MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 17/1989

FLÄCHEN IN DER GEOMETRISCHEN DATENVERARBEITUNG

16.04. bis 22.04.1989

Die Tagung fand unter der Leitung von R.E. Barnhill (Arizona State University, Tempe), W. Böhm (TU Braunschweig) und J. Hoschek (TH Darmstadt) statt.

Im Mittelpunkt des Interesses stand die Entwicklung neuer mathematischer Methoden und effizienter Algorithmen zur Darstellung von Kurven und Flächen des CAD. Aus der Vielfalt der vorgestellten Forschungsergebnisse seien folgende Schwerpunkte herausgegriffen: Qualitätsanalyse, Glättungsalgorithmen, Scattered Data-Interpolation über ebenen und gekrümmten Bereichen, multivariate Splines, Einsatz von Methoden der algebraischen Geometrie, Repräsentation spezieller Flächen, sphärische Splines zum Design zwangläufiger Bewegungen, geometrisch stetige Übergänge, rationale Splines, Interpolation mit Berücksichtigung geometrischer Nebenbedingungen, Polarformen zum Studium polynomieller Kurven und Flächen, Algorithmen zur graphischen Darstellung. Vorträge von Anwendern, in denen auch auf offene Fragen und Probleme der Praxis hingewiesen wurde, stellten eine wichtige Ergänzung dar.

Die regen Diskussionen im Anschluß an die Vorträge sowie die zahlreichen Gespräche im Verlauf der Tagung lieferten eine Fülle interessanter Anregungen. Als besonders wertvoll für die Befruchtung der aktuellen Forschung erwies sich der Kontakt zwischen den an Universitäten lehrenden Wissenschaftlern und den in der Industrie tätigen Mathematikern.

Vortragsauszüge

R. E. BARNHILL

Geometry Processing and Surfaces on Surfaces

Geometry processing is the calculation of geometric properties of already constructed curves and surfaces. We present two geometry processing topics: Surface-surface intersections and curvature analysis. We compare our marching algorithm for surface -surface intersection with a divide and conquer algorithm. Our curvature analysis concludes that three surface curvatures, Gaussian curvature, mean curvature and absolute curvature, are useful for surface interrogation. Our surfaces defined on surfaces presentation includes a comparison of the distant-weighted method of Bruce Piper with the curved triangular interpolant of Henry Ou. Each of the above topics is illustrated by color computer graphics.

C. M. HOFFMANN

Surface Operations in Higher Dimensions

Operations such as offsets, Voronoi surfaces, and variable radius blending surfaces seem to require approximation or expensive symbolic computation. We propose to avoid both by using a higher dimensional formulation that considerably simplifies the numerical difficulties.

H. POTTMANN

Scattered Data Interpolation Based upon Generalized Minimum Norm Networks

A generalization of G.M. Nielson's method for bivariate scattered data interpolation based upon a minimum norm network is presented. The essential part of the new technique is the use of a variational principle for determination of function values as well as cross boundary derivatives over the edges of a triangulation of the data. We mainly discuss the case of C^2 interpolants and present some examples including quality control with systems of isophotes. Finally, extensions to spherical scattered data methods are addressed.

T. FOLEY, R. FRANKE, D. LANE, G. NIELSON, H. HAGEN

Interpolation of Scattered Data over Closed Surfaces

Given N arbitrary points p_i on a closed surface D and associated real values F_i, we address the problem of constructing a smooth function F(p) defined for all $p \in D$ which satisfies $F(p_i) = F_i$, for $i = 1,\ldots,N$. We assume that D has genus zero, that is, topologically equivalent to a sphere. The basic approach involves mapping D to a sphere, solving a corresponding scattered data interpolation problem on a sphere, and then mapping back to the domain surface D.

M. J. PRATT

Smooth Blending of Circular-section Ducts using Piecewise Cyclides

The talk provides a review of recent work by the author and others in the use of cyclide surfaces for the creation of GC^1 blends between natural quadrics, toruses or other cyclides in general. The concept of the double-cyclide blend is then introduced, and detailes are given of the construction of a smooth blend between two cylindrical ducts of different diameters, at arbitrary relative positions and orientations in three dimensions. An extension to the blending of conical or more general cyclidal ducts is indicated.

W. BOEHM

On Cyclides in Geometric Modeling

Just 125 years ago J. Clerck Maxwell gave a nice construction of Dupin's cyclides by the use of a string. Most of their properties can be derived from this construction. Although they have a very simple Bézier representation cyclides are more suitable for solid blending than for patchwork. Some examples are given: the double-cyclide blend of two cones, a solution of the so-called Cranfield-problem, and the blend of a tripod.

J. HOSCHEK

GC^{n-1}-Functional Splines for Interpolation and Approximation of Curves, Surfaces and Solids

Implicit curves and surfaces are used for interpolation, approximation, blending of curves and surfaces and for filling holes. The method is an extension of Liming's conic section splines by introducing a power $n \geq 2$ for the transversal curve. The constructed curves and surfaces can be used for functional splines which fulfill geometric continuity conditions.

H. M^C^LAUGHLIN, B. PIPER

Spiralarcs: An Interpolation Problem

Spiralarcs are planar curves with monotone curvature. Does there exist a spiral arc which interpolates two points with specified tangent lines and specified centers of curvature at the points? The answer is negative. Does there exist an arc with not more than one vertex which satisfies the above interpolation problem? The answer is also negative. It is conjectured that the interpolation problem can be solved with an arc of not more than two vertices.

M.BERCOVIER

Related Topics in CAD and FEM: Isochoric Deformations

Using the theoretical mechanics approach the notions of deformation of a body, deformation gradient and Green Lagrange strain tensor $\underset{\sim}{E}$ are introduced. Let $\underset{\sim}{A}$ be the (matrix) deformation gradient, an isochoric deformation is defined by det $\underset{\sim}{A} = 1$. Isochoric deformations happen for so-called incompressible materials. The definition implies elementary volume preserving. However, isochoric deformations cannot be built using polynomial or rational deformation functions. Next one can relax the isochoric condition. It would be good enough to control the volume of a patch for instance, without controlling infinitesimal changes. This leads to so-called "mixed" type finite element approximation, with a Lagrange multiplier dual to the patch global volume constraint. A simple triquadratic case is used to illustrate this approach. As a conclusion it is shown that some nonclassical thoughts taken from the necessity of physics can help set the proper background and thus obtain a solvable problem!

D. LIU

GC^1 Conditions between Two Rational Bézier Patches

The GC^1 necessary and sufficient conditions between two adjacent rectangular or two triangular rational Bézier surface patches are presented. Further some practical and simple sufficient conditions are developed. There are many weights in the GC^1 conditions which are useful to easily compose a GC^1 smooth surface.

T. D. DEROSE

Necessary and Sufficient Conditions for Tangent Plane Continuity of Bézier Surfaces

Sets of conditions are derived that are necessary and sufficient for tangent plane continuity between two integral or rational Bézier surfaces. The patches may be given in either triangular or rectangular form, and no assumptions are made concerning the relative degrees of the patches; the only assumption is that the patches share common boundary control points (and weights in the case of rational surfaces). The conditions are shown to be minimal in the sense that they are, in general, independent.

W. DEGEN

Supplements to the Theory of G^k Continuity of Surface Patches

In a first part, reviewing the theoretical foundations, the definition of G^k continuity along a common boundary curve, as recently given by J. Hahn,

CAGD 6(1989), is compared with the notion of "contact of order k" used in differential geometry. Especially for G^2, it will be shown that the existence of a family of curves, crossing the boundary transversally with G^2 continuity, is equivalent to both.
In a second part, the theory is applied to two adjacent rectangular Bézier surface patches. Recently, G. Farin's G^1 construction (CGIP 20(182)), was improved by D. Liu and J. Hoschek (1989, to appear). But their solution is implicit. Using algebraic methods and the prime factorization of polynomials, an explicit solution will be obtained. By similar arguments, the analogous solution for the G^2 case is derived under an additional regularity assumption.

N. LUSCHER

Calculation of Curvature Continuous Cubic Splines

The connection between the recursion formula for B-splines and the de Boor algorithm is well known. Using results of Goodman/Unsworth and Boehm the analogous connections for curvature continuous cubic splines are presented and special properties are discussed.

T.N.T. GOODMAN

Constructing Piecewise Rational Curves with Frenet Frame Continuity

A simple geometric construction is given for the Bézier points of two rational curves which join with appropriate Frenet frame continuity. This is then used to give a geometric construction, from an arbitrary sequence of control points, for the Bézier points of a sequence of rational curves of degree n which join with Frenet frame continuity of order at most n-1.

P. BRUNET

Increasing the Flexibility of VC^1 Connections of Bézier Patches

The problem of connecting a given patch to a neighbour (to be defined) in a VC^1 way is studied. In practical applications we would expect some data on the neighbour to be fixed (for example, boundaries). From a counting of degrees of freedom, it can be found that this is not possible. In this sense, the algorithm of Farin' 82 gives the most general solution inspite of the assumption of linearity on the coefficients of the linear and rectangular unions, and in polynomial or rational patches. After that, a method is proposed that relaxes the one-sidedness of sequential patching by modifying one control point in the data patch. The consequence is that a VC^1 connection with a neighbour with both endpoints completely uncoupled is always possible. Also, the modification of the data patch can be minimized.

H. NOWACKI, D. LIU, X. LÜ

Fairing Bézier Curves with Some Constraints

A planar parametric Bézier curve is constructed from a combination of interpolation conditions, end conditions, and integral constraints such as area under the curve. A variational formulation of the problem based on a second (or higher) derivative fairness criterion is presented. It leads to a nonlinear system of equations for the free set of Bézier points, resulting from a sufficiently high degree of the Bézier curve, and for the Lagrange multiplier. The resulting curve meets all constraints and minimizes the fairness criterion. The computational effort is fairly high what suggests possible improvements by relaxing some constraints into inequalities.

H. PRAUTZSCH

A Fast Algorithm to Raise the Degree of B-Spline Curves

The number of operations existing algorithms need to raise the degree of a B-spline curve $s(x) = \sum_{i=1}^{m-n} c_i N_i^n(x)$ by one is of order $O(n^2m)$ where m denotes the number of knots the spline s(x) depends on and n the degree of the spline. A new algorithm is presented which is faster than the known algorithms for any degree and where the nubmer of operations needed is of order O(nm).

J. A. GREGORY, M. SARFAZ

A Rational Cubic Spline with Tension

A rational cubic spline curve is described which has tension control parameters for manipulating the shape of the curve. The spline is presented in both interpolatory and rational B-spline forms, and the behaviour of the resulting representations is analysed with respect to variation of the control parameters.

G. FARIN

Surfaces over Dirichlet Tessellations

We develop a class of surfaces that are based on the concept of Sibson's interpolant. This is a generalization of one-dimensional piecewise linear interpolation to the case of two or more variables. The interpolant is obtained as the ratio of certain areas arising in the recursive generation of Dirichlet tesselations. We reinterpret Sibson's interpolant as the projection of higher dimensional Bézier simplices and generalize to arbitrary degrees of those simplices. An application is a C^1 scattered data interpolant with local control and quadratic precision.

G. GEISE, U. LANGBECKER

Finite Quadric Segments with Four Conic Boundary Segments

For suitable segments the problem is solved how to get a representation as rational TP Bézier surface which is smooth in the sense of differential geometry and which has the boundary curves as u and v lines. The intuitive idea of sweeping out the surface by one conic may be realized by application of known facts concerning rational Bézier representation of conics in the view of stereographic projection. The resulting representation $x(u,v)$ is of degree $m \leq 6$ in u and of degree $n \leq 2$ in v. Some special problems are considered too.

A.WORSEY

Contouring Quadratics for Surface Analysis

We consider the problem of robustly contouring a trivariate quadratic polynomial defined over a tetrahedron. We show how the contour can always be described by a collection of rational quadratic patches. These patches are easily parametrized after considering the contouring problem on faces of the tetrahedorn. A completely robust method for solving this problem is also developed which, to within machine accuracy, describes these contour curves as rational quadratics with non-negative weights.

M. LUCIAN

Convexity Preserving and Curvature Continuous Interpolating Quadratic Rational B-Spline

An algorithm, symmetric in the input, is provided which interpolates a planar set of data without introducing inflections extraneous to the data. The two main features of the output (convexity and curvature continuity) are independent. A simple extension allows for interpolating data with associated curvatures.

R. KLASS

Solved and Unsolved Surface Problems in Car Body Design

- Intersection problems and solutions
- Offset surfaces and singular points
- Collisions problems between algebraic and parametric surfaces
- Approximation with boundary conditions
- Controlling surface shape
- Design and milling problems with "Multisurfaces"

N. PFEIFF

Using CAGD Methods in the System CASS for Styling Applications

C. de BOOR

Multivariate Polynomial Interpolation

With M a subset of d-space and Q a polynomial space, call the pair (M,Q) c o r r e c t in case interpolation from Q at M is possible and uniquely so, i.e. in case the map on Q which carries the funcion f to its restriction to M is 1-1 and onto.
The joint work with Amos Ron reported on provides a map which associates with each finite subset M of d-space a polynomial space P(M)so that (M,P(M)) is correct. The map has the following properties: (i) P(M) is defined for every finite M. (ii) P(M) depends continuously on M (to the extent possible). (iii) (acceptable) coalescence leads to osculation. (iv) P(M) is translation-invariant, hence D(ifferentiation)-invariant. (v) P(M) is scale-invariant, hence spanned by homogeneous polynomial. (vi) For any invertible matrix A, $P(AM) = P(M)A^T$. (vii) P(M)is of minimal degree, in the sense that, for any correct (M,Q) and any j, dim $Q_j \leq$ dim $P(M)_j$ (with Q_j denoting the space of all polynomials of degree $\leq j$ in Q). (viii) The map is monotone (hence a Newton form is available for the interpolating polynomial). (ix) P(M x N) = P(M) x P(N) (with M x N the cartesian product and P(M) x P(N) the tensor product). (x) P(M) is constructible from M in finitely many arithmetic steps.
Any such map must give the standard polynomial spaces in standard situations. Further, any polynomial in P(M) must be constant in any direction perpendicular to the affine hull of M.
P(M) is constructed as the 'least' of the space H := $\exp_M$:= the span of all exponentials $e_{m(x)}$:= exp(m.x) with m in M. This means that P(M) is the span of all 'least's of functions in H. One finds the 'least' of a function f as the first nontrivial term f_k in the Taylor expansion $f = f_0+f_1+\ldots+f_k+\ldots$ in which f_k contains all terms of (exact) degree k.

P. ALFELD

Multivariate Splines

Splines (i.e. smooth piecewise polynomial functions) are used universally throughout problems involving functions of one variable. It is natural to contemplate the use of similar functions in the case of several variables. However, problems that are trivial for one independent variable turn out to be extremely difficult in the case of two or more variables. In this talk, some unsolved problems concerning multivariate splines will be described and some new results will be given.

L. L. Schumaker

Data Dependent Triangulation

We consider the standard problem of fitting a surface to scattered data. Our method is based on using C^1 piecewise cubics on the Clough-Tocher split of a triangulation of the data. The main idea is to define a swap test for changing the triangulation based on reducing the energy of the surface. By using this swap thest, one can create data-dependent triangulations which provide surfaces with improved smoothness and better fits than those obtained using the usual Delaunay triangulation. The energy expressions are obtained in terms of the Bernstein-Bézier representation of the splines.

R. WALTER

Differential Geometry and Surface Rendering

We develop a method for generating and employing contours and (self)intersections of arbitrary smooth surfaces for the purpose of visibility clarifying. The result relies on differential geometric integer invariants, called sight indices which describe the change of visibility. These are the means to render distinguished curves like contours, (self)intersections, and boundaries, including visibility. In the same manner, but with different types of index formulas, arbitrary curves can be rendered. Also, more than one surface is allowed. One advantage is that the (visible parts of) curves are drawn in one stroke. Thus the method is especially suitable for gnerating high quality plots on plotters.

W. DAHMEN

Some Remarks on Convexity Preserving Interpolation

Given any finite set of points in $\mathbb{R}^3$ a method for constructing a tangent plane continuous piecewise quadratic surface with prescribed topology is described. This method interpolates the given data with prescribed normal directions. In particular when the data come from a convex surface, i.e. when a convex piecewise triangular interpolant exists, conditions are discussed that ensure the piecewise quadratic interpolant to be convex as well. Specifically, the role of a certain free shape parameter in this context is illustrated by some graphical examples. Efficient ways of rendering the quadrics are indicated.

N. DYN

Recursive Subdivision Curves and Surfaces

A family of interpolatory subdivision schemes with tension control for the design of curves and surfaces is reviewed. Smoothness properties of the limit curves/surfaces are discussed and the performance of the schemes is demonstrated by several pictures. In particular, the effect of the tension parameters is tested. All the schemes above are shown to be perturbations of known schemes, with the tension parameter controlling the size of the perturbation. Small positive values of the tension parameter yield a scheme which produces a limit curve/surface with one more degree of smoothness.

R. N. GOLDMAN

Algorithms in the Style of Boehm and Sablonniere

The de Boor algorithm can be extended to curves which are not strictly B-splines by allowing either infinite or decreasing knots. Blossoming can then be applied recursively to these curves to compute the dual functionals of the corresponding polynomial bases. This observation leads to change of basis formulas which are the analogues of knot insertion techniques for B-splines. These methods are applied to generate transformations between the B-spline, Bézier, monomial, and power forms of a curve which are the direct analogues of Boehm's knot insertion algorithm and Sablonniere's algorithm for converting from B-spline to Bézier form.

H.-P. SEIDEL

Symmetric Recursive Algorithms for Curves and Surfaces

We introduce the concept of a symmetric recursive algorithm and show that in the case of curves this leads to B-splines. We then apply this concept to surfaces and construct a new patch-representation for bivariate polynomials: The B-Patches share many properties with B-spline segments: Besides their control points they are influenced by a 3-parameter family of knots. If all knots coincide, we obtain the Bézier representation of a bivariate polynomial w.r.t. $\Delta(R,S,T)$. Therefore B-Patches are a generalization of Bézier patches. B-Patches have a de Boor like evaluation algorithm, and, as in the case of B-spline curves, the control points of a B-Patch can be computed by simply inserting a sequence of knots into the corresponding polar form. In particular this implies linear independence of the blending functions. Furthermore, B-Patches can be joined smoothly and they have an algorithm for knot insertion that is completely similar to Boehm's algorithm for curves.
After studying B-Patches we go back to curves and somewhat relax our symmetry conditions. This leads to geometric spline curves. In particular we show that β- and γ-splines have a de Casteljau type evaluation algorithm that starts with the given spline control points and computes the function value by repeated linear interpolation. This is surprising since it has been previously conjectured that no such algorithm for β- and γ-splines exists.

L. RAMSHAW

The Progressive Case of the de Casteljau Algorithm for Surfaces

The de Casteljau Algorithm for curves has two variants. The plain case evaluates a plynomial curve segment given the coefficients of its Bernstein expansion, that is, its Bézier points. The progressive case evaluates a spline curve given the coefficients of its B-spline expansion. It turns out that the de Casteljau Algorithm for surfaces can also be extended from the plain case to an analogous progressive case. Unfortunately, the resulting algorithm cannot be applied in any straightforward way to evaluate --say--a box spline surface. Both polar forms and the symmetric variant of the tensor-product construction are useful in studying this situation.

M. DÆHLEN, T. DOKKEN, T. LYCHE, K. MØRKEN

Almost Best Approximation of Circles by Curvature-continuous Bézier Curves

We provide a surprisingly simple cubic Bézier curve which gives a 6-th order accurate approximation to a segment of a circle. Joining the Bézier segments we obtain a G^2-continuous approximation to the circle. The error is approximately onetenth of what is obtained by applying the general method of de Boor, Höllig and Sabin to the case of a circle.

T. LYCHE

Exponential B-splines in Tension

Splines in tension were introduced by Schweikert in 1966 as a means of eliminating wiggles in cubic spline interpolation. We will construct a B-spline representation for tension splines allowing multiple knots and different tension parameters ρ_i and linear B-splines for large values of ρ_i. The result is a local representation for a class of functions which has the smoothness of a cubic spline and the shape of a piecewise linear approximation.

K. HÖLLIG, H. MÖRGELE

G-Splines

We introduce a new type of spline spaces for constructing GC^{n-1} surfaces for (general) networks of n-th degree Bézier patches. In particular, "G-Splines" allow to incorporate singular vertices (the number of coincident patches does not equal 4) into a tensor product network. Our approach is based on an idea of Goodman for constructing biquadratic GC^1 tensor product patch networks and a special choice of the Geometric Continuity constraints.

J. PETERS

On Smoothness and Compatibility

B. PIPER

Fill-in Regions for Curves and Surfaces

The problem of designing surfaces by using sectional curves motivates the consideration of 'curves' and this can be 'shaped' automatically by use of fill-in regions. Results of an interpolation-like algorithm that produces 'fat' planar curves will be presented along with a preliminary algorithm to use "fat" curves in the creation of surfaces.

R. T. FAROUKI

Analytic, Algebraic and Topological Properties of Plane Offset Curves

Although the notion of a "parallel" curve dates back to Leibnitz, such loci merit only passing mention in contemporary differential geometry books. Recently, however, they have enjoyed a resurgence of interest motivated by their applications in areas such as N.C. machining, geometry optics, tolerance analysis, and path-planning. We examine certain fundamental properties of offset curves form an analytical, algebraical, and topological perspective. Irregular points arise on an offset whenever the generator curvature attains a certain critical value, an they may be cusps (sudden tangent inversions) or extraordinary points (tangent-continuous points of infinite curvature). Further, there exists a one-to-one correspondence between certain regular "characteristic" points (turning points, inflections, and vertices) on a given generator curve and each of its offsets.
From the algebraic perspective, we show that the "interior" and "exterior" offsets to a polynomial generator of degree n, taken together, constitute an algebraic curve of degree 4n- in general. A simple closed-form expression for the implicit equation of the offset is presented, and it is seen that the degree is reduced by 2 for each "cusp" of the generator. Algebraic methods also furnish an algorithmic basis for identifying all self-intersections of the offset which are required when "trimming" it.

G. M. NIELSON

Sphere Splines with Application

Based upon control points on the surface of a sphere, we introduce a new piecewise, C^2, curve which remains on the sphere. The construction of this curve is analogous to cubic B-splines where the individual cubic segments are represented as Bernstein/Bézier curves. The spherical analogs of the Bernstein/Bézier curves are defined by means of a spherical version of the de Casteljau algorithm based upon geodesic interpolation on the sphere rather than linear interpolation. Several of the basic properties of these new curves are established and some examples are given. Using quarternions and a 4D extension of our spherical curve we show how to animate rotations.

R. K.E. ANDERSSON

Curves as Boundaries of Surfaces with Prescribed Shape

We will discuss the problem to generate parametric polynomial space curves, close to a set of fitting points and satisfying constraints. The constraints are derived from desired properties of the surfaces, to be bounded by a network of these curves. The need to satisfy the conditions while keeping the curve close to the fitting points gives rise to quadric minimization problems with nonlinear constraints. The solution process for these problems, however, is facilitated by benefitting from their separable character.

D. FERGUSON

Directions in Curve and Surface Design

Curves and surfaces which are appropriate for manufacturing are difficult to define. In this talk a brief review of the problems will be given. Then, two approaches will be described. The first approach is constrained fitting of data and can be described as a success. The second is a variant of the TFI method of curve blending and represents work in progress.

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