTH Aachen
Templergraben 55

5100 Aachen

Dr. R. Schäfke
FB 6 - Mathematik
Universität-GH Essen
Universitätsstr. 1-3
Postfach 103 764

4300 Essen 1

Prof.Dr. R. Mennicken
Fakultät für Mathematik
der Universität Regensburg
Universitätsstr. 31
Postfach 397

8400 Regensburg

Prof.Dr. W. Schempp
Lehrstuhl für Mathematik I
der Universität - GH Siegen
Postfach 10 12 40
Hölderlinstr. 3

5900 Siegen

Prof.Dr. J. P. Ramis
Institut de Mathematiques
Universite Louis Pasteur
7, rue Rene Descartes

F-67084 Strasbourg Cedex

Prof.Dr. D. Schmidt
FB 6 - Mathematik
Universität-GH Essen
Universitätsstr. 1-3
Postfach 103 764

4300 Essen 1

Prof.Dr. Y. Sibuya
Dept. of Mathematics, DRB 306
University of Southern California
University Park

Los Angeles , CA 90089-1113
USA

Dr. H. Volkmer
FB 6 - Mathematik
Universität-GH Essen
Universitätsstr. 1-3
Postfach 103 764

4300 Essen 1

Prof.Dr. E. Wagenführer
Fakultät für Mathematik
der Universität Regensburg
Universitätsstr. 31
Postfach 397

8400 Regensburg

Prof.Dr. M. Yoshida
Department of Mathematics
Faculty of Science
Kyushu University
Hakozaki, Higashi-ku

Fukuoka 812
JAPAN